Kaon Decays in the Standard Model

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Review: Kaon Decays in the Standard Model

V. Cirigliano, G. Ecker, H. N., A. Pich, J. Portoles, Rev. Mod. Phys. 84 (2012) 399

Discussion of all decay modes with BRs $> 10^{-11}$

- Theoretical framework
- Estimates of low energy constants
- Leptonic and semileptonic decays
- Nonleptonic decays
- Rare and radiative decays

Theoretical Framework

Short-distance description

SM $\rightarrow$ strangeness-changing transitions via W exchange

$M_K \ll M_W \rightarrow$ integrate out W $\rightarrow$ effective Lagrangian at $E = M_K$

- W exchange between quark current and leptonic current

$$\mathcal{L}_{\text{eff}}^{\text{semilept.}} = -\frac{G_F}{\sqrt{2}} S_{\text{EW}}^{1/2}\, \bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell\; \bar{u}_i\gamma^\mu(1-\gamma_5)V_{ij}d_j + h.c.$$

- W exchange between two quark currents

$$\mathcal{L}_{\text{eff}}^{\Delta S=1} = -\frac{G_F}{\sqrt{2}} V_{ud}V_{us}^* \sum_i C_i(\mu)Q_i(\mu)$$

e.g. $Q_2 = \bar{s}\gamma^\mu(1-\gamma_5)u\; \bar{u}\gamma_\mu(1-\gamma_5)d$

gluonic corrections $\rightarrow Q_1,\ Q_3$-Q_6

virtual electromagnetic interactions $\rightarrow Q_7$-Q_{10}

Theoretical Framework

Short-distance description ctd.

- leptons in final state $\rightarrow$ Q_{11}-Q_{13}

$$Q_{13} \equiv Q(\bar{\nu}\nu) = \bar{s}\gamma^{\mu}(1-\gamma_5)d \; \bar{\nu}\gamma_{\mu}(1-\gamma_5)\nu$$

- exchange of two W bosons (box diagram)

$$\mathcal{L}_{\text{eff}}^{\Delta S=2} = -\frac{G_F^2 M_W^2}{(4\pi)^2} \, C^{\Delta S=2}(\mu) Q^{\Delta S=2}(\mu)$$

$$Q^{\Delta S=2} = \bar{s}\gamma^{\mu}(1-\gamma_5)d \; \bar{s}\gamma_{\mu}(1-\gamma_5)d$$

Low energy effective theory of the SM

Construct the most general QFT consistent with the symmetries of the SM in terms of the asymptotic fields relevant at low energies

- chiral symmetry of QCD in the limit $m_u = m_d = m_s = 0$

$$\mathrm{SU(3)_L} \times \mathrm{SU(3)_R} \xrightarrow{\mathrm{SSB}} \mathrm{SU(3)_V}$$

  $\rightarrow$ 8 Goldstone bosons π, K, η

  interaction of the pseudoscalars vanishes for $p \rightarrow 0$

- $m_u,\, m_d, m_s \neq 0 \rightarrow M^2 \sim B m_q$
- chiral counting: orders of $p^2 \sim M^2 \rightarrow$ LECs
- add addition degrees of freedom: photon, light leptons
- LECs not restricted by chiral symmetry

Chiral Lagranian

$\mathcal{L}_{\text{chiral order}}$ (# of LECs)	loop order
$\mathcal{L}_{p^2}(2) + \mathcal{L}^{\text{odd}}_{p^4}(0) + \mathcal{L}^{\Delta S=1}_{G_8 p^2}(1) + \mathcal{L}^{\Delta S=1}_{G_{27} p^2}(1)$	$L = 0$
$+ \mathcal{L}^{\text{emweak}}_{G_8 e^2 p^0}(1) + \mathcal{L}^{\text{em}}_{e^2 p^0}(1) + \mathcal{L}^{\text{lepton}}_{\text{kin}}(0)$	
$+ \mathcal{L}_{p^4}(10) + \mathcal{L}^{\text{odd}}_{p^6}(23) + \mathcal{L}^{\Delta S=1}_{G_8 p^4}(22) + \mathcal{L}^{\Delta S=1}_{G_{27} p^4}(28)$	$L \leq 1$
$+ \mathcal{L}^{\text{emweak}}_{G_8 e^2 p^2}(14) + \mathcal{L}^{\text{em}}_{e^2 p^2}(13) + \mathcal{L}^{\text{lepton}}_{e^2 p^2}(5)$	
$+ \mathcal{L}_{p^6}(90)$	$L \leq 2$

Estimates of Low-Energy Constants

Matching between CHPT and the underlying SM

- LECs from low-energy data
- Resonance saturation
- Lattice determinations

Leptonic and Semileptonic Decays

- $K_{\ell 2}$ (and $\pi_{\ell 2}$) decays
  - Electromagnetic corrections
  - Extraction of V_{us}/V_{ud}
  - The ratio $R_{e/\mu}^{(K,\pi)}$
- $K_{\ell 2 \gamma}$
- $K_{\ell 3}$
  - Electromagnetic effects in $K_{\ell 3(\gamma)}$ decays
  - Quark mass ratios and $K_{\ell 3}$ decays
  - Form factors and phase space integrals
  - SU(3) breaking effects in $f_+^{K^0 \pi^-}(0)$
  - Determination of V_{us} and CKM unitarity tests
  - T violation in $K_{\mu 3}$ decays
- $K_{\ell 3 \gamma}$
- $K_{\ell 4}$
- $K_{e 5}$

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  - T violation in $K_{\mu 3}$ decays
- $K_{\ell 3\gamma}$
- $K_{\ell 4} \to \pi\pi$ scattering lengths from K_{e4} data
- K_{e5}

Nonleptonic decays

- $K \to \pi\pi$
  - $\pi\pi$ phase shifts from $K \to \pi\pi$ decays
  - ϵ'/ϵ
  - CP violation in K^0-$\overline{K^0}$ mixing
- $K \to 3\pi$
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  - $\pi\pi$ scattering lengths from $K \to 3\pi$ decays near threshold

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Rare and radiative decays

- $K \to \pi\nu\bar{\nu}$
- $K \to \pi\pi\nu\bar{\nu}$
- $K \to \gamma^{(*)}\gamma^{(*)}$
  - $K_S \to \gamma\gamma$
  - $K_L \to \gamma\gamma$
  - $K_S \to \gamma\ell^+\ell^-$
  - $K_L \to \gamma\ell^+\ell^-$
  - $K_L \to \gamma\ell_1^+\ell_1^-\ell_2^+\ell_2^-$
- $K \to \ell^+\ell^-$
  - $K_S \to \ell^+\ell^-$
  - $K_L \to \ell^+\ell^-$

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- $K \to \ell^+\ell^-$
  - $K_S \to \ell^+\ell^-$
  - $K_L \to \ell^+\ell^-$

Rare and radiative decays ctd.

- $K \to \pi\pi\gamma^{(*)}$
  - $K^+ \to \pi^+\pi^0\gamma$
  - $K_L \to \pi^+\pi^-\gamma$
  - $K_S \to \pi^+\pi^-\gamma$
  - $K \to \pi\pi\ell^+\ell^-$

- Other decays
  - $K^0 \to \gamma\gamma\gamma$
  - $K_L \to \gamma\gamma\ell^+\ell^-$
  - $K_L \to \gamma\nu\bar{\nu}$
  - $K_S \to \ell_1^+\ell_1^-\ell_2^+\ell_2^-$
  - $K_L,\, K_S \to \pi^0\pi^0\gamma$
  - $K_L,\, K_S \to \pi^0\pi^0\gamma\gamma$
  - $K \to 3\pi\gamma$

$K_{\ell 2}$ (and $\pi_{\ell 2}$) decays

V_{us}/V_{ud} from decay widths and lattice data, V. Cirigliano, H. N., PLB 700 (2011) 7

$$\frac{\Gamma_{K_{\ell 2(\gamma)}}}{\Gamma_{\pi_{\ell 2(\gamma)}}} = \frac{|V_{us}|^2}{|V_{ud}|^2}\,\frac{F_K^2}{F_\pi^2}\,\frac{M_{K^\pm}\left(1-m_\ell^2/M_{K^\pm}^2\right)^2}{M_{\pi^\pm}\left(1-m_\ell^2/M_{\pi^\pm}^2\right)^2}\left(1+\delta_{\rm EM}+\delta_{\rm SU(2)}\right)$$

F_K/F_π in the isospin limit ($m_u = m_d$, $e = 0$)

$$\begin{aligned}\delta_{\rm EM} &= \frac{\alpha}{\pi}\left(F(m_\ell/M_K) - F(m_\ell/M_\pi) + \frac{3-Z}{4}\ln\frac{M_K^2}{M_\pi^2}\right)\\ &= -0.0069(17), \quad M_{\pi^\pm}^2 - M_{\pi^0}^2 = 8\pi\alpha Z F_0^2\end{aligned}$$

$$\begin{aligned}\delta_{\rm SU(2)} &= \sqrt{3}\,\varepsilon\left[-\frac{4}{3}\left(F_K/F_\pi - 1\right) + \frac{1}{3(4\pi)^2F_0^2}\left(M_K^2 - M_\pi^2 - M_\pi^2\ln\frac{M_K^2}{M_\pi^2}\right)\right]\\ &= -0.0044(5)(11),\end{aligned}$$

$\varepsilon = \frac{\sqrt{3}}{4}\frac{m_d - m_u}{m_s - \widehat{m}} = 0.0116(13)$, $F_K/F_\pi = 1.193(6)$ from FLAG

Update of $|V_{us}/V_{ud}|$

$$
\begin{aligned}
\frac{|V_{us}|}{|V_{ud}|}\,\frac{F_K}{F_\pi} &= \left(\frac{\Gamma_{K_{\ell 2(\gamma)}} M_{\pi^\pm}}{\Gamma_{\pi_{\ell 2(\gamma)}} M_{K^\pm}}\right)^{1/2} \frac{1-m_\ell^2/M_{\pi^\pm}^2}{1-m_\ell^2/M_{K^\pm}^2}\,(1-\delta_{\mathrm{EM}}/2-\delta_{\mathrm{SU(2)}}/2) \\
&= 0.23922(25)\times\left(\frac{\Gamma_{K_{\ell 2(\gamma)}}}{\Gamma_{\pi_{\ell 2(\gamma)}}}\right)^{1/2} \\
&= 0.2765(5)
\end{aligned}
$$

Experimental input:

$\Gamma_{\pi_{\mu 2(\gamma)}} = 38.408(7)\,(\mu s)^{-1}$, $\Gamma_{K_{\mu 2(\gamma)}} = 51.33(13)\,(\mu s)^{-1}$ PDG 2012

Lattice input:

$F_K/F_\pi = 1.193(6)$ (FLAG) $\rightarrow |V_{us}/V_{ud}| = 0.2318(12)$

Isospin Breaking on the Lattice

G. M. de Divitiis et al. (RM123 Collaboration), JHEP (2012) 1204

Method: expand path integral in powers of $m_d - m_u$

- kaon masses
- kaon decay constant
- $K_{\ell 3}$ form factor
- neutron-proton mass splitting

$\delta^{\text{lattice}}_{\text{SU(2)}} = -0.0078(7)$ to be compared with $\delta^{\text{CHPT}}_{\text{SU(2)}} = -0.0044(12)$

The ratio $R^K_{e/\mu}$

V. Cirigliano, I. Rosell, JHEP 10 (2007) 005; Phys. Rev. Lett. 99 (2007) 231801

$$R^K_{e/\mu} = \frac{\Gamma_{K_{e2(\gamma)}}}{\Gamma_{K_{\mu2(\gamma)}}} = \left(\frac{m_e}{m_\mu}\right)^2 \left(\frac{M_K^2 - m_e^2}{M_K^2 - m_\mu^2}\right)^2 (1 + \delta_{\text{EM}})$$

$$= 2.477(1) \times 10^{-5}$$

- Two-loop calculation in CHPT with virtual photons and leptons
- Large-N_C calculation of an associated counterterm
- Summation of leading logarithms $\alpha^n \ln^n(m_\mu/m_e)$ (Marciano, Sirlin 1993)

Most recent NA62 result: $R^K_{e/\mu} = 2.488(10) \times 10^{-5}$

Full 2007 data set: article to be submitted in 2012, G. Ruggiero, CKM 2012

$K \to \pi \ell \nu$ ($K_{\ell 3}$) decays

$$\Gamma_{K_{\ell 3(\gamma)}} = \frac{G_F^2\,|V_{us}|^2\,M_K^5\,C_K^2}{128\,\pi^3}\,S_{\rm EW}\,|f_+^{K^0\pi^-}(0)|^2 I_{K\ell}^{(0)}(\lambda_i)\,\left(1+\delta_{\rm EM}^{K\ell}+\delta_{\rm SU(2)}^{K\pi}\right)$$

$C_K = 1$ for $K_{\ell 3}^0$, $\quad C_K = 1/\sqrt{2}$ for $K_{\ell 3}^+$

$I_{K\ell}^{(0)}$ phase space integral

$\delta_{\rm SU(2)}^{K\pi} = \left(f_+^{K\pi}(0)/f_+^{K^0\pi^-}(0)\right)^2 - 1$

Long-distance EM corrections $\quad \delta_{\rm EM}^{K\ell} = \delta_{\rm EM}^{K\ell}(\mathcal{D}_3) + \delta_{\rm EM}^{K\ell}(\mathcal{D}_{4-3})$

Short-distance factor $\quad S_{\rm EW} = 1 + \frac{2\alpha}{\pi}\left(1 - \frac{\alpha_s}{4\pi}\right)\ln\frac{M_Z}{M_\rho} + \mathcal{O}\left(\frac{\alpha\alpha_s}{\pi^2}\right) = 1.0223(5)$

A. Sirlin, 1977; 1981; W.J. Marciano, A. Sirlin, 1993

Electromagnetic effects in $K_{\ell 3(\gamma)}$ decays

V. Cirigliano, M. Giannotti, H. N., JHEP 11 (2008) 006

	$\delta_{\rm EM}^{K\ell}(\mathcal{D}_3)(\%)$	$\delta_{\rm EM}^{K\ell}(\mathcal{D}_{4-3})(\%)$	$\delta_{\rm EM}^{K\ell}(\%)$
K_{e3}^0	0.50	0.49	0.99 ± 0.22
$K_{e3}^\pm$	-0.35	0.45	0.10 ± 0.25
$K_{\mu 3}^0$	1.38	0.02	1.40 ± 0.22
$K_{\mu 3}^\pm$	0.007	0.009	0.016 ± 0.25

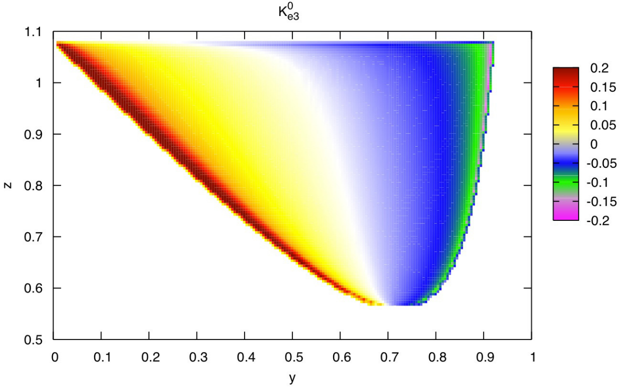
Electromagnetic effects in $K_{\ell 3(\gamma)}$ decays ctd.

Differential decay distribution

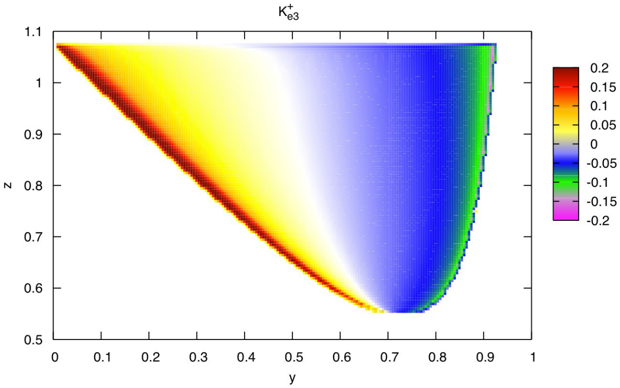
$$\frac{d^2\Gamma}{dy\,dz} = \frac{G_F^2\,|V_{us}|^2\,M_K^5\,C_K^2}{128\,\pi^3}\,S_{\text{EW}}\,|f_+^{K\pi}(0)|^2\left[\bar{\rho}^{(0)}(y,z) + \delta\bar{\rho}^{\text{EM}}(y,z)\right]$$

$$y = 2p_K \cdot p_\ell / M_K^2 = 2E_\ell / M_K\,, \qquad z = 2p_\pi \cdot p_K / M_K^2 = 2E_\pi / M_K$$

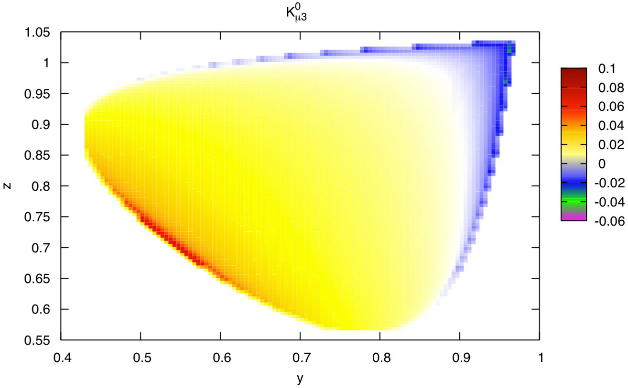
Ratio $\delta\bar{\rho}^{\text{EM}}(y,z)/\bar{\rho}^{(0)}(y,z)$ for K_{e3}^0



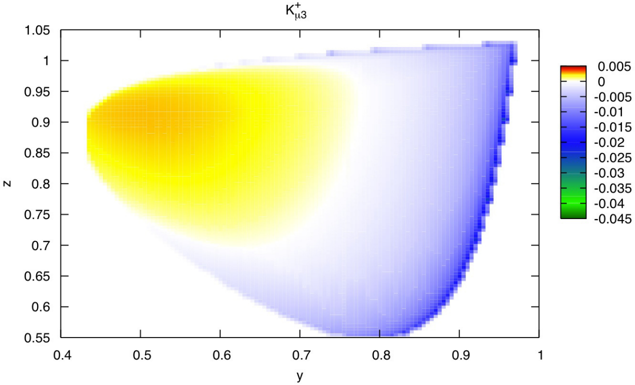
Ratio $\delta\bar{\rho}^{\mathrm{EM}}(y,z)/\bar{\rho}^{(0)}(y,z)$ for K_{e3}^{+}



Ratio $\delta\bar{\rho}^{\text{EM}}(y,z)/\bar{\rho}^{(0)}(y,z)$ for $K^0_{\mu 3}$



Ratio $\delta\bar{\rho}^{\text{EM}}(y,z)/\bar{\rho}^{(0)}(y,z)$ for $K^+_{\mu 3}$



Isospin Violation in $K_{\ell 3}$ decays

CHPT (quark mass ratios & deviation from Dashen's limit):

$$\delta_{\mathrm{SU}(2)}^{K^\pm\pi^0} = 0.058(8)$$

A. Kastner, H. N., 2008

electromagnetic LECs from B. Ananthanarayan, B. Moussallam, 2004

Combine recent $K_{\ell 3}$ data with

$$\delta_{\mathrm{SU}(2)}^{K^\pm\pi^0} = \frac{2\Gamma_{K^+_{\ell 3}}}{\Gamma_{K^0_{\ell 3}}} \frac{I_{K^0\ell}}{I_{K^+\ell}} \left(\frac{M_{K^0}}{M_{K^+}}\right)^5 - 1 - \left(\delta_{\mathrm{EM}}^{K^+\ell} - \delta_{\mathrm{EM}}^{K^0\ell}\right)$$

$$\rightarrow \quad \delta_{\mathrm{SU}(2)}^{K^\pm\pi^0} = 0.054(8)$$

Determination of V_{us} and CKM unitarity tests

Experimental average: $|V_{us}| f_+(0) = 0.2163(5)$

Recent update M. Moulson, CKM 2012, Cincinnati

Lattice (RBC/UKQCD 2010): $f_+(0) = 0.9599(34)(^{+31}_{-47})(14)$

$$\rightarrow |V_{us}| = 0.2255(5)_{\rm exp}(12)_{\rm th}$$

Fit of V_{us}, V_{ud}

V. Cirigliano, H. N., PLB 700 (2011) 7

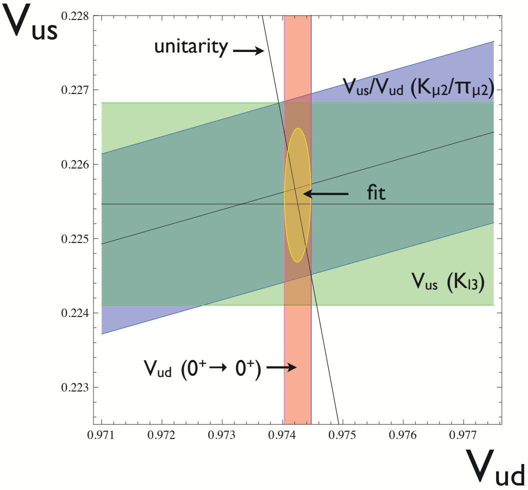
Input:

$|V_{ud}| = 0.97425(22)$ from $0^+ \to 0^+$, J.C. Hardy, I.S. Towner, 2009

$|V_{us}/V_{ud}|$ from $K_{\ell 2}/\pi_{\ell 2}$

$|V_{us}|_{K_{\ell 3}} = 0.2255(5)_{\rm exp}(12)_{\rm th}$ from $K_{\ell 3}$ decays and $f_+(0)$ from lattice

$$\to |V_{ud}| = 0.97425(22)\ , \quad |V_{us}| = 0.2256(9)$$



Vus
0.228
0.227
0.226
0.225
0.224
0.223
unitarity
Vus/Vud (Kμ2/πμ2)
fit
Vus (Kl3)
Vud (0+→ 0+)
0.971
0.972
0.973
0.974
0.975
0.976
0.977
Vud

CKM unitarity test

$$\Delta_{\text{CKM}} = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = 0.0001(6)$$

$\rightarrow$ strong bound on effective scale Λ of dimension-six operators violating quark-lepton universality of charged current weak interactions: $\Lambda > 11\,\text{TeV}\,(90\%\,\text{CL})$

V. Cirigliano, J.P. Jenkins, M. González-Alonso, 2010

$K \to \pi\pi\ell\nu$ ($K_{\ell 4}$) decays

$\pi\pi$ scattering lengths from K_{e4} data

$\pi\pi$ s-wave scattering lengths:

$$a_0 = \underbrace{\frac{7M_\pi^2}{32\pi F_\pi^2} = 0.159}_{\text{tree}} \to \underbrace{0.200}_{\text{one-loop}} \to \underbrace{0.216}_{\text{two-loop}} \to \underbrace{0.220(5)}_{\text{disp. th.} + \text{CHPT}}$$

$$a_2 = \underbrace{-\frac{M_\pi^2}{16\pi F_\pi^2} = -0.0454}_{\text{tree}} \to \underbrace{-0.0445}_{\text{one-loop}} \to \underbrace{-0.0445}_{\text{two-loop}} \to \underbrace{-0.0444(10)}_{\text{disp. th.} + \text{CHPT}}$$

most precise prediction: G. Colangelo, J. Gasser, H. Leutwyler, 2001

Results from NA48/2 K_{e4} data

J. R. Batley et al., 2010

two-parameter fit:

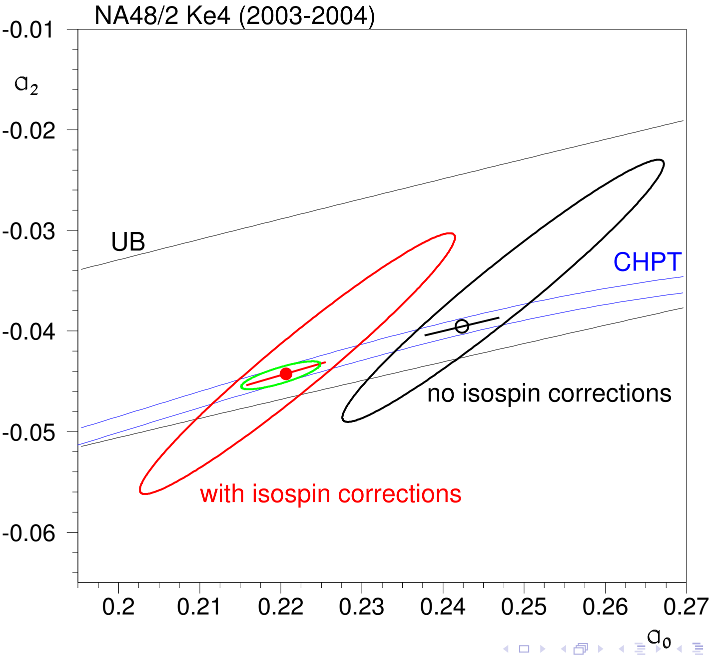
$$a_0 = 0.2220(128)_{\text{stat}}(50)_{\text{syst}}(37)_{\text{th}} \qquad a_2 = -0.0432(86)(34)(28)$$

alternative: using additional theoretical constraint

$\rightarrow$ one-parameter fit: $a_0 = 0.2206(49)(18)(64)$

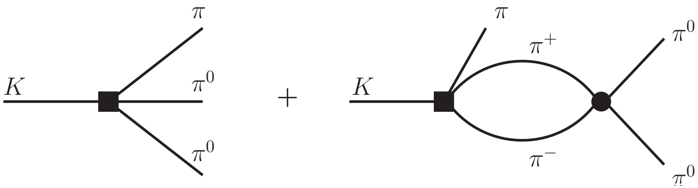
$(a_2 = -0.0442)$ from constraint

to be compared with theory: $a_0 = 0.220(5)$, $\quad a_2 = -0.0444(10)$


NA48/2 Ke4 (2003-2004)
a_2
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
UB
CHPT
no isospin corrections
with isospin corrections
0.2
0.21
0.22
0.23
0.24
0.25
0.26
0.27
a_0

$\pi\pi$ scattering lengths from $K \to 3\pi$ decays near threshold

π^+-π^0 mass difference $\to$ cusp in the $M_{\pi^0\pi^0}$ distribution at $M_{\pi^0\pi^0} = 2M_{\pi^+}$



seen in:

- $K^\pm \to \pi^\pm\pi^0\pi^0$ J.R. Batley et al. (NA48/2), 2006,2009
- $K_L \to 3\pi^0$ E. Abouzaid et al. (KTeV), 2008

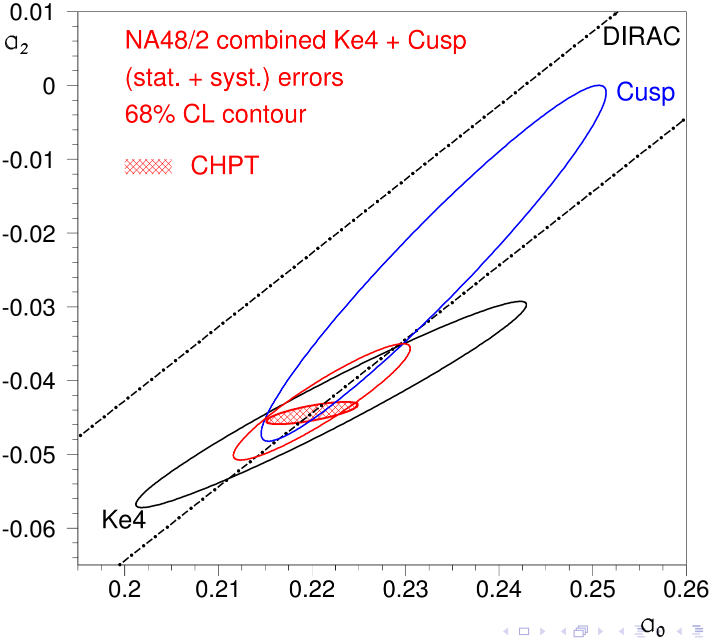
Cusp in $K \to 3\pi$ (theory)

$$a_0 - a_2 \sim A(\pi^+\pi^- \to \pi^0\pi^0)_{\text{threshold}}$$

- original idea: P. Budini, L. Fonda, 1961; N. Cabibbo, 2004
- unitarity analyticity $\to$ expansion of singular term in $M_{\pi^0\pi^0}$ distribution in powers of scattering lengths: N. Cabibbo, G. Isidori 2005
- unitarity, analyticity, CHPT: E. Gámiz, 2007
- two-loop dispersive representation of $K \to 3\pi$ amplitudes in the presence of isospin breaking: K. Kampf et al., 2009
- most advanced approach: nonrelativistic effective field theory (NRQFT)

  $K \to 3\pi$ amplitudes expanded in powers of the scattering lengths and of the pion momenta in the K rest frame: G. Colangelo et al., 2006; M. Bissegger et al., 2007; J. Gasser et al., 2011

  inclusion of electromagnetic corrections: M. Bissegger et al., 2009

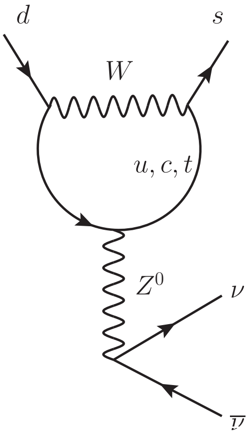

NA48/2 combined Ke4 + Cusp
(stat. + syst.) errors
68% CL contour
CHPT
DIRAC
Cusp
Ke4
a_2
a_0
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
0.2
0.21
0.22
0.23
0.24
0.25
0.26

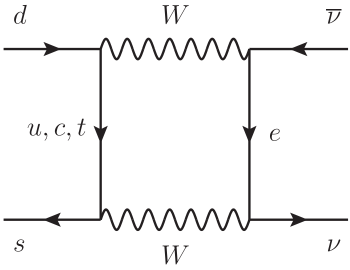
Results for the s-wave $\pi\pi$ scattering lengths

	$a_0 - a_2$	a_0	a_2
$K \to 3\pi$	0.2571(48)(25)(14)		$-0.0241(129)(94)(18)$
K_{e4}		0.2220(128)(50)(37)	$-0.0432(86)(34)(28)$
theory	0.264(4)	0.220(5)	$-0.0444(10)$

Impressive agreement between theory and experiment!

$K \to \pi\nu\bar{\nu}$





Z-penguin and box contributions to $K \to \pi\nu\bar{\nu}$

$K \to \pi\nu\bar{\nu}$ ctd.

$$A(s \to d\nu\bar{\nu}) \sim \frac{m_t^2}{M_W^2}\lambda_t + \frac{m_c^2}{M_W^2}\ln\frac{M_W}{m_c}\lambda_c + \frac{\Lambda_{\mathrm{QCD}}^2}{M_W^2}\lambda_u, \qquad \lambda_q = V_{qd}V_{qs}^*$$

GIM mechanism:

- t contribution (large CP-violating phase): $\sim 68\%$
- c contribution: $\sim 29\%$
- u contribution: $\sim 3\%$

large suppression of the u-quark (long-distance) contribution and complete dominance of the CP-violating contribution by the t-quark term

$K \to \pi\nu\bar{\nu}$ decays are "short-distance" dominated

$K \to \pi\nu\bar{\nu}$ ctd.

low energies: dominated by $Q_{13} = \bar{s}\gamma^{\mu}(1-\gamma_5)d \; \bar{\nu}\gamma_{\mu}(1-\gamma_5)\nu$

hadronic matrix element of Q_{13} can be related to $K_{\ell 3}$ form factors

$\to$ very accurate theoretical prediction of $K \to \pi\nu\bar{\nu}$ rates

Contributions of many authors: M. Lu, M.B. Wise, 1994; G. Buchalla, A. Buras, 1994, 1996, 1998, 1999; W.J. Marciano, Z. Parsa, 1996; G. Buchalla, G. Isidori, 1998; M. Misiak, J. Urban, 1999; A. Falk, A. Lewandowski, A. Petrov, 2001; G. Isidori, F. Mescia, , C. Smith, 2005; A. Buras, M. Gorbahn, U. Haisch, U. Nierste, 2005, 2006; J. Brod, M. Gorbahn, 2008; F. Mescia, C. Smith, 2007; J. Brod, M. Gorbahn, E. Stamou, 2011

Present status of $K \to \pi\nu\bar{\nu}$

from: V. Cirigliano, Kaon Physics Working Group, Project X Physics Study, June 21 2012

mode	SM	experiment
$K^+ \to \pi^+\nu\bar{\nu}$	$7.81(75)(29) \times 10^{-11}$	$(1.73^{+1.15}_{-1.05}) \times 10^{-10}$ E787/949
$K_L \to \pi^0\nu\bar{\nu}$	$2.43(39)(6) \times 10^{-11}$	$< 2.6 \times 10^{-8}$ E391a

Summary

- Interplay of short- and long-distance physics
- EFT methods: operator product expansion and CHPT
- Resonance model and lattice calculations to determine LECs
- Excellent precision for semileptonic decays
- Electromagnetic contributions
- Strong isospin breaking
- Determination of V_{us} from $K_{\ell 3}$ decays
- CKM unitarity test

Summary ctd.

- Test of lepton universality in $R^K_{e/\mu}$
- Extraction of $\pi\pi$ scattering lengths from K_{e4} and $K \to 3\pi$
- Impressive interplay of high precision measurements and refined theoretical methods
- Short distance dominated decays $K \to \pi\nu\bar{\nu}$
- Very accurate theoretical predictions in the SM
- Sensitive probe for new physics