Under the spell of gauge theory

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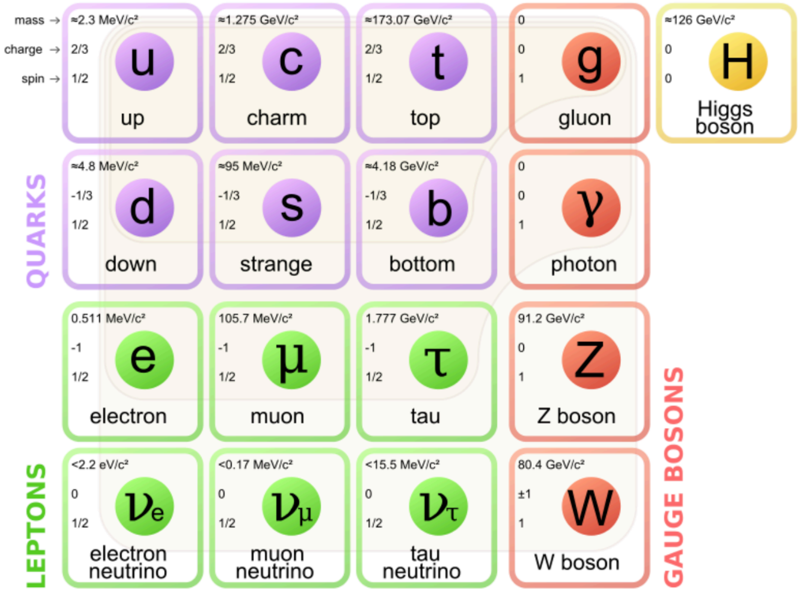
DESY colloquium
27.03.2018

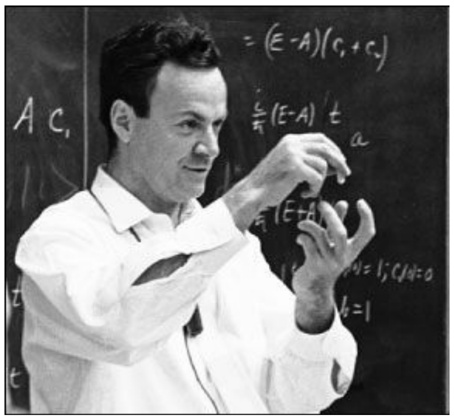
Gauge theories

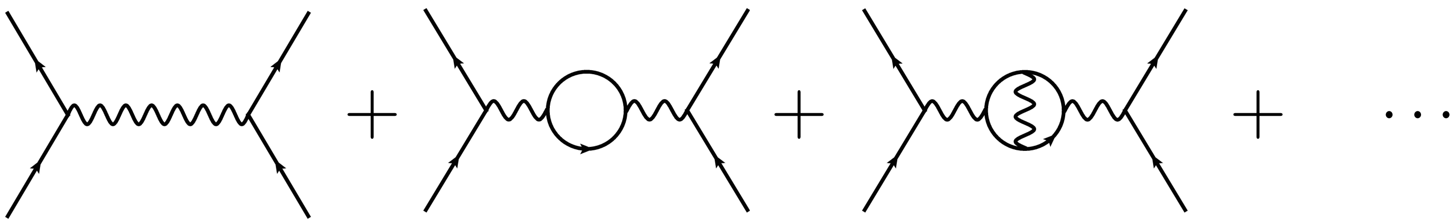


U(N) or SU(N) gauge group
gauge bosons force carriers

Matter fields (quarks and leptons)

Feynman: how to calculate a process **perturbatively**:

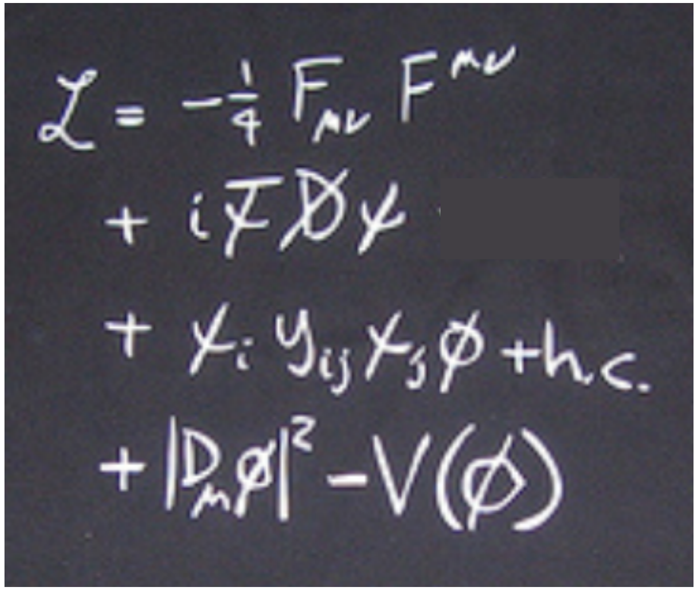




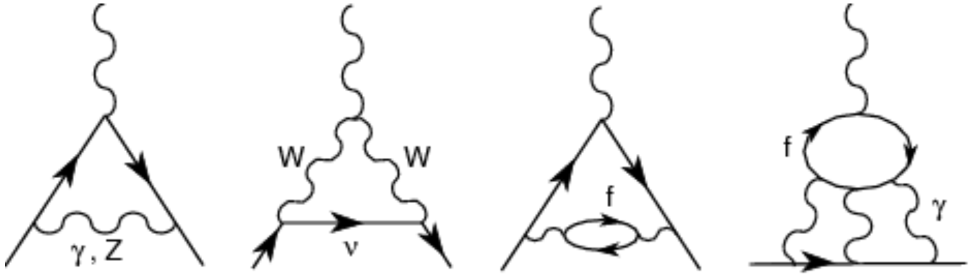
$$= c_1 \lambda + c_2 \lambda^2 + c_3 \lambda^3 + \cdots$$

$\lambda << 1$

Gauge theories



Feynman diagrams have taken us very far!



Precision tests of QED

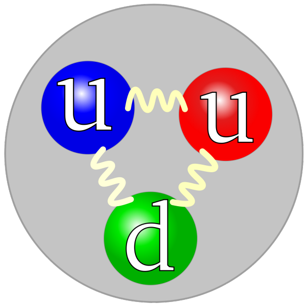
$g_e^{exp.} = 2.0023193043617 \pm 3$

$g_e^{th.} = 2.00231930436...$

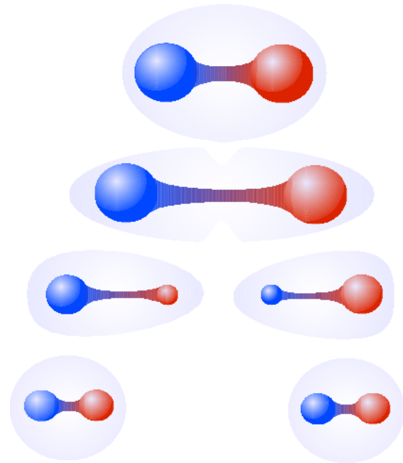
Precision electroweak/pQCD:

Very important for the search of new physics!

A big open problem



Confinement:
Quarks cannot be isolated in Nature, and cannot be directly observed.



Find an analytic proof that quantum chromodynamics (QCD) should be confining.

Cannot be done pertubatively

Millennium Prize Problems ($1,000,000)
21st century analogue of Hilbert's problems

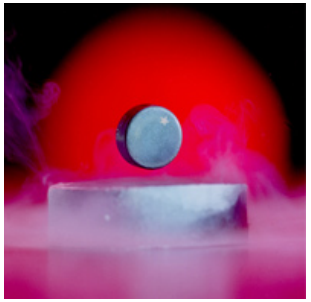
CLAY MATHEMATICS INSTITUTE

Many big open problems

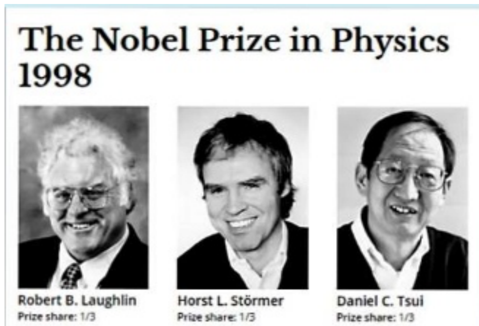
One big question

What is the correct description?

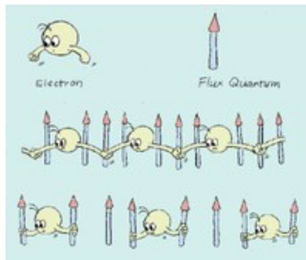
QCD: From quarks and gluons to baryons and mesons?



Superconductivity: Cooper pair: **high-T_C?**



Hall effect: composite fermions



Emmy Noether

Symmetry → *Conservation law*

Use the Symmetry to solve the problem.

The more symmetry the easier it is to solve the problem.

Gauge theories are very hard to understand:
Let's add **Supersymmetry.**

boson ↔ **fermion**

Adding supersymmetry

Help understanding and solving the problem

Supersymmetric Gauge theories in 4D

$\mathcal{N} = 4$ Super Yang-Mills (SYM)

No matter fields are allowed! Only “gluons”!

Conformal: $\lambda \neq \lambda(E)$!

More susy easier to calculate

$\mathcal{N} = 2$

b
f ↔ b ↔ f
b

Supersymmetric QCD (SQCD)

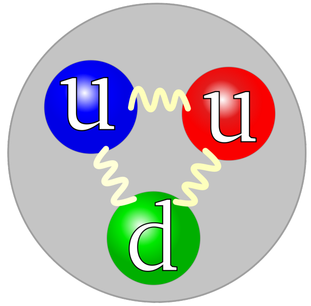
$\mathcal{N} = 1$ b ⟷ f

Less susy more realistic

$\mathcal{N} = 0$ QCD

The real world

Plan of attack



Understand non-perturbative phenomena

✱ *Add more symmetry:* ***supersymmetry****!*

✱ $\mathcal{N}=4$ *SYM is the harmonic oscillator of our century!*

✱ *Remove supersymmetry (more realistic theories).*

Before my time

Symmetry alone

*Can we **uniquely fix the dynamics** of QFT*
*by using **only symmetry** plus **general principles**?*

<u>1960's</u> S-matrix approach did not go far

<u>1970's and 1980's</u> **The Conformal Bootstrap!**

The Conformal Bootstrap

Conformal symmetry = the scale doesn't matter!

$$\lambda \neq \lambda(E)$$

*For conformal theories by using **symmetry** plus **general principles** we can derive an*

Infinite set of consistency relations

Symmetry alone

Can we ***uniquely fix the dynamics*** *of QFT*
by using ***only symmetry*** *plus* ***general principles****?*

1960's S-matrix approach did not succeed

1970's and 1980's **The Conformal Bootstrap!**

Great success! Exactly solve **2D** theories with Conformal symmetry.

No supersymmetry needed!

Did success stop only in 2D?

What about 4D?

Supersymmetry + EM duality

Duality

1D

$$H = \frac{1}{2m}p^2 + \frac{k}{2}x^2$$

maps to itself

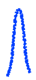
$$x \to \frac{p}{\sqrt{mk}}$$
$$p \to -\sqrt{mk}x$$

Quantum mechanically: Fourier transform!

small fluctuations x ⟷ *large fluctuations p*

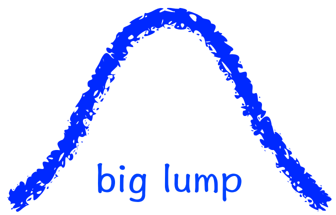
2D

Theory A equivalent to theory B



small QM ripple

elementary field ⟷ *soliton solution*



big lump of energy

Two different descriptions of the same physics!

Intrinsically Quantum Mechanical phenomenon!

Electromagnetic Duality

$$\nabla \cdot E = 0$$

$$\nabla \cdot B = 0$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times B = \frac{\partial E}{\partial t}$$

$$E \rightarrow B$$

$$B \rightarrow -E$$

Electromagnetic Duality

$$\nabla \cdot E = \rho_e$$

$$\nabla \cdot B = 0 + \rho_m$$

$$\nabla \times E = -\frac{\partial B}{\partial t} + J_m$$

$$\nabla \times B = \frac{\partial E}{\partial t} + J_e$$

$$E \to B$$

$$B \to -E$$

$\rho_m \leftrightarrow \rho_e$

Where are the magnetic monopoles?

Electromagnetic Duality

$$\nabla \cdot E = \rho_e$$

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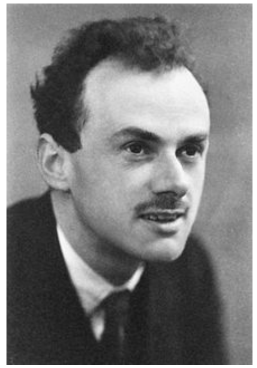
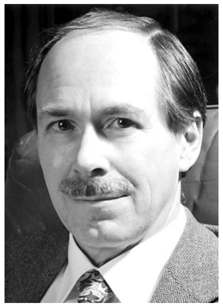
Where are the magnetic monopoles?



[Dirac 1931]

consistent with Q.M. : $$e \cdot m = 2\pi\hbar n$$

['tHooft,Polyakov 1974]



The magnetic monopoles are solitons!

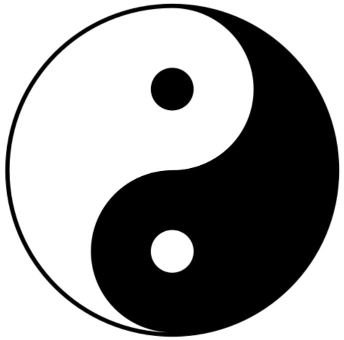
Electromagnetic Duality

[Montonen,Olive1977]

[Seiberg,Witten1994]

Together with supersymmetry it becomes powerful!

Theory A with coupling constant g
is equivalent to
Theory B with coupling constant 1/g



Weak - strong coupling duality!

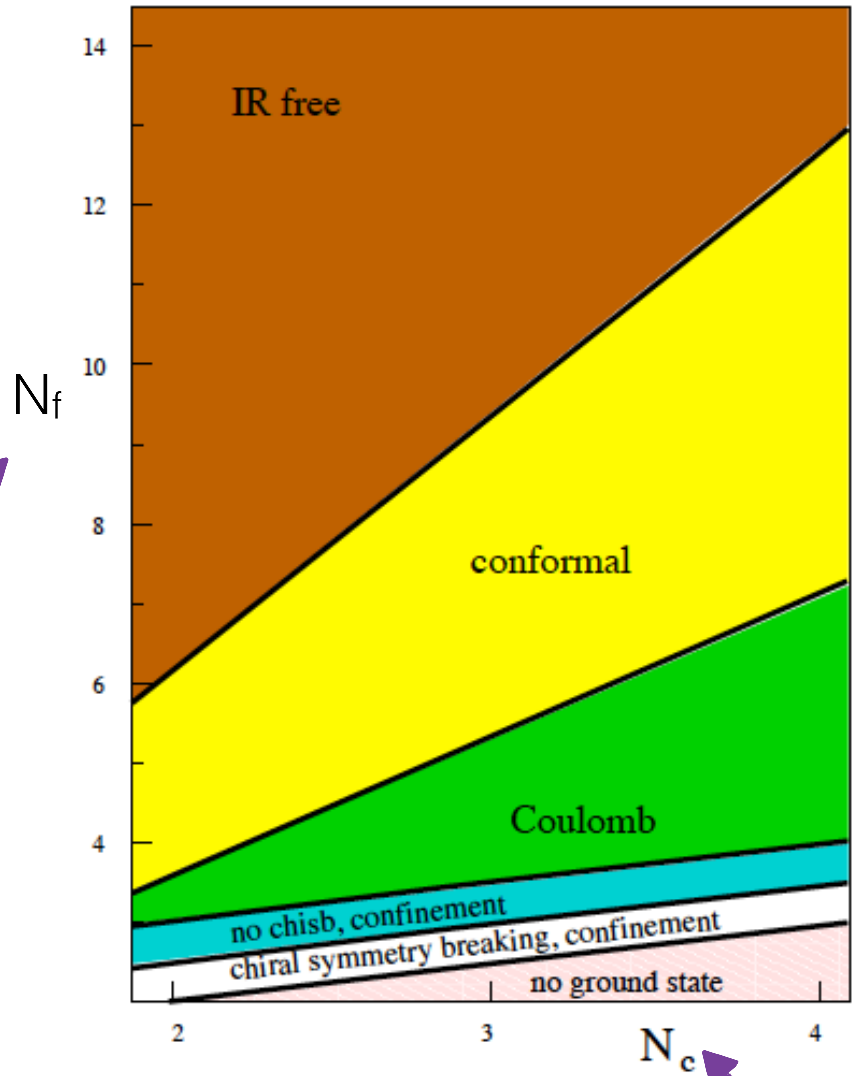
The elementary particles of A are magnetic monopoles of B

We can use it to solve for the **low energy spectrum** of theories with $\mathcal{N}=4$ and $\mathcal{N}=2$ supersymmetry!

Electromagnetic Duality

[Seiberg 1994]

EM duality plus $\mathcal{N}=1$ supersymmetry: phase structure of SQCD



number of flavors

number of colors

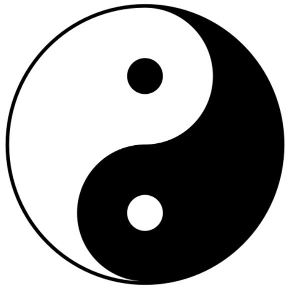
$$V_{q\bar{q}}(r) \sim C$$

$$V_{q\bar{q}}(r) \sim \frac{G_C}{r}$$

$$V_{q\bar{q}}(r) \sim k\,r$$

For QCD a lot of guesswork

More can be done!



Yet another duality!

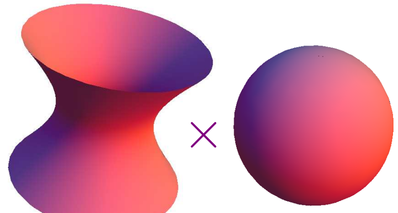
AdS/CFT correspondence

['t Hooft 1993, Susskind 1995]

A relation: Gravity theories in d+1 and gauge theories d-dimensions.

[Maldacena 1998]

The 4D $\mathcal{N} = 4$ SYM = *10D string theory on*



AdS_5 S^5

A duality:

*Use **weakly** coupled gravity: **strongly** coupled gauge theory.*

Holographic models: strong coupling regime: geometry

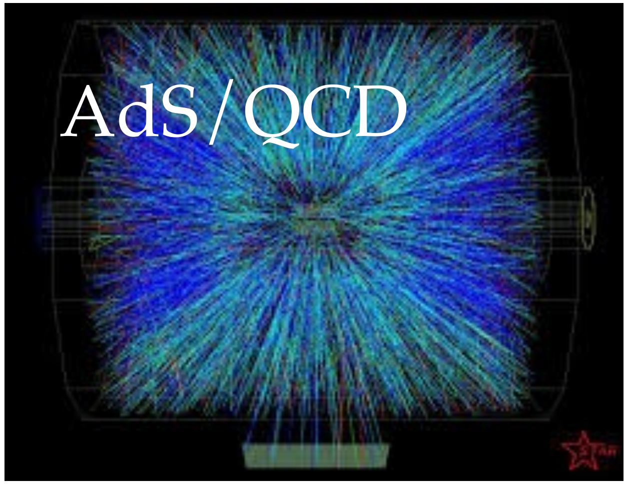
- Confinement
- Chiral symmetry breaking
- Ultrahot QuarkQluonPlasma/ Ultracold atoms
- Applications of Condensed matter systems

Before the AdS/CFT

Transport coefficients in high temperature gauge theories

$$\frac{\eta}{s} \sim \frac{1}{g_{\rm YM}^4 \log g_{\rm YM}^{-1}}$$

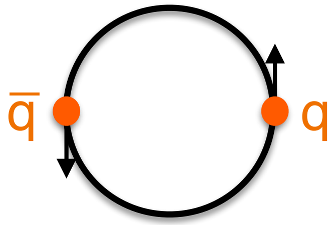
Small viscosity: hydrodynamic modeling of data from heavy ion collisions.
In fact, maybe hydro won't work.



After the AdS/CFT

The shear viscosity of strongly coupled $\mathcal{N} = 4$ supersymmetric Yang-Mills plasma

$$\frac{\eta}{s} = \frac{1}{4\pi}$$



Wilson Loop

example observable

$$\log W(\lambda) \sim V_{q\bar{q}}(r)$$

Measures the strength of the interaction between quark - antiquark

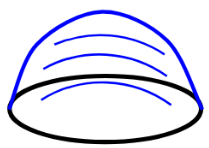
With AdS/CFT can be computed via the **area** of the **string world sheet!**

0 → λ

SYM perturbation theory

String perturbation theory

$1 +$  $+ \ldots$



Modern developments

Exact results

Everybody was doing AdS/CFT! There were two options:

- ✱ Trying to check the AdS/CFT correspondence for $\mathcal{N}=4$ SYM
- ✱ Search for gravity duals for more realistic theories (less supersymmetry)

[EP PhD]

On the way we discovered that it is possible to obtain

Exact results for many observables

$$c_1\,\lambda + c_2\,\lambda^2 + c_3\,\lambda^3 + \cdots = F(\lambda)$$

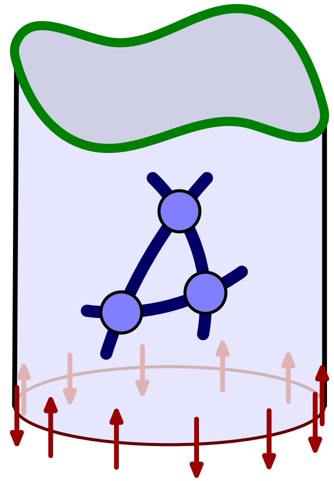
due to:

- ✱ *Integrability*
- ✱ *Localization*
- ✱ *4D/2D relations*

Integrability

= as many conservations laws as d.o.f.

$\mathcal{N}=4$ SYM is integrable in the planar limit for **any coupling**



✱ *Perturbation theory*: mapped to an **integrable spin chain**

✱ *Strong coupling*: integrable 2D theory on the string world-sheet

Powerful integrability toolkit

- **Exactly**: all energies of all states for **any coupling!**

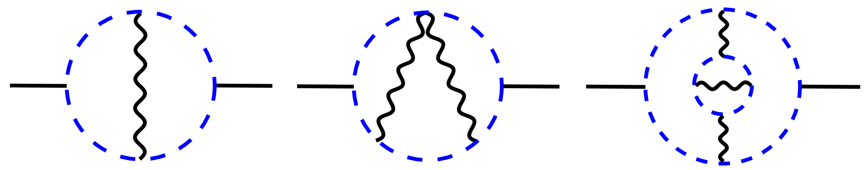
Integrability now is applied to **other observables.**

Removing supersymmetry

A long list of **Observables** in $\mathcal{N}=2$ theories is obtained from their $\mathcal{N}=4$ counterparts by replacing: $\lambda \to f(\lambda)$ [EP 2013]

Relative renormalization: Can calculate it with Feynman diagrams

$$f(\lambda) = \lambda + \lambda\left(Z_{\mathcal{N}=2} - Z_{\mathcal{N}=4}\right)$$



Also, compute it exactly using localization. And check with AdS/CFT. [Mitev,EP]

Lesson: *Gauge symmetry is important: not supersymmetry!*

Similar sector for N=1 theories [Carstensen,EP]

Localization

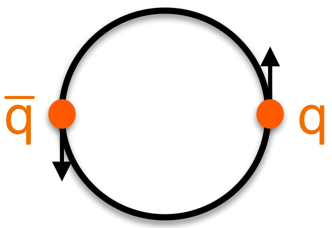
= like saddle point approximation but exact!

$$Z_{S^4} = \int [D\Phi]\, e^{-S[\Phi]} = \int da\, |\mathcal{Z}(a)|^2$$

The **path integral** localizes to an **ordinary integral**
(*Cancelations due to supersymmetry*)

We can do an ordinary integral.
Compute the path integral exactly.
For **any value of the coupling constant.**

Example of exact observable
for planar N=4:



$$W(\lambda) = 2\,\frac{I_1(\sqrt{\lambda})}{\sqrt{\lambda}} = \begin{cases} 1 + \frac{\lambda}{8} + \frac{\lambda^2}{192} + \frac{\lambda^3}{9216} + \cdots\,, & \lambda << 1 \\ \sqrt{\frac{2}{\pi}}\lambda^{-\frac{3}{4}} e^{\sqrt{\lambda}} + \cdots\,, & \lambda >> 1 \end{cases}$$

4D/2D relations

Breakthrough for theories with $\mathcal{N} = 2$ supersymmetry

[Alday,Gaiotto,Tachikawa 2009] [Gadde,EP,Rastelli,Razamat 2009]

*Study how different **observables** transform under **EM duality**.*

*Discover that this is the same as the **bootstrap equation** of a **2D** theory.*

4D observable = 2D observable

Enlarge the list of observables we can calculate exactly!

Removing supersymmetry

Can we have 4D/2D relations for $\mathcal{N}=1$ *theories?*

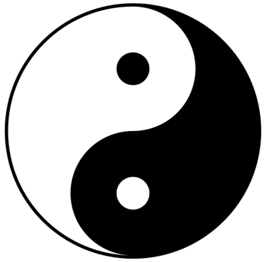
4D observable = 2D observable

Developing a method for finding such relations.

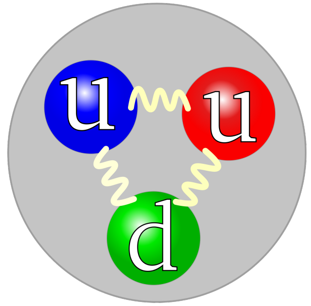
A large class of N=1 theories

- *2D symmetry algebra and representations*
- *Exact results for* $\mathcal{N}=1$ *(instantons)*
- *More observables*

[Coman,EP,Taki,Yagi 2015]
[Mitev,EP 2017]
[Bourton, EP 2017]
[work in progress]

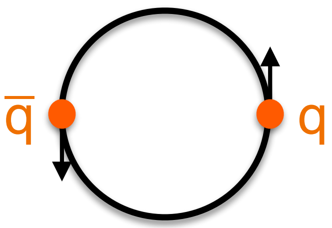


Summary



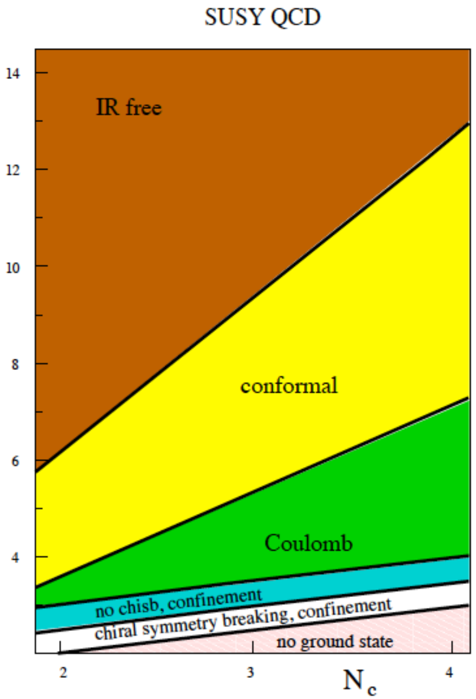
Understand non-perturbative phenomena

- *Add more symmetry:* ***supersymmetry****!*
- $\mathcal{N}=4$ *SYM is the harmonic oscillator of our century!*
- *Remove supersymmetry (more realistic theories).*



$$W(\lambda) = W_{\mathcal{N}=4}(f(\lambda))$$

Vision for the future

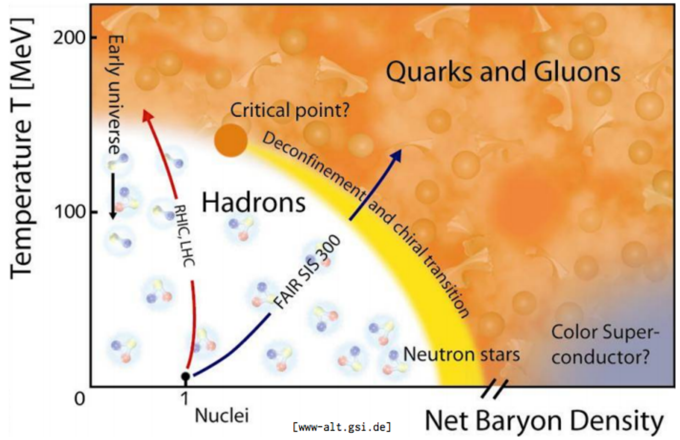


* *Exact results for $\mathcal{N} = 1$ SQCD.*

$$V_{q\bar{q}}(r) \sim C$$

$$V_{q\bar{q}}(r) \sim \frac{G_C}{r}$$

$$V_{q\bar{q}}(r) \sim k\, r$$



* *Exact results at finite temperature?*

* *No supersymmetry? With the bootstrap!*

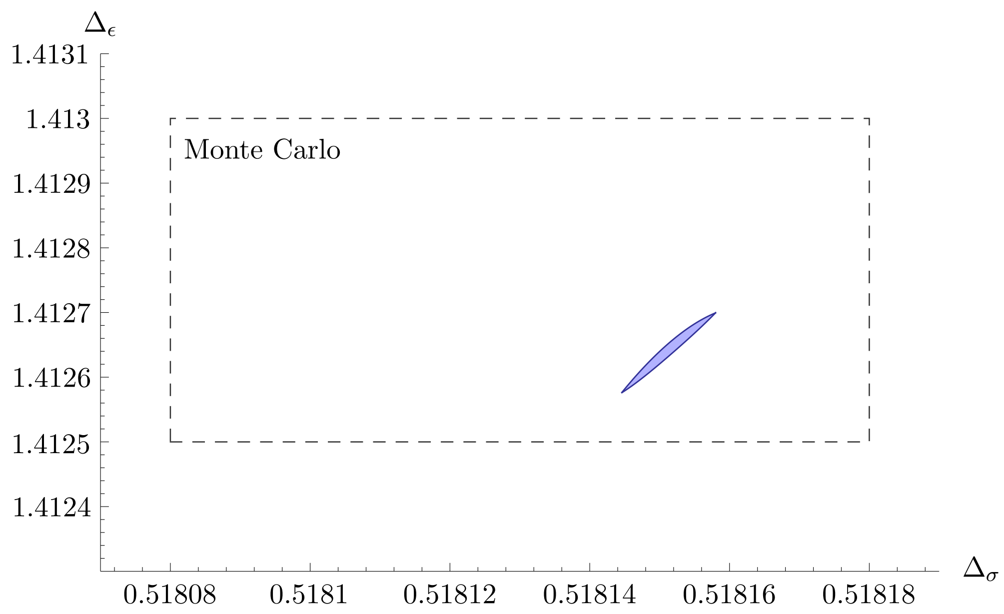
* *Quantum gravity? Gauge theory provides a reformulation of QG!*


N=4 SYM
Vielen Dank für Ihre Aufmerksamkeit!
Real world QCD

Backup slides

Back to the Bootstrap

*Can we **only** by using **symmetry** plus **general principles** **uniquely fix the dynamics** of QFT?*



1980's Exactly solved **2D** theories

Great success!

2014 Solution of the **3D** Ising model!

[El-Showk et al 2014]

In **4D** only **with supersymmetry** for now!

What about QCD?

[work in progress]