Under the spell of gauge theory

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Gauge theories

U(N) or SU(N) gauge group
gauge bosons force carriers

Matter fields (quarks and leptons)

Feynman: how to calculate a process perturbatively:

\[ \frac{\mathcal{L}}{\sqrt{2}} = c_1 \lambda + c_2 \lambda^2 + c_3 \lambda^3 + \cdots \]

\( \lambda << 1 \)
Gauge theories

Feynman diagrams have taken us very far!

Precision tests of QED

\[ g_e^{\text{exp.}} = 2.0023193043617 \pm 3 \]
\[ g_e^{\text{th.}} = 2.00231930436... \]

Precision electroweak/pQCD:
Very important for the search of new physics!
A big open problem

Confinement:
Quarks cannot be isolated in Nature, and cannot be directly observed.

Find an analytic proof that quantum chromodynamics (QCD) should be confining.

*Cannot be done pertubatively*

Millennium Prize Problems ($1,000,000)
21st century analogue of Hilbert's problems
Many big open problems

One big question

What is the correct description?

**QCD:** From quarks and gluons to baryons and mesons?

**Superconductivity:** Cooper pair: high-$T_c$?

**Hall effect:** composite fermions
Emmy Noether

Symmetry $\rightarrow$ Conservation law

Use the Symmetry to solve the problem.

The more symmetry the easier it is to solve the problem.

Gauge theories are very hard to understand:
Let’s add Supersymmetry.

boson $\leftrightarrow$ fermion
Adding supersymmetry

Help understanding and solving the problem
Supersymmetric Gauge theories in 4D

\[ \mathcal{N} = 4 \quad \text{Super Yang-Mills (SYM)} \]

No matter fields are allowed! Only “gluons”!
Conformal: \( \lambda \neq \lambda(E) \)!

\[ \mathcal{N} = 2 \quad \begin{array}{ccc}
\text{f} & \leftrightarrow & \text{f} \\
\text{b} & \leftrightarrow & \text{b}
\end{array} \]

Supersymmetric QCD (SQCD)

\[ \mathcal{N} = 1 \quad \text{b} \leftrightarrow \text{f} \]

\[ \mathcal{N} = 0 \quad \text{QCD} \]

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More susy easier to calculate

Less susy more realistic

The real world
Plan of attack

Understand non-perturbative phenomena

★ Add more symmetry: *supersymmetry*!

★ $\mathcal{N} = 4$ SYM *is the harmonic oscillator of our century!*

★ Remove supersymmetry (more realistic theories).
Before my time
Symmetry alone

Can we *uniquely fix the dynamics* of QFT by using *only symmetry* plus *general principles*?

1960’s  S-matrix approach  did not go far

1970's and 1980’s  The Conformal Bootstrap!
The Conformal Bootstrap

Conformal symmetry = the scale doesn’t matter!

\[ \lambda \neq \lambda(E) \]

*For conformal theories by using symmetry plus general principles we can derive an*

**Infinite set of consistency relations**
Symmetry alone

Can we *uniquely fix the dynamics* of QFT by using *only symmetry* plus *general principles*?

1960’s S-matrix approach did not succeed

1970’s and 1980’s The Conformal Bootstrap!

Great success! Exactly solve 2D theories with Conformal symmetry. No supersymmetry needed!
Did success stop only in 2D?

What about 4D?

Supersymmetry + EM duality
Duality

\[ H = \frac{1}{2m}p^2 + \frac{k}{2}x^2 \]

maps to itself

Quantum mechanically: Fourier transform!

small fluctuations \( x \) \( \leftrightarrow \) large fluctuations \( p \)

2D

Theory A equivalent to theory B

elementary field \( \leftrightarrow \) soliton solution

Two different descriptions of the same physics!

Intrinsically Quantum Mechanical phenomenon!
Electromagnetic Duality

\[ \nabla \cdot \bar{E} = 0 \]
\[ \nabla \cdot \bar{B} = 0 \]
\[ \nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \]
\[ \nabla \times \bar{B} = \frac{\partial \bar{E}}{\partial t} \]

\[ E \rightarrow B \]
\[ B \rightarrow -E \]
Electromagnetic Duality

\[ \nabla \cdot E = \rho_e \]
\[ \nabla \cdot B = 0 + \rho_m \]
\[ \nabla \times E = -\frac{\partial B}{\partial t} + J_m \]
\[ \nabla \times B = \frac{\partial E}{\partial t} + J_e \]

\[ E \rightarrow B \]
\[ B \rightarrow -E \]

\[ \rho_m \leftrightarrow \rho_e \]

Where are the magnetic monopoles?
Electromagnetic Duality

\[ \nabla \cdot E = \rho_e \]
\[ \nabla \cdot B = 0 + \rho_m \]
\[ \nabla \times E = -\frac{\partial B}{\partial t} + J_m \]
\[ \nabla \times B = \frac{\partial E}{\partial t} + J_e \]

Where are the magnetic monopoles?

consistent with Q.M.: \[ e \cdot m = 2\pi \hbar n \]

\[ E \rightarrow B \]
\[ B \rightarrow -E \]
\[ \rho_m \leftrightarrow \rho_e \]

[\text{'t Hooft, Polyakov 1974}]

[Dirac 1931]

The magnetic monopoles are solitons!
Electromagnetic Duality

Together with supersymmetry it becomes powerful!

Theory A with coupling constant \( g \)

is equivalent to

Theory B with coupling constant \( 1/g \)

Weak - strong coupling duality!

The elementary particles of A are

magnetic monopoles of B

We can use it to solve for the low energy spectrum

of theories with \( \mathcal{N} = 4 \) and \( \mathcal{N} = 2 \) supersymmetry!
Electromagnetic Duality

EM duality plus $\mathcal{N} = 1$ supersymmetry: phase structure of SQCD

For QCD a lot of guesswork

$r \cdot E = \varepsilon (1)$

$r \cdot B = 0 (2)$

$r \cdot E = \frac{\partial B}{\partial t} (3)$

$r \cdot B = \frac{\partial E}{\partial t} + J (4)$

$E \rightarrow B, B \rightarrow E (5)$

$e \cdot m = 2 \pi \sim n$

$W_N = 4 (f)$

$V_{q\bar{q}}(r) \sim C$

$V_{q\bar{q}}(r) \sim \frac{GC}{r}$

$V_{q\bar{q}}(r) \sim kr$
More can be done!

Yet another duality!
AdS/CFT correspondence

[’t Hooft 1993, Susskind 1995]

A relation: Gravity theories in d+1 and gauge theories d-dimensions.

[Maldacena 1998]
The 4D $\mathcal{N} = 4$ SYM = 10D string theory on $\text{AdS}_5 \times \text{S}^5$

A duality:

Use weakly coupled gravity: strongly coupled gauge theory.

Holographic models: strong coupling regime: geometry

- Confinement
- Chiral symmetry breaking
- Ultrahot QuarkQuonPlasma/ Ultracold atoms
- Applications of Condensed matter systems
Before the AdS/CFT

Transport coefficients in high temperature gauge theories

\[ \frac{\eta}{s} \sim \frac{1}{g_{YM}^4 \log g_{YM}^{-1}} \]

Small viscosity: hydrodynamic modeling of data from heavy ion collisions. In fact, maybe hydro won’t work.

After the AdS/CFT

The shear viscosity of strongly coupled \( \mathcal{N} = 4 \) supersymmetric Yang-Mills plasma

\[ \frac{\eta}{s} = \frac{1}{4\pi} \]
Wilson Loop

\[
\log W(\lambda) \sim V_{q\bar{q}}(r)
\]

Measures the strength of the interaction between quark - antiquark

With AdS/CFT can be computed via the area of the string world sheet!

[Diagram showing Wilson Loop and related mathematical expressions]
Modern developments
Exact results

Everybody was doing AdS/CFT! There were two options:

* Trying to check the AdS/CFT correspondence for $\mathcal{N} = 4$ SYM
* Search for gravity duals for more realistic theories (less supersymmetry)

On the way we discovered that it is possible to obtain

**Exact results** for many observables

$$c_1 \lambda + c_2 \lambda^2 + c_3 \lambda^3 + \cdots = F(\lambda)$$

due to:

* Integrability
* Localization
* 4D/2D relations
Integrability

$\mathcal{N} = 4$ SYM is integrable in the planar limit for any coupling!

- Perturbation theory: mapped to an integrable spin chain
- Strong coupling: integrable 2D theory on the string world-sheet

Powerful integrability toolkit

- Exactly: all energies of all states for any coupling!

Integrability now is applied to other observables.
Removing supersymmetry

A long list of Observables in $\mathcal{N} = 2$ theories is obtained from their $\mathcal{N} = 4$ counterparts by replacing: $\lambda \rightarrow f(\lambda)$ [EP 2013]

Relative renormalization: Can calculate it with Feynman diagrams

$f(\lambda) = \lambda + \lambda(Z_{\mathcal{N}=2} - Z_{\mathcal{N}=4})$

Also, compute it exactly using localization. And check with AdS/CFT. [Mitev,EP]

Lesson: Gauge symmetry is important: not supersymmetry!

Similar sector for $\mathcal{N}=1$ theories [Carstensen,EP]
Localization

The path integral localizes to an ordinary integral (Cancelations due to supersymmetry)

We can do an ordinary integral. Compute the path integral exactly. For any value of the coupling constant.

Example of exact observable for planar $N=4$:

\[ W(\lambda) = 2 \frac{I_1(\sqrt{\lambda})}{\sqrt{\lambda}} = \begin{cases} 1 + \frac{\lambda}{8} + \frac{\lambda^2}{192} + \frac{\lambda^3}{9216} + \cdots, & \lambda << 1 \\ \sqrt{\frac{2}{\pi}} \lambda^{-\frac{3}{4}} e^{\sqrt{\lambda}} + \cdots, & \lambda >> 1 \end{cases} \]
4D/2D relations

Breakthrough for theories with $\mathcal{N} = 2$ supersymmetry

[Alday,Gaiotto,Tachikawa 2009] [Gadde,EP,Rastelli,Razamat 2009]

Study how different observables transform under EM duality.

Discover that this is the same as the bootstrap equation of a 2D theory.

\[
\begin{array}{c}
\text{4D observable} = \text{2D observable}
\end{array}
\]

Enlarge the list of observables we can calculate exactly!
Removing supersymmetry

Can we have 4D/2D relations for $\mathcal{N} = 1$ theories?

4D observable = 2D observable

Developing a method for finding such relations.

A large class of $\mathcal{N}=1$ theories

★ 2D symmetry algebra and representations

★ Exact results for $\mathcal{N} = 1$ (instantons)

★ More observables

[Coman,EP,Taki,Yagi 2015]
[Mitev,EP 2017]
[Bourton, EP 2017]
[work in progress]
Summary

Understand non-perturbative phenomena

* Add more symmetry: supersymmetry!

* $\mathcal{N} = 4$ SYM is the harmonic oscillator of our century!

* Remove supersymmetry (more realistic theories).

$W(\lambda) = W_{\mathcal{N}=4}(f(\lambda))$
**Vision for the future**

* Exact results for $\mathcal{N} = 1$ SQCD.

$$V_{q\bar{q}}(r) \sim C$$

$$V_{q\bar{q}}(r) \sim \frac{G_C}{r}$$

$$V_{q\bar{q}}(r) \sim k r$$

* Exact results at finite temperature?

* No supersymmetry? With the bootstrap!

* Quantum gravity? Gauge theory provides a reformulation of QG!
$N=4$ SYM

Vielen Dank für Ihre Aufmerksamkeit!

Real world QCD
Backup slides
Back to the Bootstrap

Can we only by using symmetry plus general principles uniquely fix the dynamics of QFT?

1980’s  Exactly solved 2D theories  Great success!

2014  Solution of the 3D Ising model!  [El-Showk et al 2014]

In 4D only with supersymmetry for now!

What about QCD?  [work in progress]