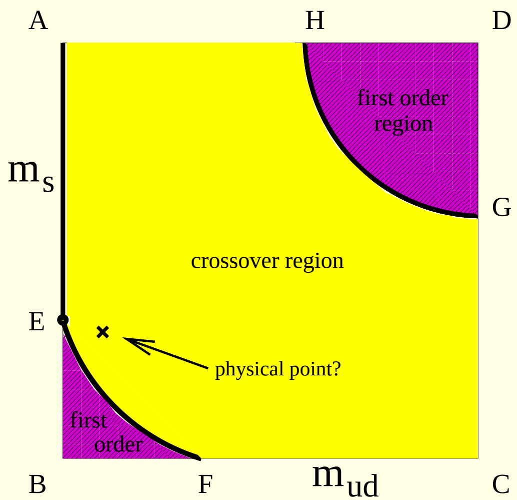
QCD THERMODYNAMICS

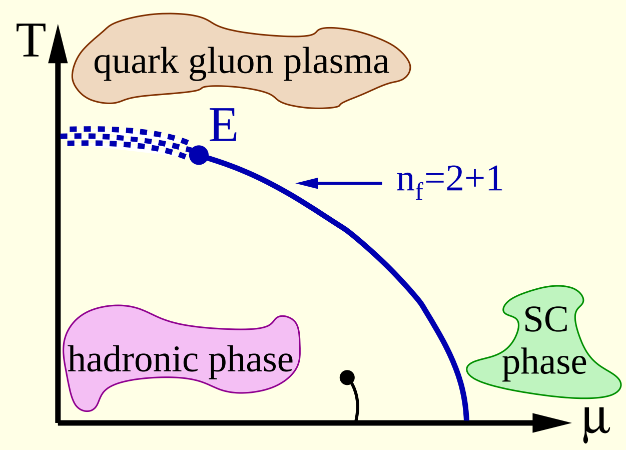
(Approaching the continuum limit: N_t=4,6,8,10 or $a \approx$ 0.3, 0.2, 0.15, 0.12 fm)

Z. Fodor

1. Introduction
2. The nature of the transition: broad cross-over
3. The transition temperature: T_c
4. Static potential as a function of T and r
5. The equation of state at large temperatures
6. Conclusions

Standard picture of the phase diagram and its uncertainties





physical quark masses: important for the nature of the transition

n_f=2+1 theory with m_q=0 or ∞ gives a first order transition

for intermediate quark masses we have an analytic cross over (no χPT)

F. Karsch et al., Nucl. Phys. Proc. 129 (2004) 614; Lattice'07 G. Endrodi, O. Philipsen

continuum limit is important for the order of the transition:

n_f=3 case (standard action, N_t=4): critical $m_{ps}\approx$300 MeV

with different discretization error (p4 action, N_t=4): critical $m_{ps}\approx$70 MeV

the physical pseudoscalar mass is just between these two values

what happens for physical quark masses, in the continuum, at what T_c?

N_t=4,6,8,10 lattices correspond to $a\approx$ 0.3 fm, 0.2 fm, 0.15 fm, 0.12 fm

CPU: $\approx N_t^{13}$ (thermodynamics): N_t=10 needs 50-100$\times$ more than N_t=6


IBM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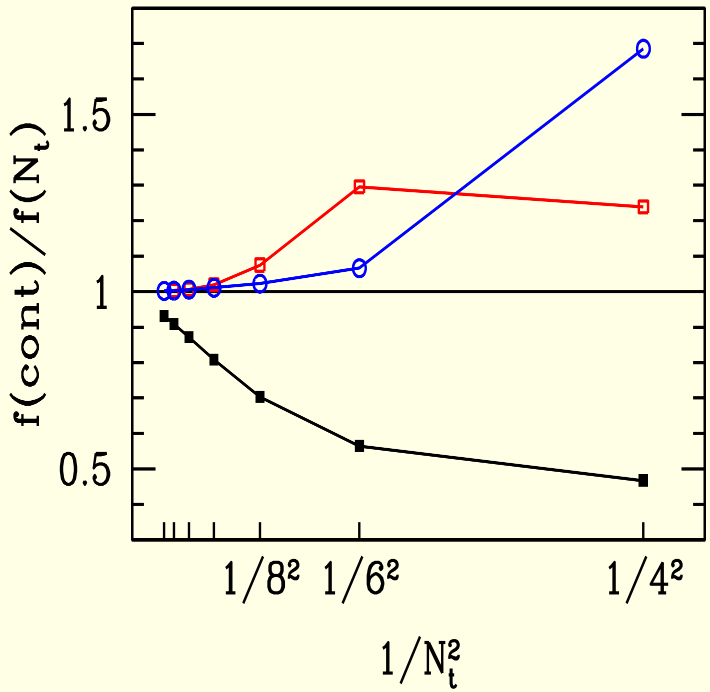
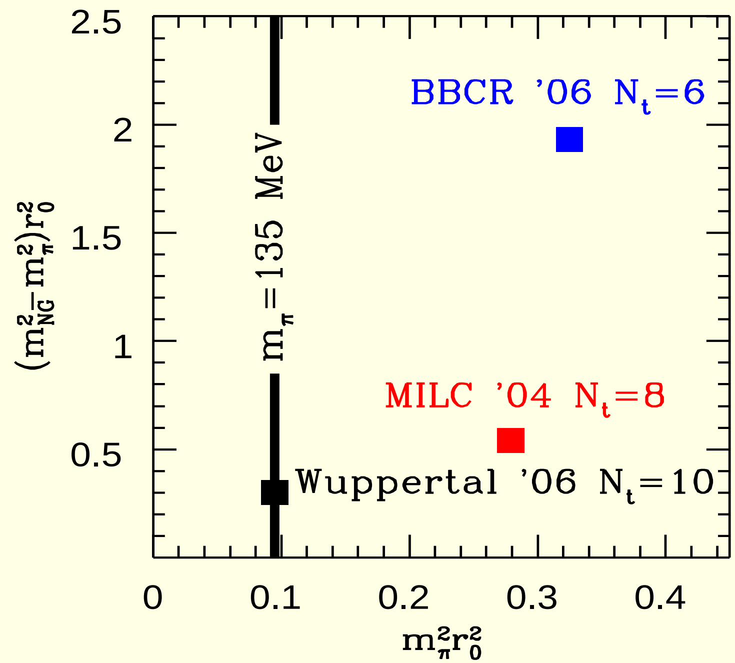
The nature of the QCD transition

Y.Aoki, G.Endrodi, Z.Fodor, S.D.Katz, K.K.Szabo, Nature, 443 (2006) 675 [hep-lat/0611014]
F. Wilczek, Nature, 443 (2006) 675

Symanzik improved gauge, stout improved n_f=2+1 staggered fermions
simulations along the line of constant physics: m_π=135 MeV, m_K=500 MeV

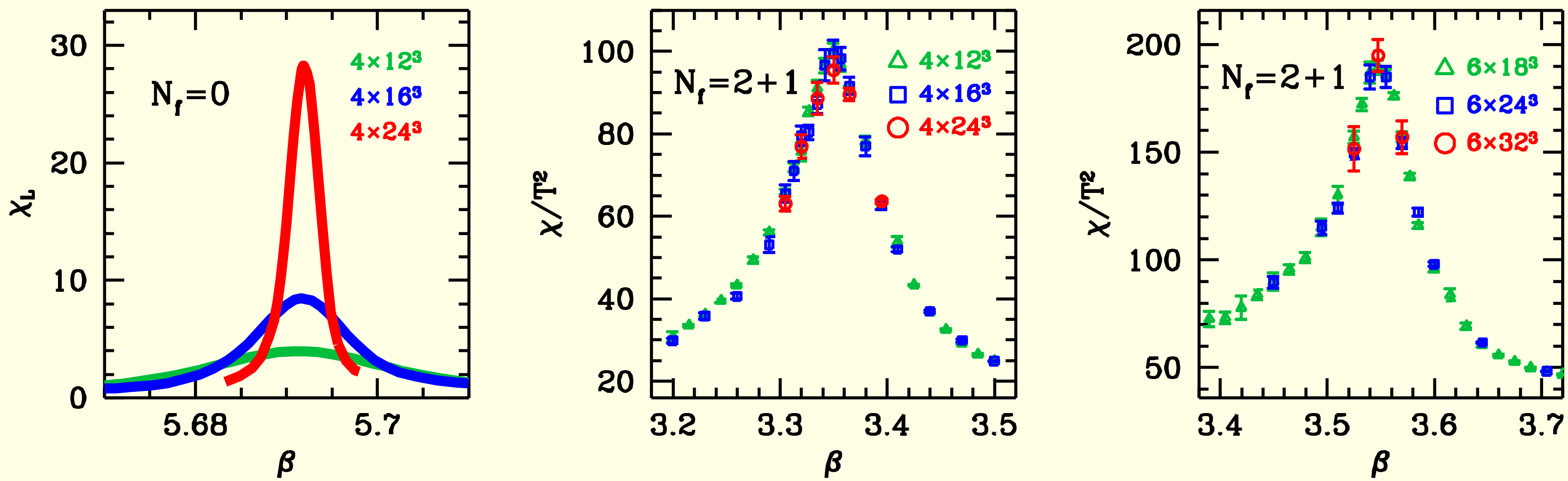




extrapolation from N_t and N_t+2 (standard action) ≈ as good as N_t with p4
N_t=8,10 gives ≈ ±1%, but a<0.15, 0.12 fm needed to set the scale (±1%)
thermodynamic quantities are obtained "more precisely" than the scale
(p4 independent config. is >10× more CPU ⇒ instead balance: a→0)

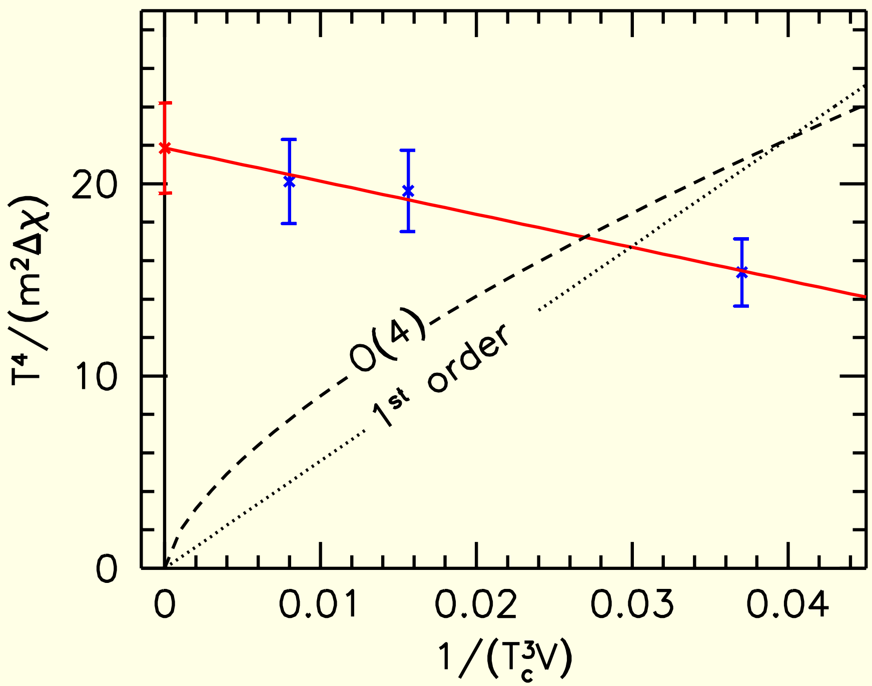
• finite size scaling for the chiral susceptibility: $\chi=(T/V)\partial^2 \log Z/\partial m^2$

first order transition $\Longrightarrow$ peak width $\propto$ 1/V, peak height $\propto$ V
cross-over $\Longrightarrow$ peak width $\approx$ constant, peak height $\approx$ constant



eight times larger volumes: volume independent scaling ⇒ cross-over
do we get the same result (cross-over) in the continuum limit?
one might have the unlucky case as we had in n_f=3 QCD:
discretization errors changed the nature of the transition for physical m_{ps}

• finite size scaling analysis with continuum extrapolated $m^2\Delta\chi$



the result is consistent with an approximately constant behavior for a factor of 5 difference within the volume range

chance probability for 1/V is 10^{-19} for O(4) is $7 \cdot 10^{-13}$

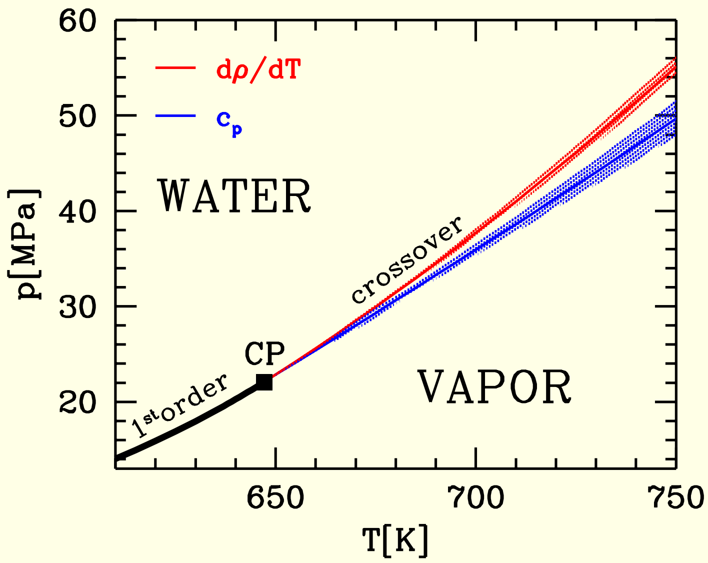
continuum result with physical quark masses in staggered QCD:

the QCD transition at μ=0 is a cross-over

The transition temperature (N_t=4,6,8,10)

Y. Aoki, Z. Fodor, S.D. Katz, K.K. Szabo, Phys. Lett. B. 643 (2006) 46 [hep-lat/0609068]

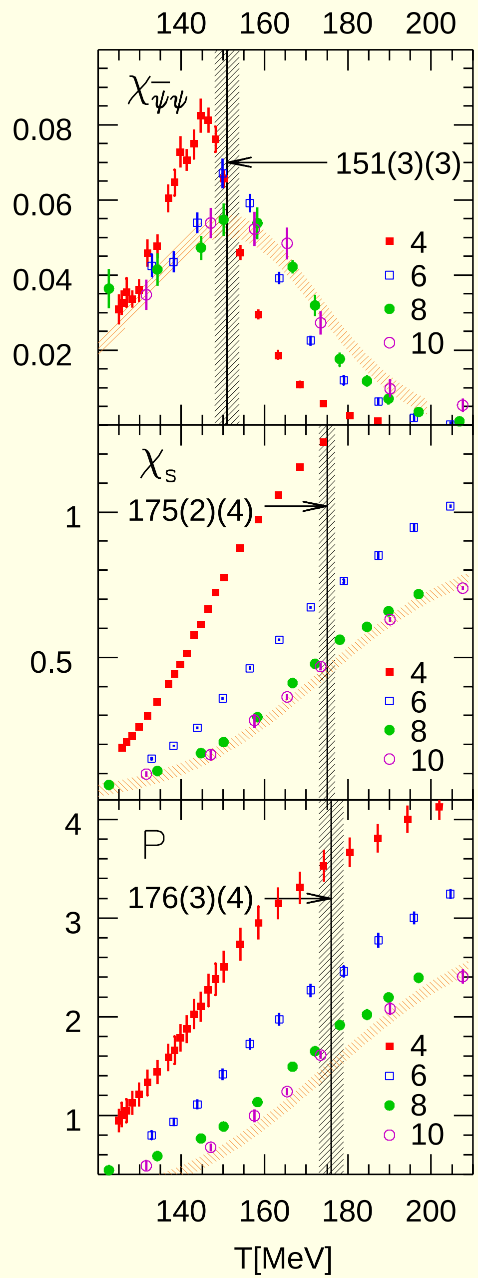
- a cross-over has no unique T_c: example of water-steam transition



above the critical point c_p and $d\rho/dT$ give different T_cs.

QCD: chiral & quark number susceptibilities or Polyakov loop
they result in different T_c values $\Rightarrow$ physical difference

extrapolations from large a: σ, m_ρ, r_0, m_N or f_K give different a (in fm)
this lead to different T_c values $\Rightarrow$ non-physical ambiguity
will be removed in the continuum limit (most precise scale is set by f_K)



Chiral susceptibility

T_c=151(3)(3) MeV

ΔT_c=28(5)(1) MeV

Quark number susceptibility

T_c=175(2)(4) MeV

ΔT_c=42(4)(1) MeV

Polyakov loop

T_c=176(2)(4) MeV

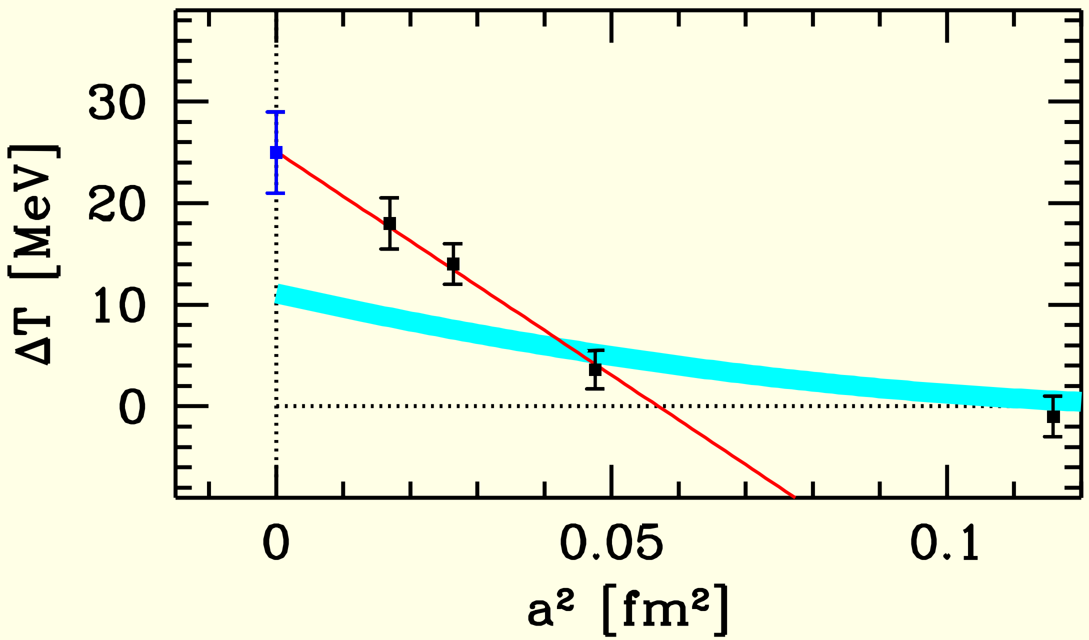
ΔT_c=38(5)(1) MeV

N_t=6,8,10 are in the a^2 scaling regime, N_t=8,10 are practically the same

$\Rightarrow$ 25(4) MeV difference between the chiral & the deconfinement transitions

normalization changes T_c (multiply a Gaussian by $T^2 \Rightarrow$ peak shifts)
continuum: e.g. $\Delta\chi/T^2$ gives $\approx$10 MeV higher T_c than $m^2\Delta\chi/T^4$ (blue curve)

the difference can be seen only at small lattice spacings
C. De Tar hotQCD N_t=8 (asqtad): T_c from χ tends to be at smaller values



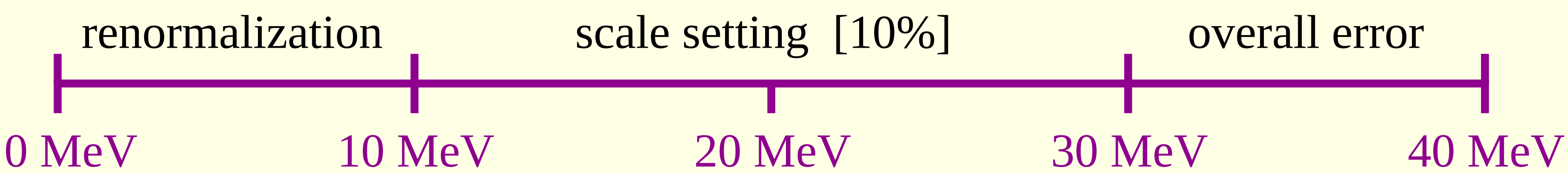
precise data at N_t=8 and 10 are needed to see the difference

• $T_c(\chi_{\bar{\psi}\psi})$ consistent with MILC '2004: $T_c = 169(12)(4)$ MeV

• BBCR collaboration: recent result [M. Cheng et.al, Phys. Rev. D74 (2006) 054507]
Transition temperature from $\chi_{\bar{\psi}\psi}$ and Polyakov loop, from both quantities
T_c=192(7)(4) MeV, $\Longrightarrow$ for $\chi_{\bar{\psi}\psi}$ contradicts our result (≈40 MeV)

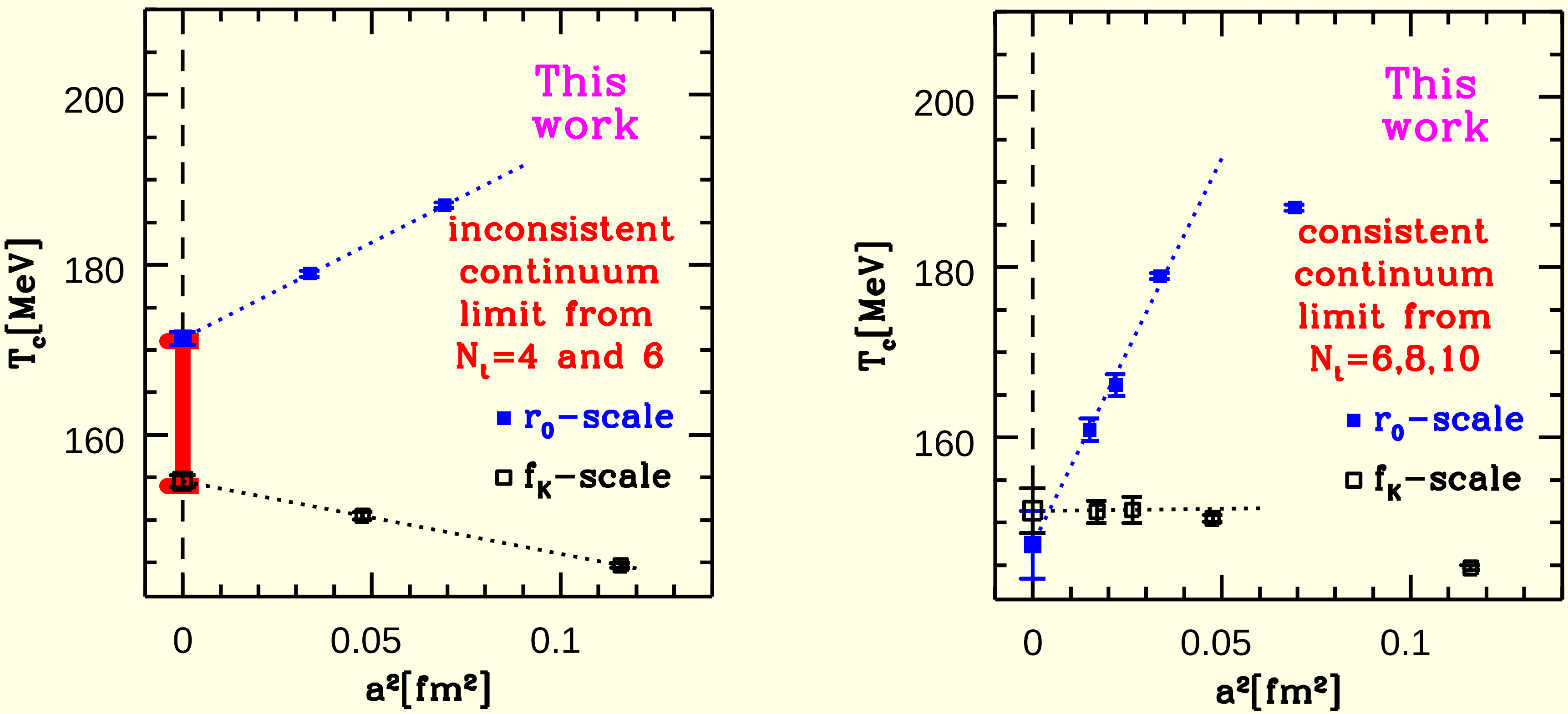
Main differences to our work

no renormalization, χ/T^2 is used: explains only ≈ 10 MeV difference
only $N_t = 4$ & 6 (cutoff: a ≈0.3 fm & 0.2 fm or a^{-1}≈700 MeV & 1 GeV)
scale is set by r_0 instead of f_K (influences only the overall accuracy)



What is the reason for this discrepancy?
Their last concluding remark: it is desirable to
"obtain a reliable independent scale setting for the transition temperature from an observable not related to properties of the static potential".

What if one used the static potential (r_0) and f_K to set the scale?
compare N_t=4,6 and 4,6,8,10 extrapolations with different scale settings



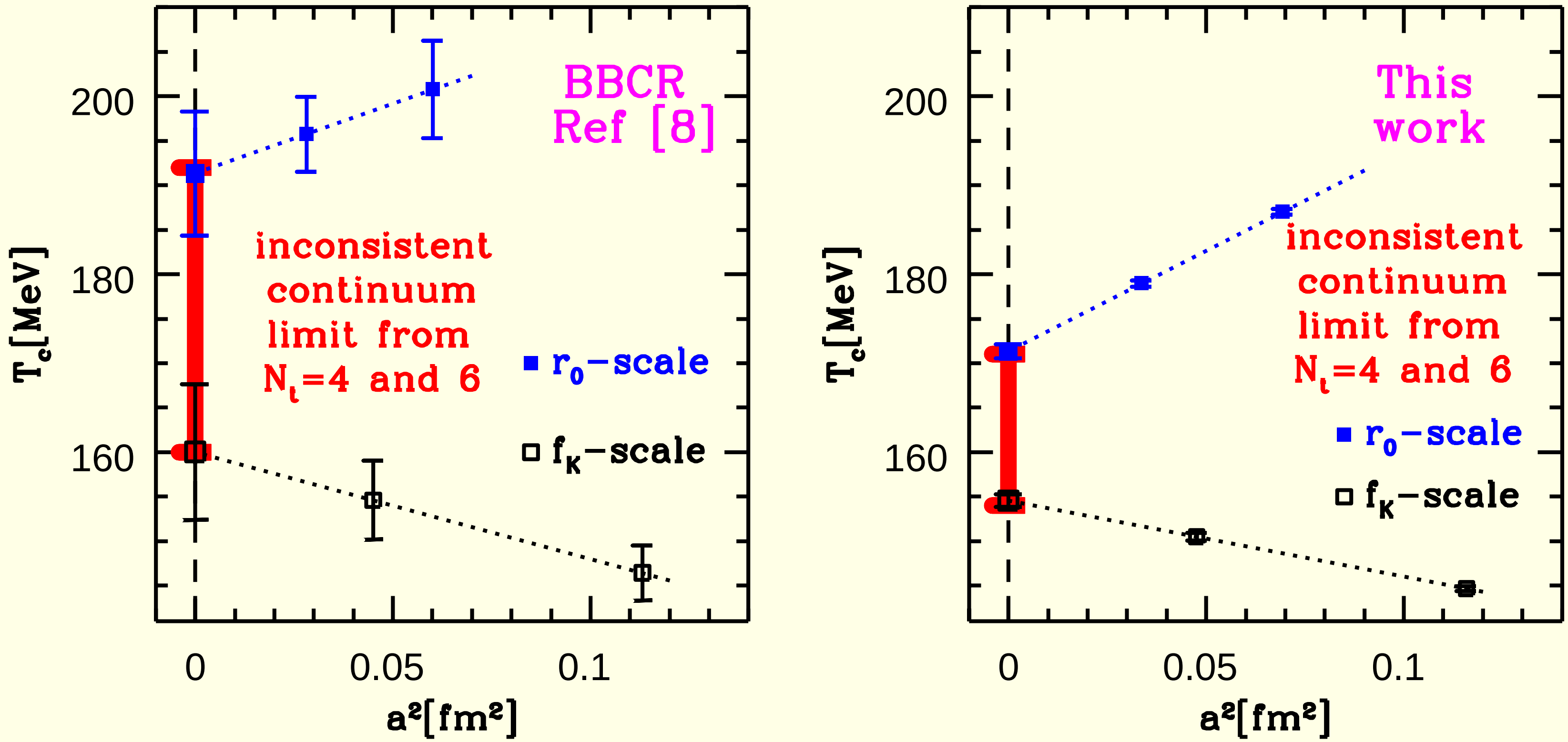
Continuum limits from $N_t = 4,6$ are inconsistent, from $N_t = 6,8,10$ consistent
not surprising: eg. asqtad at $N_t \approx$10 has $\approx$10% scale difference between r_1 & f_K
Lüscher (Dublin) & DelDebbio et al: a=.06fm $\approx$20% difference between r_0 & m_{K^*}

one needs 3 points in the scaling regime (2 points are always on a line)

What if they used f_K to set the scale?

We repeated some of their $T = 0$ simulations to determine f_K

Alternatively:
We can use r_0 and only N_t=4,6



Continuum extrapolations from $N_t = 4, 6$ are inconsistent!

not surprising: eg. asqtad at N_t≈10 has ≈10% scale difference between r_1 & f_K
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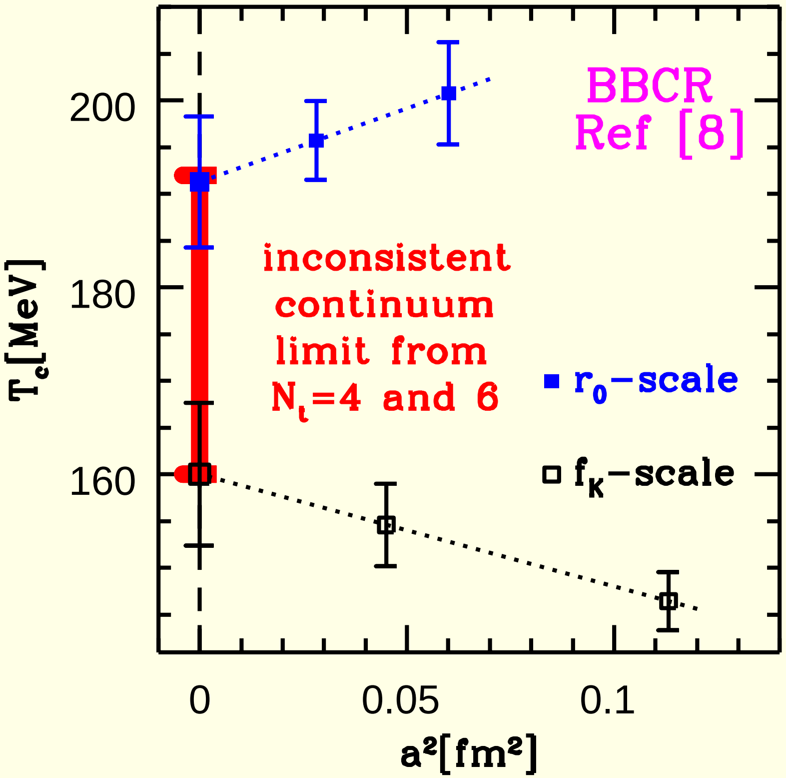

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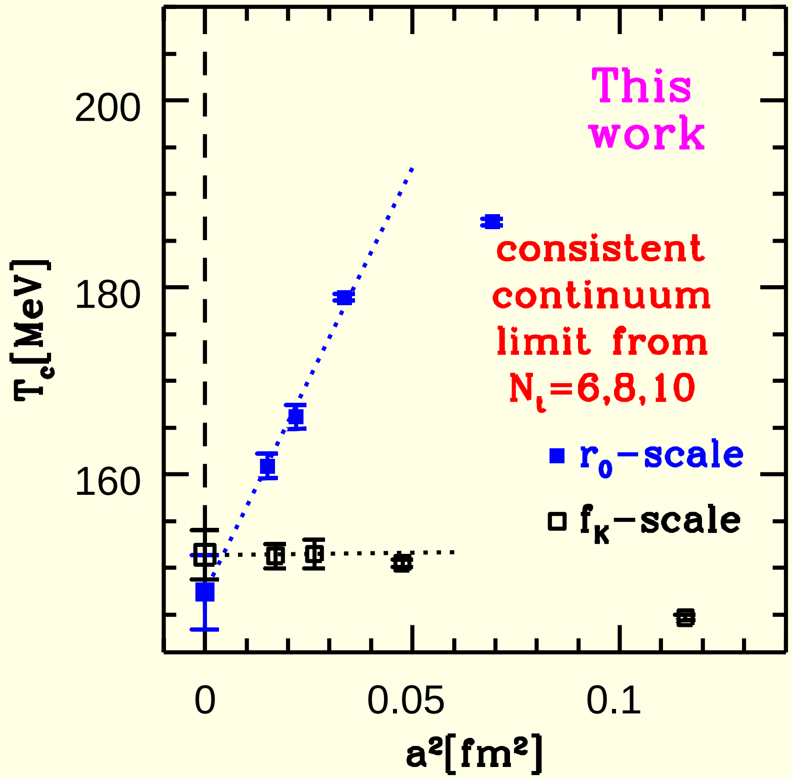


Continuum extrapolations from $N_t = 6, 8, 10$ are consistent!

Conclusion: continuum limit from N_t=4,6 isn't safe ($a\approx$0.3, 0.2 fm or 0.7, 1GeV)

● transition temperature depends on the geometry

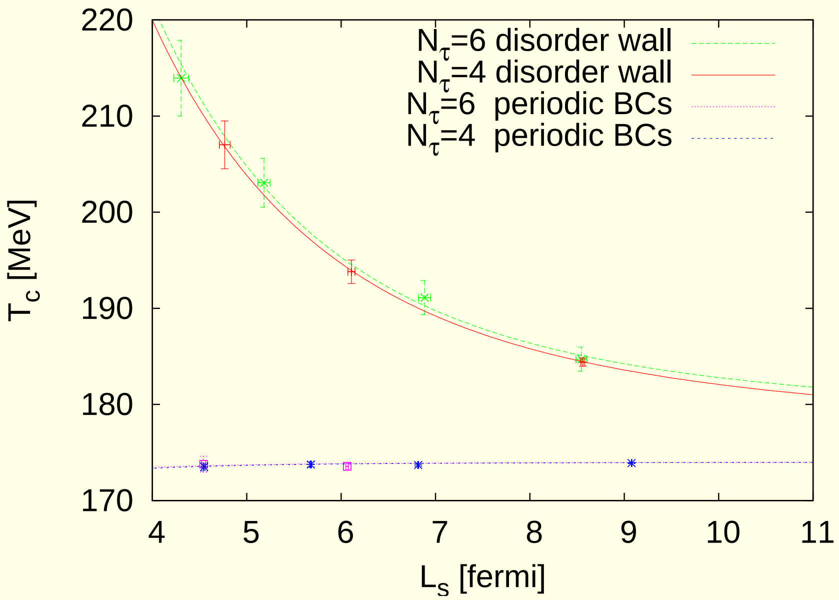
eg. nanotube-water didn't freeze, even at hundreds of degrees below 0^oC

A. Bazavov and B. Berg, Phys.Rev. D76 014502 (2007)

determined the transition temperature pure SU(3), no qarks

with "confined" spatial boundary conditions: more like experiments

instead of periodic one (which we use to reach V$\longrightarrow\infty$ fast)



large deviation (upto 30 MeV) from the infinite volume limit

$\Longrightarrow$ calculate it in full QCD (cross-over) for different geometries

• several observables can be determined with N_t=4,6,8,10 configurations

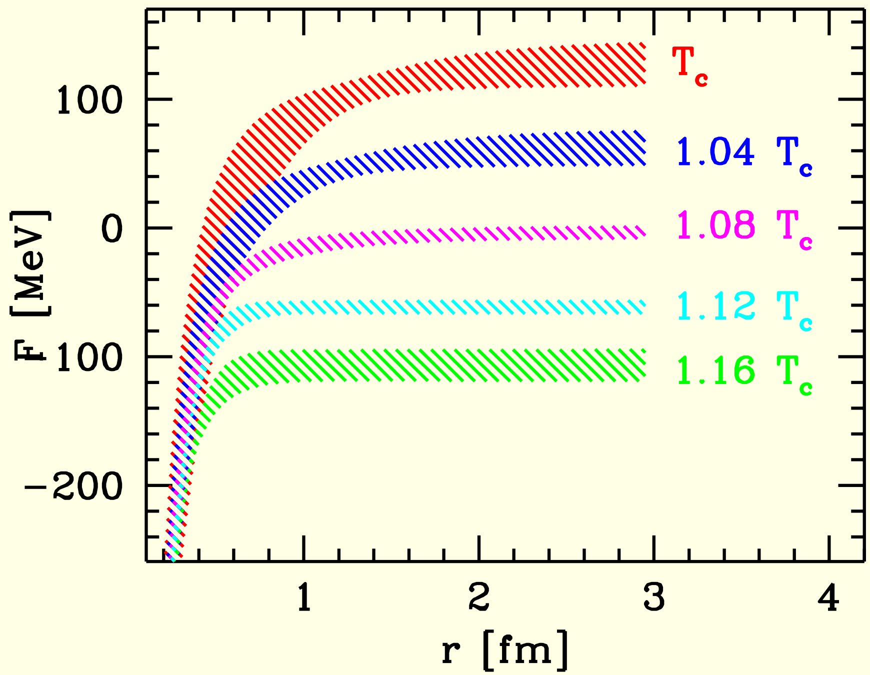
a. curvature on the μ–T diagram (ongoing analysis)

b. static quark free energies

Polyakov loop correlator:

$$\sum_x \langle \mathrm{TrP}(x)\mathrm{TrP}(x+r)\rangle \propto e^{-F_{q\bar{q}}(r)/T}$$

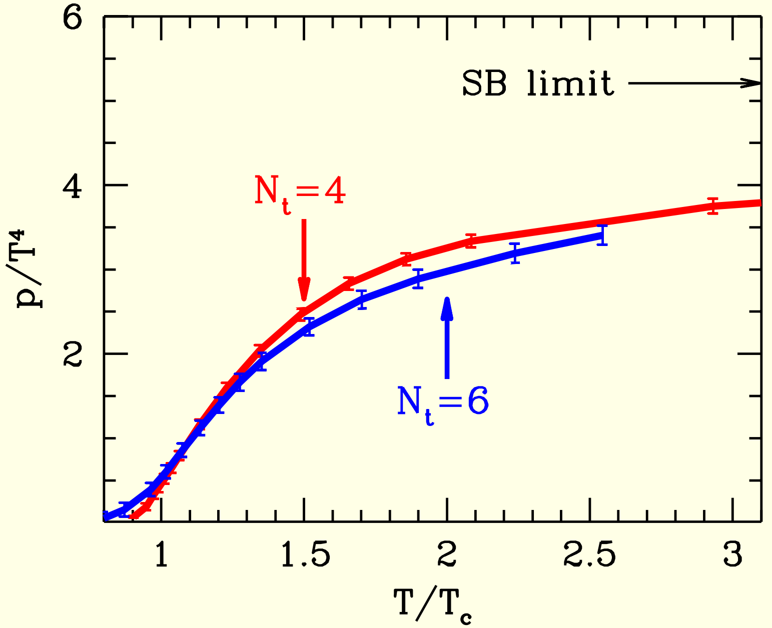
measure correlators on N_t=4,6,8,10 lattices $\Longrightarrow$ continuum limit of $F_{q\bar{q}}$

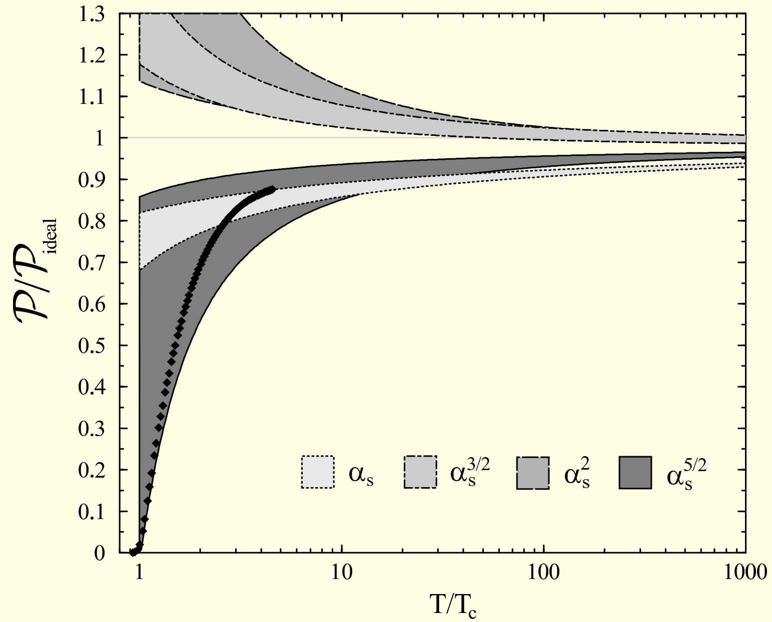


Link to perturbation theory: equation of state at large temperatures

lattice results for the EoS extend upto a few times T_c

perturbative series “converges” only at asymptotically high T





- the standard technique is the integral method:

$\bar{p}$=T/V·log(Z), but Z is difficult ⇒ $\bar{p}$ integral of (∂log(Z)/∂β,∂log(Z)/∂m)
substract the T=0 term, the pressure is given by: $p(T)=\bar{p}(T)-\bar{p}(T=0)$

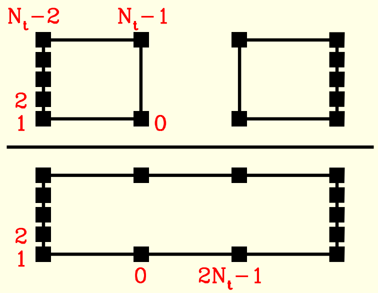
- back of an envelope estimate:

$T_c\approx$150–200 MeV, m_π=135 MeV and try to reach T=20·T_c for N_t=8 (a=0.0075 fm)
⇒N_s > 4/m_π ≈ 6/T_c = 6·20/T = 6·20·N_t ≈ 1000 ⇒ completely out of reach

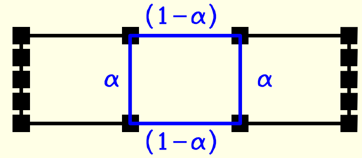
a. substract successively: $p(T)=\bar{p}(T)-\bar{p}(T=0)=[\bar{p}(T)-\bar{p}(T/2)]+[\bar{p}(T/2)-\bar{p}(T/4)]+...$

$\Longrightarrow$ for substractions at most twice as large lattices are needed

b. instead of the integral method calculate: $\bar{p}(T)-\bar{p}(T/2)=T/(2V)\cdot\log[Z^2(N_t)/Z(2N_t)]$



$$\frac{Z^2(N_t)}{Z(2N_t)} =$$



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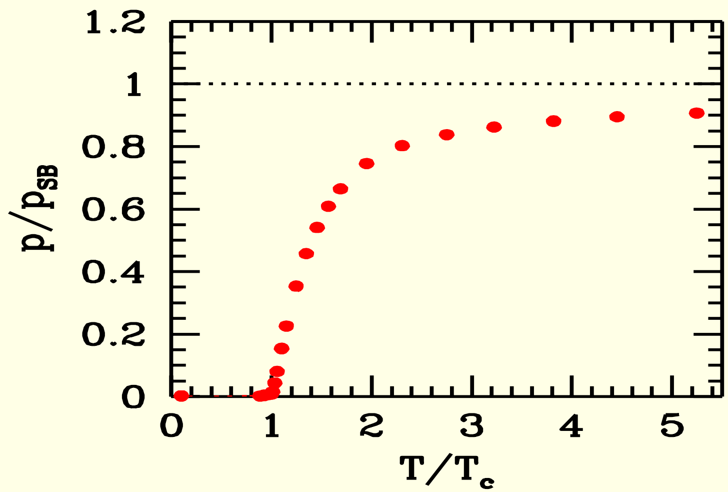

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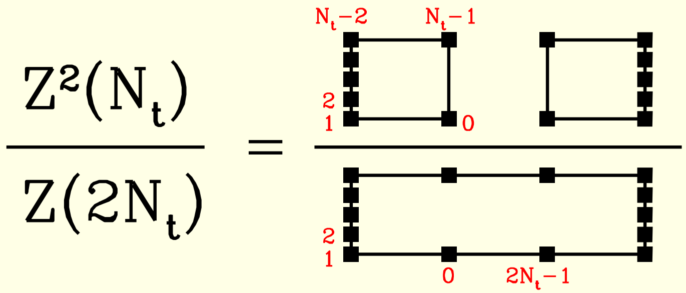

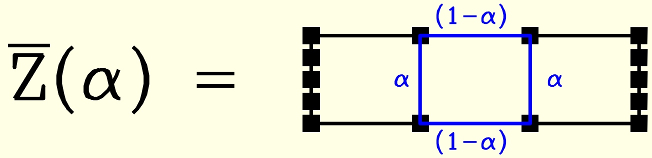

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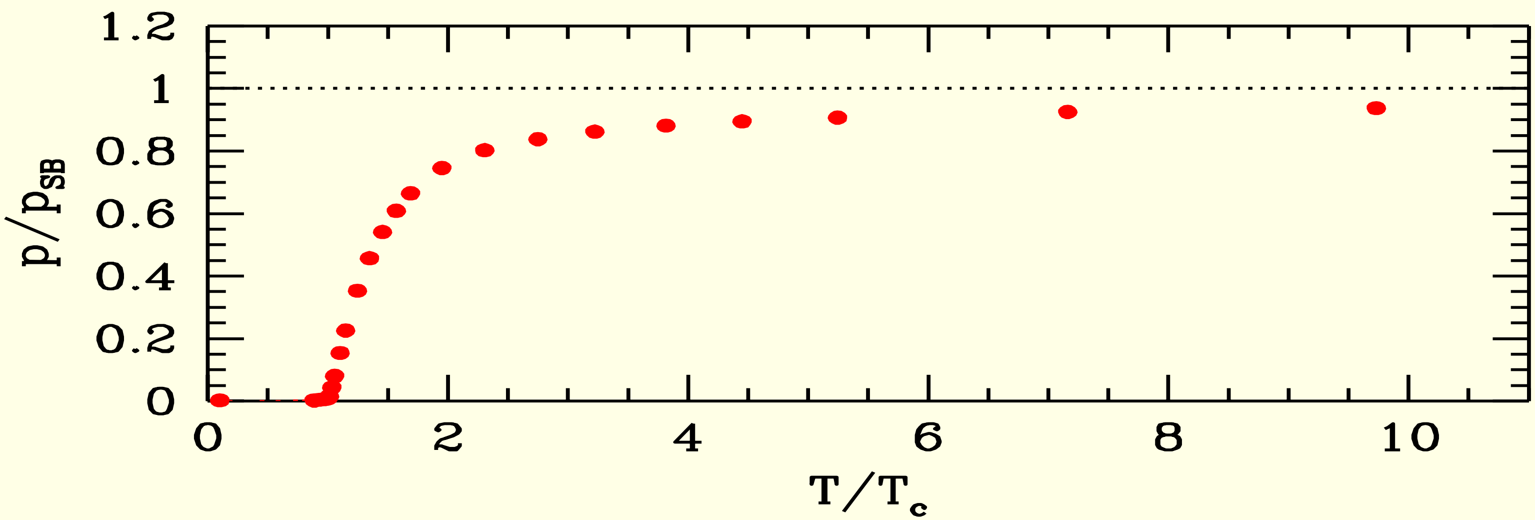

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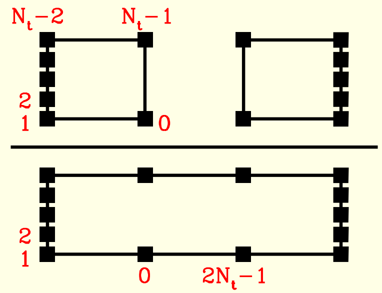

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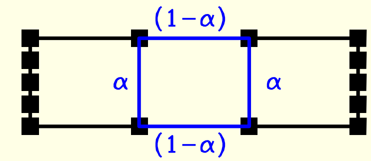

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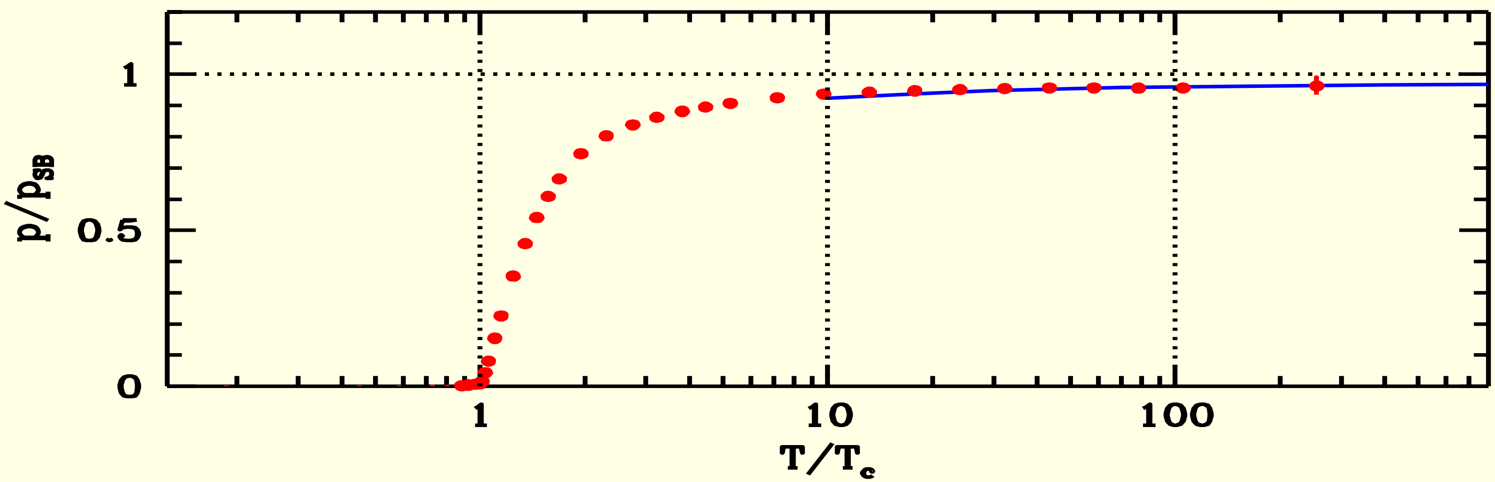

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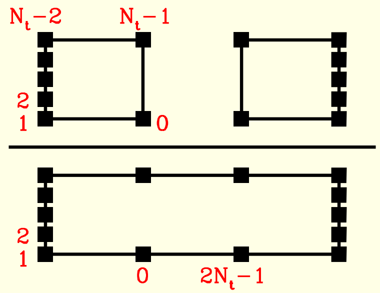

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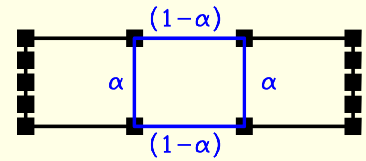

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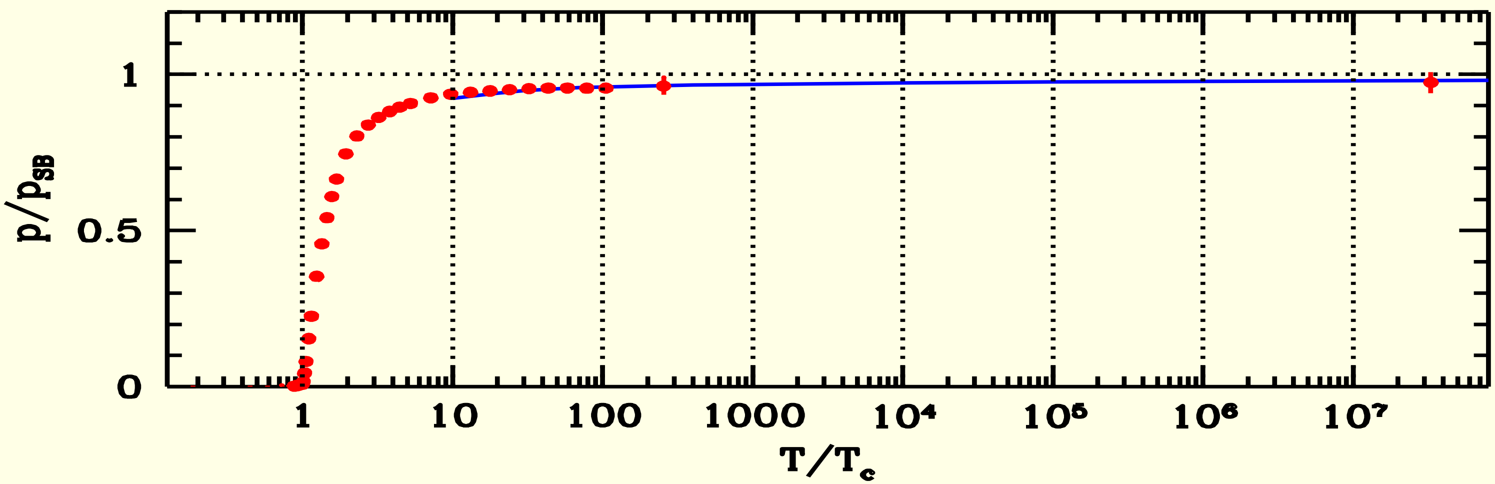

long awaited link between lattice thermodynamics and pert. theory is there

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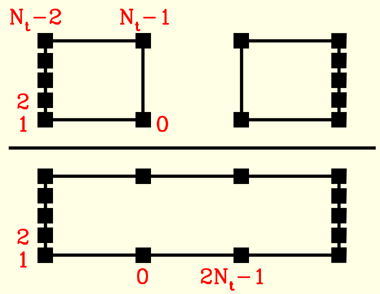

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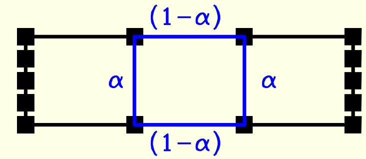

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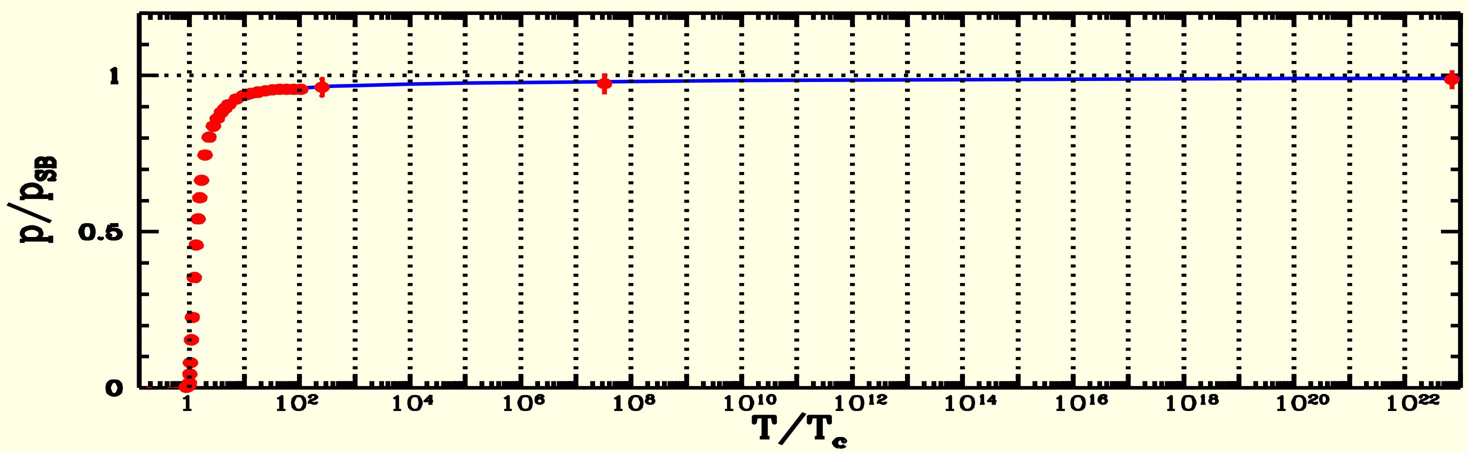

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G.Endrodi, Z.Fodor, S.D.Katz, K.K.Szabo, arXiv:0710.4197

Conclusions

- The QCD transition is a cross-over
- finite size scaling analysis of the chiral susceptibility
- continuum limit using $N_t = 4, 6, 8, 10$
- no volume dependence $\Rightarrow$ no real phase transition scenario

- The transition temperature is determined

- Chiral susceptibility:

  T_c=151(3)(3) MeV, ΔT_c=28(5)(1) MeV

- Quark number susceptibility:

  T_c=175(2)(4) MeV, ΔT_c=42(4)(1) MeV

- Polyakov loop:

  T_c=176(2)(4) MeV, ΔT_c=38(5)(1) MeV

- Use finer lattices ($N_t = 8, 10$) for the proper continuum limit
  new results are expected from the hotQCD Collaboration
  important to gain a consitent picture (independent calculations)

- Other observables with N_T=4,6,8,10 configurations

- ongoing continuum analysis for the curvature on the μ–T plane

- continuum result for the static quark free energies

- Gap between lattice and perturbative bulk thermodynamics

- two new methods to reach (arbitrary) high temperatures

- connection to perturbation theory is established

• an even more often experienced example

melting of ice shows singular behavior: ice $\longrightarrow$ water

melting of butter shows analytic behaviour (broad transition, cross-over)
natural fats are mixed triglycerids of fatty acids from C_4 to C_{24}
these are saturated or unsaturated of even carbon numbers

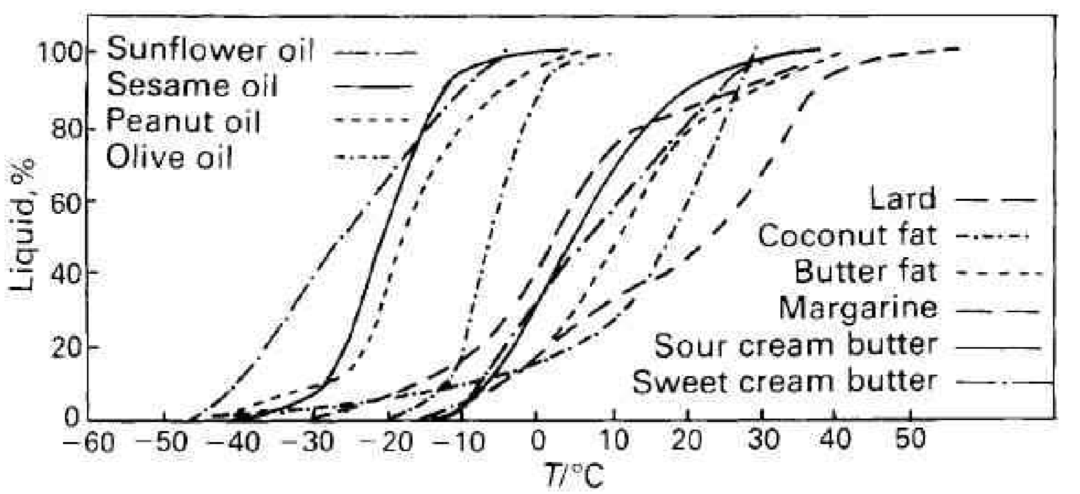


Fig. 4. Liquid proportions of various fats and oils

since in QCD we have an analytic cross-over
we will see very similar temperature dependence for all quantities
e.g. chiral condensate, strange quark susceptibility or Polyakov loop

The transition temperature

Y. Aoki, Z. Fodor, S.D. Katz, K.K. Szabo, Phys. Lett. B. 643 (2006) 46 [hep-lat/0609068]

$T = 0$:

set the physical scale and locate the physical point

Three quantities are needed (m_π and m_K for the quark masses)

Several possibilities for the third quantity

- string tension (not existing in full QCD)
- static quark potential at intermediate distances ($r_0^2 \cdot dV/dr$=1.65)
- directly measurable quantities (e.g. f_K)

Further quantities are predictions (e.g. r_0, f_π, m_{K^*})

$T > 0$:

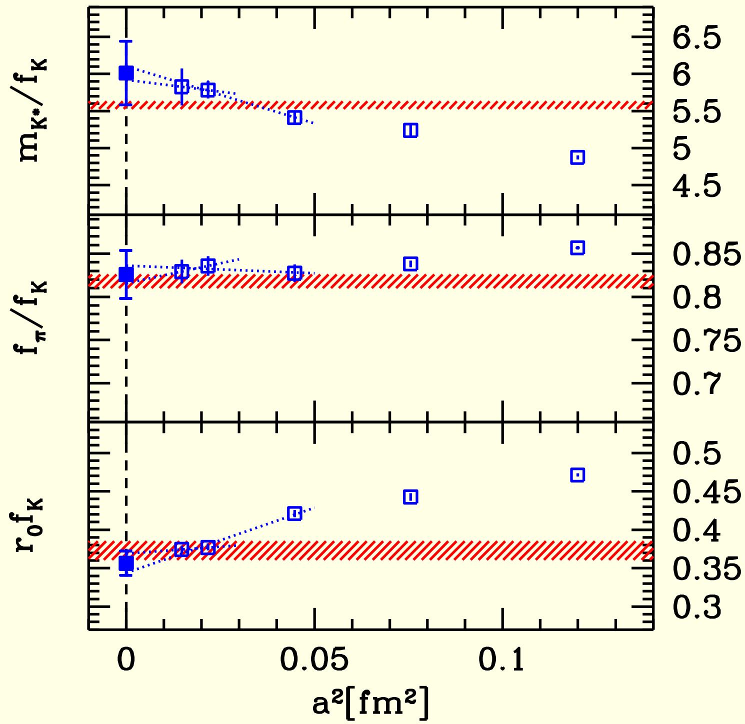
cross-over $\rightarrow$ different definitions give different T_c

Possible choices:

- Chiral susceptibility
- Quark number susceptibility
- Polyakov-loop

T=0 Simulations

- m_π, m_K and f_K was used to set the quark masses and scale
- $m_{ud} \approx 3, 5, 7, 9 \times m_{ud,phys}$ together with chiral extrapolation
- lattices from $12^3 \cdot 24$ up to $24^3 \cdot 32$



Predictions for m_{K^*}, f_π and consistent with experimental values

r_0 is consistent with MILC measurement

• How to get rid of the discretization errors?

a. susceptibility for fixed physical volumes in the continuum
b. finite size analysis of the continuum extrapolated values

renormalize the susceptibility the same way as the free energy

$$f(T) \propto \log Z(T\neq 0)/V_4 - \log Z(T=0)/\overline{V}_4$$

$p(T)$ has a continuum limit and we can use $m_r = Z_m \cdot m$

$$\chi_r(T) = \partial^2/(\partial m_r^2)\,[\log Z(T\neq 0)/V_4 - \log Z(T=0)/\overline{V}_4]$$

construct a quantity in continuum: Z_m drops out from $m^2 \partial^2/\partial m^2$

$$\Longrightarrow m_r^2 \cdot \chi_r(T) = m^2 \cdot [\chi(T\neq 0) - \chi(T=0)]$$

Chiral susceptibility:

Renormalization: seen before

Quark number susceptibility:

$$\frac{\chi_s}{T^2} = \frac{1}{TV} \frac{\partial^2 \log Z}{\partial \mu_s^2}\bigg|_{\mu_s=0}$$

No renormalization necessary

Polyakov loop:

$$P = \frac{1}{N_s^3} \sum_{\mathbf{x}} \mathrm{tr}[U_4(\mathbf{x},0)U_4(\mathbf{x},1)\ldots U_4(\mathbf{x},N_t-1)]$$

Related to the static quark free energy:

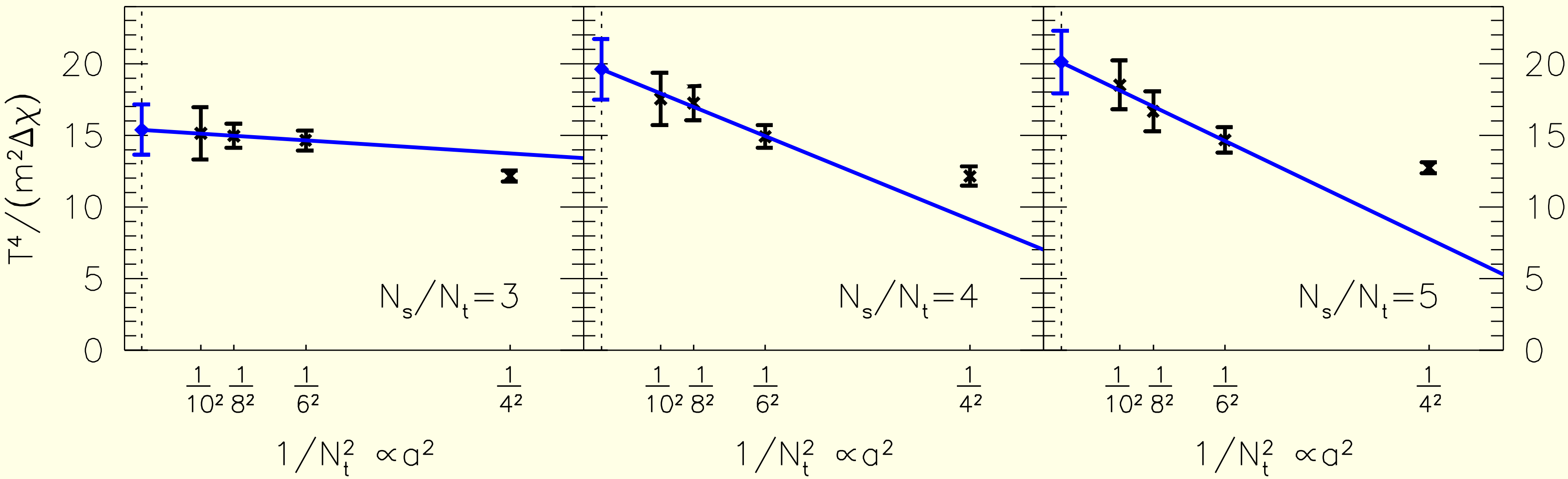
$$|\langle P \rangle|^2 = \exp(-\Delta F_{q\bar{q}}(r \longrightarrow \infty)/T)$$

Renormalization condition for the potential: $V_R(r_0) = 0$

$$|\langle P_R \rangle| = |\langle P \rangle| \exp(V(r_0)/(2T))$$

• we need a continuum extrapolation (width, height)

calculate $m^2\Delta\chi = m^2[\chi(T\neq 0)-\chi(T=0)]$ at the transition point



continuum limit values are obtained from N_t=4,6,8,10

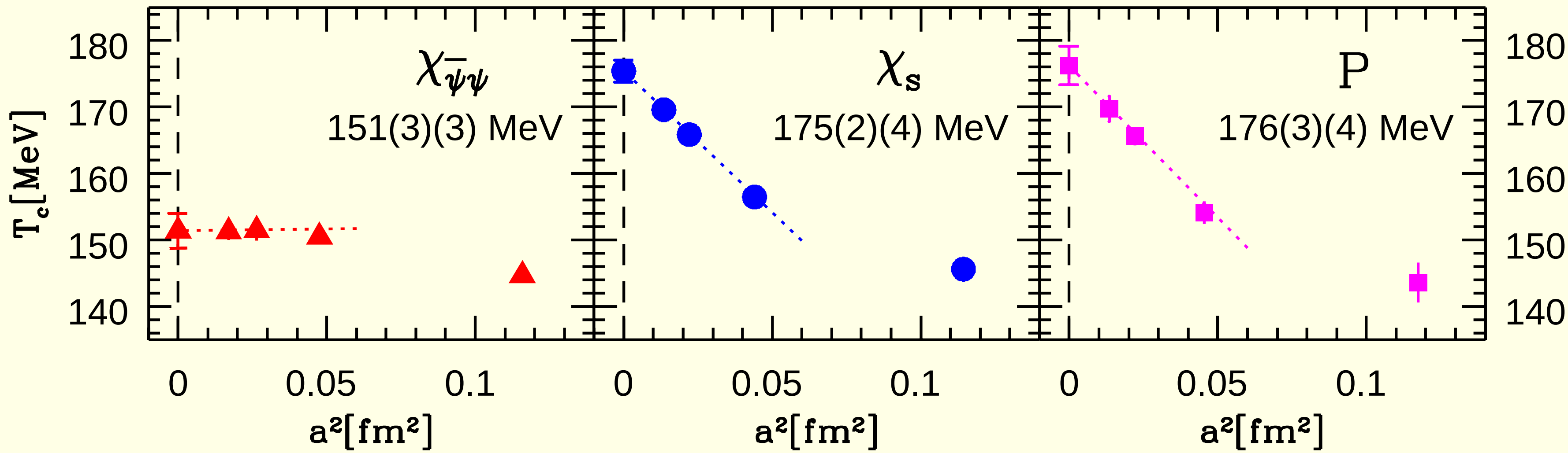
N_t=6,8,10 temporal sizes are already in the a^2 scaling region

choice of the action or the line of constant physics is ambiguous

this choice has influence only on the slope, not on the value

the three continuum extrapolated values do not show 1/V scaling

Continuum extrapolations



N_t=4 is off, N_t=6,8 and 10 show nice scaling for all quantities

Chiral and de-confinement transitions at different locations
25(4) MeV difference

Note: different normalization leads to different T_c
(e.g. $\Delta\chi/T^2$ leads to ≈10 MeV higher T_c)

→ $T_c(\Delta\chi)$ consistent with MILC '2004: $T_c = 169(12)(4)$
Their analysis used coarser lattices, non-physical quark masses, smaller aspect ratios and inexact R algorithm