

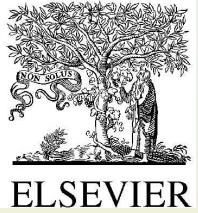
Tetraquarks - Evidence Grows!

Ahmed Ali (DESY), Tuesday, 29.06.2010

underlying work



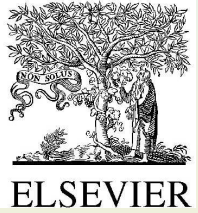
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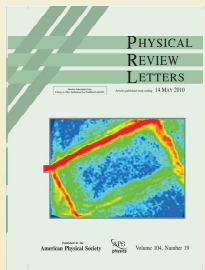
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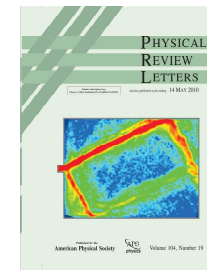
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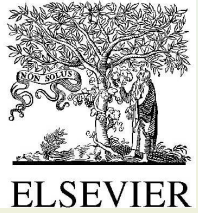
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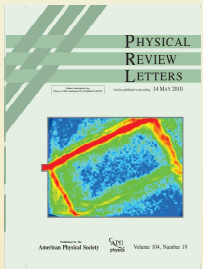
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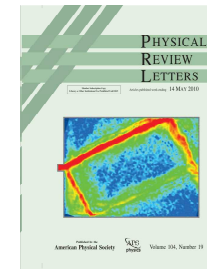
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(work in progress)



Overview

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- what are tetraquarks?

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- conclusion and outlook

what are tetraquarks?

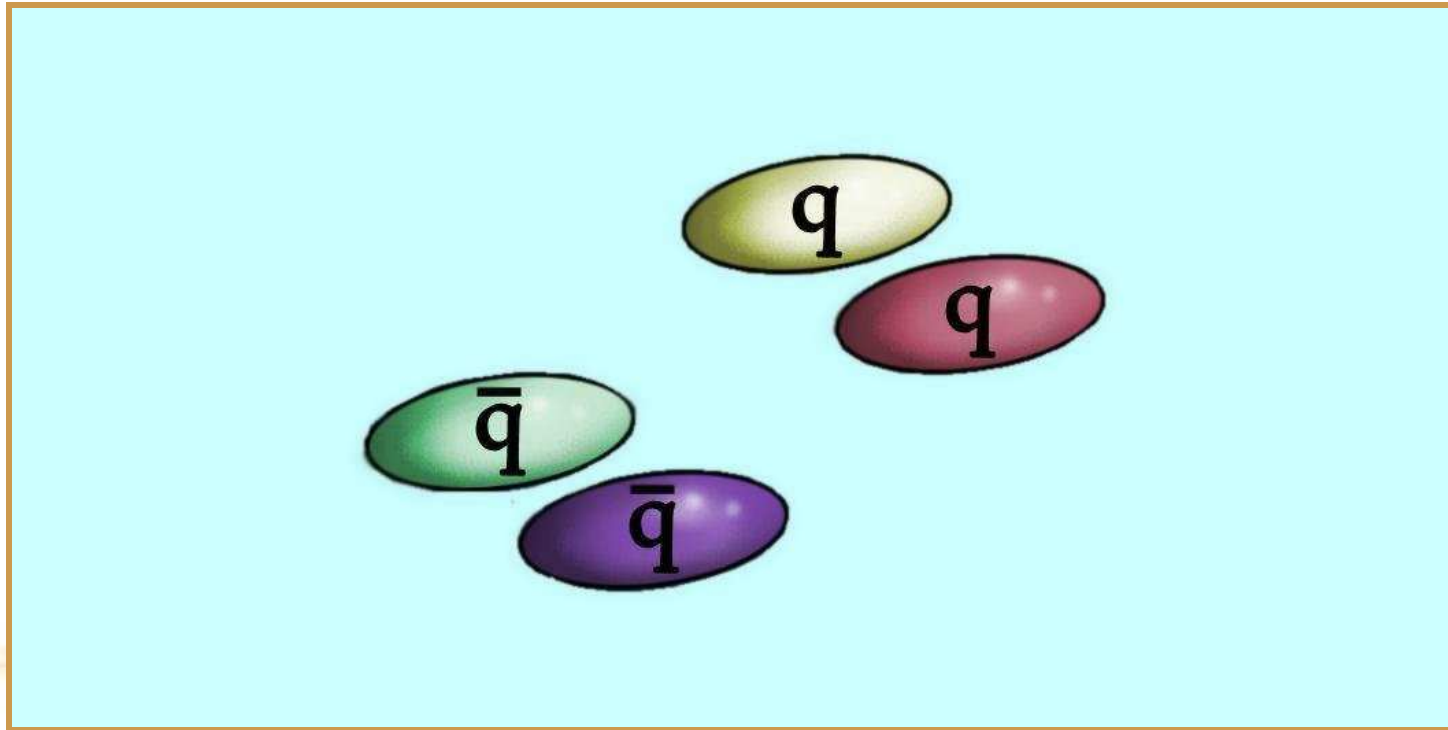
tetraquark constituents

the building blocks of tetraquarks are



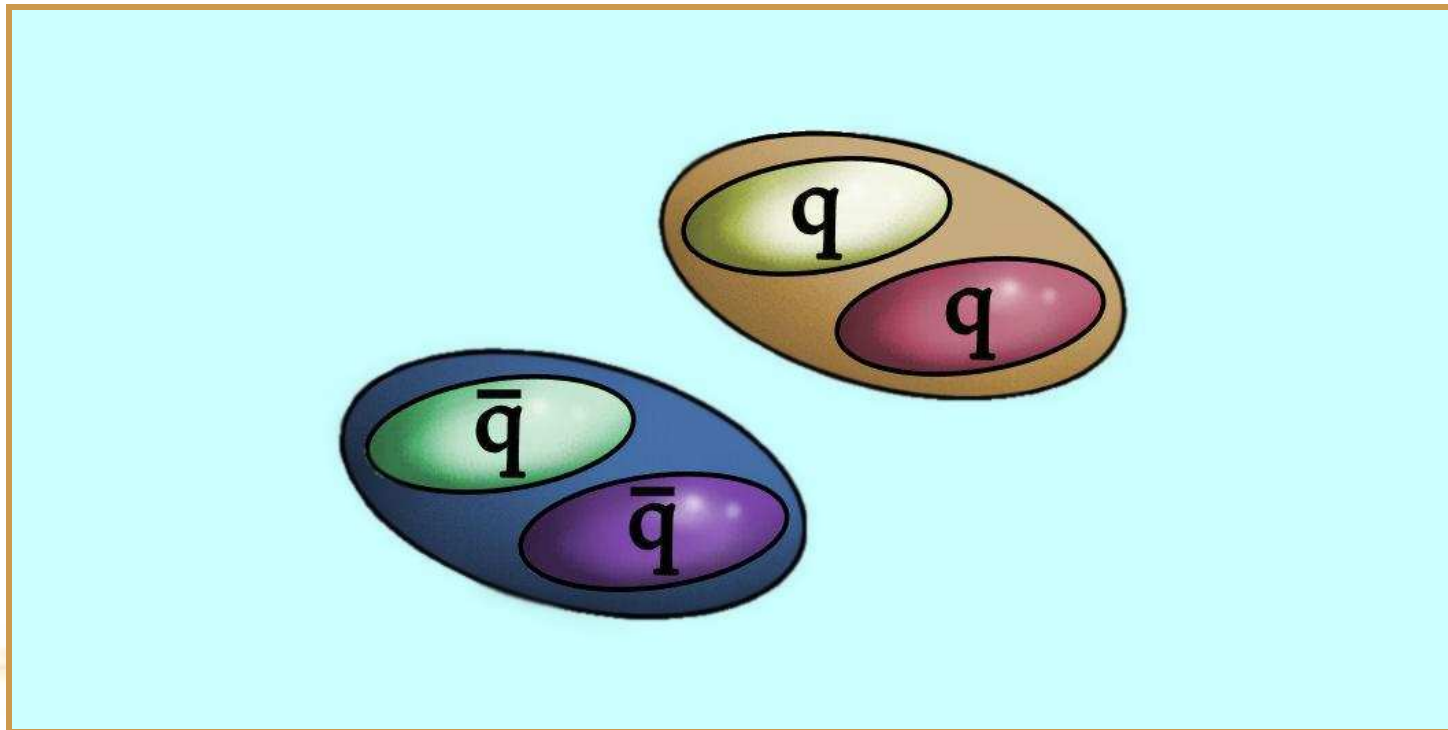
tetraquark constituents

4 quarks



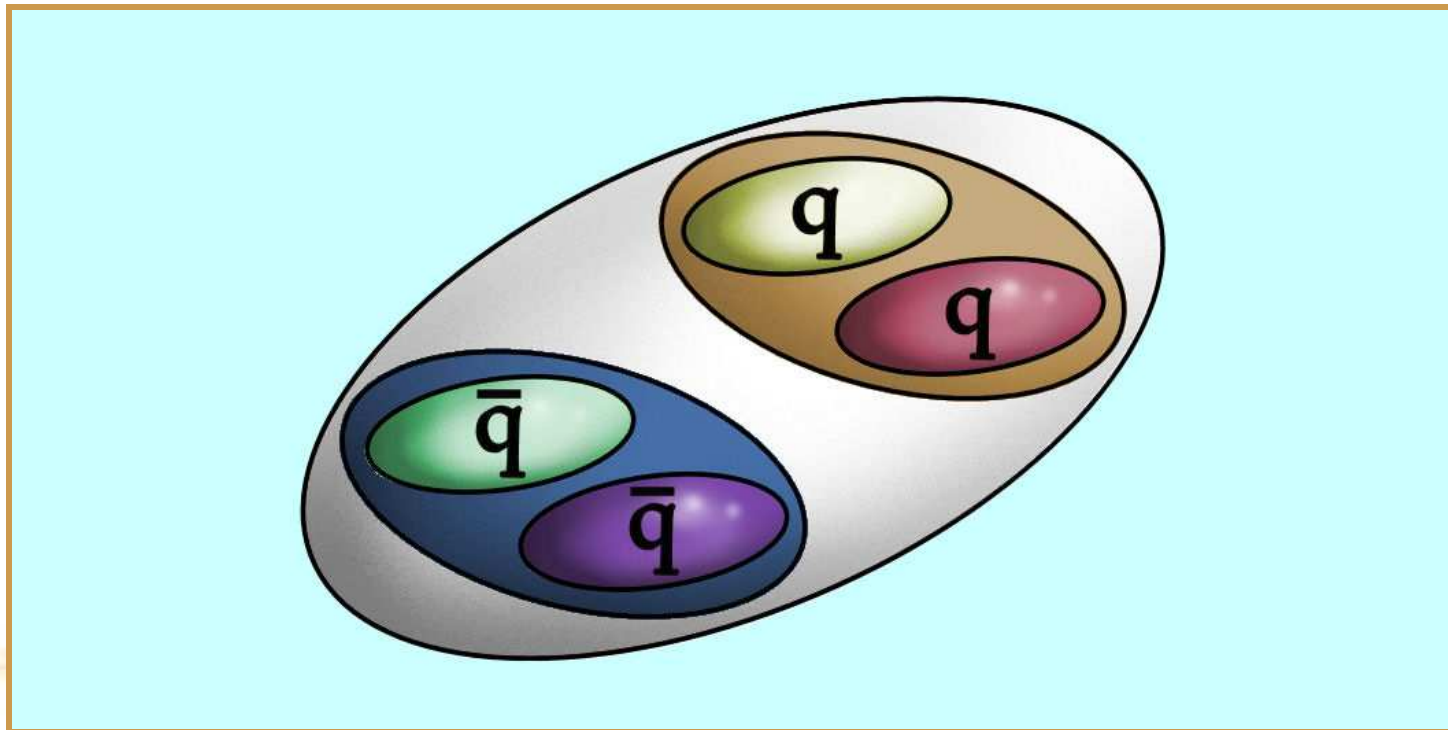
tetraquark constituents

forming a pair of diquarks and antiquarks

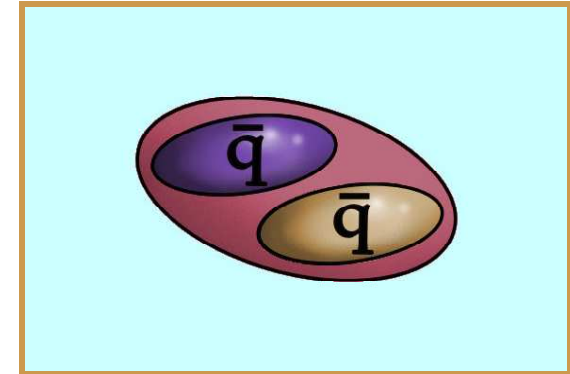


tetraquark constituents

the diquarks and antidiquarks form a tetraquark

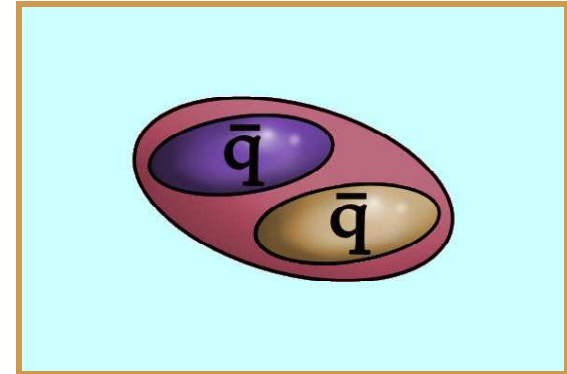


diquarks: color representation



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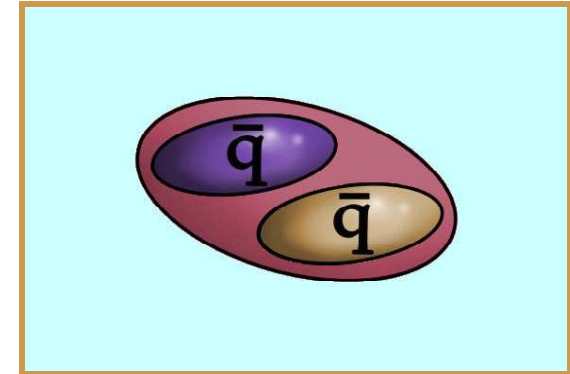
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From one gluon exchange model
results [R. Jaffe, Phys. Rep., 409 (2005) 1]:



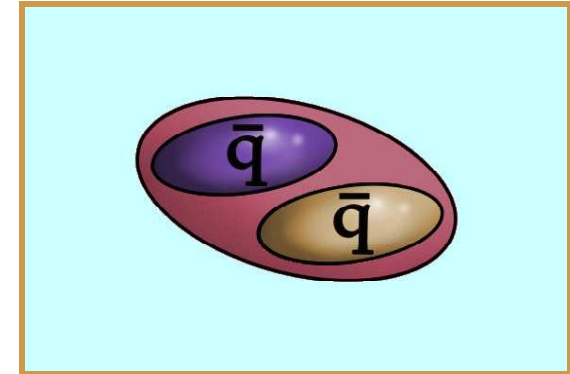
$D = A \otimes B$	$\mathbf{1}$	$\mathbf{3}$	$\bar{\mathbf{6}}$	$\mathbf{8}$
I	$-4/3$	$-2/3$	$1/3$	$1/6$



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The discriminator I

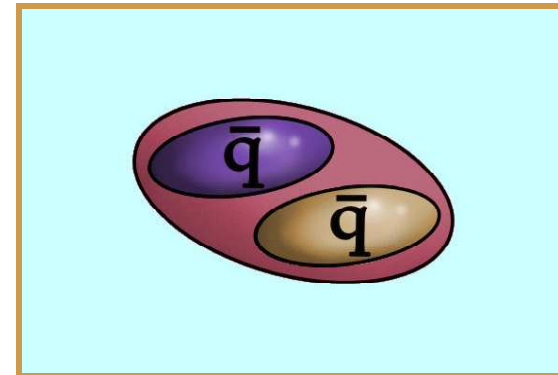
$$I = \frac{1}{2}(C(D) - C(A) - C(B)),$$

is the sum of the product of $SU(3)_C$ charges. **Attractive forces** (like in electromagnetism) exist for **negative signs** ($C(X)$: casimir invariant of representation X).

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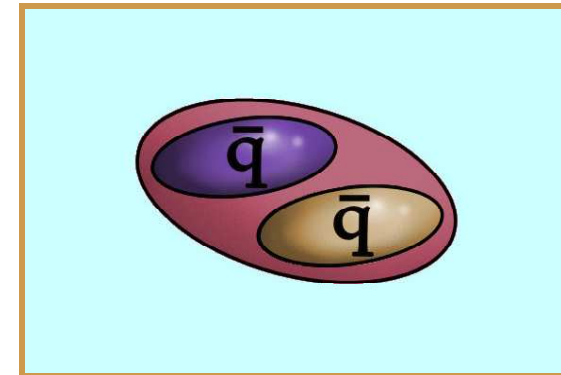
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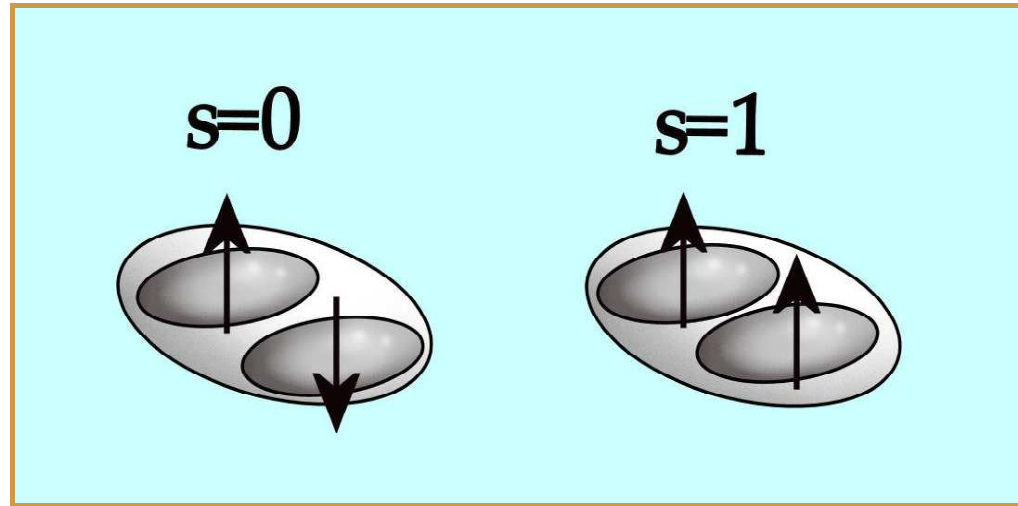
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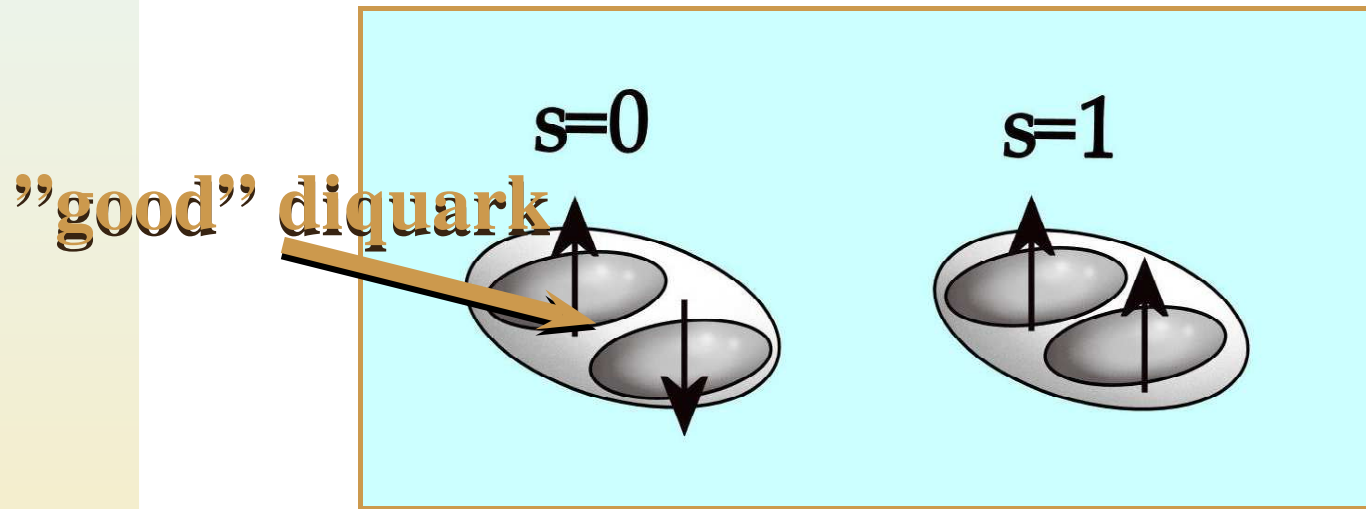
diquarks: spin representation



lattice simulations [Alexandrou et al., Phys. Rev. Lett. **97**, 222002 (2006)]
show in the light quark sector:



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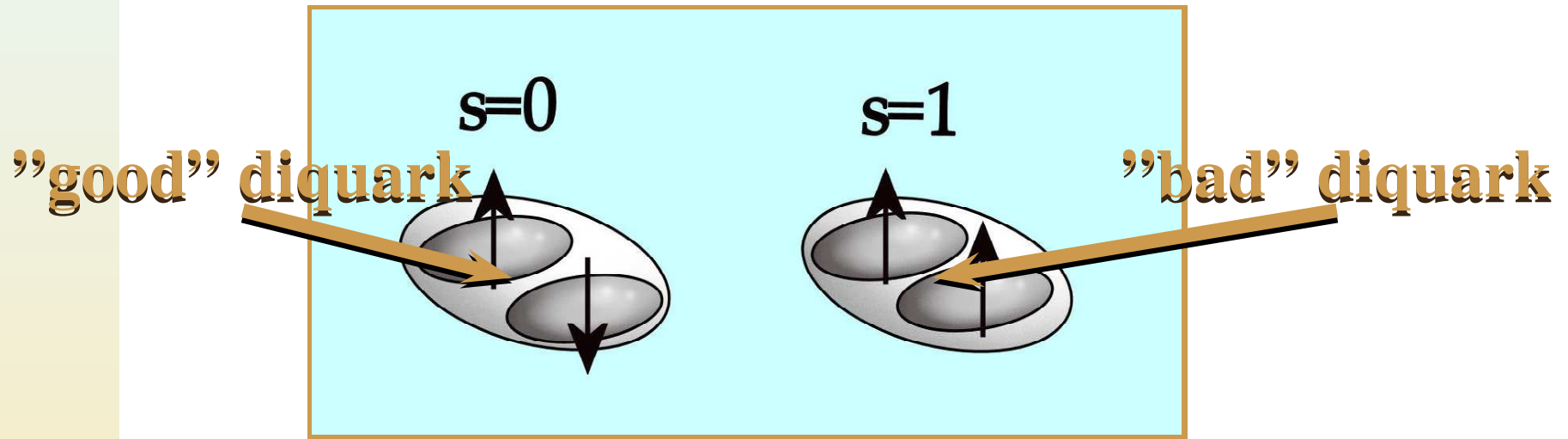


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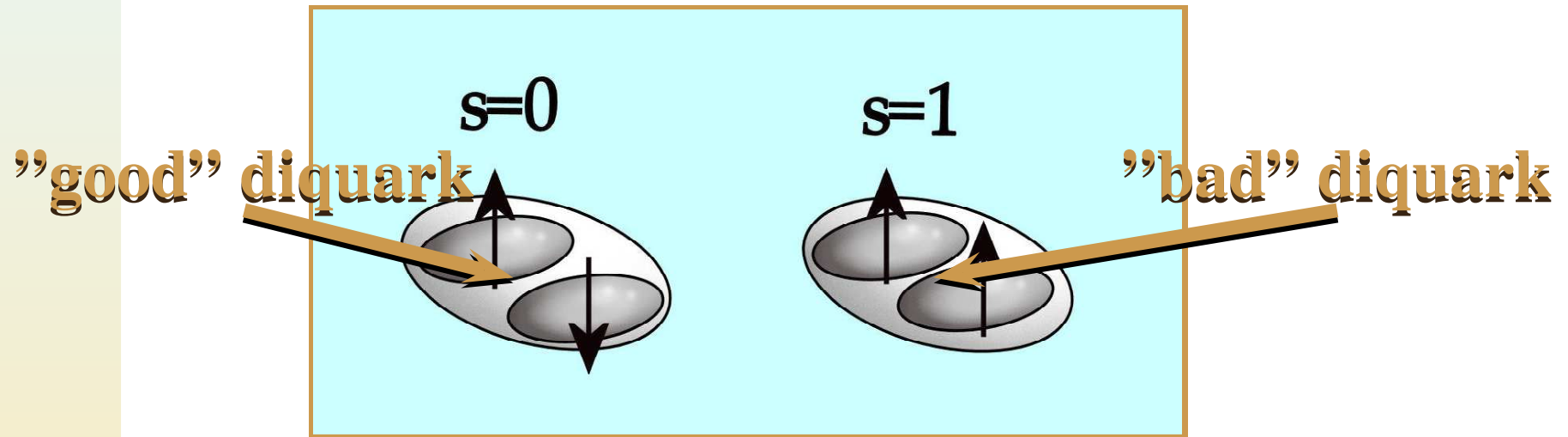


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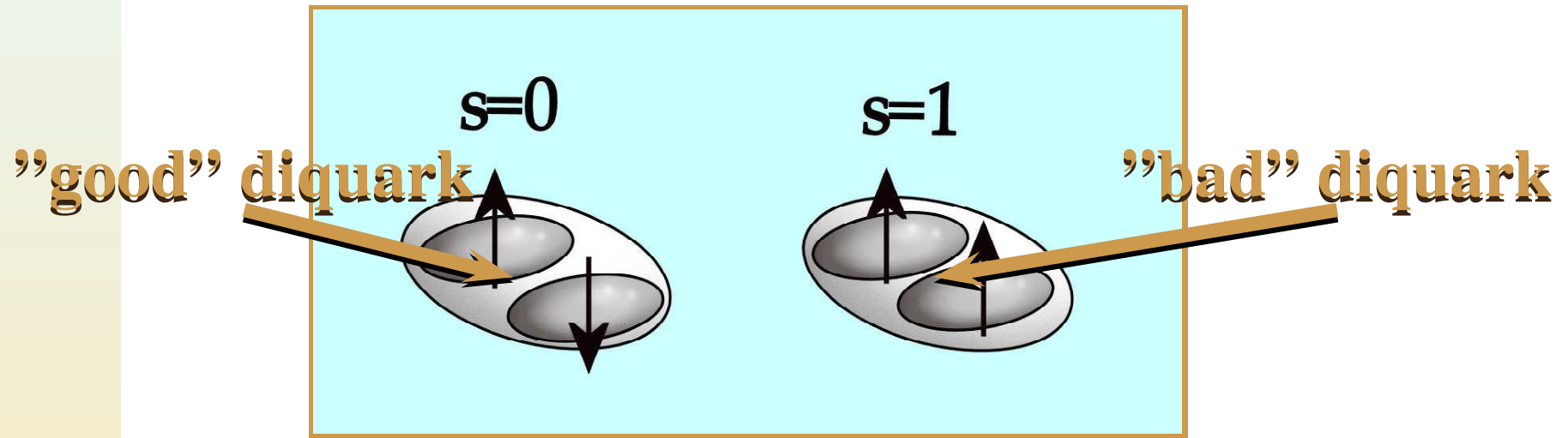
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In the Heavy-Quark-Limit **spin decoupling** should allow
for both cases to exist.



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- binding **now: closer look at** lattice results
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diquarks: evidence in lattice QCD

gauge invariant two-density correlators:

$$C_{\Gamma}(\mathbf{r}_u, \mathbf{r}_d, t) \equiv \langle 0 | J_{\Gamma}(\mathbf{0}, 2t) J_0^u(\mathbf{r}_u, t) J_0^d(\mathbf{r}_d, t) J_{\Gamma}^{\dagger}(\mathbf{0}, 0) | 0 \rangle$$



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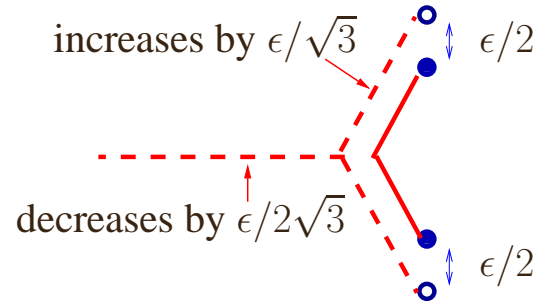
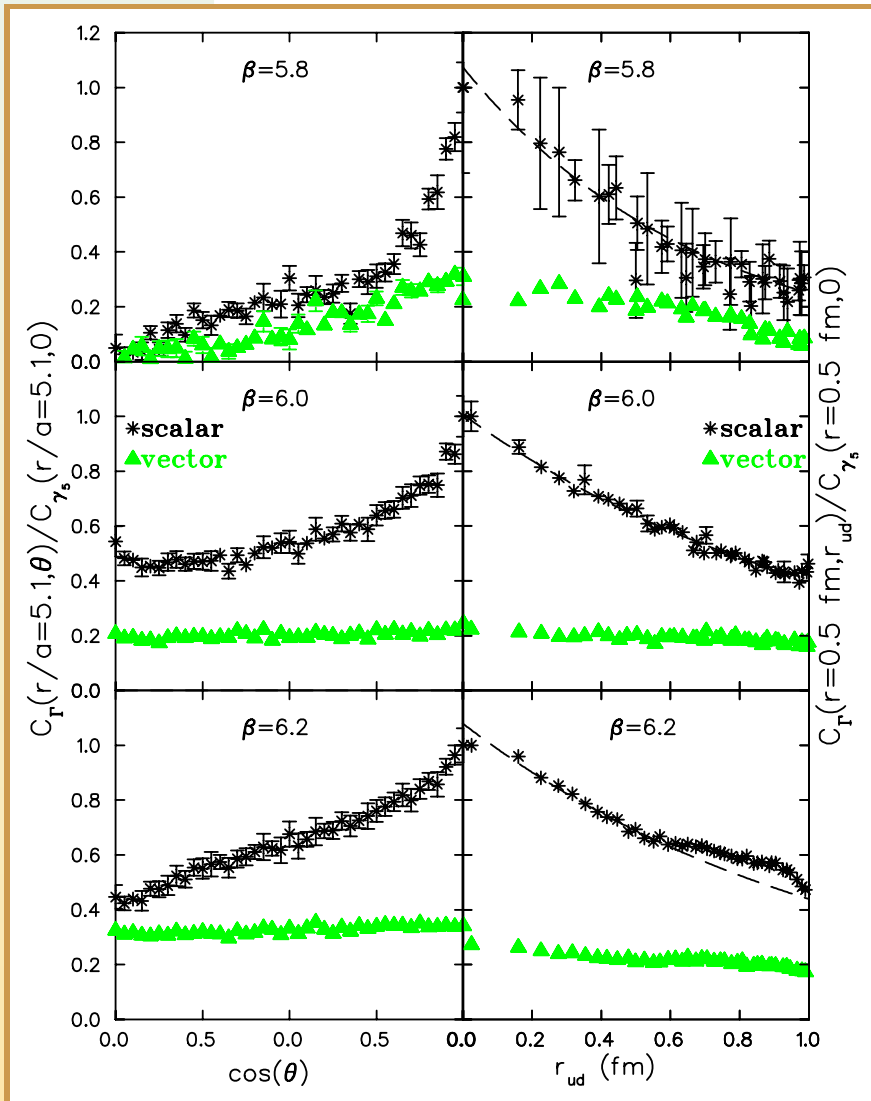
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1^+ : $\Gamma = \gamma_i$ diquark size: $r_0(r)$



diquarks: evidence in lattice QCD



$$\delta L = (2/\sqrt{3} - 1/2\sqrt{3})\epsilon = \sqrt{3}\epsilon/2$$

$\kappa = \frac{K_{[qq][\bar{q}\bar{q}]}}{K_{q\bar{q}}} = \sqrt{3}/2,$
 $K_{[qq][\bar{q}\bar{q}]}$ ($K_{q\bar{q}}$) being
 the string tension of the
 tetraquark (meson).

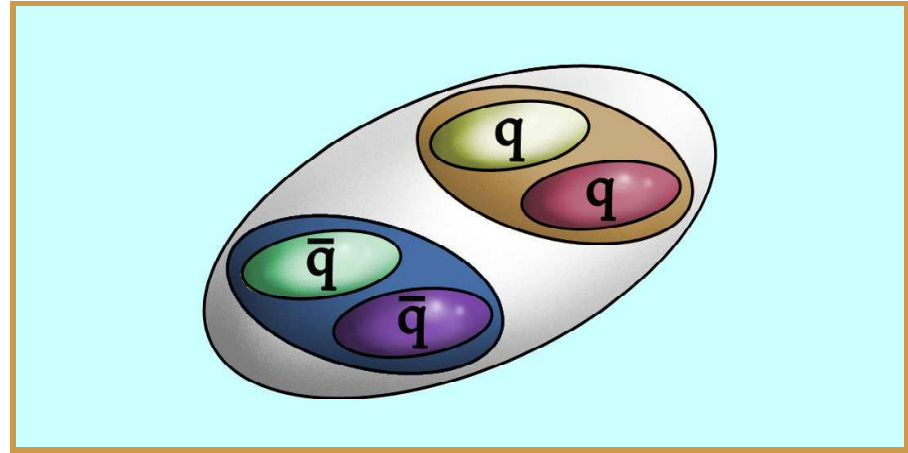


tetraquark vs. hadronic molecule



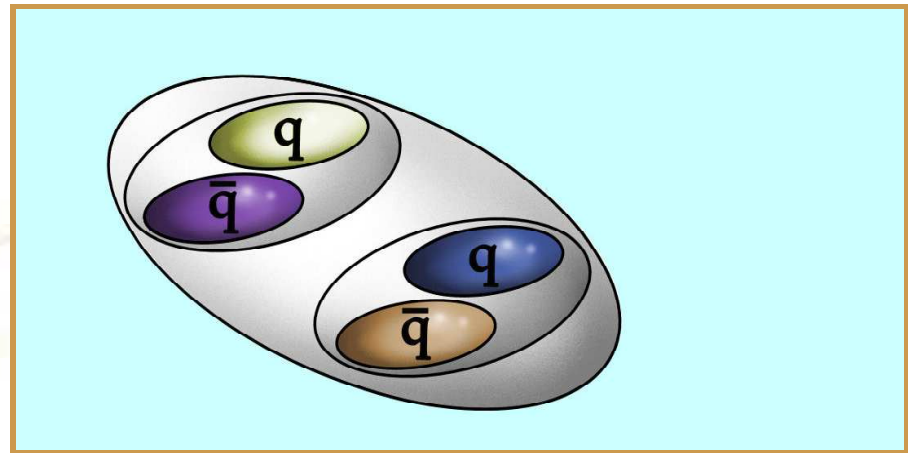
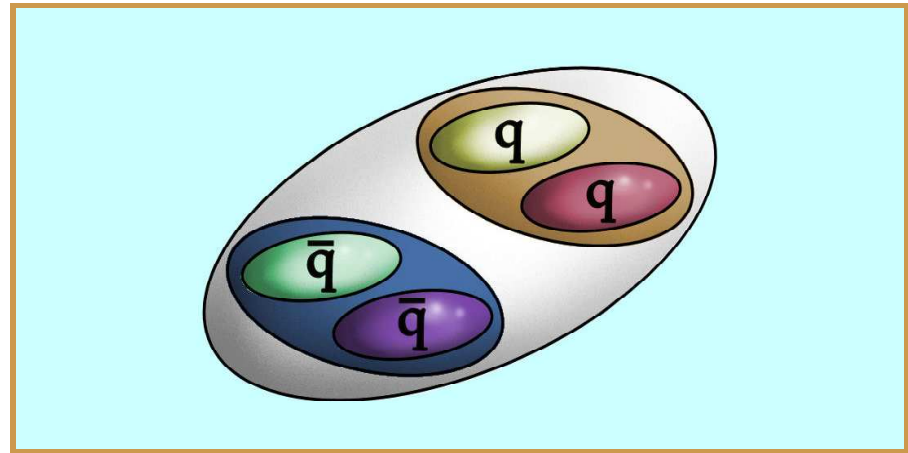
tetraquark vs. hadronic molecule

- A tetraquark is a **strongly bound** QCD object consisting of colored constituents, the diquarks and antiquarks:



tetraquark vs. hadronic molecule

- A tetraquark is a **strongly bound** QCD object consisting of colored constituents, the diquarks and antiquarks:
- In contrast to a hadronic molecule, which is bound by the **exchange of pions** :



states observed by Belle

Summary of new states observed by Belle. [arXiv:0910.3404 [hep-ex]]

State	M (MeV)	Γ (MeV)	J^{PC}	Decay Modes	Production Modes	Also observed by
$Y_s(2175)$	2175 ± 8	58 ± 26	1^{--}	$\phi f_0(980)$ $\pi^+ \pi^- J/\psi,$	$e^+ e^-$ (ISR) $J/\psi \rightarrow \eta Y_s(2175)$	BaBar, BESII BaBar
$X(3872)$	3871.4 ± 0.6	< 2.3	1^{++}	$\gamma J/\psi, D\bar{D}^*$	$B \rightarrow K X(3872), p\bar{p}$	CDF, D0,
$X(3915)$	3914 ± 4	28^{+12}_{-14}	$0/2^{++}$	$\omega J/\psi$	$\gamma\gamma \rightarrow X(3915)$	
$Z(3930)$	3929 ± 5	29 ± 10	2^{++}	$D\bar{D}$ $D\bar{D}^*$ (not $D\bar{D}$)	$\gamma\gamma \rightarrow Z(3940)$	
$X(3940)$	3942 ± 9	37 ± 17	$0^{?+}$	or $\omega J/\psi$	$e^+ e^- \rightarrow J/\psi X(3940)$	
$Y(3940)$	3943 ± 17	87 ± 34	$?^{?+}$	$\omega J/\psi$ (not $D\bar{D}^*$)	$B \rightarrow KY(3940)$	BaBar
$Y(4008)$	4008^{+82}_{-49}	226^{+97}_{-80}	1^{--}	$\pi^+ \pi^- J/\psi$	$e^+ e^-$ (ISR)	
$X(4160)$	4156 ± 29	139^{+113}_{-65}	$0^{?+}$	$D^* \bar{D}^*$ (not $D\bar{D}$)	$e^+ e^- \rightarrow J/\psi X(4160)$	
$Y(4260)$	4264 ± 12	83 ± 22	1^{--}	$\pi^+ \pi^- J/\psi$	$e^+ e^-$ (ISR)	BaBar, CLEO
$Y(4350)$	4361 ± 13	74 ± 18	1^{--}	$\pi^+ \pi^- \psi'$	$e^+ e^-$ (ISR)	BaBar
$X(4630)$	4634^{+9}_{-11}	92^{+41}_{-32}	1^{--}	$\Lambda_c^+ \Lambda_c^-$	$e^+ e^-$ (ISR)	
$Y(4660)$	4664 ± 12	48 ± 15	1^{--}	$\pi^+ \pi^- \psi'$	$e^+ e^-$ (ISR)	
$Z(4050)$	4051^{+24}_{-23}	82^{+51}_{-29}	?	$\pi^\pm \chi_{c1}$	$B \rightarrow K Z^\pm(4050)$	
$Z(4250)$	4248^{+185}_{-45}	177^{+320}_{-72}	?	$\pi^\pm \chi_{c1}$	$B \rightarrow K Z^\pm(4250)$	
$Z(4430)$	4433 ± 5	45^{+35}_{-18}	?	$\pi^\pm \psi'$	$B \rightarrow K Z^\pm(4430)$	
$Y_b(10890)$	$10,890 \pm 3$	55 ± 9	1^{--}	$\pi^+ \pi^- \Upsilon(1, 2, 3S)$	$e^+ e^- \rightarrow Y_b$	

recent theoretical review: [Drenska et al., arXiv:1006.2741 [hep-ph]]



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Summary of new states observed by Belle. [arXiv:0910.3404 [hep-ex]]

State	M (MeV)	Γ (MeV)	J^{PC}	Decay Modes	Production Modes	Also observed by
$Y_s(2175)$	2175 ± 8	58 ± 26	1^{--}	$\phi f_0(980)$ $\pi^+ \pi^- J/\psi,$	$e^+ e^-$ (ISR) $J/\psi \rightarrow \eta Y_s(2175)$	BaBar, BESII BaBar
$X(3872)$	3871.4 ± 0.6	< 2.3	1^{++}	$\gamma J/\psi, D\bar{D}^*$	$B \rightarrow K X(3872), p\bar{p}$	CDF, D0,
$X(3915)$	3914 ± 4	28^{+12}_{-14}	$0/2^{++}$	$\omega J/\psi$	$\gamma\gamma \rightarrow X(3915)$	
$Z(3930)$	3929 ± 5	29 ± 10	2^{++}	$D\bar{D}$ $D\bar{D}^*$ (not $D\bar{D}$)	$\gamma\gamma \rightarrow Z(3940)$	
$X(3940)$	3942 ± 9	37 ± 17	$0^{?+}$	or $\omega J/\psi$	$e^+ e^- \rightarrow J/\psi X(3940)$	
$Y(3940)$	3943 ± 17	87 ± 34	$?^{?+}$	$\omega J/\psi$ (not $D\bar{D}^*$)	$B \rightarrow KY(3940)$	BaBar
$Y(4008)$	4008^{+82}_{-49}	226^{+97}_{-80}	1^{--}	$\pi^+ \pi^- J/\psi$	$e^+ e^-$ (ISR)	
$X(4160)$	4156 ± 29	139^{+113}_{-65}	$0^{?+}$	$D^* \bar{D}^*$ (not $D\bar{D}$)	$e^+ e^- \rightarrow J/\psi X(4160)$	
$Y(4260)$	4264 ± 12	83 ± 22	1^{--}	$\pi^+ \pi^- J/\psi$	$e^+ e^-$ (ISR)	BaBar, CLEO
$Y(4350)$	4361 ± 13	74 ± 18	1^{--}	$\pi^+ \pi^- \psi'$	$e^+ e^-$ (ISR)	BaBar
$X(4630)$	4634^{+9}_{-11}	92^{+41}_{-32}	1^{--}	$\Lambda_c^+ \Lambda_c^-$	$e^+ e^-$ (ISR)	
$Y(4660)$	4664 ± 12	48 ± 15	1^{--}	$\pi^+ \pi^- \psi'$	$e^+ e^-$ (ISR)	
$Z(4050)$	4051^{+24}_{-23}	82^{+51}_{-29}	?	$\pi^\pm \chi_{c1}$	$B \rightarrow K Z^\pm(4050)$	
$Z(4250)$	4248^{+185}_{-45}	177^{+320}_{-72}	?	$\pi^\pm \chi_{c1}$	$B \rightarrow K Z^\pm(4250)$	
$Z(4430)$	4433 ± 5	45^{+35}_{-18}	?	$\pi^\pm \psi'$	$B \rightarrow K Z^\pm(4430)$	
$Y_b(10890)$	$10,890 \pm 3$	55 ± 9	1^{--}	$\pi^+ \pi^- \Upsilon(1, 2, 3S)$	$e^+ e^- \rightarrow Y_b$	

recent theoretical review: [Drenska et al., arXiv:1006.2741 [hep-ph]]

our (hidden bottom) tetraquark candidate



calculating tetraquark masses

- QCD Sum Rules [Wang, Eur. Phys. J. C **67**, 411 (2010)]
- Relativistic Quasipotential model [Ebert et al., Mod. Phys. Lett. A **24**, 567 (2009)]
- Constituent quark model [Drenska et al., arXiv:1006.2741 [hep-ph]]
- Estimates presented here are based on [Ali et al., Phys. Lett. B **684**, 28 (2010)]

tetraquarks as mathematical objects

- interpolating operators of the diquarks:

”good”: 0^+ $Q_{i\alpha} = \epsilon_{\alpha\beta\gamma} (\bar{b}_c^\beta \gamma_5 q_i^\gamma - \bar{q}_{i_c}^\beta \gamma_5 b^\gamma)$



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states

Tetraquark states are characterized by the spin s_Q of the diquark coupling to total angular momentum J :

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hamiltonian

The previously defined states need to diagonalize the hamiltonian:

$$H = 2m_Q + H_{SS}^{(qq)} + H_{SS}^{(q\bar{q})} + H_{SL} + H_{LL}$$



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$$H_{LL} = B_Q \frac{L_{Q\bar{Q}}(L_{Q\bar{Q}} + 1)}{2}$$



example 1^{++}

- giving the 1^{++} , $[\bar{b}\bar{q}][bq]$ state as example:

$$|1^{++}\rangle = \frac{1}{\sqrt{2}} (|0_Q, 1_{\bar{Q}}; 1_J\rangle + |1_Q, 0_{\bar{Q}}; 1_J\rangle)$$



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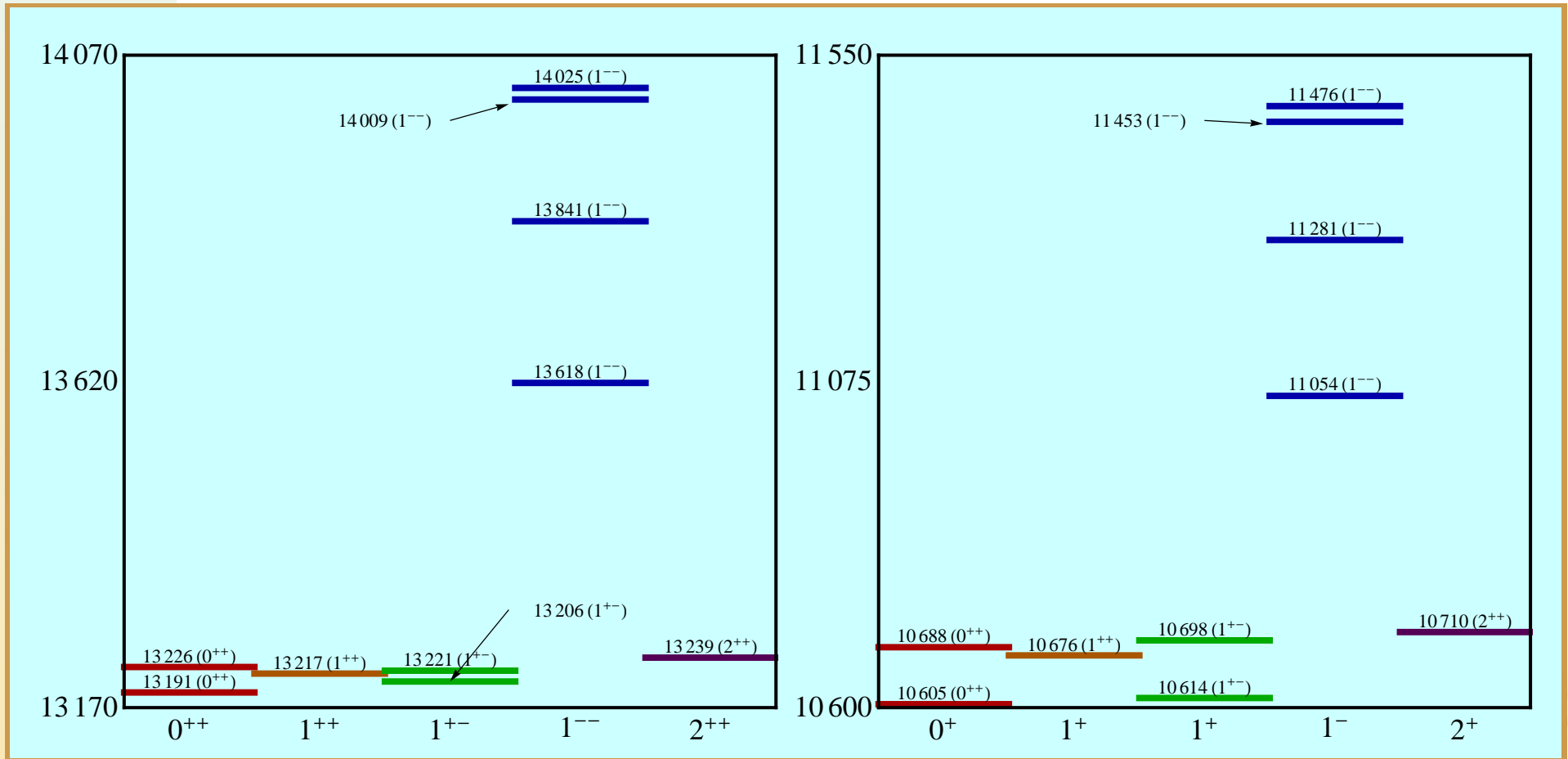
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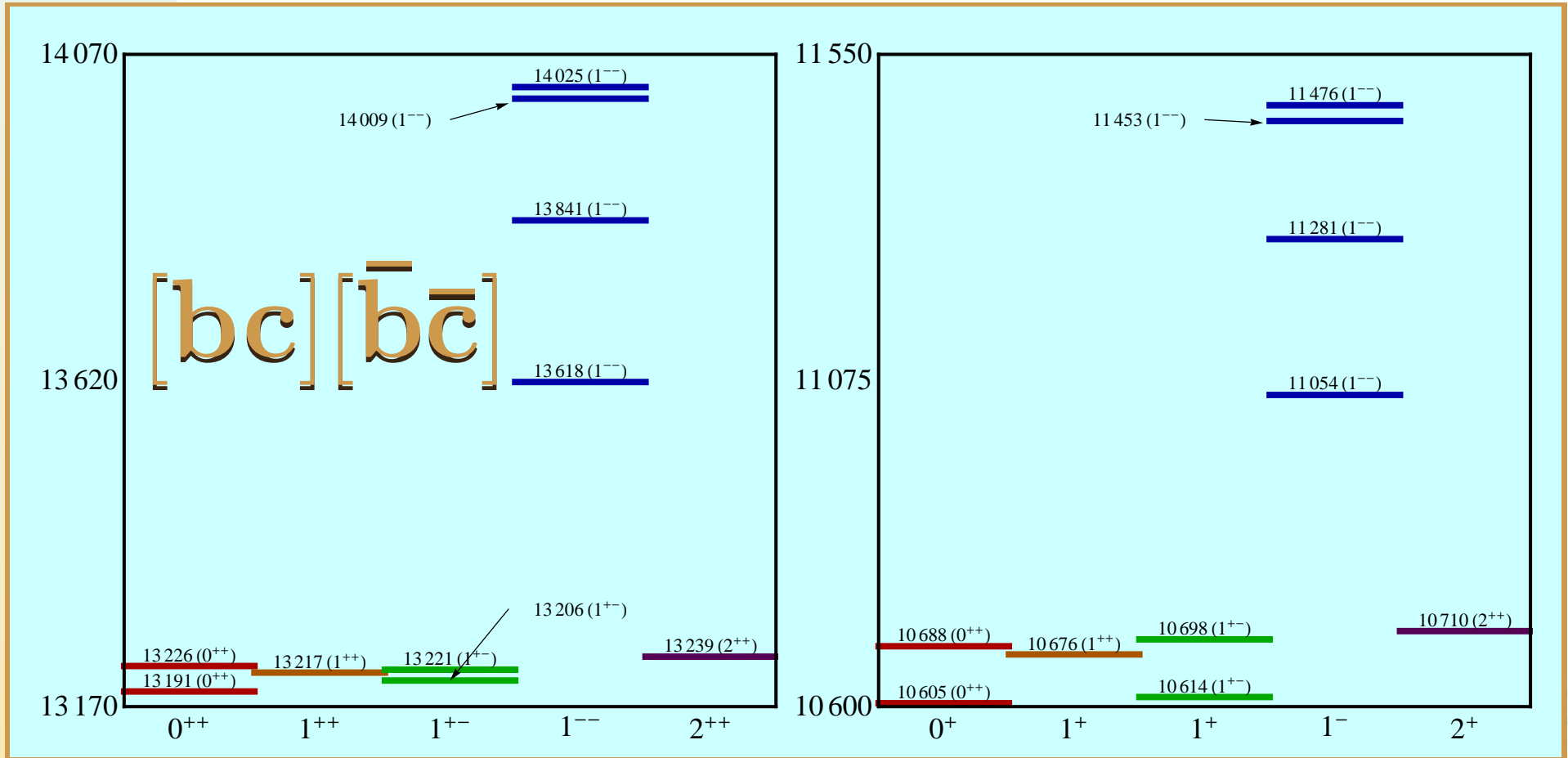
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$m_{[bq]}$	$(\mathcal{K}_{bq})_{\bar{3}}$	$\mathcal{K}_{b\bar{q}}$	$\mathcal{K}_{q\bar{q}}$	$\mathcal{K}_{b\bar{b}}$
5250 MeV	6 MeV	6 MeV	80 MeV	9 MeV

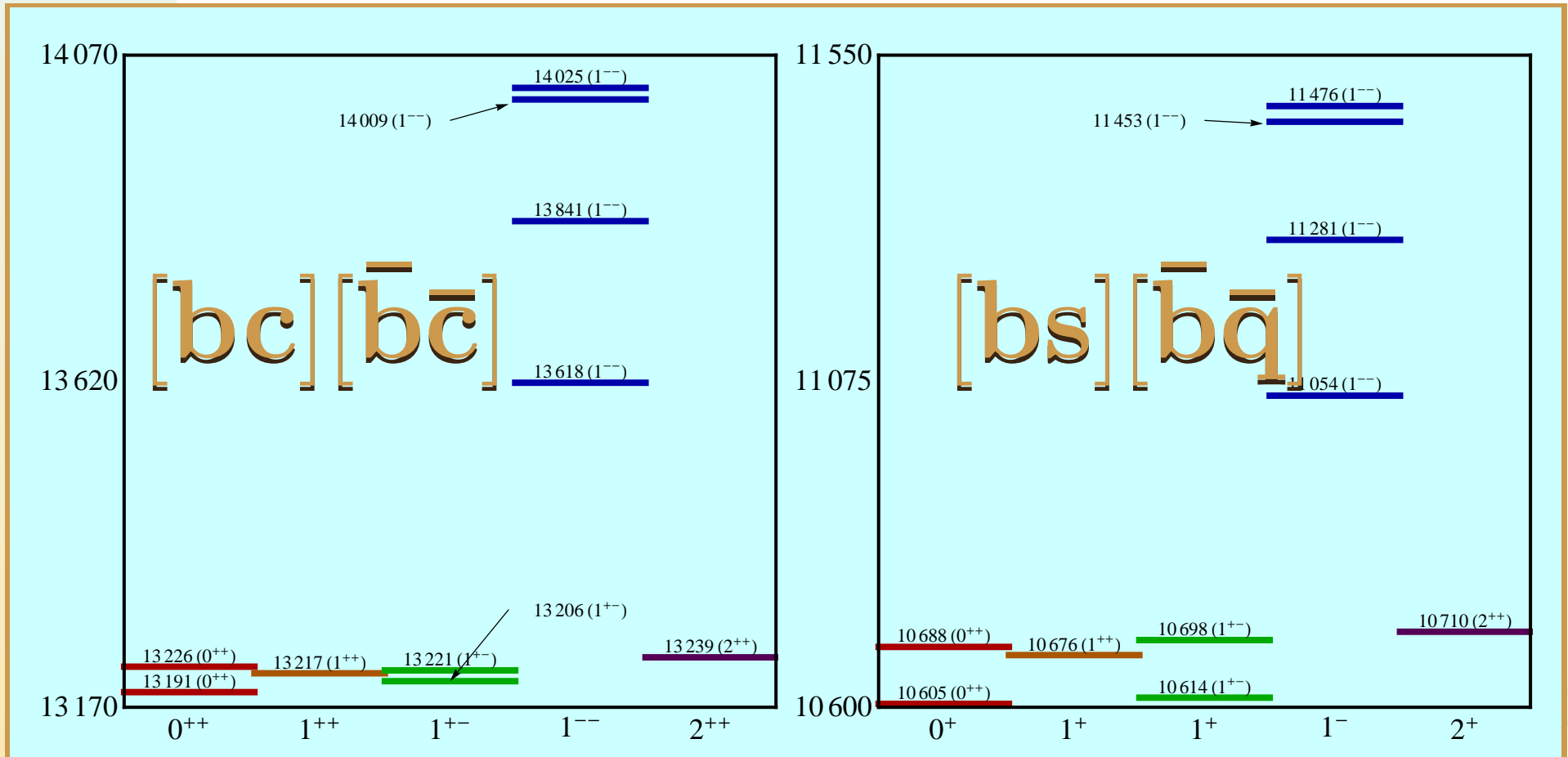
tetraquark mass spectrum



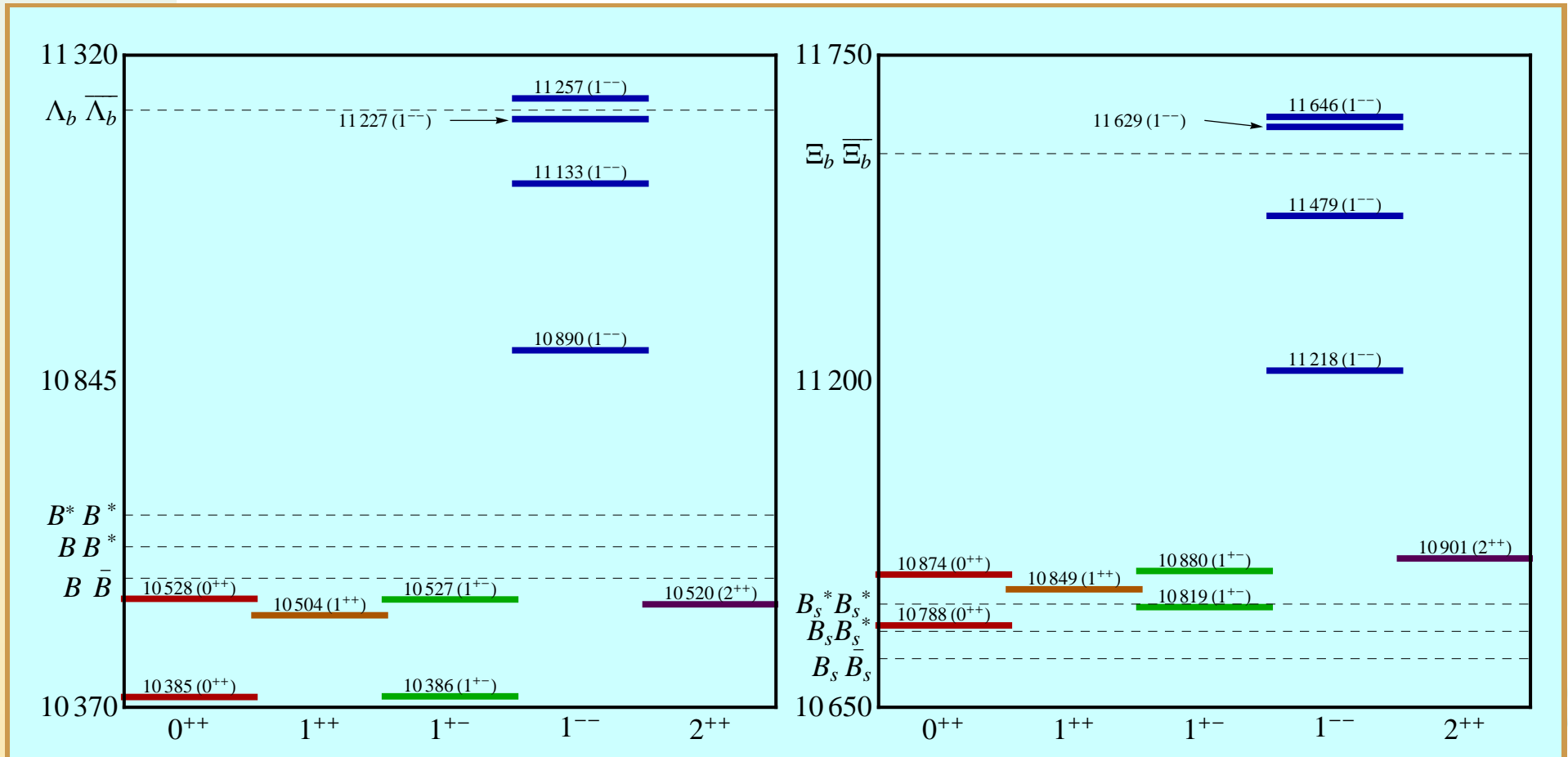
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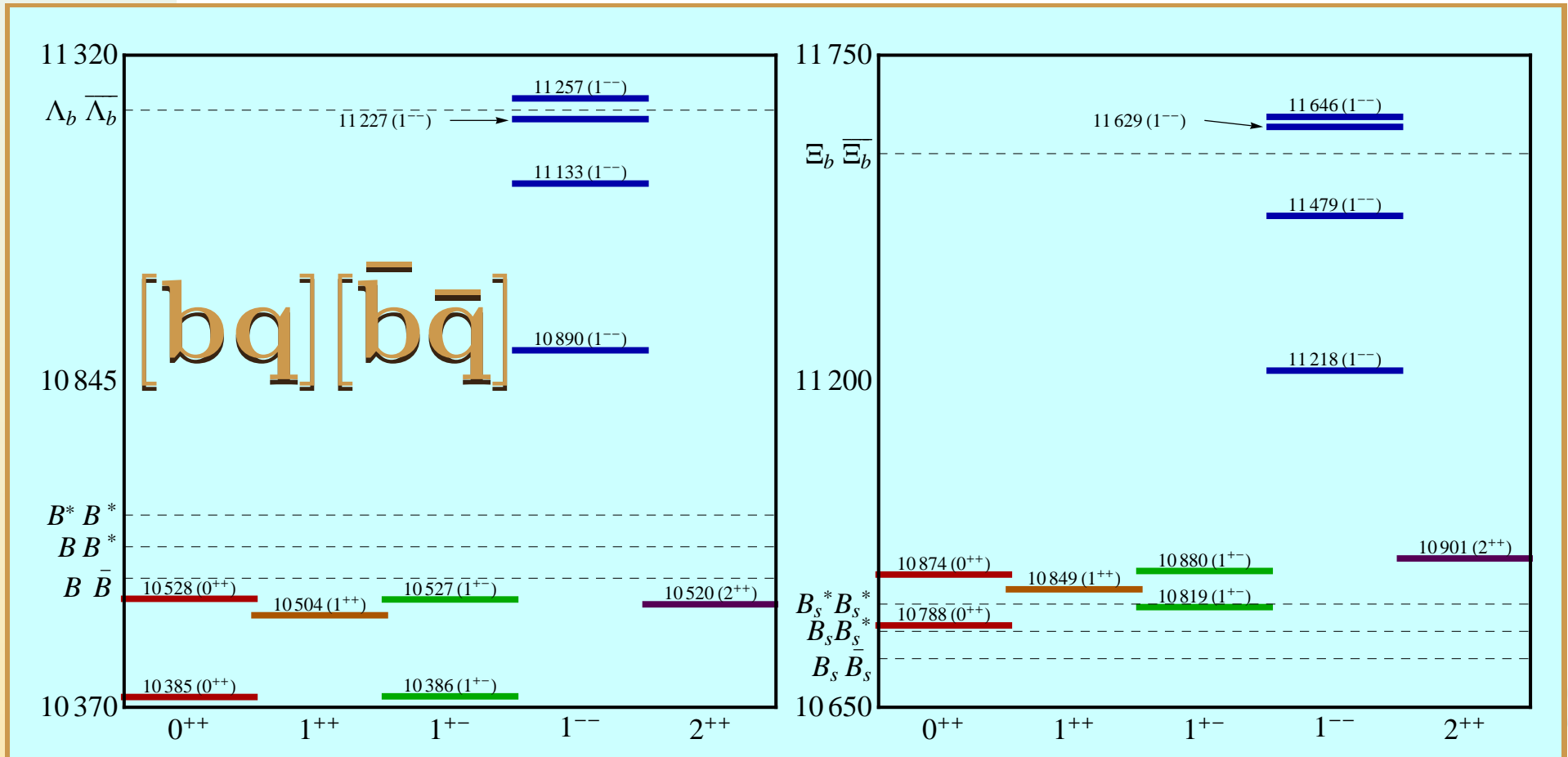
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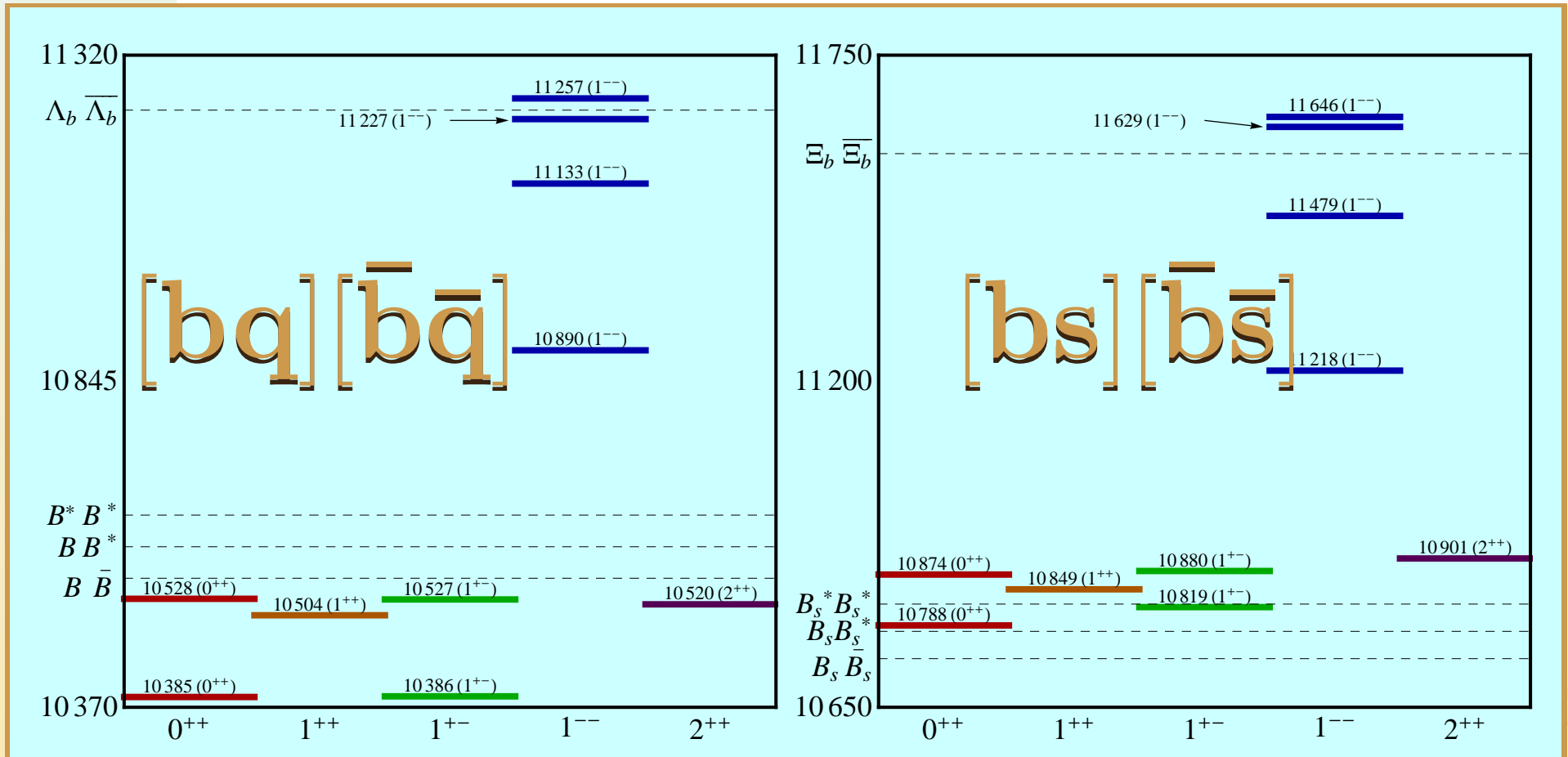
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- they will give rise to **80 mass eigenstates** if isospin breaking is taken into account
- most of them are **hard to find** due to typically large widths and mixing with the ordinary hadrons

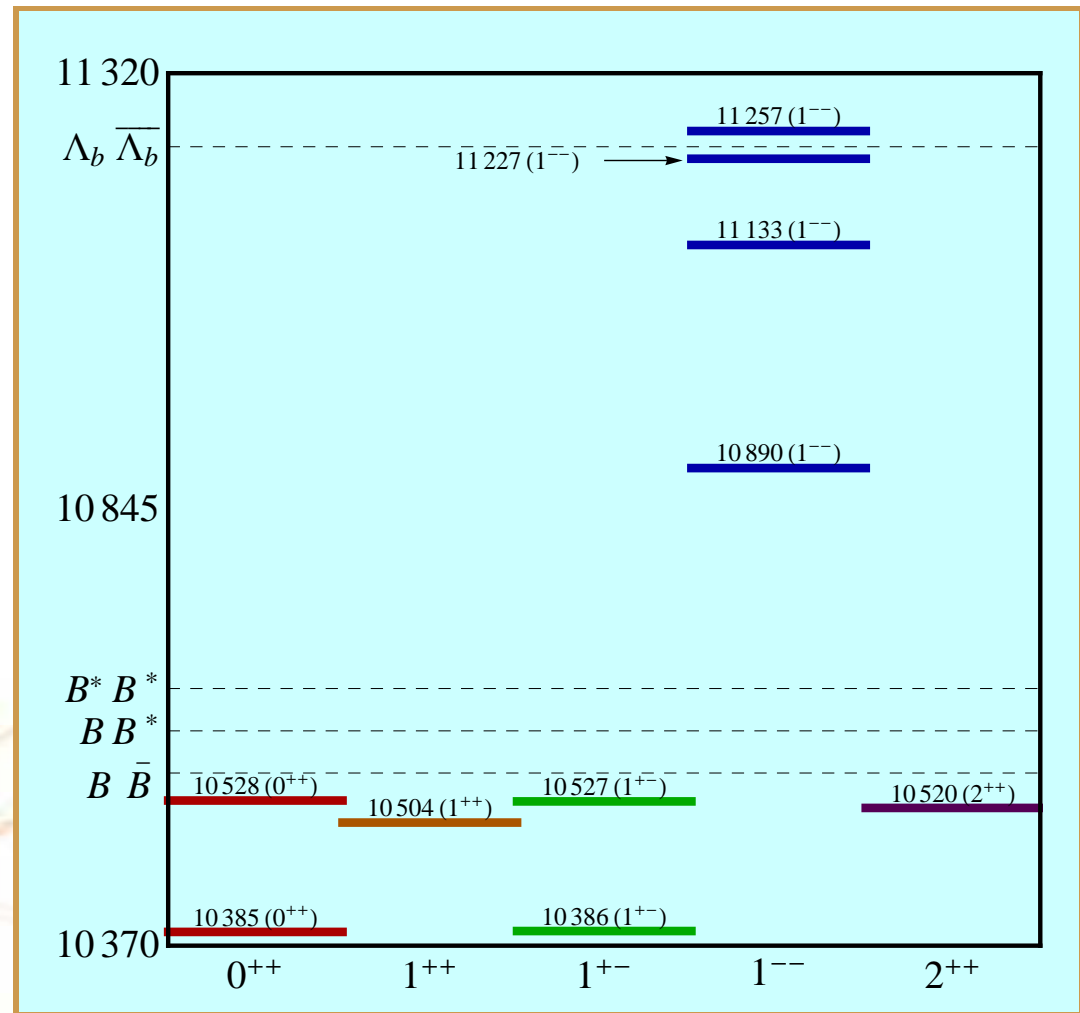
possible observable states

which states are accessible in today's experiments?



possible observable states

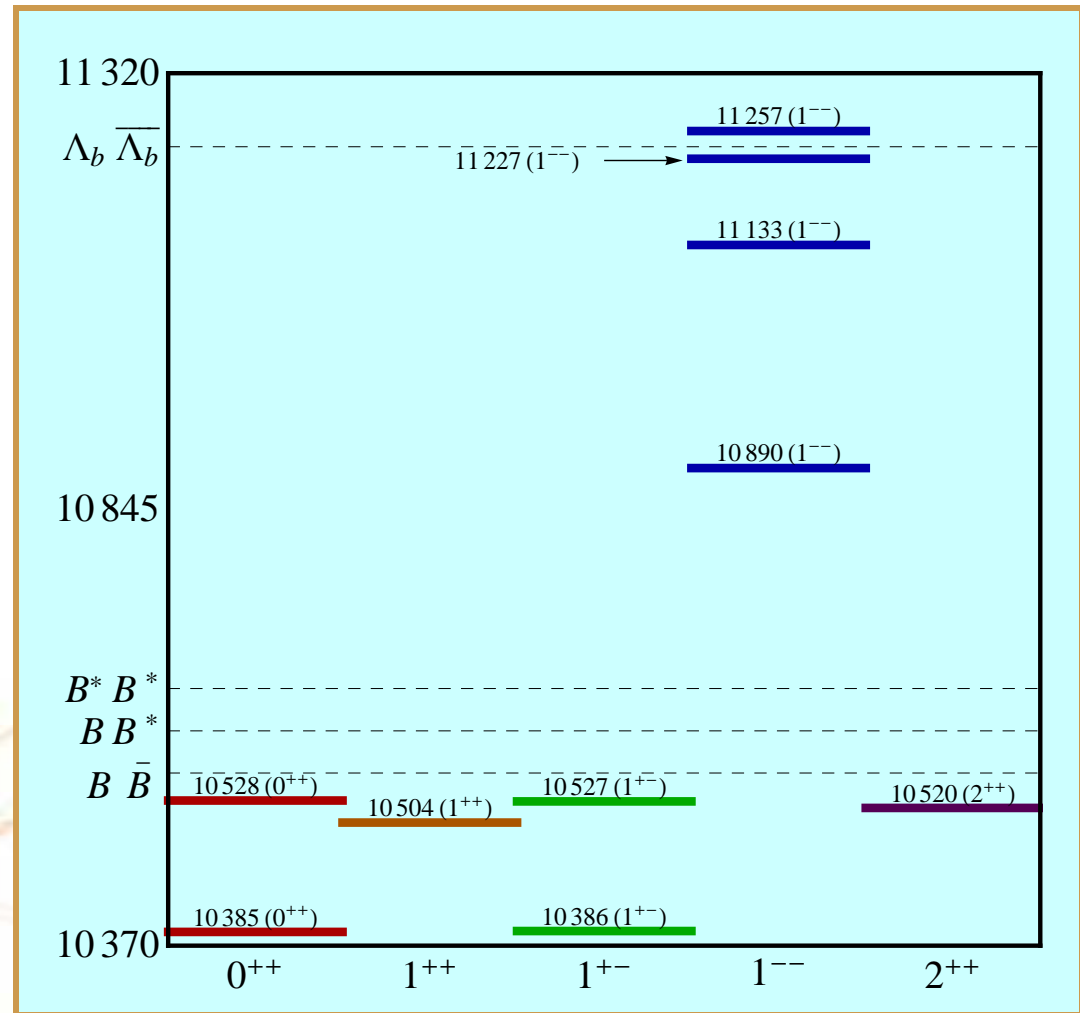
closer look at the light $[bq][\bar{b}\bar{q}]$ tetraquarks:



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- The 1^{--} states, we call Y_b , have the right quantum numbers to be produced in e^+e^- annihilation.

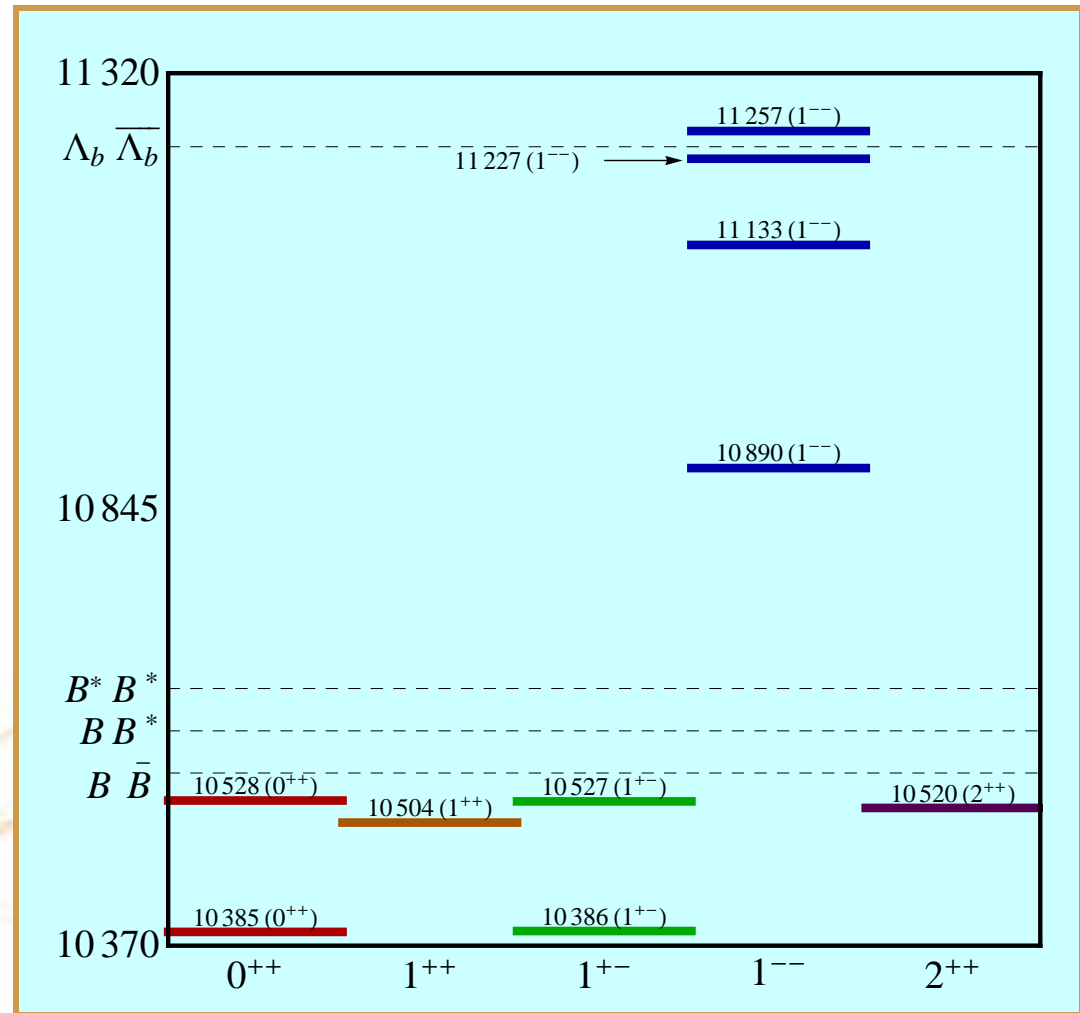


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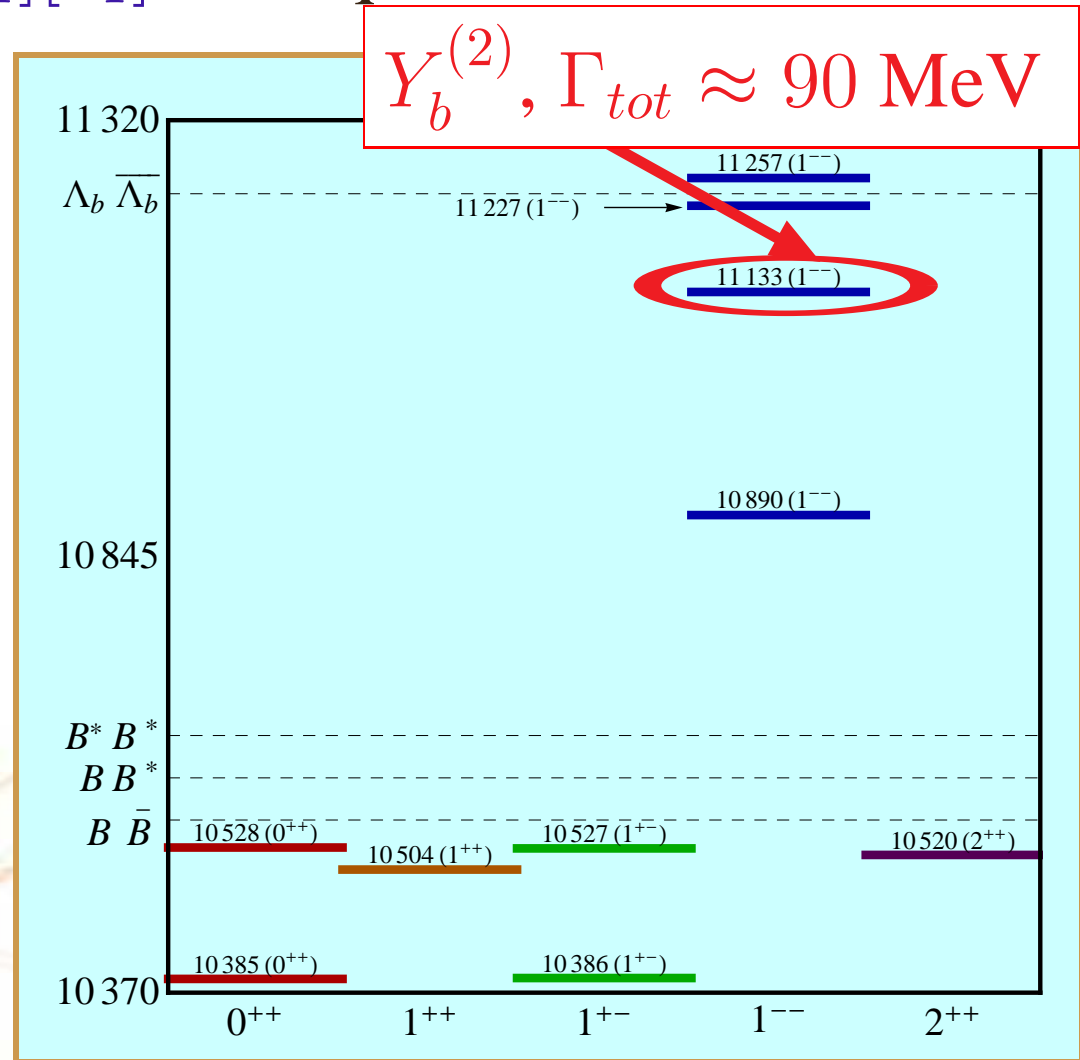


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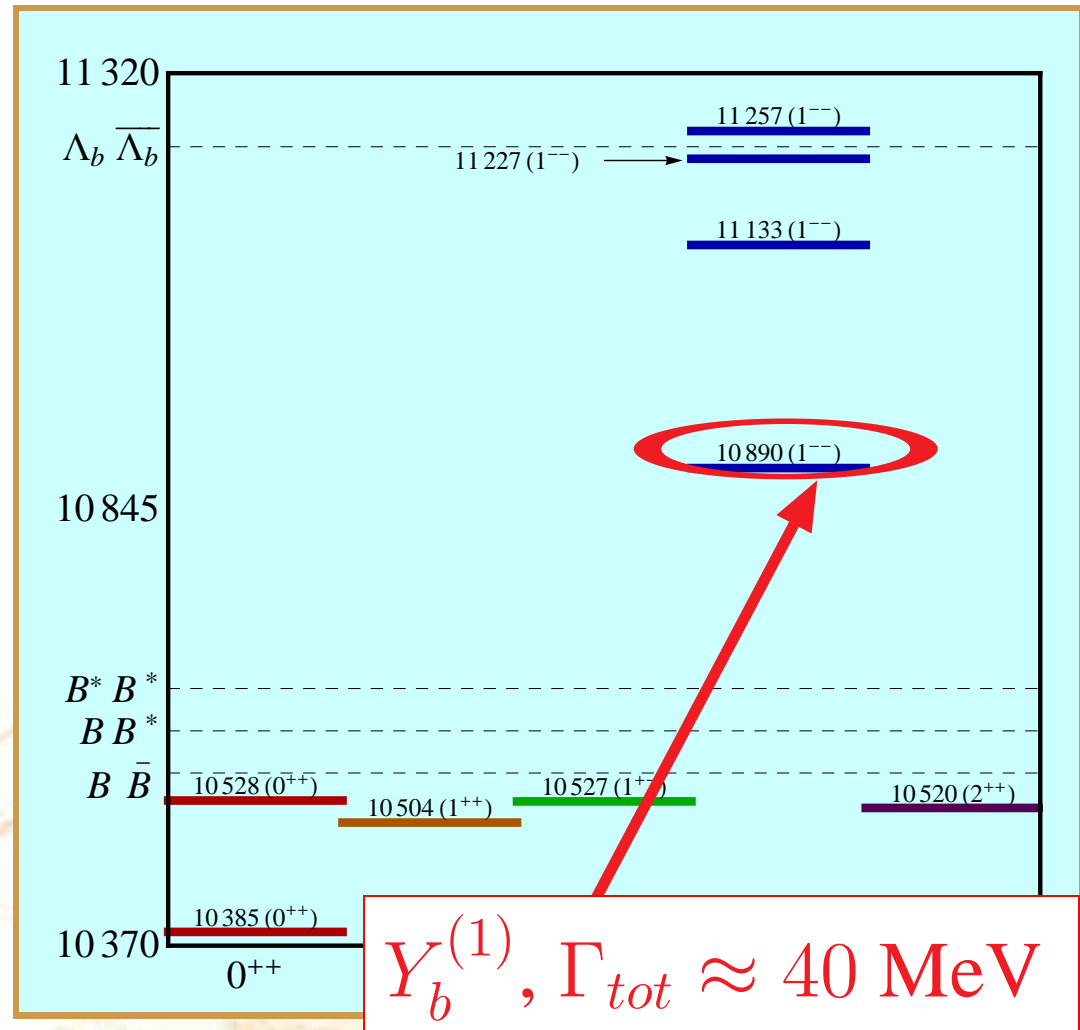
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isospin breaking

Y_b mass eigenstates

$$Y_{[b,l]} = \cos \theta Y_{[bu]} + \sin \theta Y_{[bd]},$$
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and charge

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Y_b production

- We have derived the Van Royen-Weisskopf formula for the electronic widths of the 1^{--} tetraquark, made up of point-like diquarks [A. Ali, C. Hambrock and S. Mishima] :



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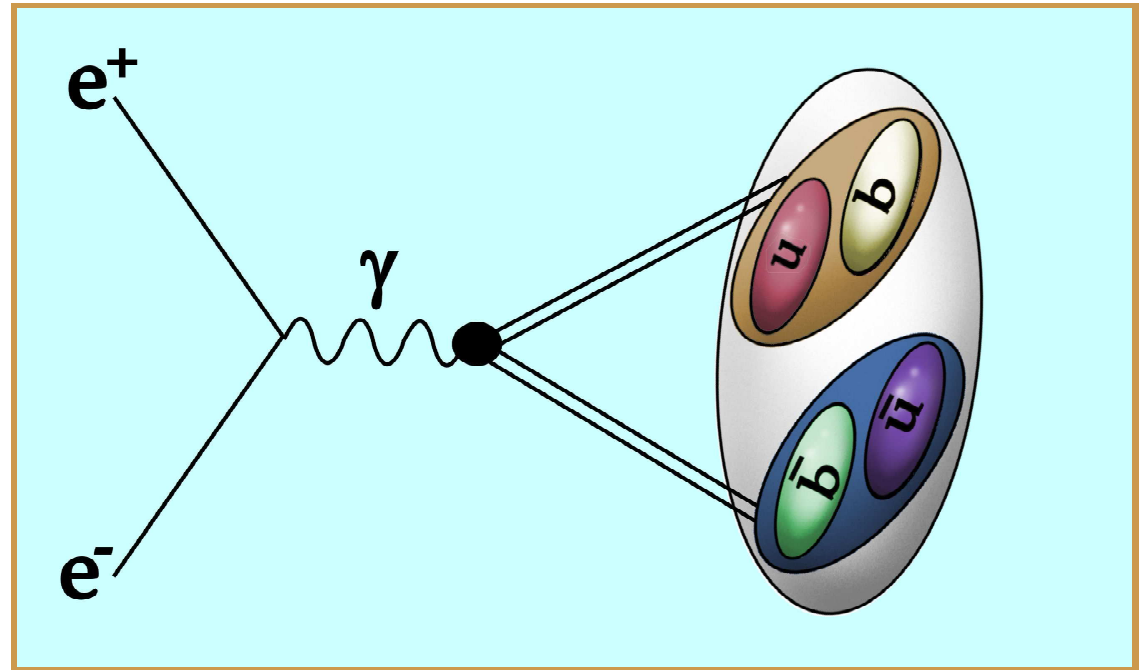
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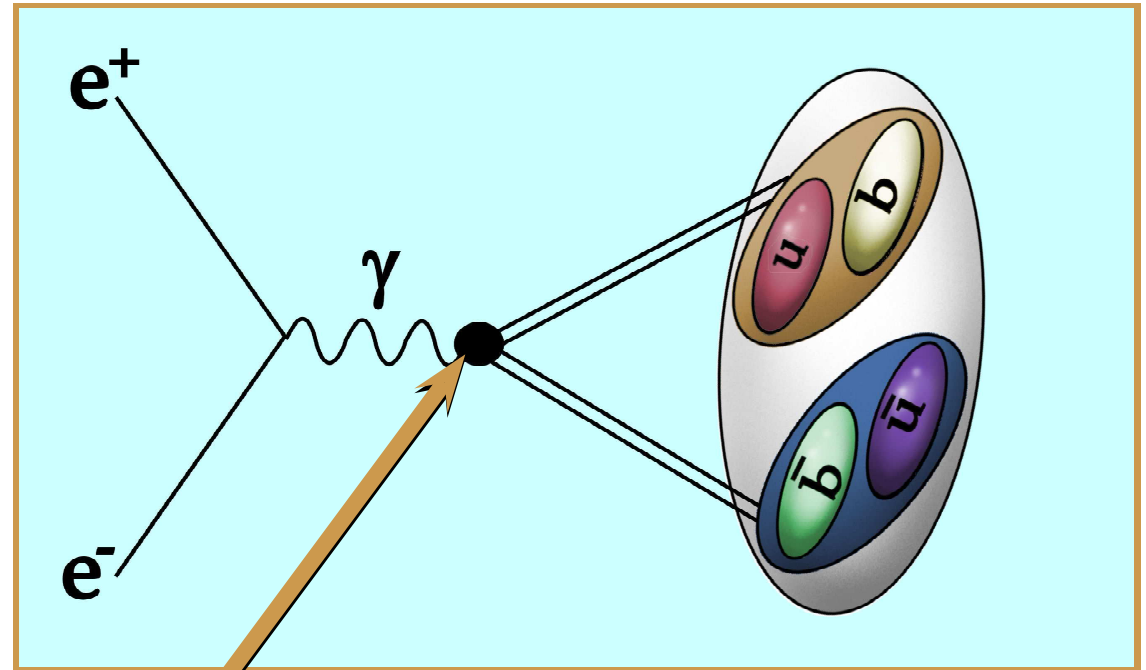


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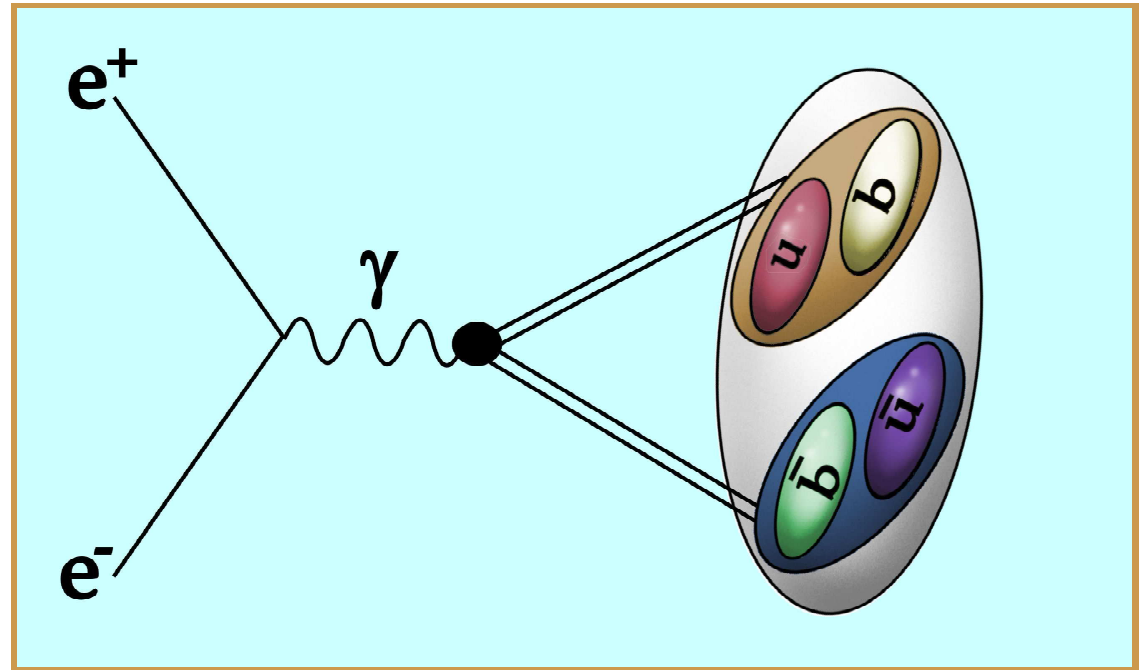
diquark charge

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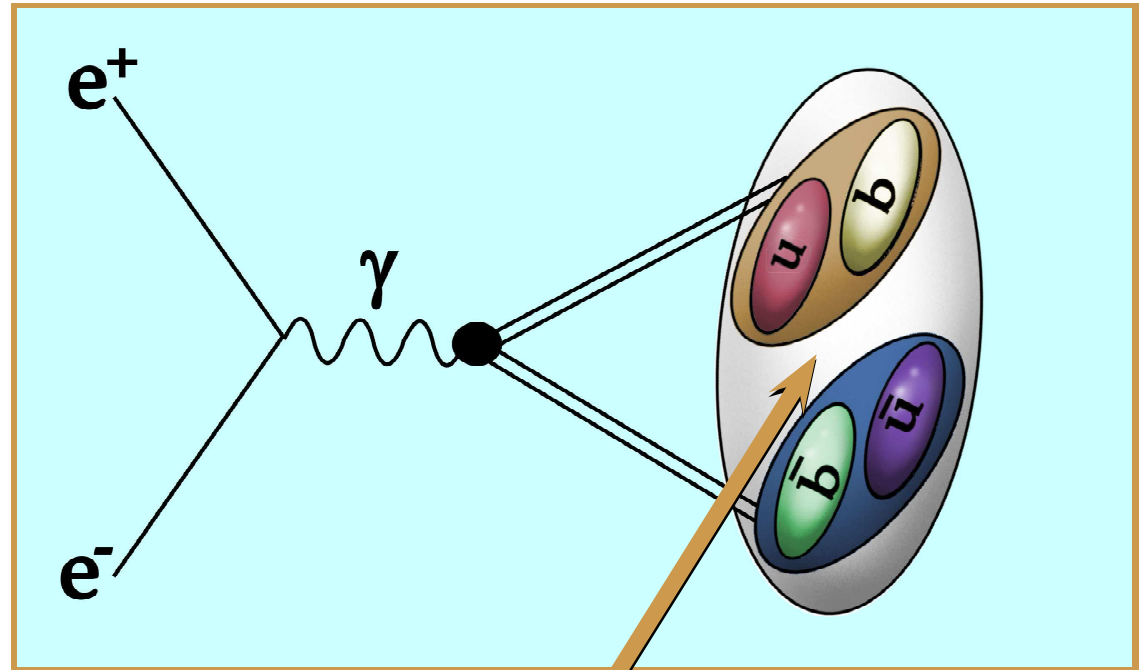
hadronic size parameter

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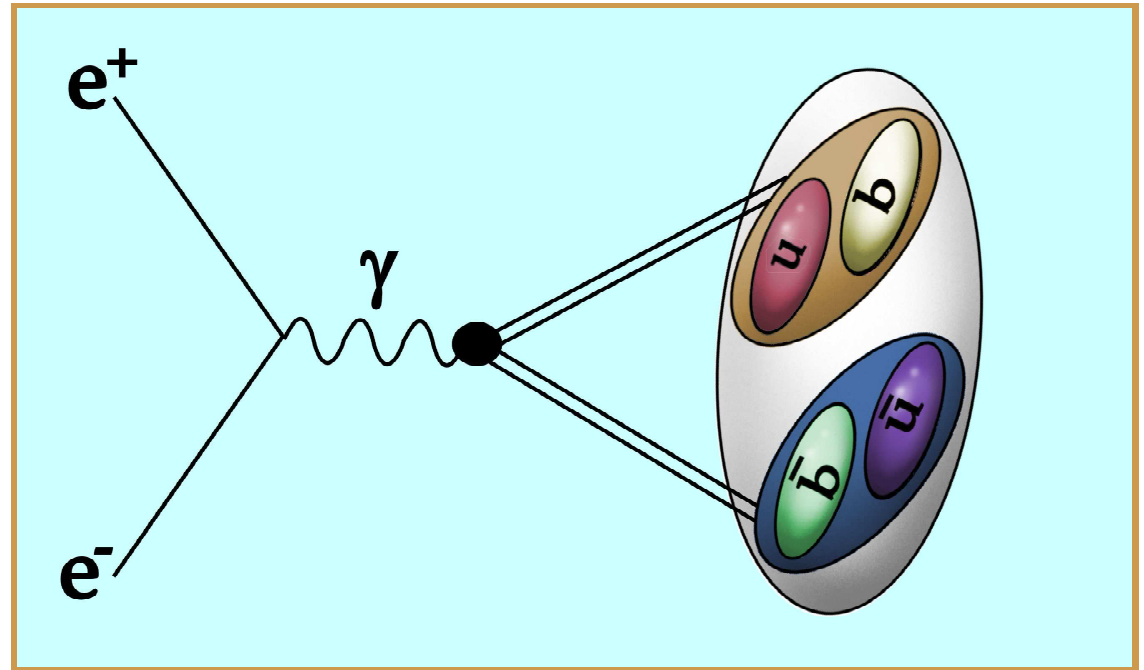
radial tetraquark wave function at origin

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- Production ratio: $R_{ee} = \frac{\Gamma_{Y_{[b,l]}}}{\Gamma_{Y_{[b,h]}}} = \left(\frac{1-2\tan\theta}{2+\tan\theta} \right)^2$ ($1/4 \leq R_{ee} \leq 4$).



Y_b decay

Γ_{tot} is dominated by two-body decays ($B\bar{B}$, $B\bar{B}^*$, $B^*\bar{B}^*$):



Y_b decay

channel

$B\bar{B}$

$B\bar{B}^*$

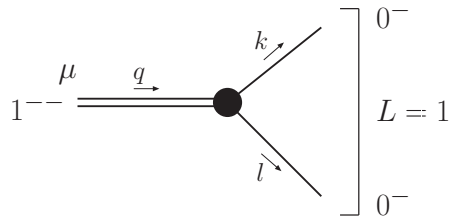
$B^*\bar{B}^*$



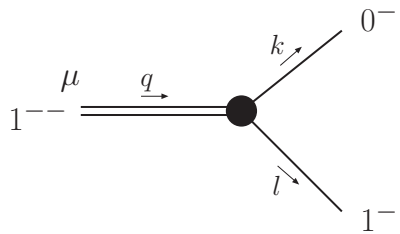
Y_b decay

channel diagram

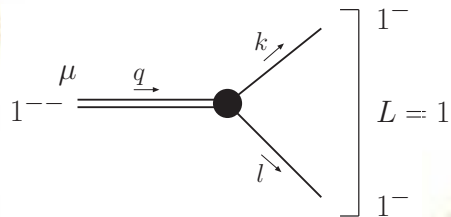
$B\bar{B}$



$B\bar{B}^*$



$B^*\bar{B}^*$



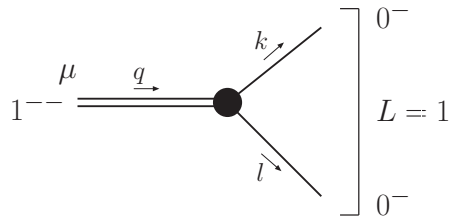
Y_b decay

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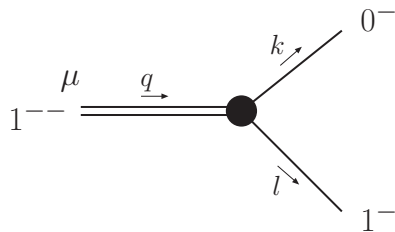
vertex

$B\bar{B}$



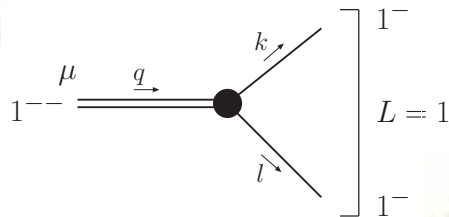
$$\cong F(k^\mu - l^\mu)$$

$B\bar{B}^*$



$$\cong \frac{F}{M} \epsilon^{\mu\nu\rho\sigma} k_\rho l_\sigma$$

$B^*\bar{B}^*$



$$\cong \begin{aligned} &F(g^{\mu\rho}(q+l)^\nu \\ &-g^{\mu\nu}(k+q)^\rho \\ &+g^{\rho\nu}(q+k)^\mu) \end{aligned}$$



Y_b decay

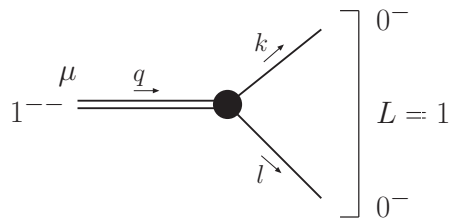
channel

diagram

vertex

width

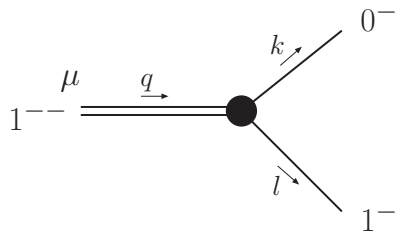
$B\bar{B}$



$$\cong F(k^\mu - l^\mu)$$

$$\Rightarrow \Gamma = \frac{F^2 |\vec{k}|^3}{2M^2 \pi}$$

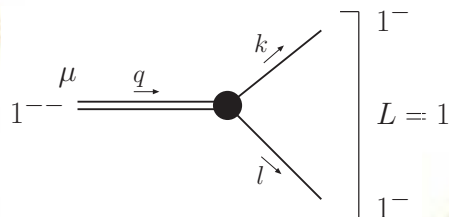
$B\bar{B}^*$



$$\cong \frac{F}{M} \epsilon^{\mu\nu\rho\sigma} k_\rho l_\sigma$$

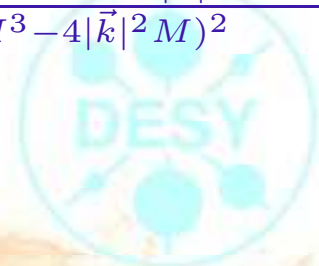
$$\Rightarrow \Gamma = \frac{F^2 |\vec{k}|^3}{4M^2 \pi}$$

$B^* \bar{B}^*$



$$\cong \begin{aligned} & F(g^{\mu\rho}(q+l)^\nu \\ & -g^{\mu\nu}(k+q)^\rho \\ & +g^{\rho\nu}(q+k)^\mu) \end{aligned}$$

$$\Rightarrow \Gamma = \frac{F^2 |\vec{k}|^3 (48|\vec{k}|^4 - 104M^2 |\vec{k}|^2 + 27M^4)}{2\pi (M^3 - 4|\vec{k}|^2 M)^2}$$



Y_b decay

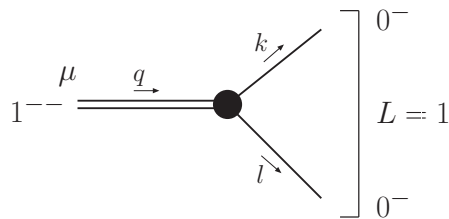
channel

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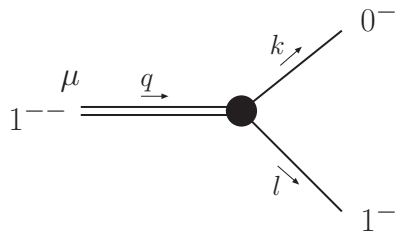
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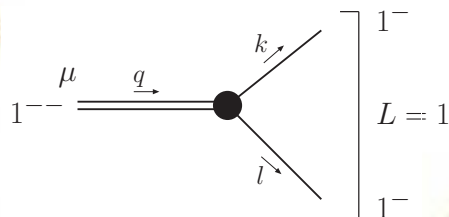
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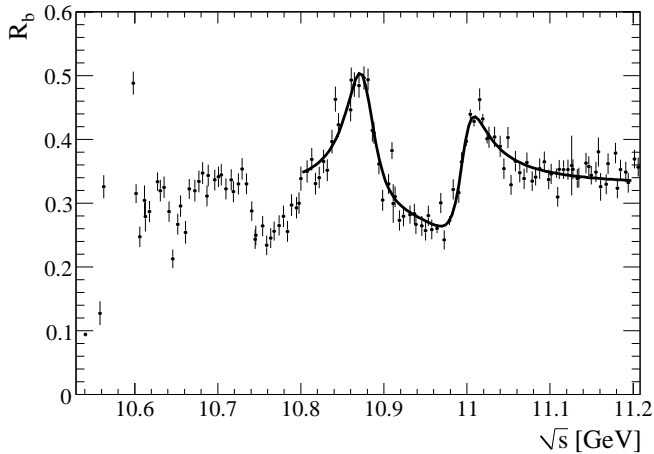
$$\cong \begin{aligned} & F(g^{\mu\rho}(q+l)^\nu \\ & -g^{\mu\nu}(k+q)^\rho \\ & +g^{\rho\nu}(q+k)^\mu) \end{aligned}$$

$$\Rightarrow \Gamma = \frac{F^2 |\vec{k}|^3 (48|\vec{k}|^4 - 104M^2 |\vec{k}|^2 + 27M^4)}{2\pi (M^3 - 4|\vec{k}|^2 M)^2}$$

- The couplings F are estimated from the measured widths of the $\Upsilon(5S)$ decays ($\Gamma_{tot}(Y_b^{(1)}) \approx 40$ MeV, $\Gamma_{tot}(Y_b^{(2)}) \approx 90$ MeV, ...)

fit to BaBar data

BaBar fit



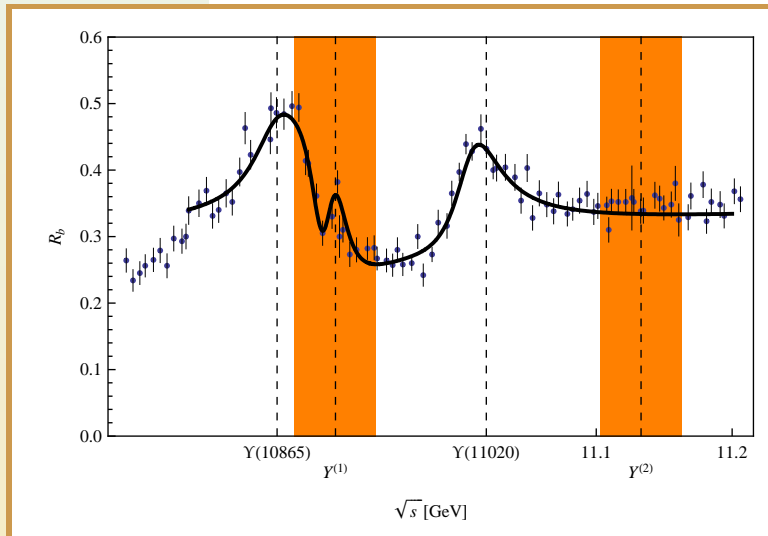
$$\begin{aligned} \sigma(e^+e^- \rightarrow b\bar{b}) = & |A_{nr}|^2 + |A_r + A_{10860}e^{i\phi_{10860}} \\ & \times BW(M_{10860}, \Gamma_{10860}) + A_{11020}e^{i\phi_{11020}} \\ & \times BW(M_{11020}, \Gamma_{11020})|^2 \end{aligned}$$

$$\chi^2/\text{d.o.f.} \approx 2$$

[Phys. Rev. Lett. **102**, 012001 (2009)]



BaBar fit



$\chi^2/\text{d.o.f.} = 88/67$

$$\sigma(e^+e^- \rightarrow b\bar{b}) =$$

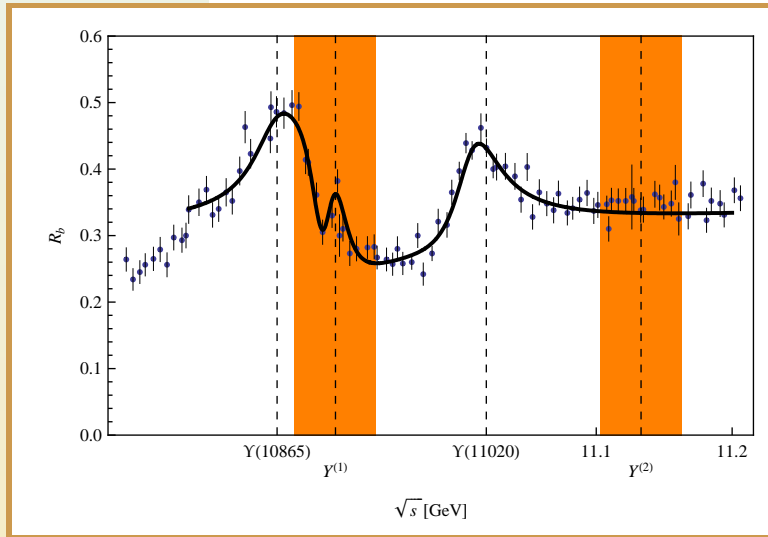
$$|A_{nr}|^2 + |A_r + A_{10860}e^{i\phi_{10860}} \times BW(M_{10860}, \Gamma_{10860}) + A_{11020}e^{i\phi_{11020}} \times BW(M_{11020}, \Gamma_{11020})|^2$$

add $A_{Y_{[b,l]}} e^{i\phi_{Y_{[b,l]}}} BW(M_{Y_{[b,l]}}, \Gamma_{Y_{[b,l]}})$

and $A_{Y_{[b,h]}} e^{i\phi_{Y_{[b,h]}}} BW(M_{Y_{[b,h]}}, \Gamma_{Y_{[b,h]}})$



BaBar fit



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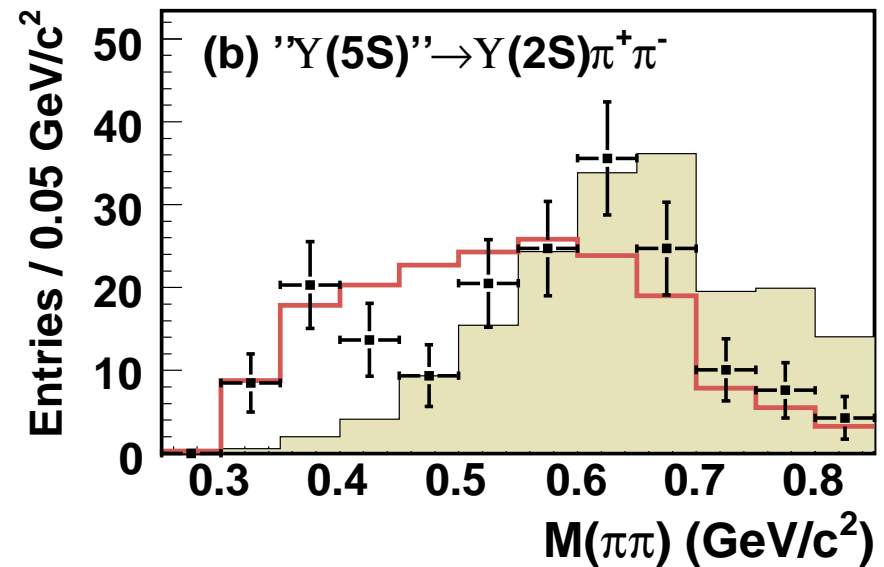
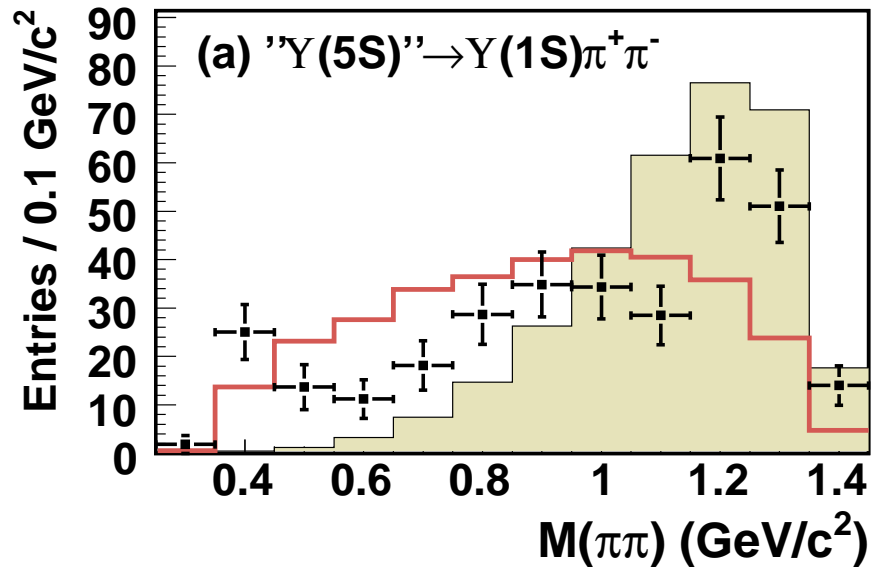
	$M[\text{MeV}]$	$\Gamma[\text{MeV}]$	$\varphi [\text{rad.}]$
$\Upsilon(5S)$	10864 ± 5	46 ± 8	1.3 ± 0.3
$\Upsilon(6S)$	11007 ± 0.3	40 ± 2	0.88 ± 0.06
$Y_{[b,l]}$	$10900 - \Delta M/2 \pm 2$	28 ± 2	1.3 ± 0.2
$Y_{[b,h]}$	$10900 + \Delta M/2 \pm 2$	28 ± 2	1.9 ± 0.2

$\Delta M = 5.6 \pm 2.8 \text{ MeV}, \Gamma_{ee}(Y_{[b,l]}) = 0.045 \pm 0.015 \text{ keV}, \Gamma_{ee}(Y_{[b,h]}) = 0.04 \pm 0.015 \text{ keV}$



Belle data, explanation and fitting

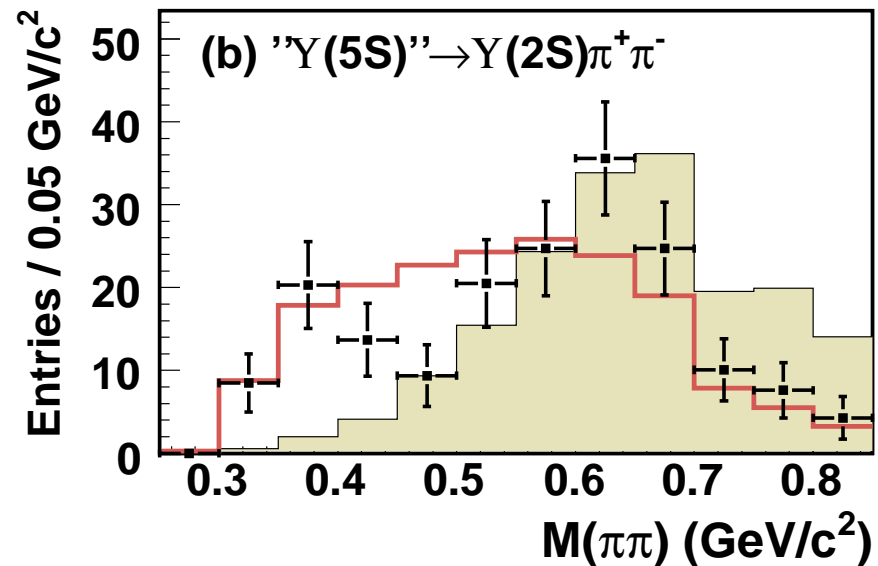
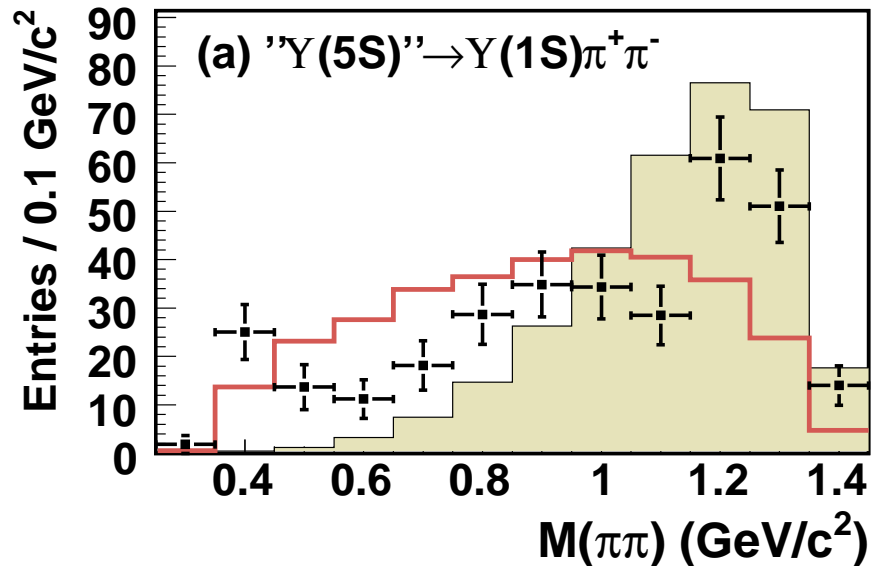
enigmatic Belle data



Phys. Rev. Lett. **100**, 112001 (2008)



enigmatic Belle data



Phys. Rev. Lett. **100**, 112001 (2008)

reported partial decay width:

$$\Gamma(''\Upsilon(5S)'' \rightarrow \Upsilon(1S)\pi^+\pi^-) = 0.59 \pm 0.04 \pm 0.09 \text{ MeV}$$

$$\Gamma(''\Upsilon(5S)'' \rightarrow \Upsilon(2S)\pi^+\pi^-) = 0.85 \pm 0.07 \pm 0.16 \text{ MeV}$$

why is the belle data puzzling?

- typical $\Upsilon(nS) \rightarrow \Upsilon(1S)\pi\pi$ partial widths:



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$$\Gamma(\Upsilon(2S) \rightarrow \Upsilon(1S)\pi\pi) \approx 0.0060 \text{ MeV}$$

$$\Gamma(\Upsilon(3S) \rightarrow \Upsilon(1S)\pi\pi) \approx 0.0009 \text{ MeV}$$

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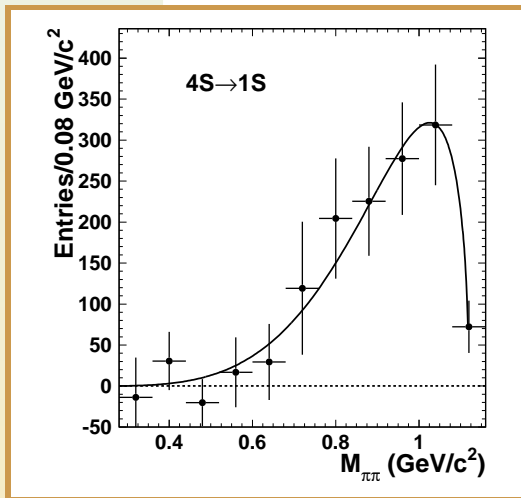
$$\Gamma(\Upsilon(5S) \rightarrow \Upsilon(1S)\pi^+\pi^-) \approx 0.59 \text{ MeV}$$

differs by two orders of magnitude!



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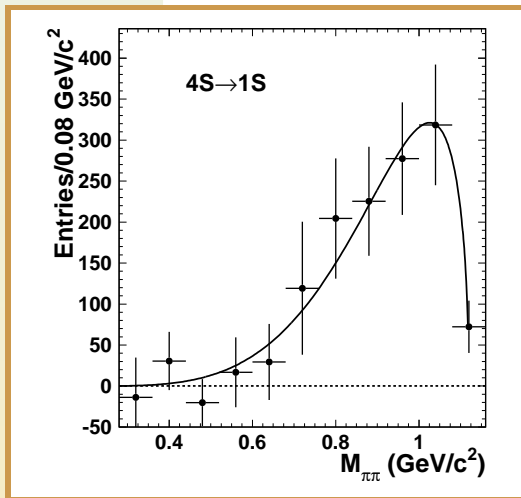


[Phys. Rev. D **79** (2009) 051103]

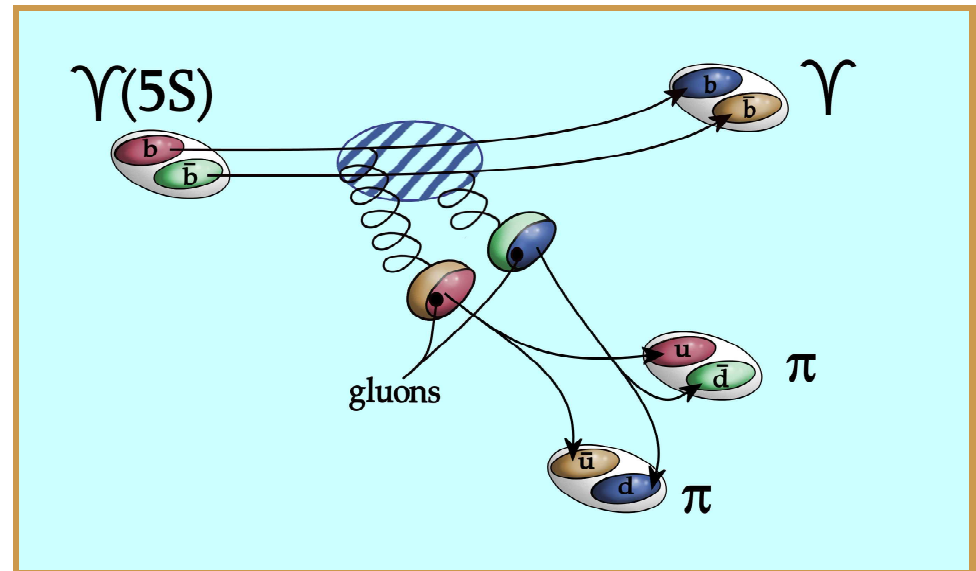


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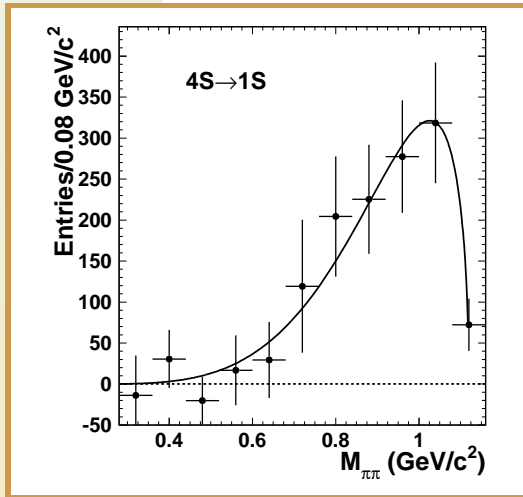


[Phys. Rev. D **79** (2009) 051103]

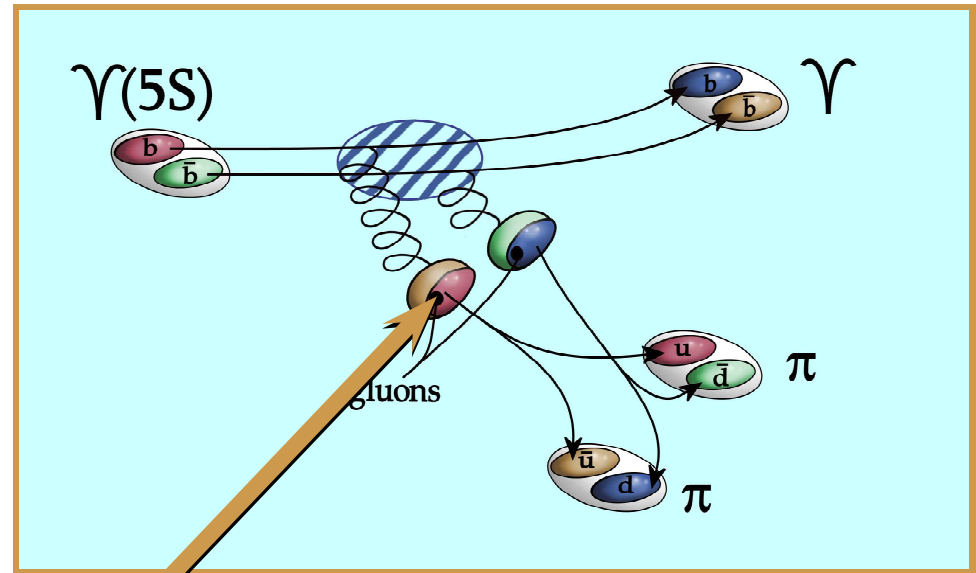


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[Phys. Rev. D **79** (2009) 051103]



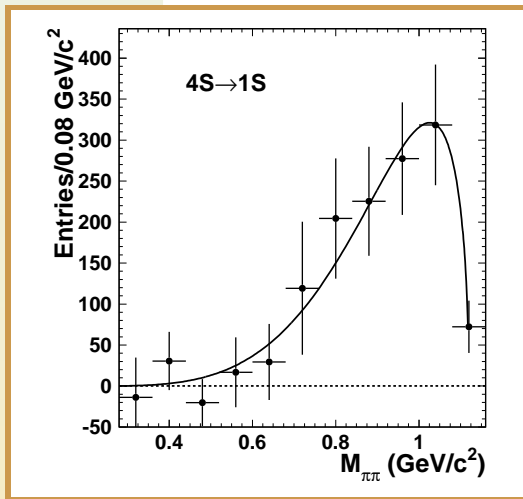
underlying process is

Zweig forbidden

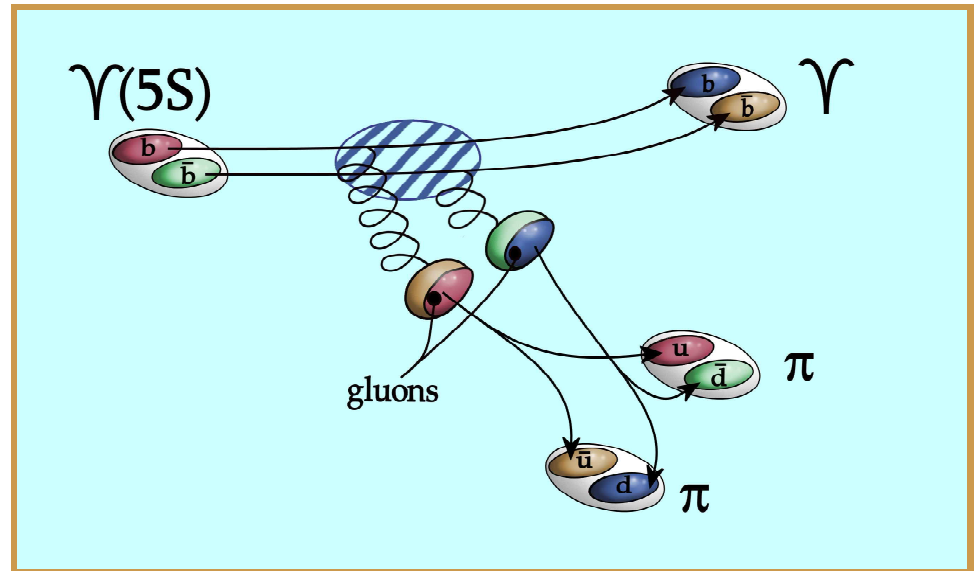


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[Phys. Rev. D **79** (2009) 051103]

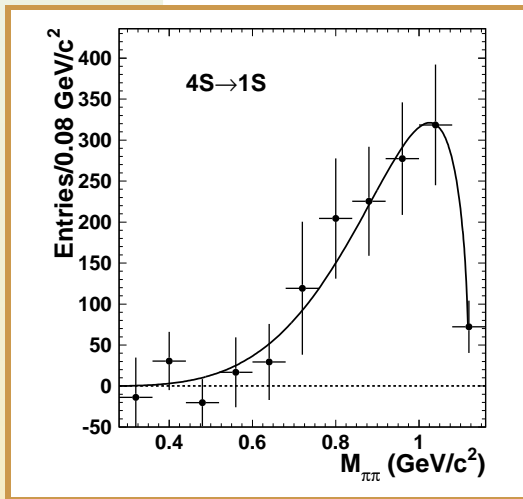


$$\mathcal{M}_a^{\mu\nu} = g^{\mu\nu} \frac{F}{F_\pi^2} \left[m_{\pi\pi}^2 - \beta(\Delta M)^2 \left(1 + \frac{2m_\pi^2}{m_{\pi\pi}^2} \right) + \frac{3}{2} \beta \left((\Delta M)^2 - m_{\pi\pi}^2 \right) \left(1 - \frac{4m_\pi^2}{m_{\pi\pi}^2} \right) \left(\cos^2 \theta - \frac{1}{3} \right) \right]$$

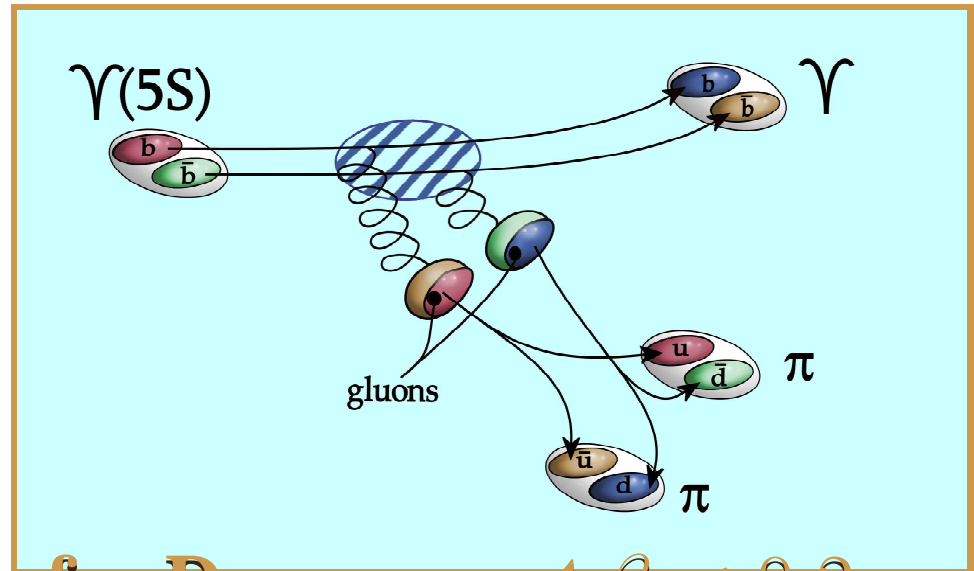
[Phys. Rev. Lett. **35**, 1 (1975)]

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[Phys. Rev. D **79** (2009) 051103]



measure for D-wave part $\beta < 0.2$

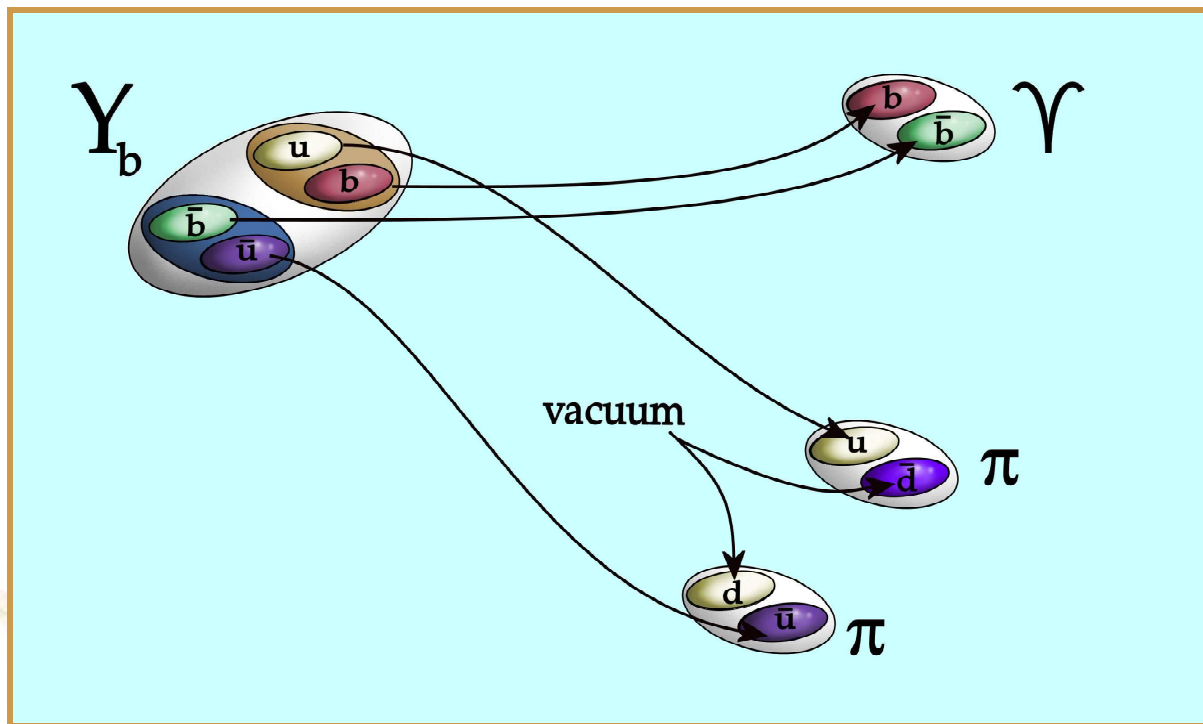
$$\mathcal{M}_a^{\mu\nu} = g^{\mu\nu} \frac{F}{F_\pi^2} \left[m_{\pi\pi}^2 - \beta (\Delta M)^2 \left(1 + \frac{2m_\pi^2}{m_{\pi\pi}^2} \right) + \frac{3}{2} \beta \left((\Delta M)^2 - m_{\pi\pi}^2 \right) \left(1 - \frac{4m_\pi^2}{m_{\pi\pi}^2} \right) \left(\cos^2 \theta - \frac{1}{3} \right) \right]$$

[Phys. Rev. Lett. **35**, 1 (1975)]



continuum contribution

- The tetraquark decay has a different **Zweig allowed** underlying process:



short intermezzo: light tetraquarks

light tetraquarks

- From the light quark sector we have a full $SU(3)_F$ nonet of tetraquark resonances [t'Hooft et al., Phys. Lett. B **662**, 424 (2008)] :



light tetraquarks

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$$\sigma^{[0]} = [ud][\bar{u}\bar{d}]$$

$$\kappa = [su][\bar{u}\bar{d}]; [sd][\bar{u}\bar{d}]$$

(+conjugate doublet)

$$f_0^{[0]} = \frac{[su][\bar{s}\bar{u}] + [sd][\bar{s}\bar{d}]}{\sqrt{2}}$$

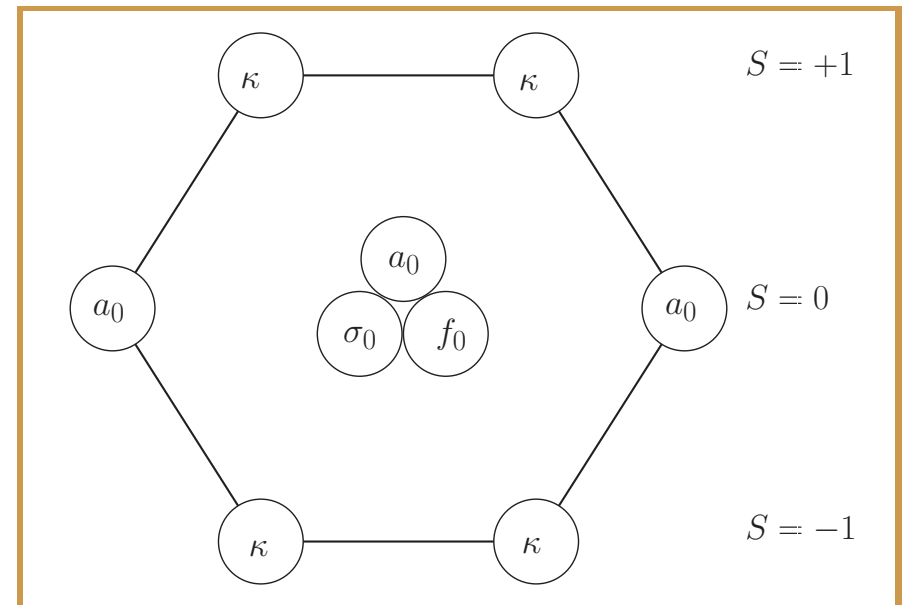
$$a_0 = [su][\bar{s}\bar{d}]; [sd][\bar{s}\bar{u}];$$
$$\frac{[su][\bar{s}\bar{u}] - [sd][\bar{s}\bar{d}]}{\sqrt{2}}$$



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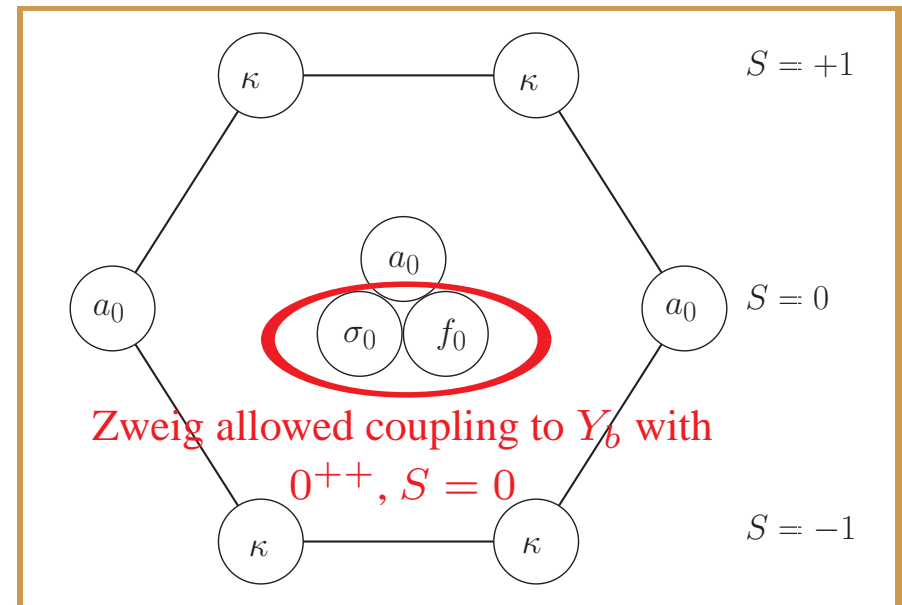
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light tetraquark interactions

The effective Lagrangian (i, j are flavor indices):

$$\mathcal{L} \propto \text{Det}(Q_{LR}) , \quad (Q_{LR})^{ij} = \bar{q}_L^i q_R^j$$



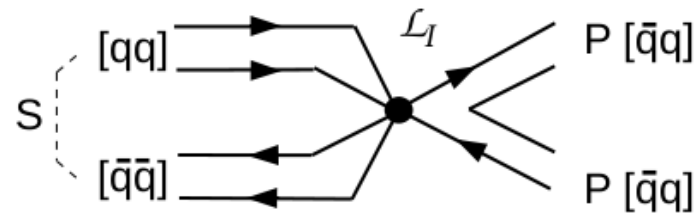
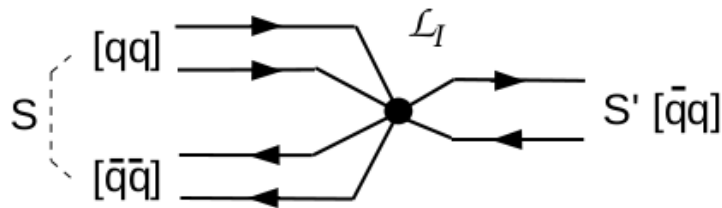
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induces tetraquark-meson 6-quark interactions via

$$\text{Tr}(J^{[4q]} J^{2q}) , \quad \text{with} \quad J_{ij}^{[4q]} = [\bar{q}\bar{q}]_i [qq]_j , \quad J_{ij}^{2q} = \bar{q}_j q_i$$



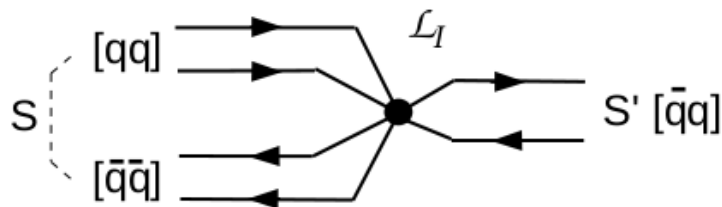
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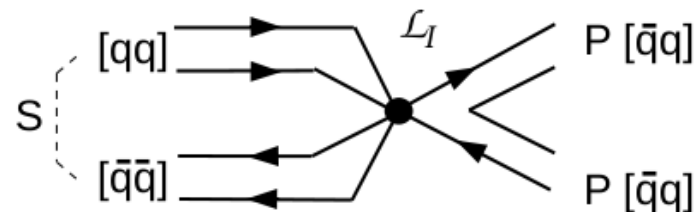
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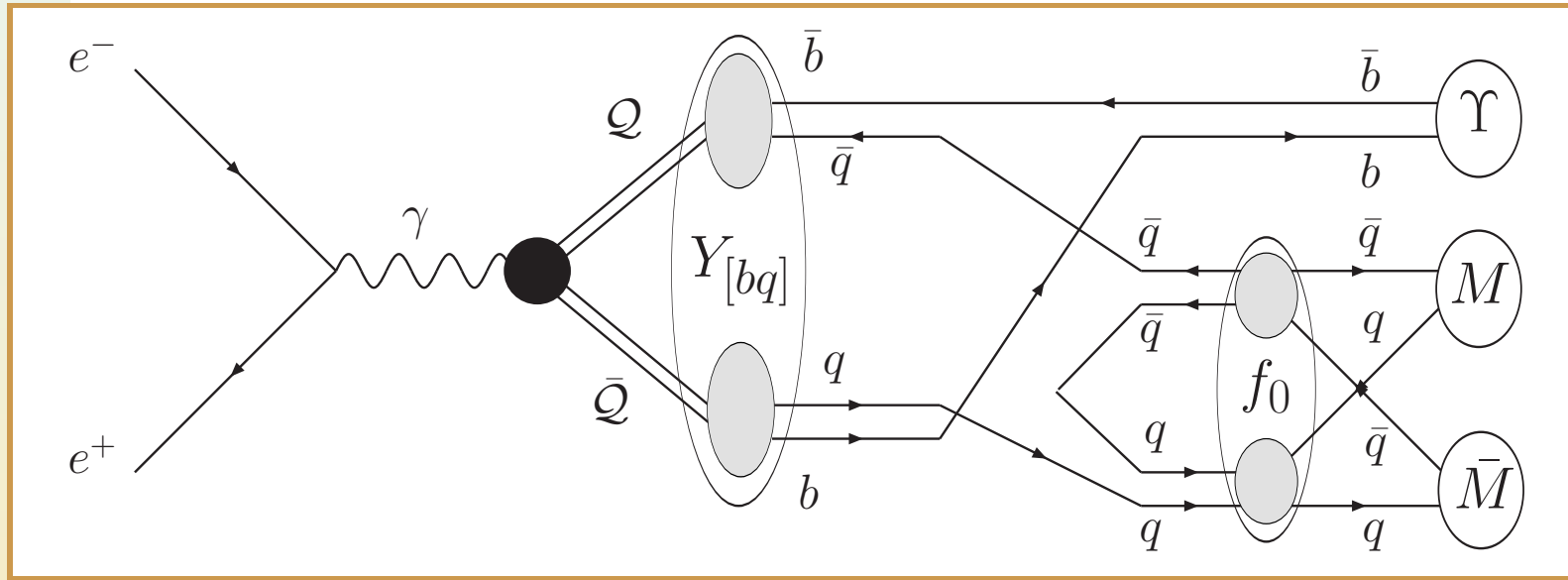
tetraquark-meson mixing



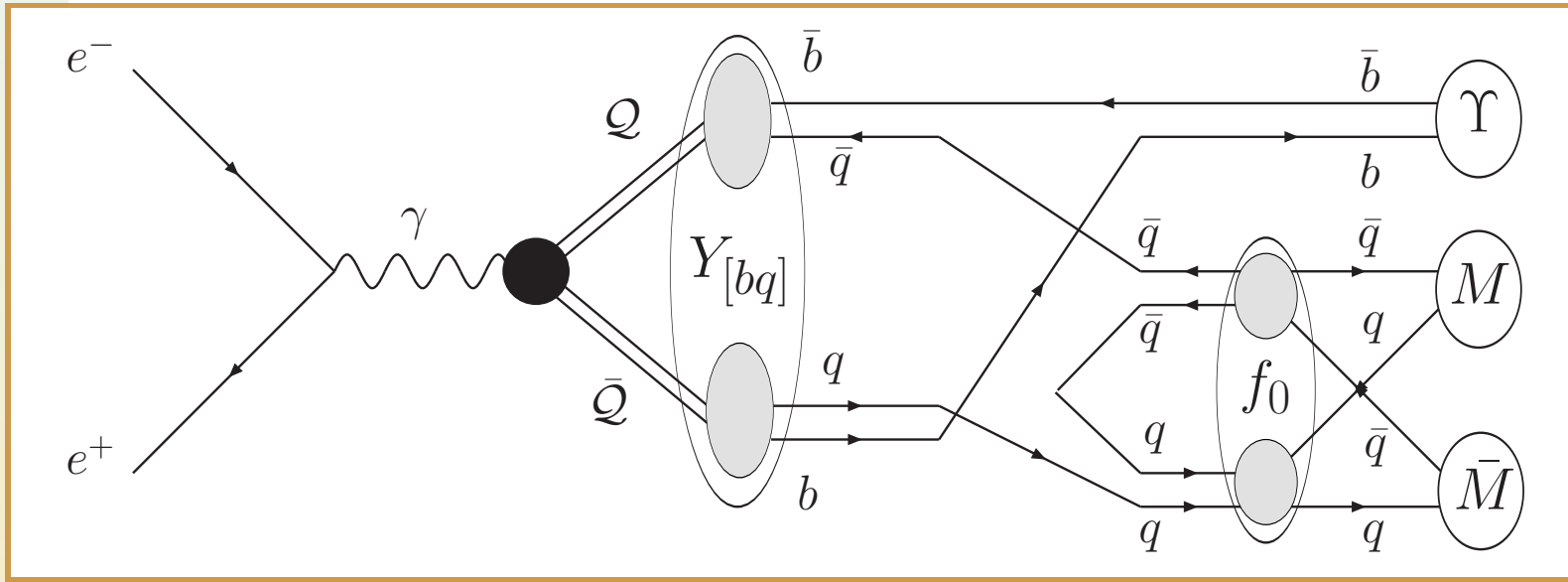
tetraquark decay

summing up contributions

0^{++} resonance contribution



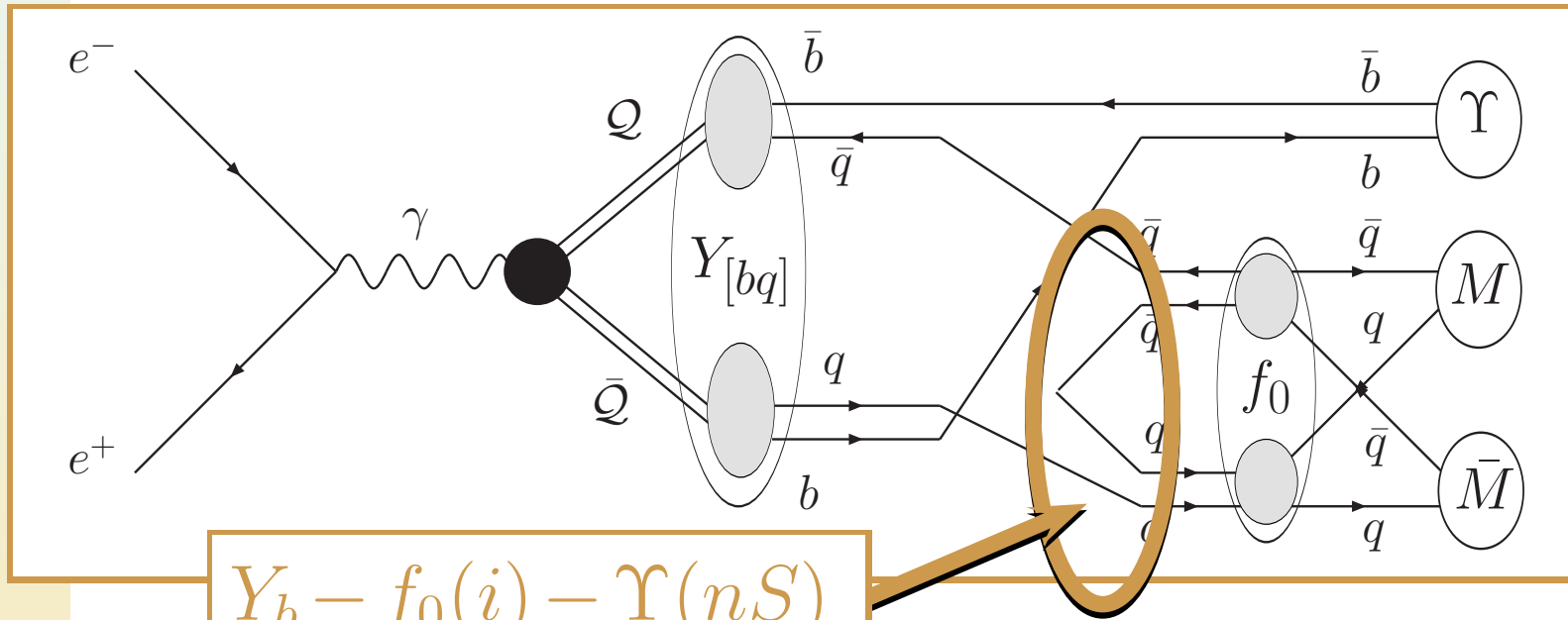
0^{++} resonance contribution



$$\mathcal{M}_{0^{++} \text{ resonance}} = \epsilon^Y \cdot \epsilon^\gamma \sum_i \frac{a_{f_0(i)} e^{i\varphi_{f_0(i)}} (m_{\pi\pi}^2 - 2m_\pi^2)/2}{m_{\pi\pi}^2 - m_{f_0(i)}^2 + im_{f_0(i)} \Gamma_{f_0(i)} (m_{\pi\pi})}$$

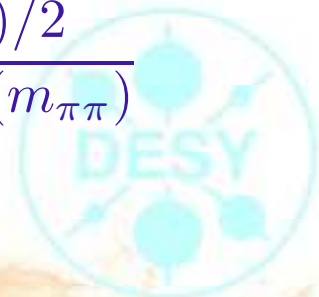


0^{++} resonance contribution

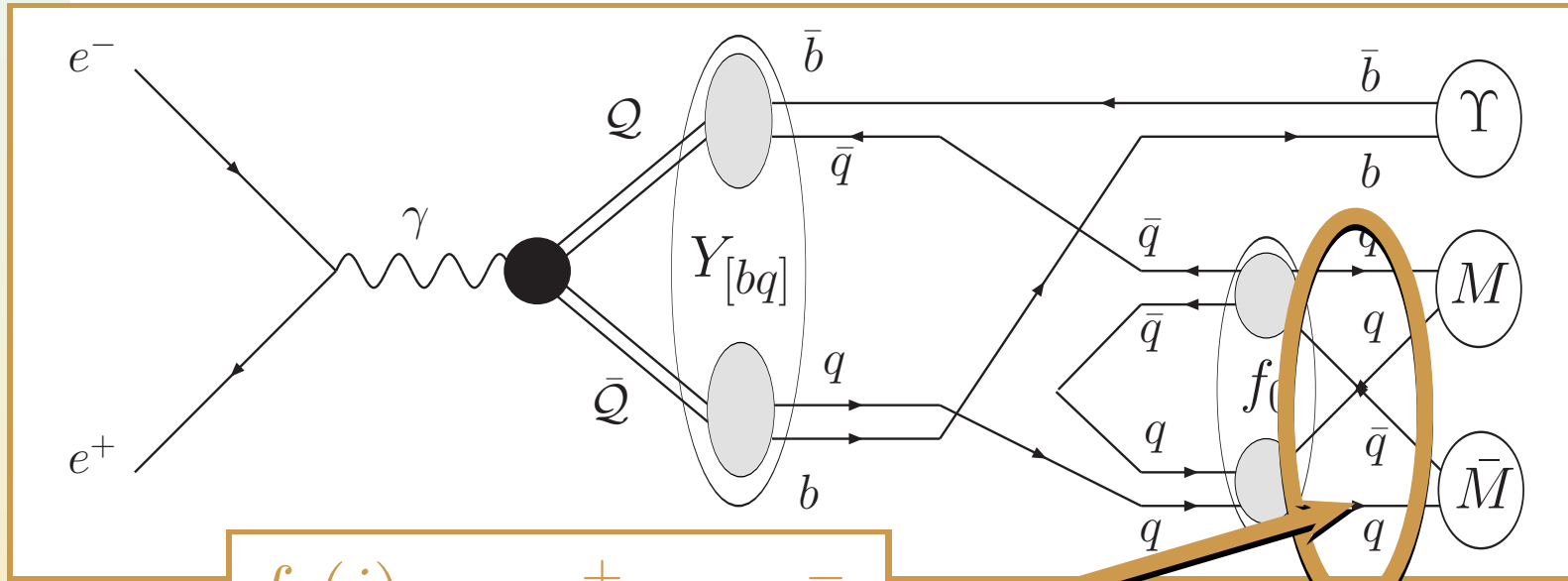


$Y_b - f_0(i) - \Upsilon(nS)$
coupling $F_{f_0(i)}$

$$\mathcal{M}_{0^{++} \text{ resonance}} = \epsilon^Y \cdot \epsilon^\Upsilon \sum_i \frac{a_{f_0(i)} e^{i\varphi_{f_0(i)}} (m_{\pi\pi}^2 - 2m_\pi^2)/2}{m_{\pi\pi}^2 - m_{f_0(i)}^2 + im_{f_0(i)}\Gamma_{f_0(i)}(m_{\pi\pi})}$$

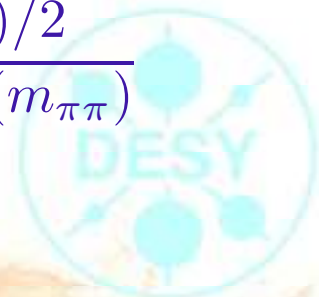


0^{++} resonance contribution

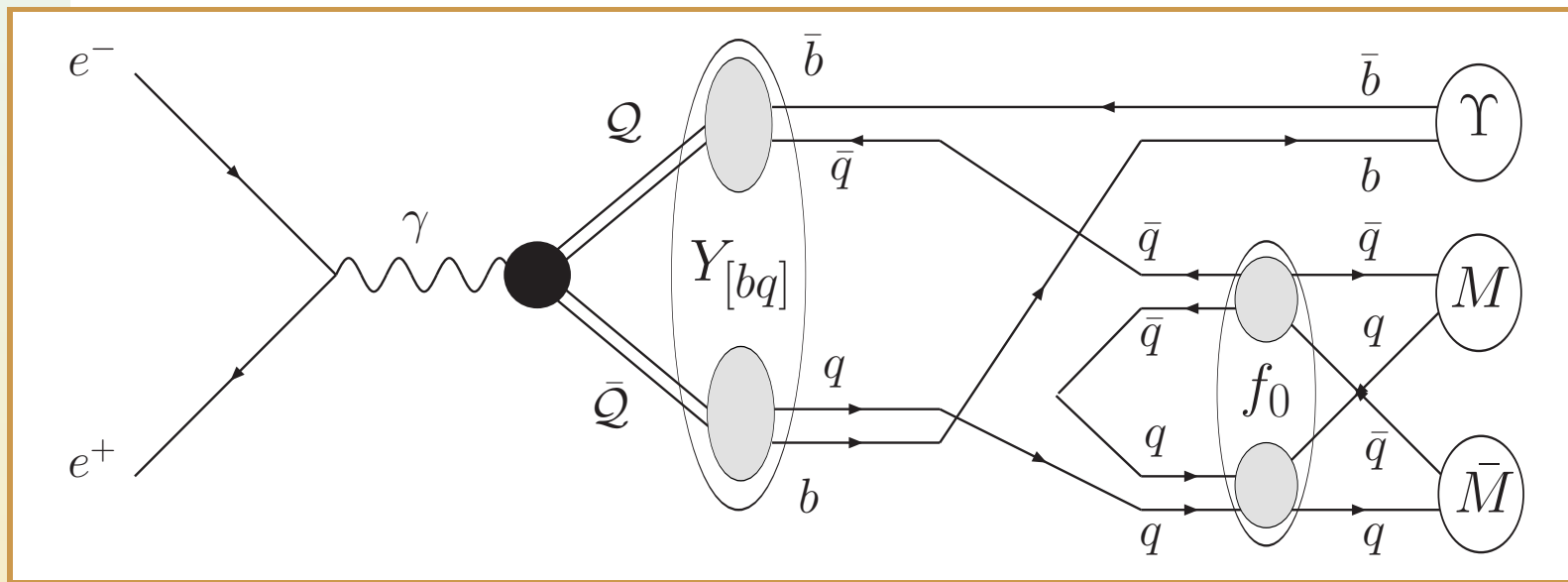


$f_0(i) - \pi^+ - \pi^-$
coupling $g_{f_0(i)\pi\pi}$

$$\mathcal{M}_{0^{++} \text{ resonance}} = \epsilon^Y \cdot \epsilon^\Upsilon \sum_i \frac{a_{f_0(i)} e^{i\varphi_{f_0(i)}} (m_{\pi\pi}^2 - 2m_\pi^2)/2}{m_{\pi\pi}^2 - m_{f_0(i)}^2 + im_{f_0(i)} \Gamma_{f_0(i)}(m_{\pi\pi})}$$



0^{++} resonance contribution

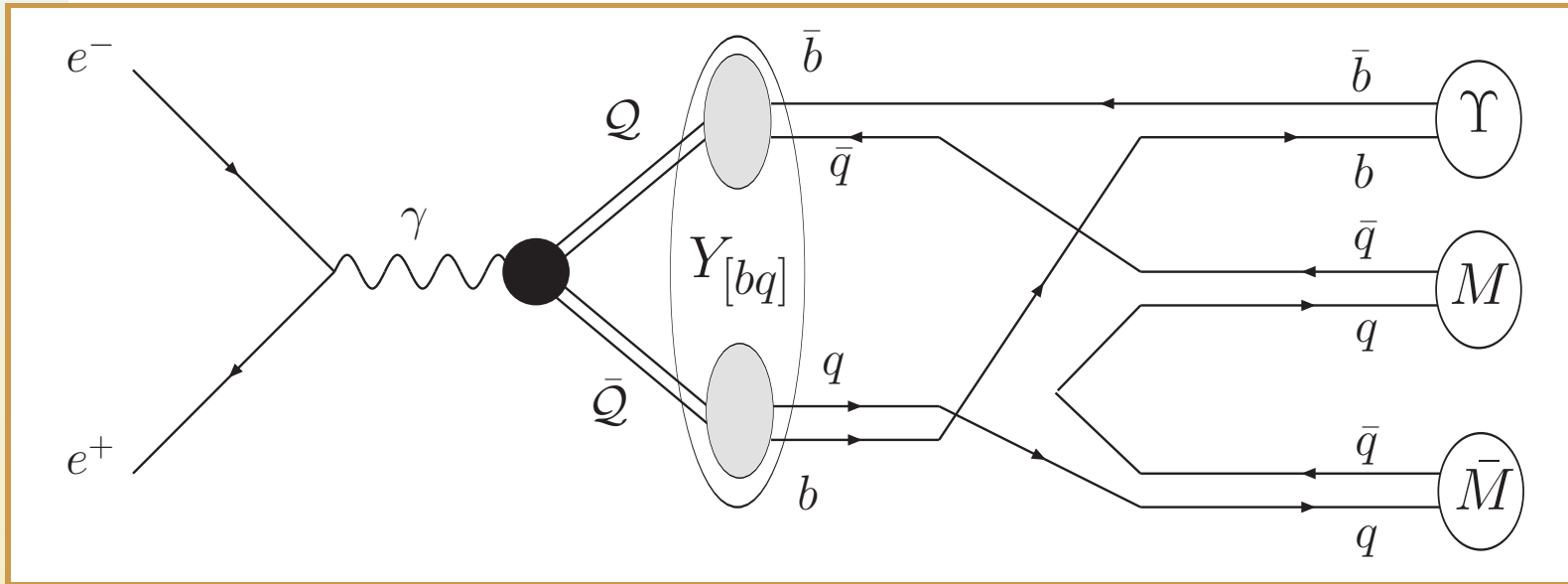


$$a_{f_0(i)} \propto F_{f_0(i)} \times g_{f_0(i)\pi\pi}$$

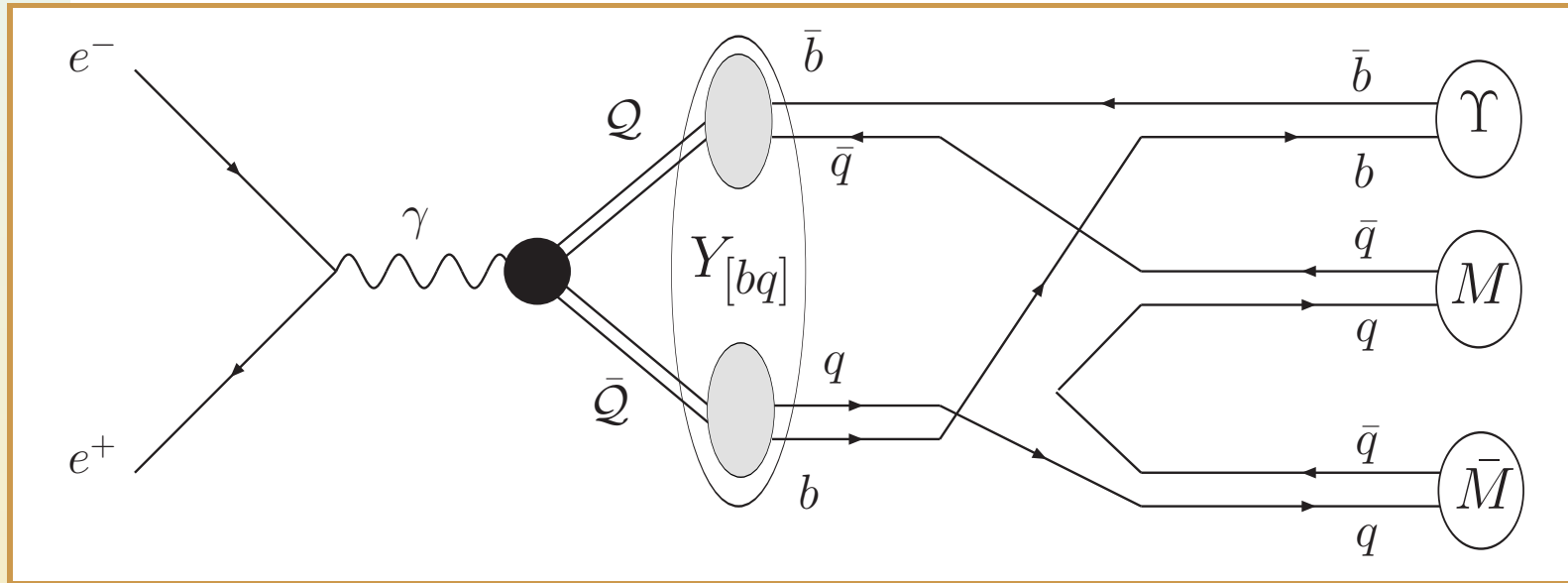
$$\mathcal{M}_{0^{++} \text{ resonance}} = \epsilon^Y \cdot \epsilon^\gamma \sum_i \frac{a_{f_0(i)} e^{i\varphi_{f_0(i)}} (m_{\pi\pi}^2 - 2m_\pi^2)/2}{m_{\pi\pi}^2 - m_{f_0(i)}^2 + im_{f_0(i)}\Gamma_{f_0(i)}(m_{\pi\pi})}$$



continuum contribution

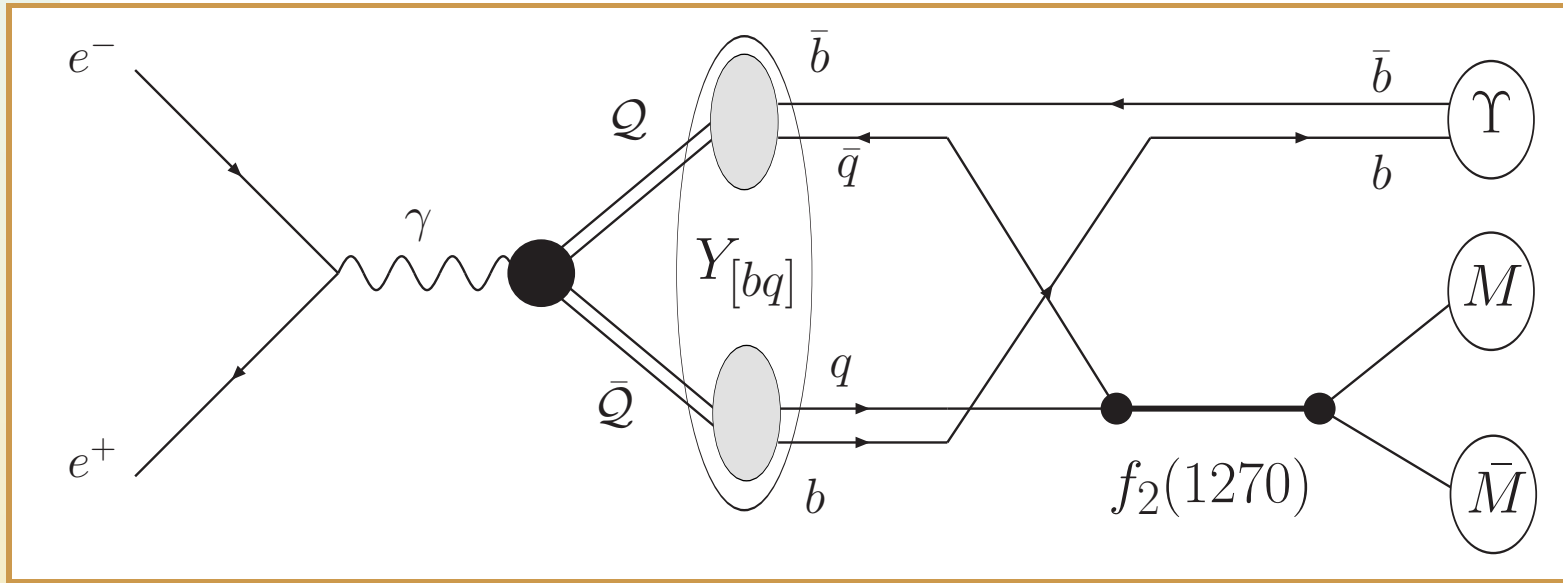


continuum contribution

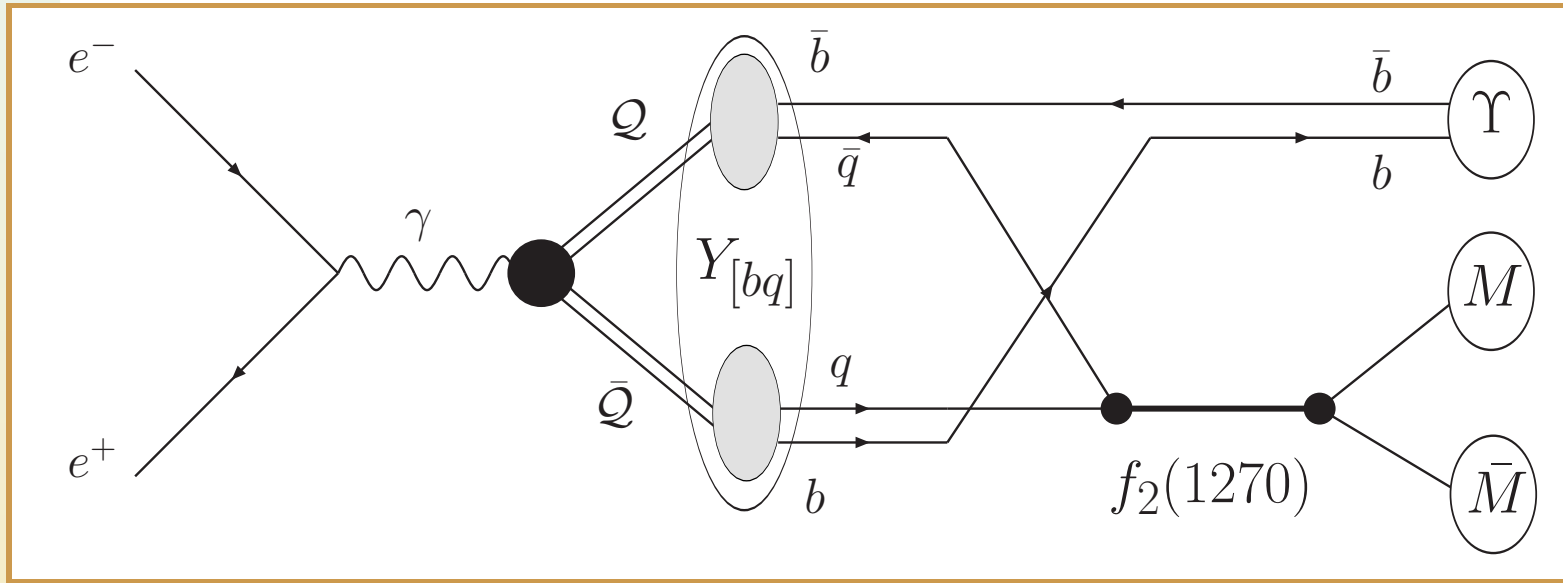


$$\mathcal{M}_{\text{continuum}} = \varepsilon^Y \cdot \varepsilon^\Upsilon \frac{F}{F_\pi^2} \left[m_{\pi\pi}^2 - \beta(\Delta M)^2 \left(1 + \frac{2m_\pi^2}{m_{\pi\pi}^2} \right) + \frac{3}{2}\beta((\Delta M)^2 - m_{\pi\pi}^2) \times \left(1 - \frac{4m_\pi^2}{m_{\pi\pi}^2} \right) (\cos^2 \theta - \frac{1}{3}) \right]$$

D-wave 2^{++} contribution



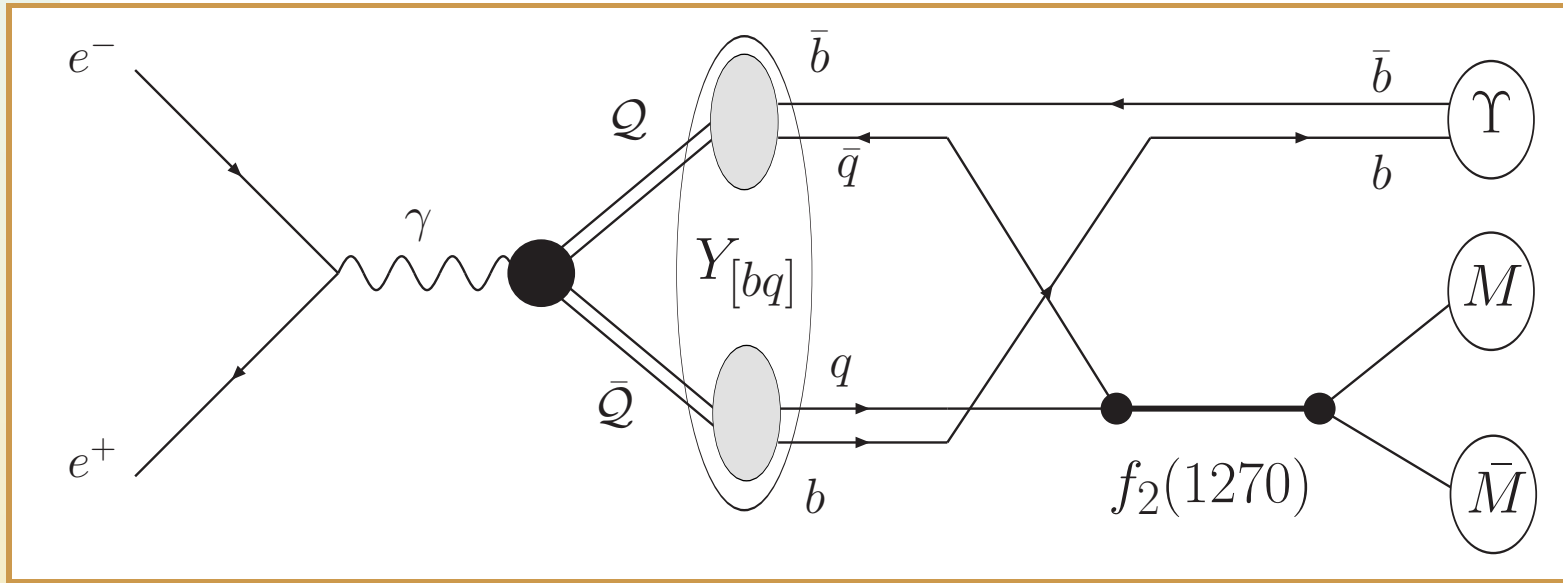
D-wave 2^{++} contribution



$$\mathcal{M}_{f_2(1270)} = \varepsilon^Y \cdot \varepsilon^\Upsilon a_{f_2(1270)} e^{i\varphi_{f_2(1270)}} A_{f_2(1270)}$$



D-wave 2^{++} contribution

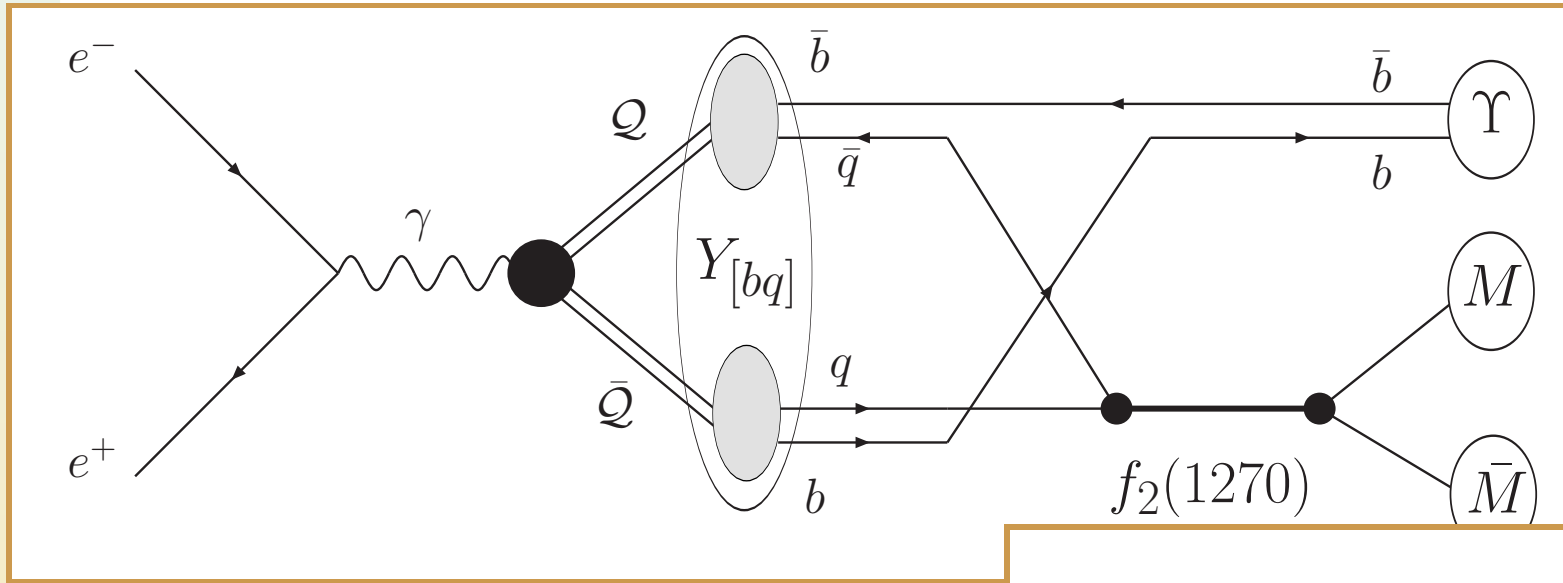


$$\mathcal{M}_{f_2(1270)} = \varepsilon^Y \cdot \varepsilon^\Upsilon a_{f_2(1270)} e^{i\varphi_{f_2(1270)}} A_{f_2(1270)}$$

$$A_{f_2(1270)} = \frac{\sqrt{8\pi(2J+1)}}{\sqrt{m_{\pi\pi}}} Y_2 \frac{a_{f_2(1270)} \sqrt{m_{f_2(1270)}}}{m_{f_2(1270)}^2 - m_{\pi\pi}^2 - im_{f_2(1270)} \Gamma_{f_2(1270)}}$$



D-wave 2^{++} contribution



spherical harmonics:

$$|Y_2^2| = \sqrt{\frac{15}{32\pi}} \sin^2 \theta$$

$$\mathcal{M}_{f_2(1270)} = \epsilon^Y \cdot \epsilon^\Upsilon a_{f_2(1270)}$$

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full amplitude

summing the single contributions leads to the full amplitude:

$$\begin{aligned} \mathcal{M} = & \varepsilon^Y . \varepsilon^\Upsilon \left[\frac{F}{F_\pi^2} \left[m_{\pi\pi}^2 - \beta(\Delta M)^2 \left(1 + \frac{2m_\pi^2}{m_{\pi\pi}^2} \right) \right. \right. \\ & \left. \left. + \frac{3}{2} \beta \left((\Delta M)^2 - m_{\pi\pi}^2 \right) \left(1 - \frac{4m_\pi^2}{m_{\pi\pi}^2} \right) \left(\cos^2 \theta - \frac{1}{3} \right) \right] \right. \\ & \left. + \sum_i \frac{a_{f_0(i)} e^{i\varphi_{f_0(i)}} (m_{\pi\pi}^2 - 2m_\pi^2) / 2}{m_{\pi\pi}^2 - m_{f_0(i)}^2 + i m_{f_0(i)} \Gamma_{f_0(i)}(m_{\pi\pi})} \right. \\ & \left. + a_{f_2(1270)} e^{i\varphi_{f_2(1270)}} A_{f_2(1270)}(m_{\pi\pi}) \right] \end{aligned}$$



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 & \left. + \sum_i \frac{a_{f_0(i)} e^{i\varphi_{f_0(i)}} (m_{\pi\pi}^2 - 2m_\pi^2)/2}{m_{\pi\pi}^2 - m_{f_0(i)}^2 + im_{f_0(i)} \Gamma_{f_0(i)}(m_{\pi\pi})} \right. \\
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 \end{aligned}$$

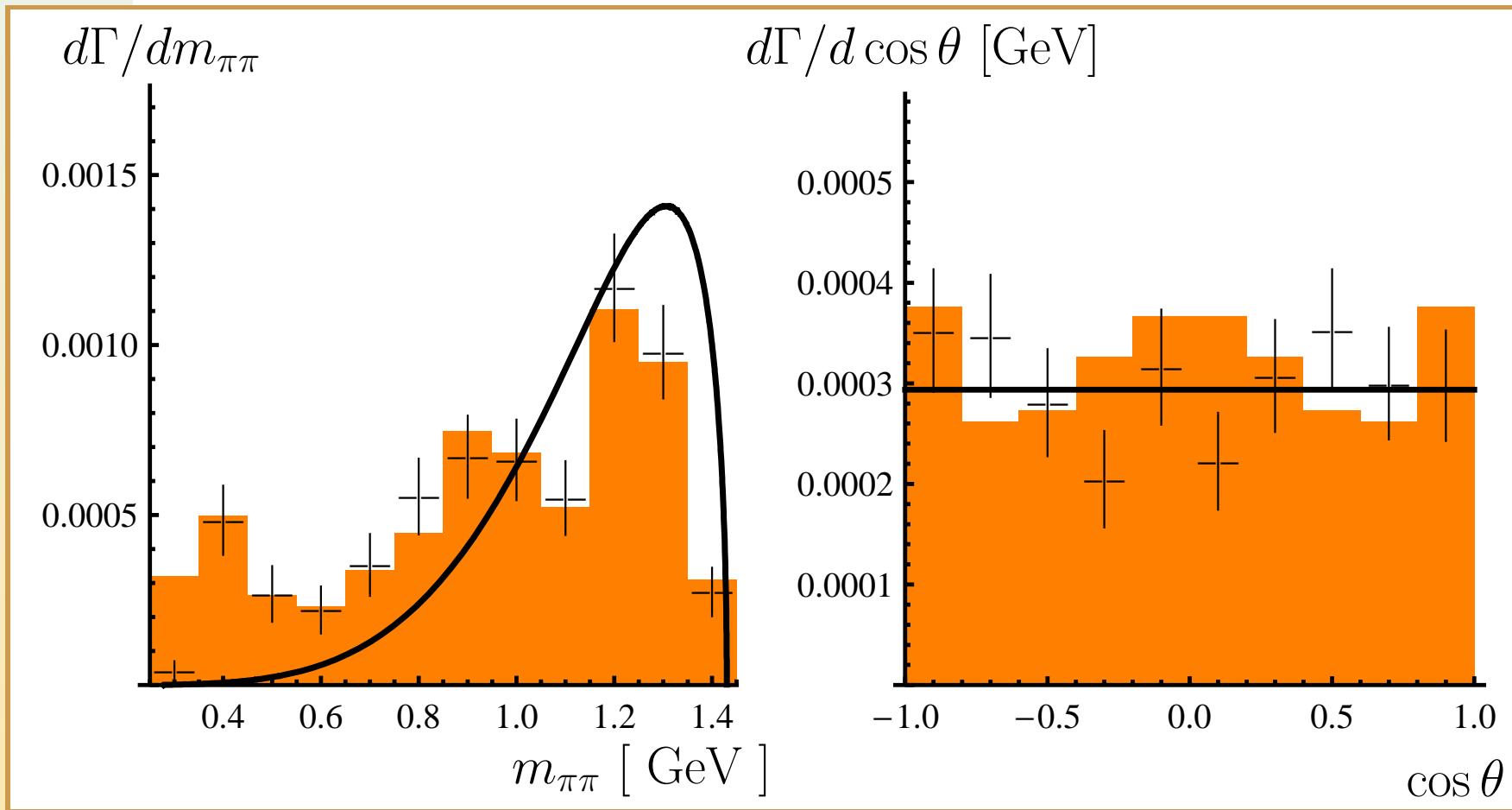
differential partial decay width:

$$d\Gamma = \frac{1}{(2\pi)^3} \frac{1}{32M_{Y_b}^3} |\mathcal{M}|^2 dm_{Y_\pi}^2 dm_{\pi\pi}^2$$



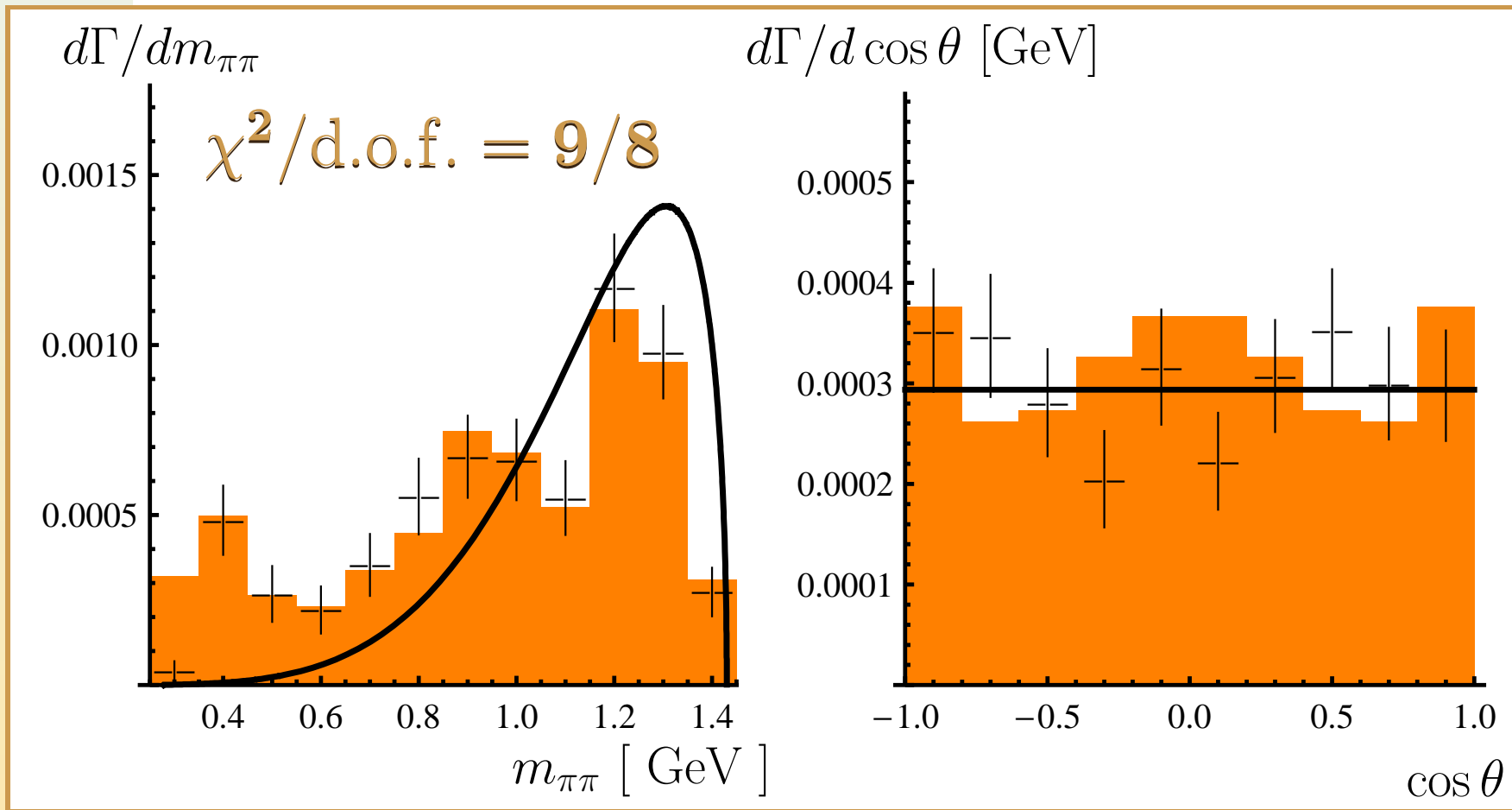
fit to Belle data

Fit for $Y_b \rightarrow \Upsilon(1S)\pi^+\pi^-$:



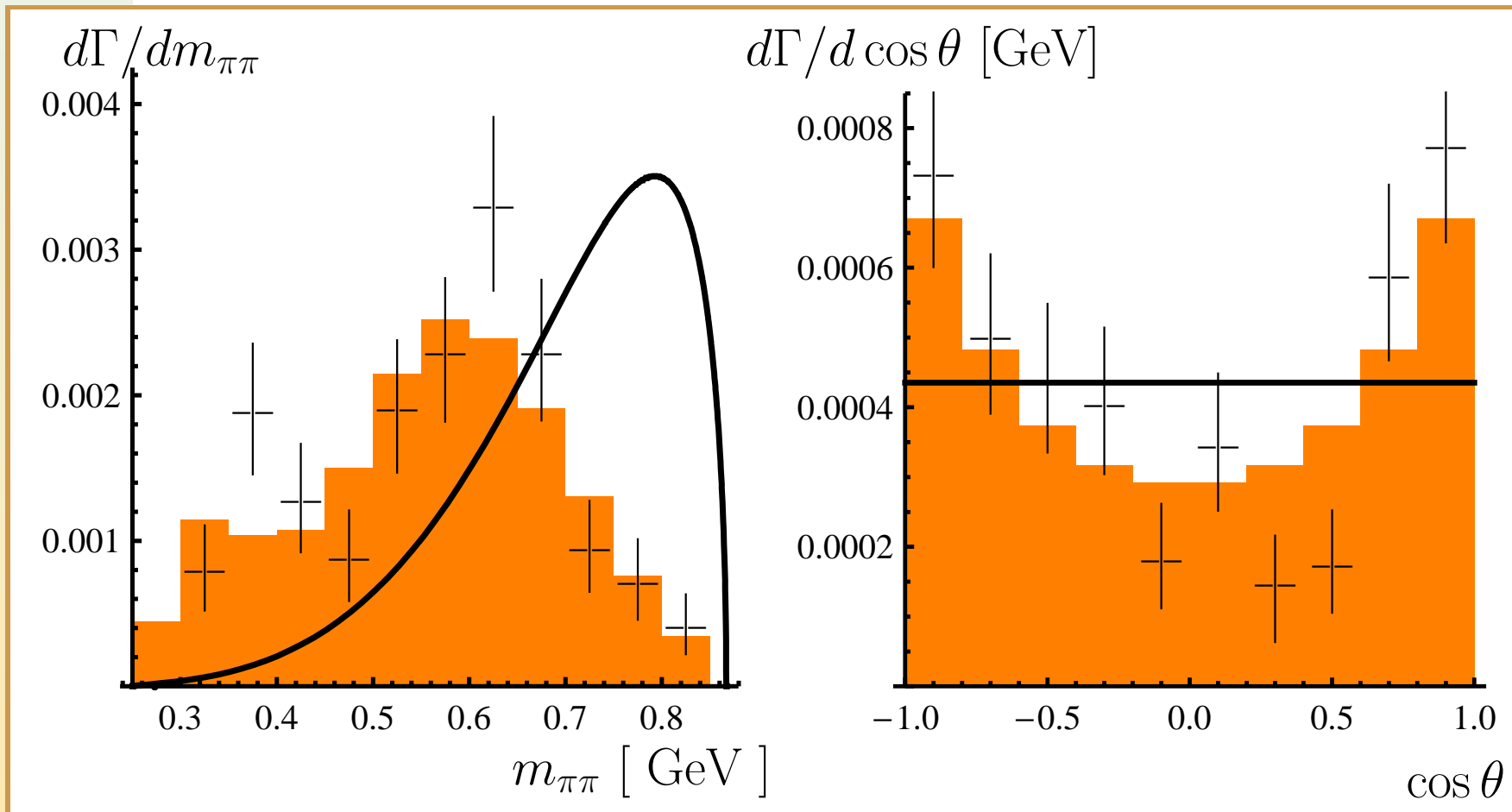
fit to Belle data

Fit for $Y_b \rightarrow \Upsilon(1S)\pi^+\pi^-$:



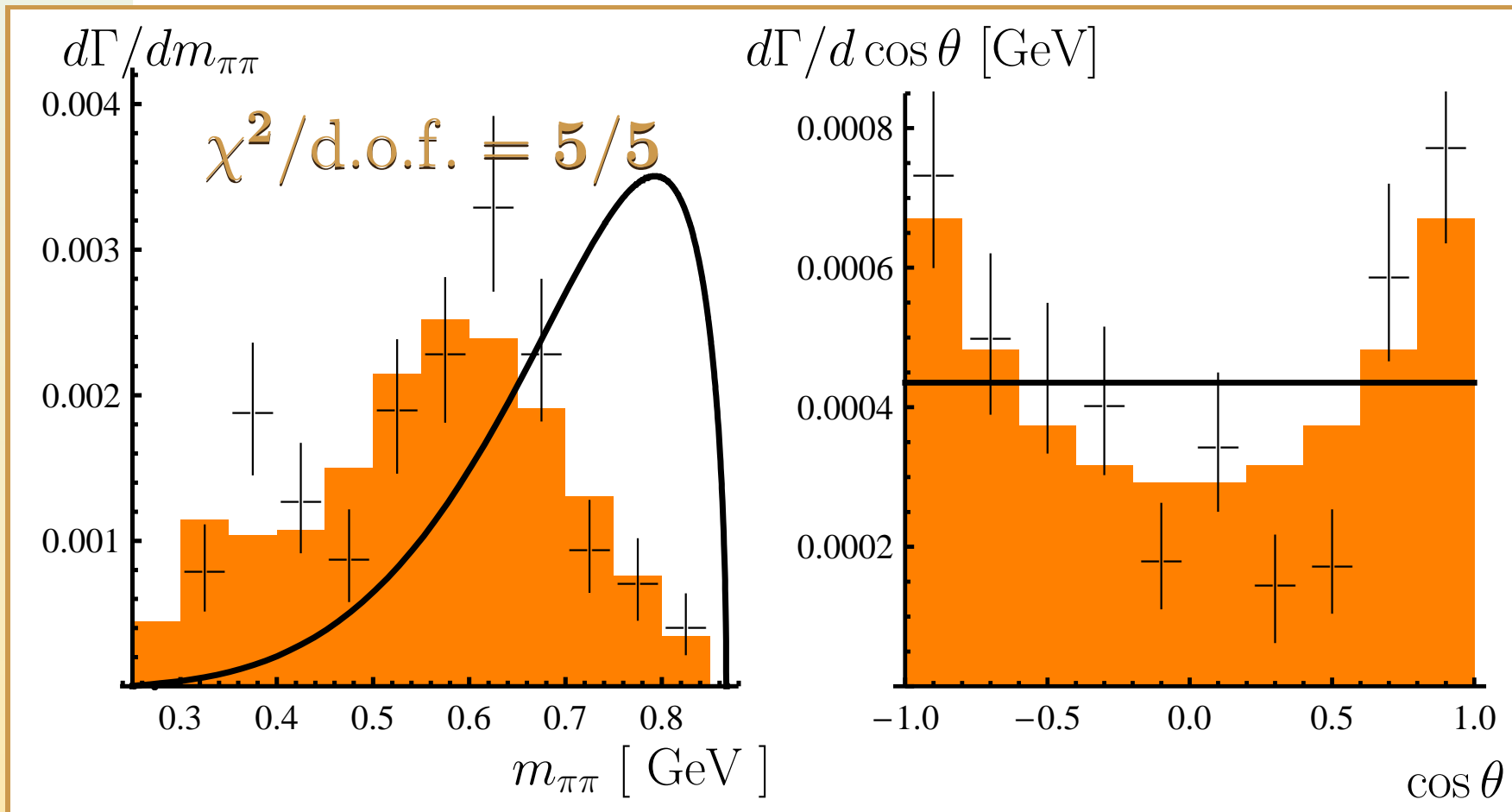
fit to Belle data

Fit for $Y_b \rightarrow \Upsilon(2S)\pi^+\pi^-$:



fit to Belle data

Fit for $Y_b \rightarrow \Upsilon(2S)\pi^+\pi^-$:



fit values

- fit values for $Y_b \rightarrow \Upsilon(1S)\pi^+\pi^-$:

$$F = 0.19 \pm 0.03, \beta = 0.54 \pm 0.12,$$

$$a_{f_2(1270)} = 0.5 \pm 0.16, \varphi_{f_2(1270)} = 3.33 \pm 0.06$$

	$a_{f_0(i)}$	$F_{f_0(i)}$	$\varphi_{f_0(i)}$ (rad.)
$f_0(600)$	3.6 ± 0.7	1.38 ± 0.27	1.14 ± 0.14
$f_0(980)$	0.47 ± 0.02	1.02 ± 0.04	4.12 ± 0.3



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- fit values for $Y_b \rightarrow \Upsilon(2S)\pi^+\pi^-$:

$$F = 0.86 \pm 0.34, \beta = 0.7 \pm 0.3$$

	$a_{f_0(i)}$	$F_{f_0(i)}$	$\varphi_{f_0(i)}$ (rad.)
$f_0(600)$	10.89 ± 2.4	4.19 ± 0.92	2.76 ± 0.22

outlook and conclusion

Summarizing:

- The $\Upsilon(5s) \rightarrow \Upsilon(1S)\pi^+\pi^-$, $\Upsilon(2S)\pi^+\pi^-$ decays are **Zweig forbidden**.

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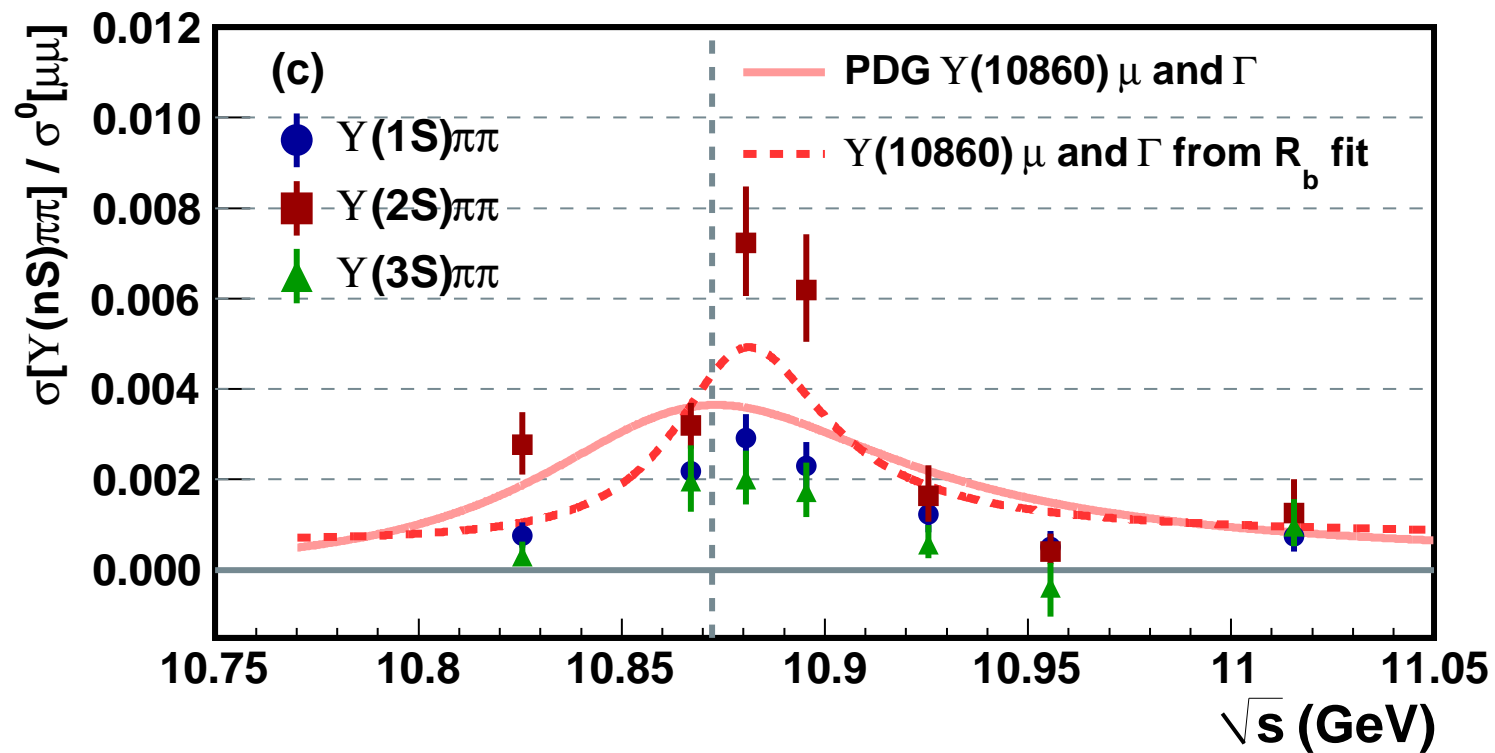
- The $\Upsilon(5s) \rightarrow \Upsilon(1S)\pi^+\pi^-$, $\Upsilon(2S)\pi^+\pi^-$ decays are **Zweig forbidden**.
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Summarizing:

- The $\Upsilon(5s) \rightarrow \Upsilon(1S)\pi^+\pi^-$, $\Upsilon(2S)\pi^+\pi^-$ decays are **Zweig forbidden**.
- The $Y_b \rightarrow \Upsilon(1S)\pi^+\pi^-$, $\Upsilon(2S)\pi^+\pi^-$ decays are **Zweig allowed** and can explain the observed decay width, larger by two orders of magnitude.
- The coupling to intermediate light quark resonances ($f_0(600)$, $f_0(980)$, $a_2(1270)$) can explain the shape of the invariant mass contribution (**scalar meson dominance**).

new Belle data

fits under preparation





Belle data taking

- Spring 2010 run
 - Last Belle data taking has started May 13 (“experiment 73”)
 - Will take until June 30 (7 weeks)
 - 1 week startup
 - 3 weeks machine study for SuperKEKB
 - 4 weeks physics ($Y(5S)$, scan to search for the Ali tetraquark [PLB684, 28-39, 2010])

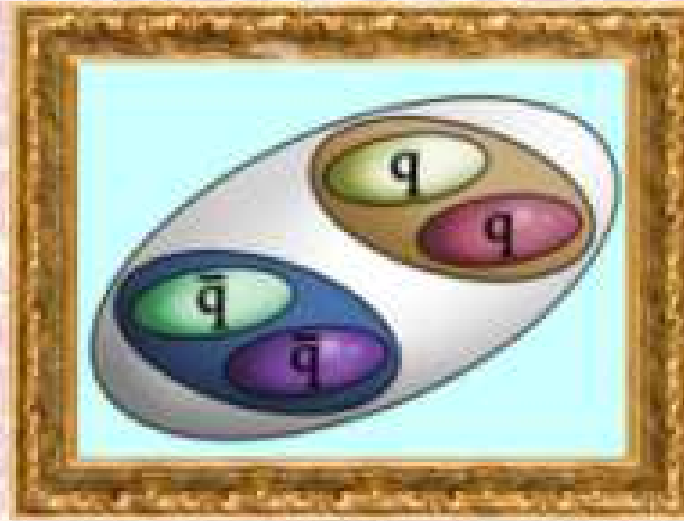
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In conclusion
we hope for good news after the World Cup





thank you!