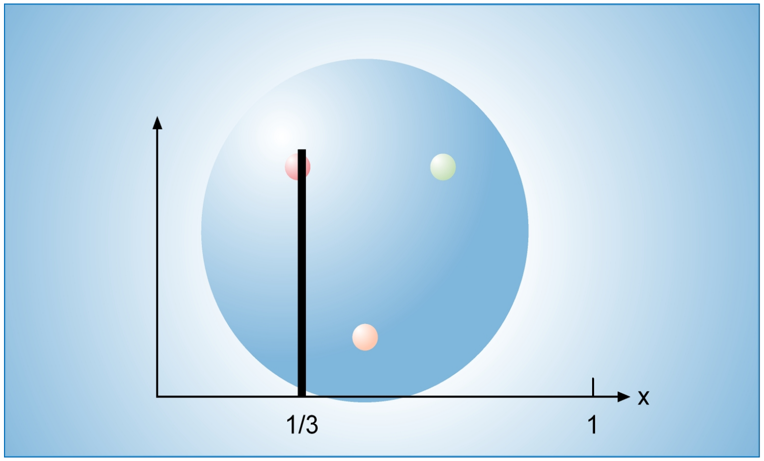
Lattice Field Theory for Pedestrians: an introductory lecture

Karl Jansen

- **Motivation**
- **Introduction to Lattice Field Theory**
- **Examples of present lattice calculations**
  - Hadron spectrum
  - Dark matter search: scalar quark content of the nucleon
  - Vacuum stability of the standard model
- **Conclusion**

Quarks are the fundamental constituents of nuclear matter



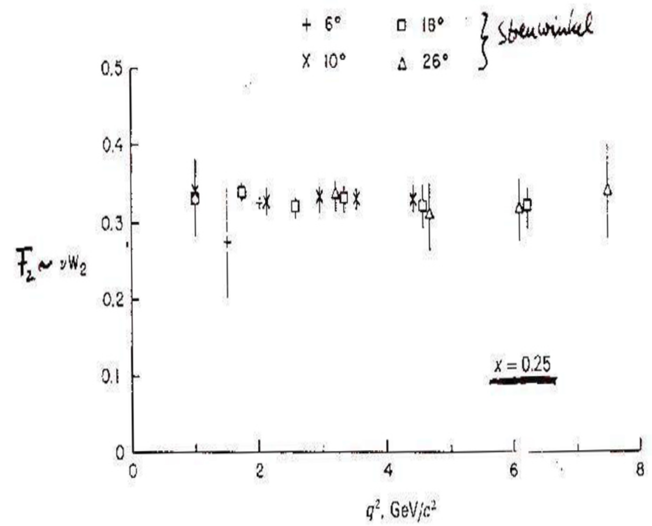


Fig. 7.17 νW_2 (or F_2) as a function of q^2 at $x = 0.25$. For this choice of x, there is practically no q^2-dependence, that is, exact "scaling". (After Friedman and Kendall 1972.)

Friedman and Kendall, 1972)

$f(x, Q^2)\big|_{x\approx 0.25, Q^2 > 10\mathrm{GeV}}$ independent of Q^2

(x momentum of quarks, Q^2 momentum transfer)

Interpretation (Feynman): scattering on single quarks in a hadron
$\rightarrow$ (Bjorken) scaling

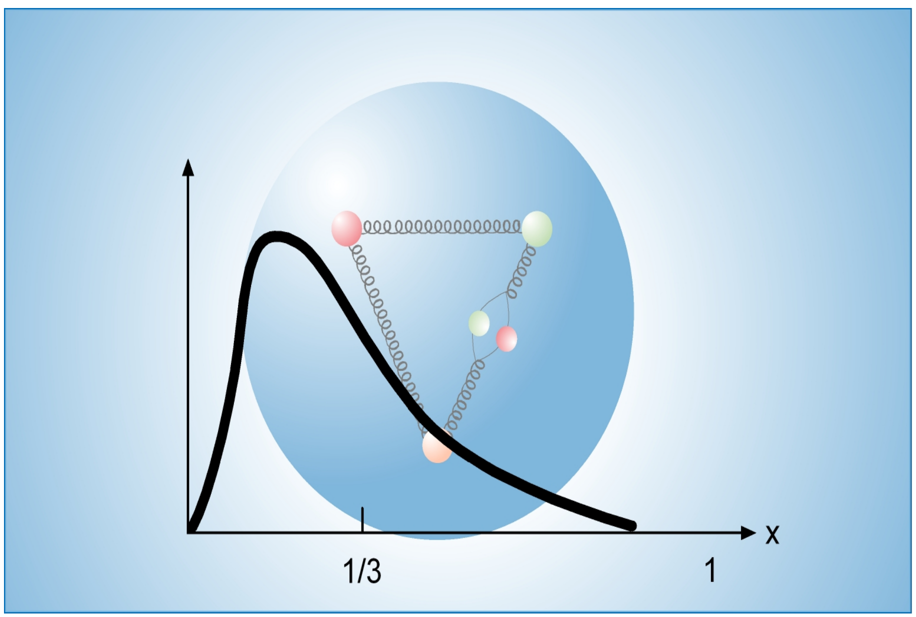
Quantum Fluctuations and the Quark Picture

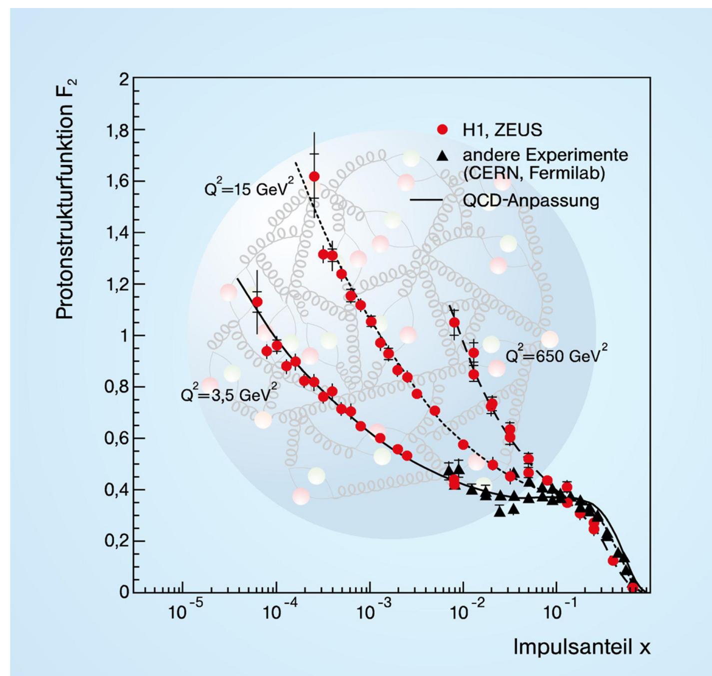
analysis in perturbation theory

$$\int_0^1 dx f(x,Q^2) = 3\left[1 - \frac{\alpha_s(Q^2)}{\pi} - a(n_f)\left(\frac{\alpha_s(Q^2)}{\pi}\right)^2 - b(n_f)\left(\frac{\alpha_s(Q^2)}{\pi}\right)^3\right]$$

– $a(n_f), b(n_f)$ calculable coefficients

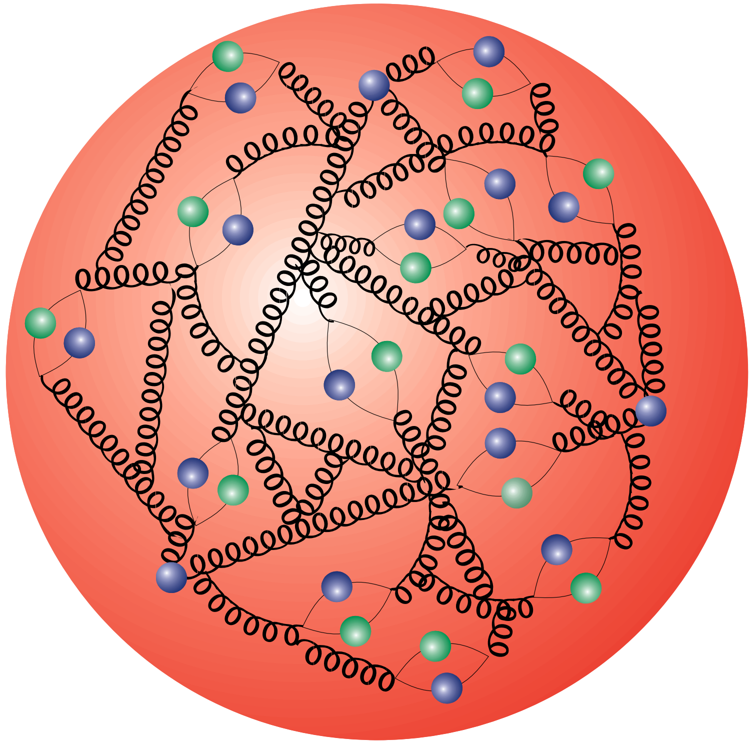
deviations from scaling $\rightarrow$ determination of strong coupling





Why we need lattice QCD

- situation becomes incredibly complicated



- value of the coupling (expansion parameter) $\alpha_{\mathrm{strong}}(1\mathrm{fm}) \approx 1$

$\Rightarrow$ need different (“exact”) method

$\Rightarrow$ has to be non-perturbative
 $\rightarrow$ more than all Feynman graphs

- Wilson’s Proposal: Lattice Quantum Chromodynamics

Lattice Gauge Theory had to be invented

$\rightarrow$ QuantumChromoDynamics

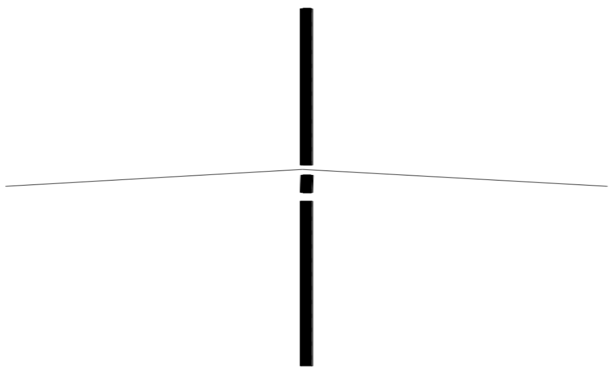
asymptotic freedom	confinement
distances $\ll$ 1fm	distances $\gtrsim$ 1fm
world of quarks and gluons	world of hadrons and glue balls
perturbative description	non-perturbative methods

Unfortunately, it is not known yet whether the quarks in quantum chromodynamics actually form the required bound states. To establish whether these bound states exist one must solve a strong coupling problem and present methods for solving field theories don't work for strong coupling.

Wilson, Cargese Lecture notes 1976

Feynman's alternative formulation of quantum mechanics

the double slit experiment



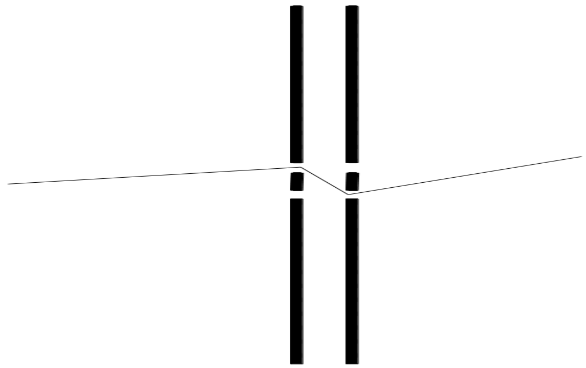
superposition principle

$\rightarrow$ interference pattern

$\rightarrow$ probability $P = |\Phi_1 + \Phi_2|^2$

Φ_i quantum mechanical amplitude

Adding slits

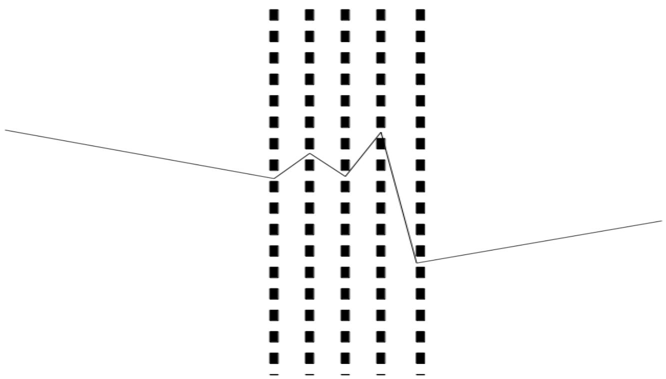


four possible paths

$\rightarrow$ probability $P = |\Phi_1 + \Phi_2 + \Phi_3 + \Phi_4|^2$

Φ_i quantum mechanical amplitude

Even more ...



→ probability $P = |\sum_i \Phi_i|^2 \equiv |\sum_{\text{paths}} \Phi_{\text{path}}|^2$

Feynman $\Phi_{\text{path}} = e^{\frac{i}{\hbar} S_{\text{cl}}(\text{path})}$

$S_{\text{cl}}(\text{path})$ *classical action of path*

Quantum mechanical oscillator in Euclidean time

Feynman path integral in quantum mechanics

$$\mathcal{Z} = \int \mathcal{D}x e^{\frac{i}{\hbar} S_{\mathrm{cl}}}$$

- S_{cl} *classical* action, e.g. quantum mechanical oscillator

$$S_{\mathrm{cl}} = \int dt \left[\tfrac{1}{2}\dot{x}^2(t) - V(x(t))\right]$$

- $x(t)$ are *classical* paths

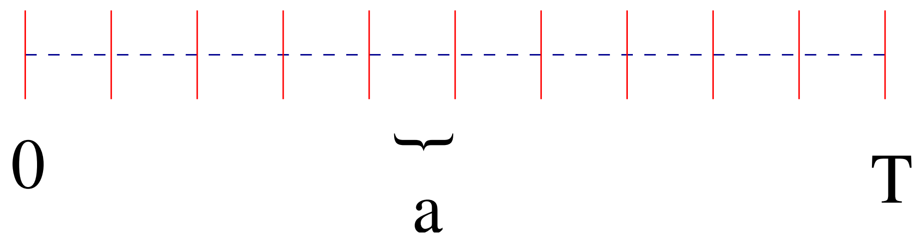
perform analytical continuation to imaginary (Euclidean) time τ

$$t \to -i\tau \qquad f(t) \to f(\tau)$$

$$\mathcal{Z} = \int \mathcal{D}x e^{-\frac{1}{\hbar} S_{\mathrm{E}}}, \; S_E = \int d\tau \left[\tfrac{1}{2}\dot{x}^2 + V(x)\right]$$

Discretizing

"time lattice" with $N = T/a$ lattice points



$x(\tau) \to x(n)$ boundary condition: $x(N+1) = x(0)$

case 1: $\dot{x} \to [x(n+a) - x(n)]/a$

$$= \frac{1}{a}\left[x(n) + \dot{x}a + \frac{1}{2}\dot{x}^2a^2 + \cdots - x(n)\right] = \dot{x} + O(a)$$

$\Rightarrow$ linear discretization effects

case 2: $\dot{x} \to [x(n+a) - x(n-a)]/2a$

$$= \frac{1}{2a}\left[x(n) + \dot{x}a + \frac{1}{2}\dot{x}^2a^2 + \cdots - x(n) + \dot{x}a - \frac{1}{2}\dot{x}^2a^2\right] = \dot{x} + O(a^2)$$

$\Rightarrow$ quadratic discretization effects

Lattice version of quantum mechanical oscillator

- discretization provides well defined path integral

$$\mathcal{Z} = \underbrace{\int \prod_{n=1}^{N} dx_n}_{\int \mathcal{D}x} \int e^{-a \sum_{n=1}^{N} \frac{(x(n+a)-x(n))^2}{a^2} + V(x(n))}$$

- measuring observables
  - average position $\langle x \rangle = \int \prod_{n=1}^{N} dx_n x e^{-S} / \mathcal{Z}$
  - average position square $\langle x^2 \rangle = \int \prod_{n=1}^{N} dx_n x^2 e^{-S} / \mathcal{Z}$

Monte Carlo Method

$$\langle f(x)\rangle = \int dx f(x) e^{-x^2} / \int dx e^{-x^2}$$

$\rightarrow$ solve numerically:

- generate succesively Gaussian random numbers x_i
- do this N-times

$$\Rightarrow \langle f(x)\rangle \approx \frac{1}{NoP} \sum_i f(x_i) \pm O(1/\sqrt{NoP})$$

- but, what if distribution $e^{-S(x)}$ is much more complicated?

find a transition probality $W(x,x')$ that brings us

$\{x\} = \{x_1, x_2, \cdots, x_N\} \rightarrow \{x'\} = \{x'_1, x'_2, \cdots, x'_N\}$
and which satisfies

- $W(x,x') > 0$ strong ergodicity ($W \geq 0$ is weak ergodicity)
- $\int dx' W(x,x') = 1$
- $W(x,x') = \int dx'' W(x,x'') W(x'',x')$ (Markov chain)
- $W(x,x')$ is measure preserving, $dx' = dx$

under these conditions, we are guaranteed

– to converge to Boltzmann distribution e^{-S}

– independent from the initial conditions

$\rightarrow$ proof: (Creutz and Freedman; Lüscher, Cargese lectures)

Detailed balance condition

- sufficient condition: detailed balance

$$\frac{W(x,x')}{W(x',x)} = \frac{P(x')}{P(x)}$$

$\rightarrow$ (most of) our conditions are fulfilled

- Metropolis algorithm choice

$$W_{\mathrm{Met}}(x,x') = \Theta(S(x) - S(x')) + \exp\left(-\Delta S(x',x)\right)\Theta(S(x') - S(x))$$

$\Delta S(x',x) = S(x') - S(x)$, $\Theta()$ Heavyside function

Metropolis Algorithms

$i)$ generate uniformly distributed new x' in a neighbourhood of x
$x'_i \in [x_i - \Omega, x_i + \Omega]$

$ii)$ if $S_{\rm new} - S_{\rm old} \equiv \Delta S(x', x) \leq 0)$ accept x'

$iii)$ if $\Delta S(x', x) > 0)$ accept with probability $\exp(-\Delta S(x', x))$

- steps $i) - iii)$ are repeated MCsteps-times

Metropolis Algorithms

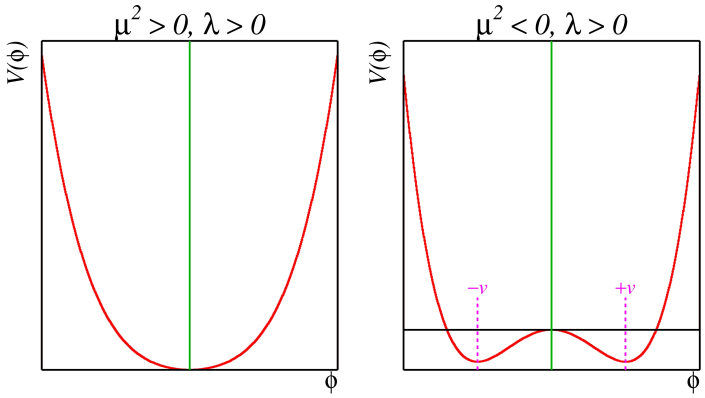
- very general algorithm, can be used for many physical systems
- shows, however, often very long autocorrelation times
- much too costly for fermionic systems (why?)

Action to be programmed

$$S = a \sum_{i=1}^{N} \frac{1}{2} M_0 \frac{(x_{i+1} - x_i)^2}{a^2} + \frac{1}{2}\mu^2 x_i^2 + \lambda x_i^4$$

periodic boundary condition: $x_{N+1} = x_1 \ , \ x_0 = x_N$

- potential $V(x)$
  - $\mu^2 > 0, \lambda > 0$: harmonic potential $\langle x \rangle = 0$
  - $\mu^2 < 0, \lambda > 0$: anharmonic potential $\langle x \rangle = \pm v \neq 0$



Observables

- average position

$$\langle x \rangle = \frac{1}{\text{MCsteps}} \sum_{\text{MCsteps}} \left[\frac{1}{N} \sum_{i=1}^{N} x_i \right] ,$$

- average position squared
  $\rightarrow$ theoretical value known for $a > 0$ and $\mu^2 > 0$

$$\langle x^2 \rangle = \frac{1}{\text{MCsteps}} \sum_{\text{MCsteps}} \left[\frac{1}{N} \sum_{i=1}^{N} x_i^2 \right]$$

- acceptance rate, should be $\approx 50\%$
- error for observable O

$$\Delta O = \sqrt{\frac{1}{(\text{MCsteps})(\text{MCsteps}-1)} \sum_{\text{MCsteps}} [\langle O^2 \rangle - \langle O \rangle^2]}$$

Schwinger model: 2-dimensional Quantum Electrodynamics

(Schwinger 1962)

Quantization via Feynman path integral (in Euclidean time)

$$\mathcal{Z} = \int \mathcal{D}A_\mu \mathcal{D}\Psi \mathcal{D}\bar{\Psi} e^{-S_{\rm gauge} - S_{\rm ferm}}$$

Fermion action

$$S_{\rm ferm} = \int d^2x \bar{\Psi}(x) \left[D_\mu + m\right] \Psi(x)$$

gauge covriant derivative

$$D_\mu \Psi(x) \equiv (\partial_\mu - i g_0 A_\mu(x)) \Psi(x)$$

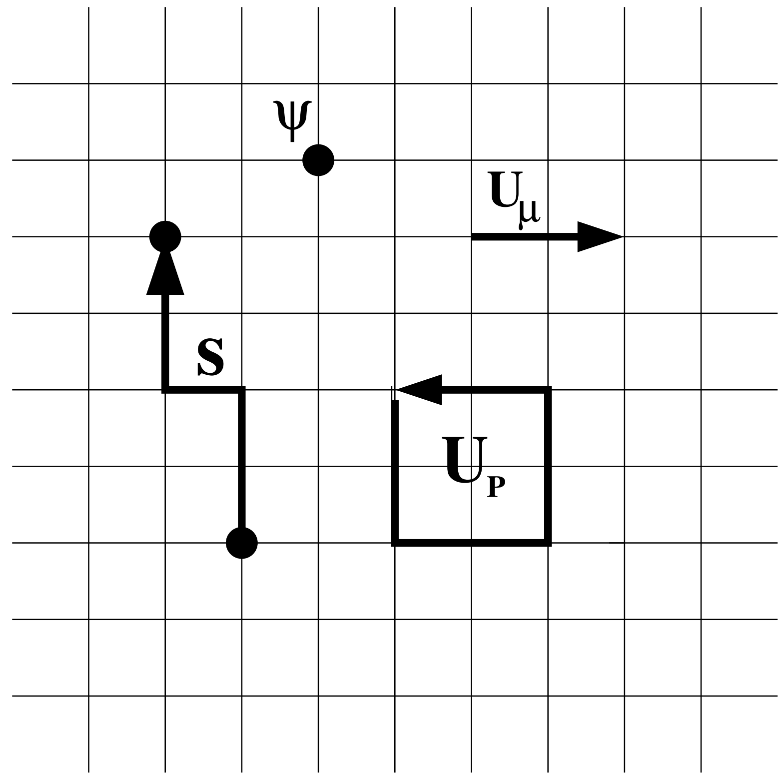
with A_μ gauge potential, g_0 bare coupling

$$S_{\rm gauge} = \int d^2x F_{\mu\nu} F_{\mu\nu} \; , \; F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$$

equations of motion: obtain classical Maxwell equations

Lattice Schwinger model

introduce a 2-dimensional lattice with lattice spacing a



fields $\Psi(x)$, $\bar{\Psi}(x)$ on the lattice sites

$x = (t, \mathbf{x})$ integers

discretized fermion action

$$S \to a^2 \sum_x \bar{\Psi} \left[\gamma_\mu \partial_\mu - r \underbrace{\partial_\mu^2}_{\nabla_\mu^* \nabla_\mu} + m\right] \Psi(x)$$

$$\partial_\mu = \frac{1}{2} \left[\nabla_\mu^* + \nabla_\mu\right]$$

discrete derivatives

$$\nabla_\mu \Psi(x) = \frac{1}{a} \left[\Psi(x + a\hat{\mu}) - \Psi(x)\right] \ , \quad \nabla_\mu^* \Psi(x) = \frac{1}{a} \left[\Psi(x) - \Psi(x - a\hat{\mu})\right]$$

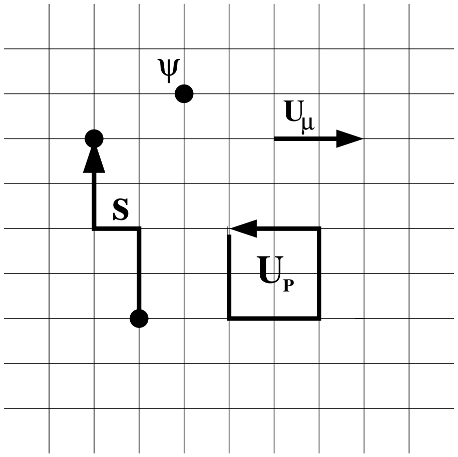
second order derivative $\rightarrow$ remove doubler $\leftarrow$ break chiral symmetry

Implementing gauge invariance

Wilson's fundamental observation: introduce parallel transporter connecting the points x and $y = x + a\hat{\mu}$:

$$U(x, \mu) = e^{iaA_\mu(x)} \in U(1)$$

$\Rightarrow$ lattice derivative: $\nabla_\mu \Psi(x) = \frac{1}{a}\left[U(x,\mu)\Psi(x+\mu) - \Psi(x)\right]$



$$U_p = U(x,\mu)U(x+\mu,\nu)U^\dagger(x+\nu,\mu)U^\dagger(x,\nu)$$

$$\rightarrow F_{\mu\nu}F^{\mu\nu}(x) \quad \text{for} \quad a \rightarrow 0$$

$$S = a^2 \sum_x \left\{ \beta(= \tfrac{1}{g_0^2}) \left[1 - \mathrm{Re}(U_{(x,p)})\right] + \bar{\psi}\left[m + \tfrac{1}{2}\{\gamma_\mu(\nabla_\mu + \nabla^\star_\mu) - a\nabla^\star_\mu\nabla_\mu\}\right]\psi \right\}$$

partition functions (path integral) with Boltzmann weight (action) S

$$\mathcal{Z} = \int_{\text{fields}} e^{-S}$$

Physical Observables

expectation value of physical observables $\mathcal{O}$

$$\underbrace{\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int_{\text{fields}} \mathcal{O} e^{-S}}$$

$\downarrow$ lattice discretization

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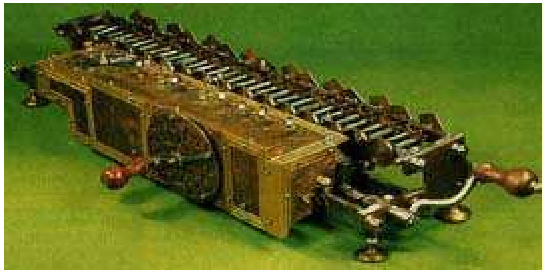
$\downarrow$

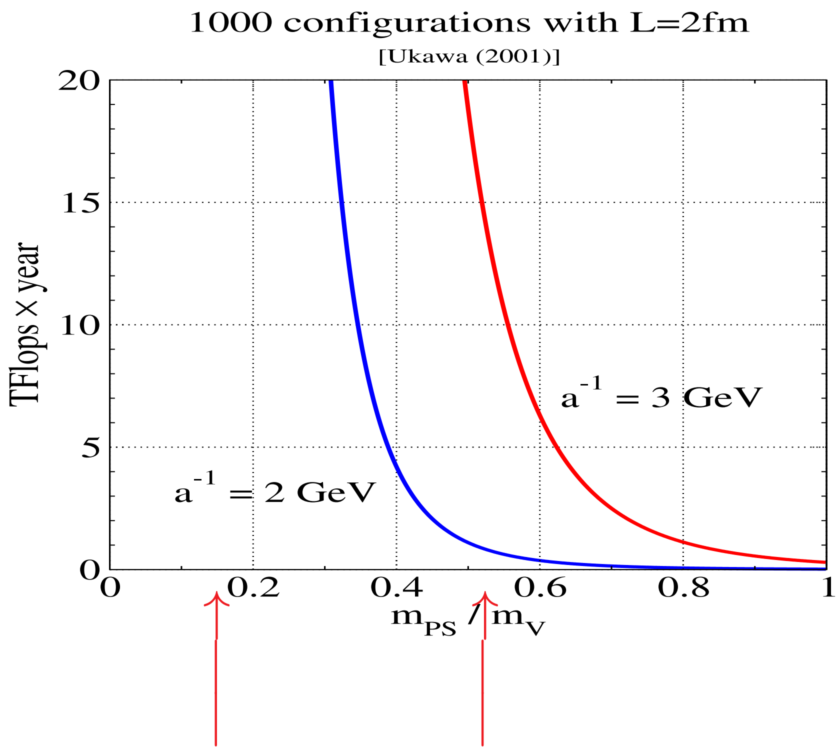


From the Schwinger model to quantum chromodynamics

- system becomes 4-dimensional:
  $[50 \cdot 50] \rightarrow [50 \cdot 50] \cdot 2500$
- gauge field $U(x,\mu) \in U(1) \rightarrow U(x,\mu) \in SU(3)$
- quarks receive 4 Dirac and 3 color components:
  $[50 \cdot 50] \rightarrow [50 \cdot 50] \cdot 30000$
- theory needs *non-perturbative* renormalization

The graph that wrote history: the "Berlin Wall"

see panel discussion in Lattice2001, Berlin, 2001



physical point

contact to χPT (?)

formula $C \propto \left(\frac{m_\pi}{m_\rho}\right)^{-z_\pi} (L)^{z_L} (a)^{-z_a}$

$$z_\pi = 6, \;\; z_L = 5, \;\; z_a = 7$$

"both a 10^8 increase in computing power AND spectacular algorithmic advances before a useful interaction with experiments starts taking place." (Wilson, 1989)

$\Rightarrow$ need of **Exaflops Computers**

Why are fermions so expensive?

- need to evaluate

$$\mathcal{Z} = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi e^{-\bar{\psi}\{D^{\text{Dirac}}_{\text{lattice}}\}\psi} \propto \det[D^{\text{Dirac}}_{\text{lattice}}]$$

- bosonic representation of determinant

$$\det[D^{\text{Dirac}}_{\text{lattice}}] \propto \int \mathcal{D}\Phi^\dagger \mathcal{D}\Phi e^{-\Phi^\dagger\{D^{-1}_{\text{lattice}}\}\Phi}$$

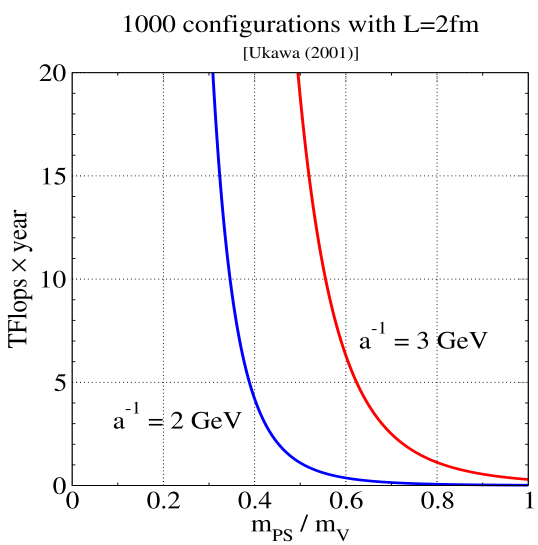
- need vector $X = D^{-1}_{\text{lattice}}\Phi$
- solve linear equation $D_{\text{lattice}}X = \Phi$

  D_{lattice} matrix of dimension $100\text{million} \otimes 100\text{million} \approx 12 \cdot 48^3 \cdot 96$
  (however, matrix is sparse)
- number of such "inversions": $O(1000 - 10000)$ for one field configuration
- want: $O(1000 - 10000)$ such field configurations

A generic improvement for Wilson type fermions

New variants of HMC algorithm
(here (Urbach, Shindler, Wenger, K.J.), see also RHMC, SAP)

- even/odd preconditioning
- (twisted) mass-shift (Hasenbusch trick)
- multiple time steps



2001

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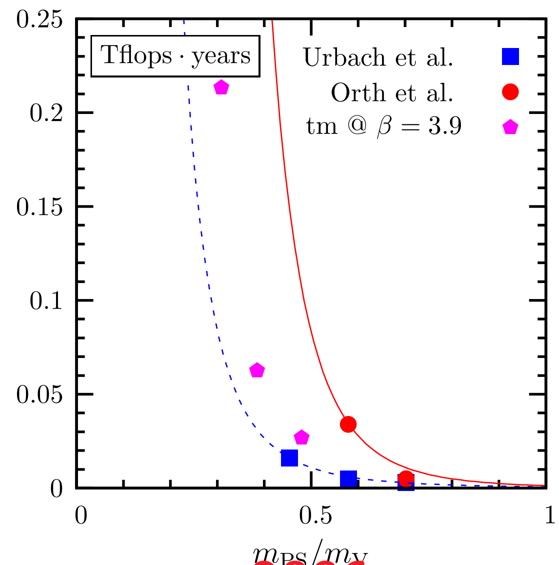
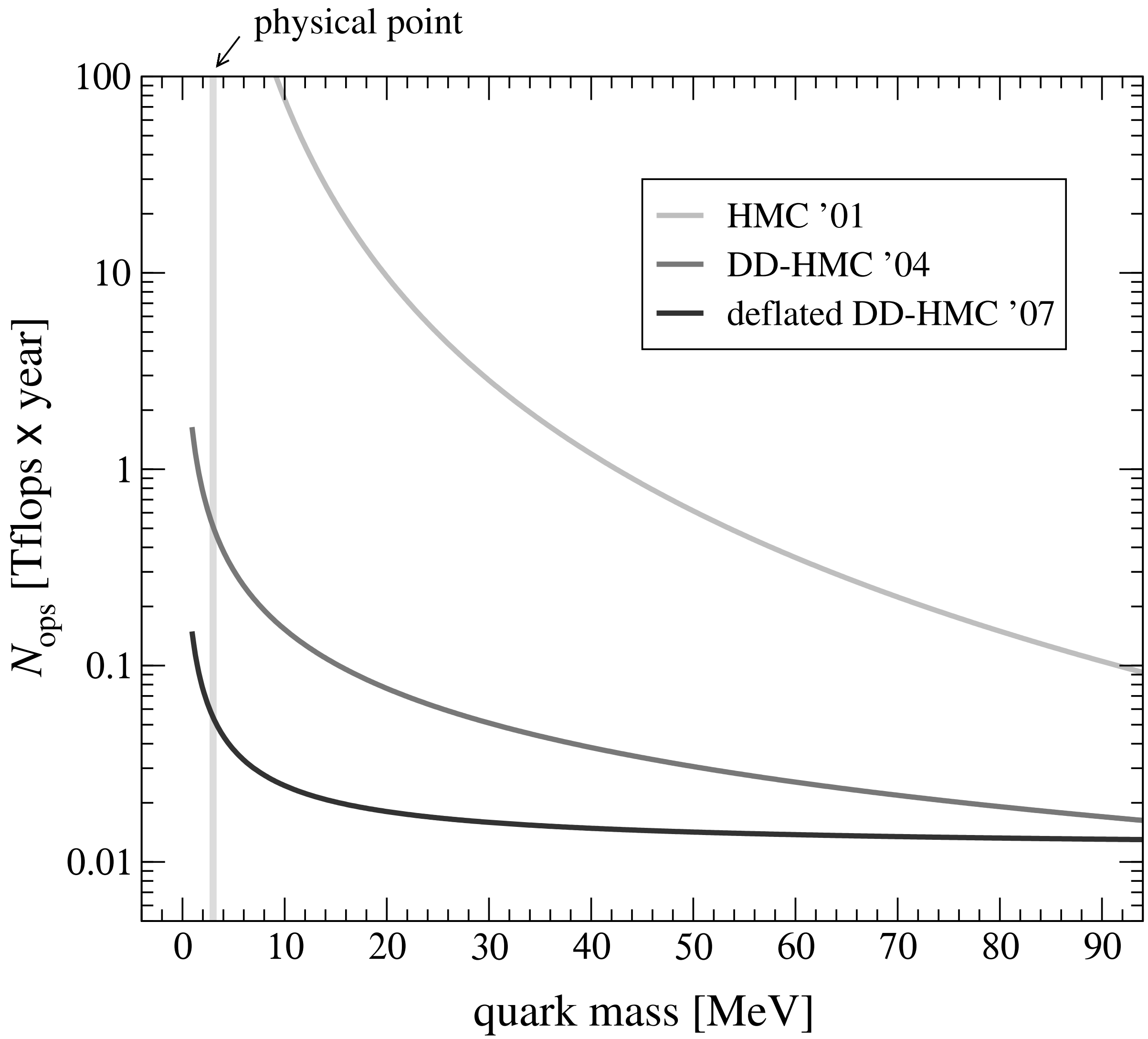


2006

– comparable to staggered

– reach small pseudo scalar masses $\approx 300\text{MeV}$

Recent pricture



German Supercomputer Infrastructure

- JUQUEEN (IBM BG/Q)
  at Supercompter center Jülich
  5 Petaflops

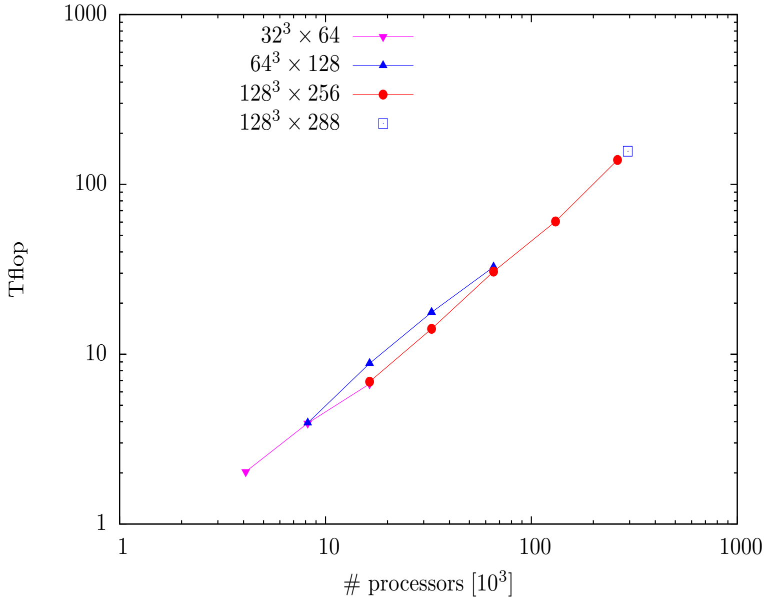
- HLRN (Hannover-Berlin)
  Gottfried and Konrad
  (CRAY XC30)
  2.6 Petaflops

- Leibniz Supercomputer center Munich
  combined IBM/Intel system SuperMUC
  3 Petaflops

Computertime: through local calls, e.g. NIC,
or Europe wide: PRACE $\longrightarrow$ peer reviewed

Strong Scaling

- Test on 72 racks BlueGene/P installation at supercomputer center Jülich
- using tmHMC code

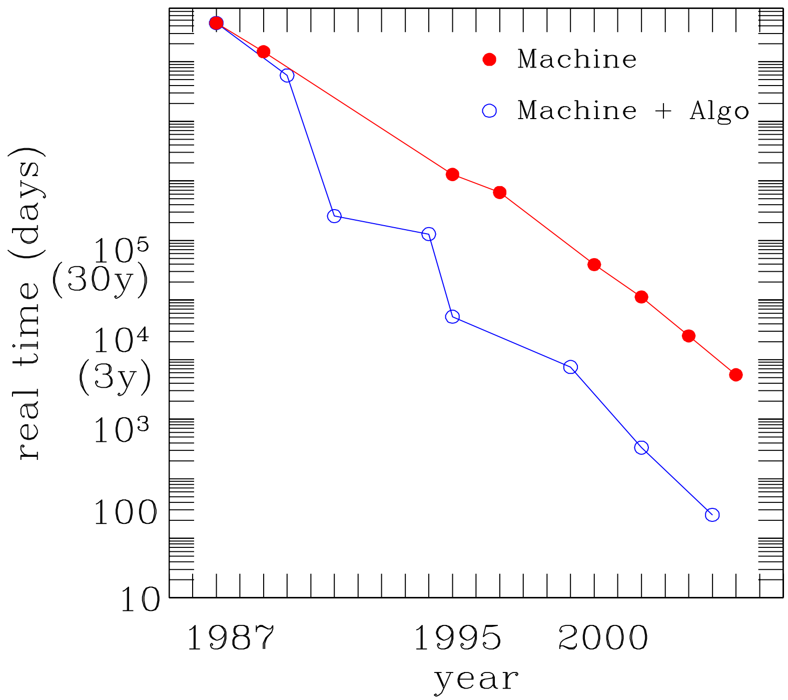


Computer and algorithm development over the years

Lattice physicists have invested a lot in algorithm development

supercomputer architectures show remarkable speedup

time estimates for simulating $32^3 \cdot 64$ lattice, 5000 configurations



$\rightarrow$ algorithm development very important

$\rightarrow$ typical architectures: **BG/L,P,Q, Intel, GPUs**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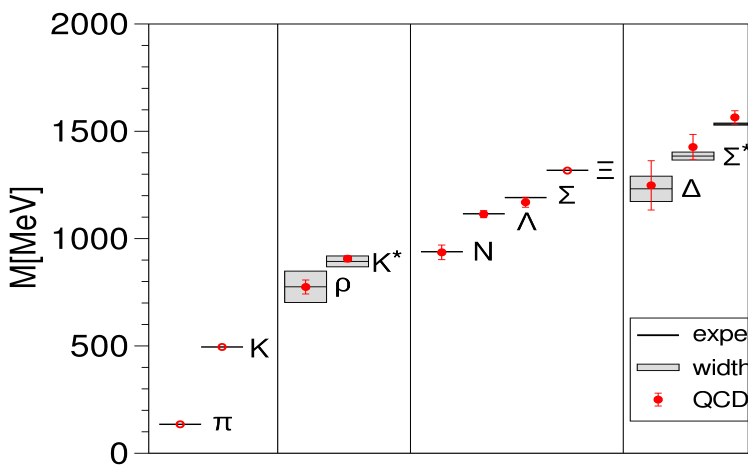
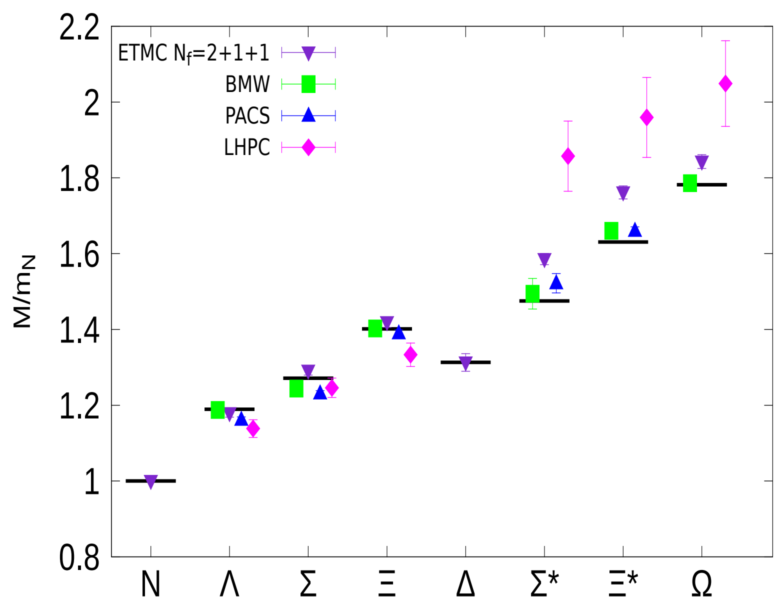
The lattice QCD benchmark calculation: the spectrum

spectrum for $N_f = 2+1$ and $2+1+1$ flavours





first spectrum calculation BMW

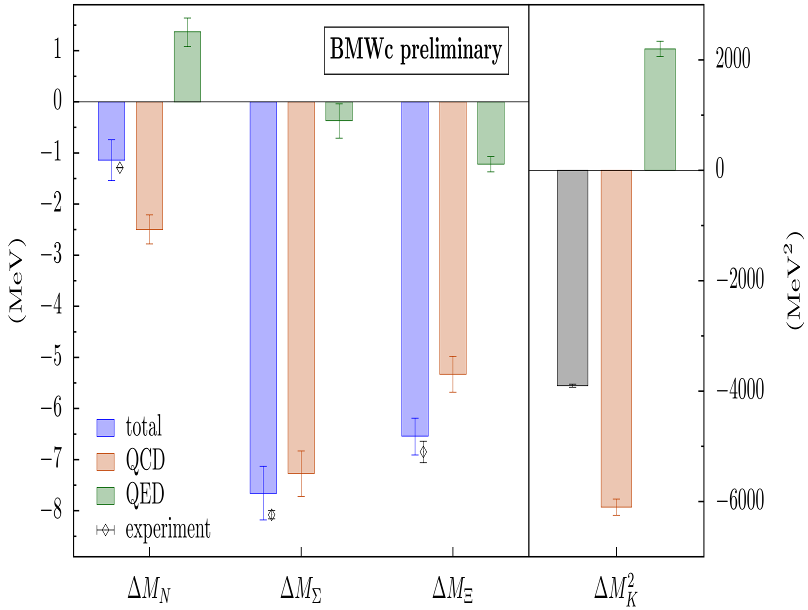
repeated by other collaborations

(ETMC: C. Alexandrou, M. Constantinou, V. Drach, G. Koutsou, K.J.)

- spectrum for $N_f = 2$, $N_f = 2+1$ and $N_f = 2+1+1$ flavours
  $\rightarrow$ no flavour effects for light baryon spectrum

Even isospin and electromagnetic mass splitting

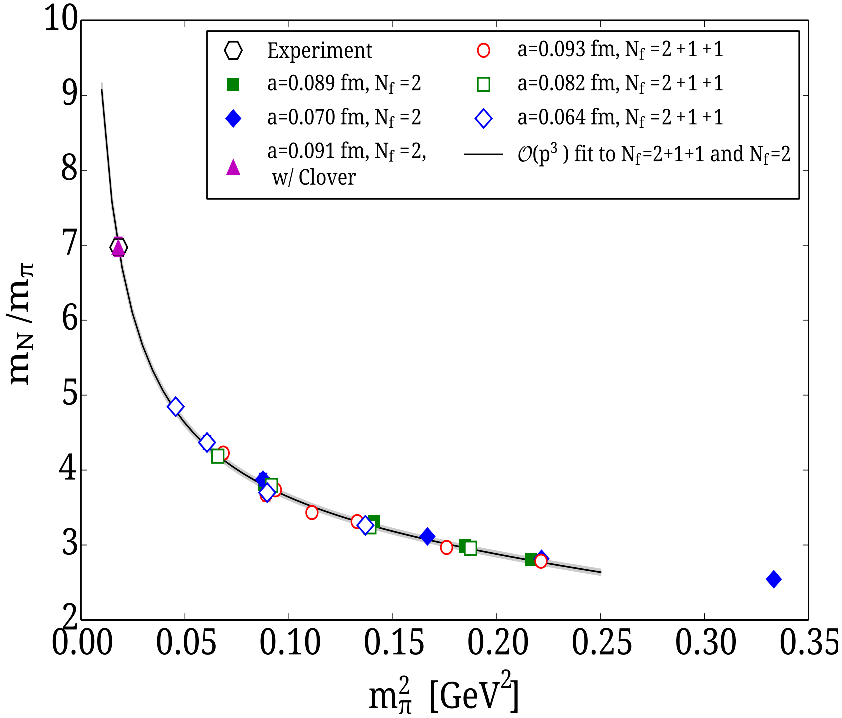
(BMW collaboration)



baryon spectrum with mass splitting

- nucleon: isospin and electromagnetic effects with opposite signs
- nevertheless physical splitting reproduced

Setting the scale



tune to $\frac{M^{\rm phys}_{\rm proton}}{M^{\rm phys}_{\rm pion}} = \frac{M^{\rm latt}_{\rm proton}}{M^{\rm latt}_{\rm pion}} = 6.95$

use proton mass: $a \cdot M^{\rm phys}_{\rm proton} = M^{\rm latt}_{\rm proton}$

$\Rightarrow$ determine lattice spacing

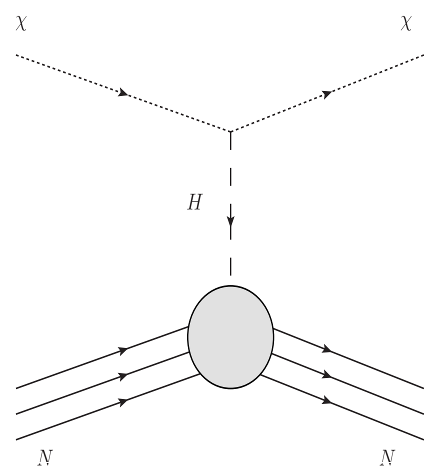
$\Rightarrow$ using value of a other quantities have to come out right up to discretization effects

	M_{Ds}/M_K	M_{Ds}/M_D	f_K/f_π	f_{Ds}/f_D
lat.	3.96(2)	1.049(4)	1.197(6)	1.19(2)
PDG	3.988	1.0556(02)	1.197(06)	1.26(6)

- with strange, charm quarks $\rightarrow$ need additional input
- repeat for smaller and smaller a $\rightarrow$ continuum limit

The strange quark content of the nucleon

- scalar candidate particles for dark matter
- interaction with nucleon: scalar quark content via the Higgs boson exchange diagram



spin independend cross section:

$$\sigma_{\mathrm{SI}} \propto \sum_q \frac{\langle N|\bar{q}q|N\rangle}{m_N} \quad ; q = u, d, s, c$$

want *sigma terms* (scalar quark content of nucleon):

$\sigma_u = \langle N|\bar{u}u|N\rangle$, $\sigma_d = \langle N|\bar{d}d|N\rangle$,

$\sigma_s = \langle N|\bar{s}s|N\rangle$, $\sigma_c = \langle N|\bar{c}c|N\rangle$,

$\sigma_{\pi N} = \frac{\langle N|\bar{u}u+\bar{d}d|N\rangle}{(m_u+m_d)/2}$

The problem

spin independent cross section strongly dependend on sigma terms

e.g., phenomenological analyses: $48\mathrm{MeV} < \sigma_{\pi N} < 80\mathrm{MeV}$

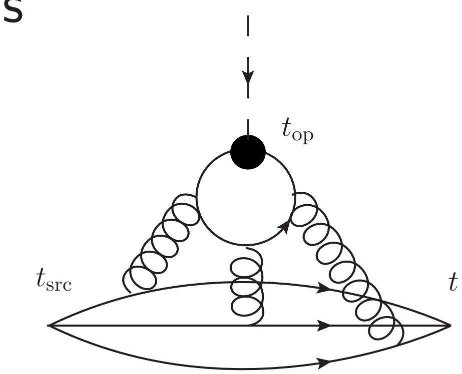
$\Rightarrow$ cross section can change by an order of magnitude

want: a first principle, non-perturbative computation of σ_q

$\rightarrow$ the lattice

Problem on the lattice

- disconnected diagrams:
  quarks lines only indirectly conneced via gluons
  $\Rightarrow$ very bad signal to noise ration
  $\Rightarrow$ often simply neglected



- High statistics analysis
  (A. Abdel-Rehim, C. Alexandrou, M. Constantinou, K. Hadjiyiannakou, Ch. Kallidonis, G. Koutsou, A. Vaquero Avilés-Casco, K.J.)

- substantial algorithm development (exact deflation, truncated solver)

- $O(200.000)$ measurements (typical: $O(1000)$ measurements)

- dedicated Gauss center project:
  $\approx$ 50 Million core hours
  on Hazel Hen at Stuttgart supercomputer center ($\approx$ 2.5 Million Euro)

Results for sigma terms

- find signal for all sigma terms:

$$\sigma_{\pi N} = 37.22(2.57)(^{+0.99}_{-0.6})\ \mathrm{MeV},$$
$$\sigma_s = 41.05(8.25)(^{+1.09}_{-0.69})\ \mathrm{MeV},$$
$$\sigma_c = 79(21)(^{+2.1}_{-1.3})\ \mathrm{MeV}$$

- strange quark content of nucleon: $y_N \equiv \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u+\bar{d}d|N\rangle} = 0.075(16)$

- remarks
  - $\sigma_{\pi N}$ compatible with other lattice works
  - σ_s most precise result so far
  - σ_c only available result

Higgs boson mass bounds from the lattice

- the Lagrangian of the scalar theory (≈ Ginzburg-Landau theory)

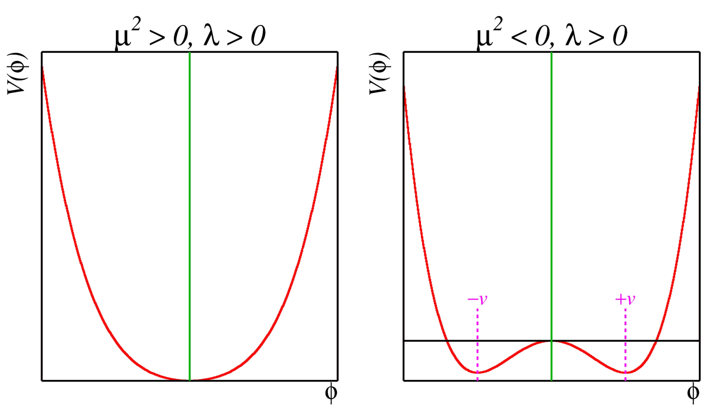
$$L_\varphi[\varphi] = \tfrac{1}{2}\partial_\mu \varphi_x^\dagger \partial_\mu \varphi_x + \tfrac{1}{2}\mu^2 \varphi_x^\dagger \varphi_x + \lambda \left(\varphi_x^\dagger \varphi_x\right)^2 ,$$

– μ^2 bare Higgs boson mass, λ bare quartic coupling

– φ 4-component real scalar field

– $O(N)$ invariance of Lagrangian

two phases $\langle\varphi\rangle \equiv v = 0$ symmetric phase
$v > 0$ spontaneously broken (Higgs) phase



$v = 0$ $v > 0$

Predicting your own death: the suicid of the standard model

- 1-loop analysis of broken phase of the scalar theory

$$\lambda^{\mathrm{ren}}(p^2 = \Lambda^2) = \frac{\lambda_0}{1 - b_N \lambda_0 \, \log\left(\frac{\Lambda^2}{m_H^2}\right)} \, , b_N = \frac{N+8}{8\pi^2}$$

Interpretation:

- for $\Lambda \to \infty : \lambda^{\mathrm{ren}} = 0 \leftarrow$ triviality of the φ^4-theory (and of the standard-model)

- cut-off cannot be removed from the theory
  $\Rightarrow$ standard model only effective theory, valid up to a certain cut-off value Λ

- intrinsic relation between cut-off and Higgs-boson mass

- interpretation of cut-off: energy scale of yet to be discovered physics beyond the standard model

Consequences of triviality

$$\lambda^{\rm ren}(p^2 = \Lambda^2) = \frac{\lambda_0}{1 - b_N \lambda_0 \, \log\left(\frac{\Lambda^2}{m_H^2}\right)} \, , b_N = \frac{N+8}{8\pi^2}$$

singularity (Landau pole): $\log\left(\frac{\Lambda^2}{m_H^2}\right) = \frac{1}{b_N \lambda_0}$

- avoid Landau pole $\rightarrow$ bound on $\lambda^{\rm ren}$
- since $m_H^2 = 2v^2\lambda^{\rm ren} \Rightarrow$ bound on Higgs boson mass

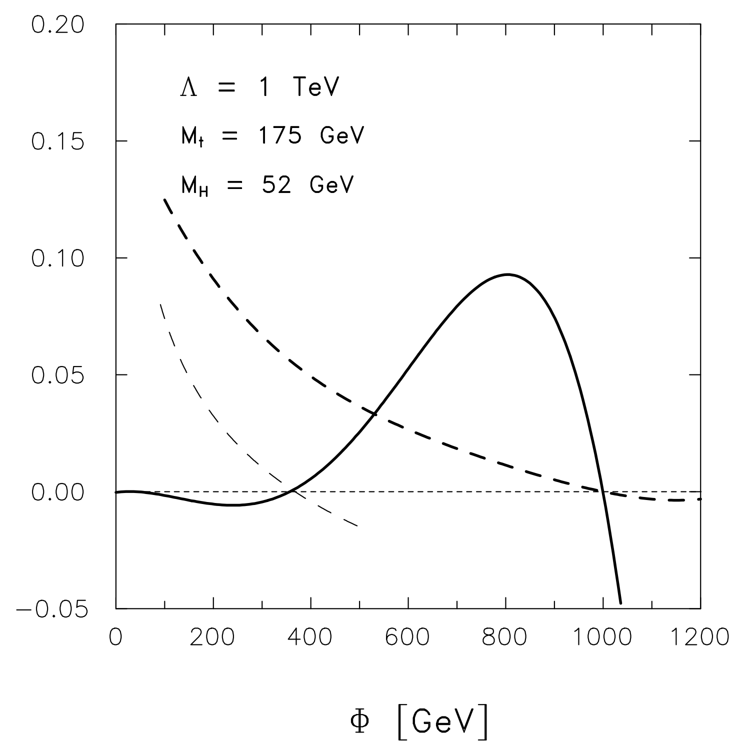
Effects of adding quarks

- effective potential

$$U_{\text{eff}} = V + 1/2 \int_k \ln[k^2 + m^2] - 2N_{\text{F}} \int_k \ln[k^2 + y^2\varphi^2]$$

$\Rightarrow$ negative contribution for fermions $\rightarrow$ theory becomes unstable

- to avoid instability
  $\rightarrow$ (lower) bound on Higgs boson mass



The holy grail: exact chiral invariant Higgs-Yukawa lattice action (Lüscher)

- the lattice fermionic and Yukawa parts $\leftarrow$ exactly same form as in continuum $\Rightarrow$ breakthrough in lattice field theory

$$(L_F + L_Y)[\bar{\psi}, \psi] = \bar{\psi} D_{\rm ov} \psi + y_b \left(\bar{t}, \bar{b}\right)_L \varphi b_R + y_t \left(\bar{t}, \bar{b}\right)_L \tilde{\varphi} t_R + c.c.$$

- change from continuum:
  - $i\gamma_\mu \partial_\mu \to D_{\rm ov}$
  - $P_\pm = \frac{1 \pm \gamma_5}{2} \to \hat{P}_\pm = \frac{1 \pm \hat{\gamma}_5}{2}, \hat{\gamma}_5 = \gamma_5 \left(1 - a D_{\rm ov}\right)$

- exact *lattice* $\mathrm{SU}(2)_L \times \mathrm{U}(1)_R$ chiral symmetry

$$\psi \to U_R \hat{P}_+ \psi + \Omega_L \hat{P}_- \psi, \bar{\psi} \to \bar{\psi} P_+ \Omega_L^\dagger + \bar{\psi} P_- U_R^\dagger$$

$$\phi \to U_R \phi \Omega_L^\dagger, \phi^\dagger \to \Omega_L \phi^\dagger U_R^\dagger.$$

with $\Omega_L \in \mathrm{SU}(2)$, $U_R \in U(1)$

The algorithm

improvements (Philipp Gerhold, K.J.):

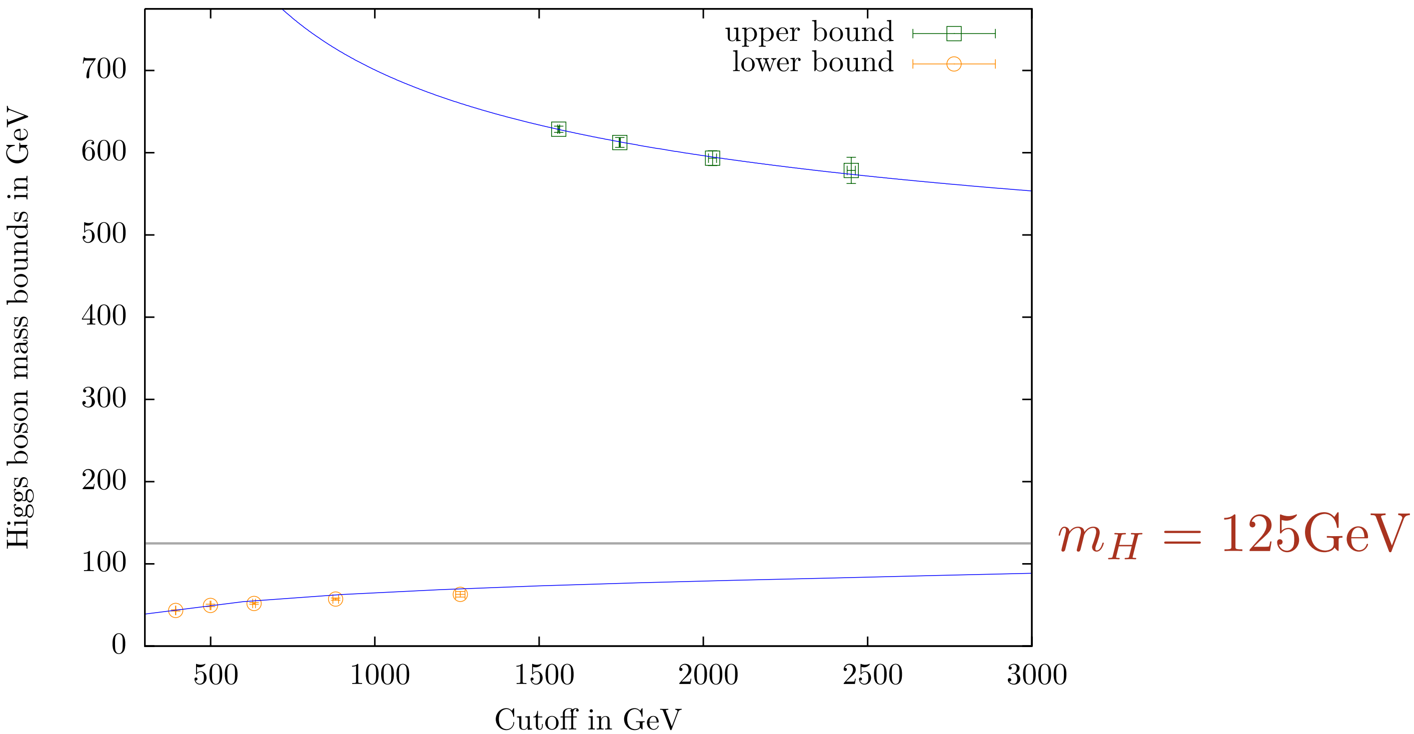
- special preconditioning techniques for fermion matrix:
  $\rightarrow$ factors of O(10)-O(100) improvement for condition number
- Fourier acceleration

	FACC	traLength	Nconf	ACtime	cost
$\kappa = 0.12313$	No	2.0	2020	132.1 ± 6.4	2662 ± 129
$\kappa = 0.12313$	Yes	2.0	21780	1.1 ± 0.1	37 ± 1
$\kappa = 0.30400$	No	1.0	2580	34.9 ± 2.1	450 ± 28
$\kappa = 0.30400$	Yes	1.0	22360	3.8 ± 0.2	171 ± 8

- exact Krylow space reweighting
- multiple time scale integrators

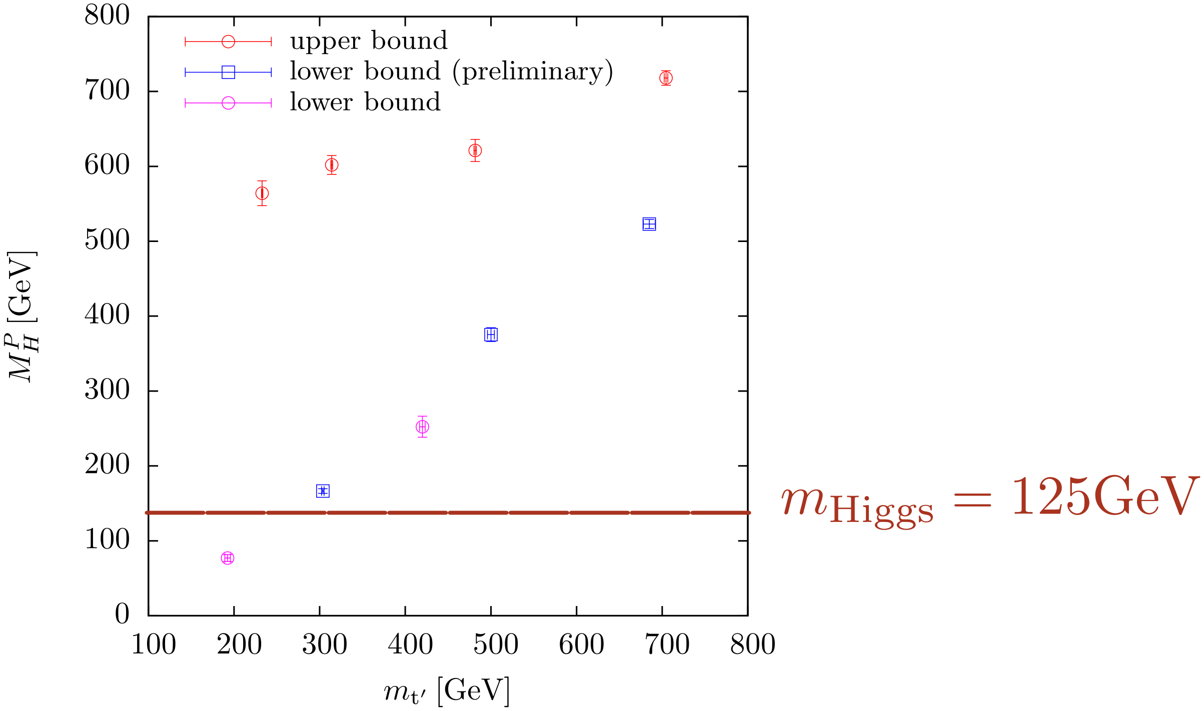
Bounds on Higgs boson mass from the lattice

(J. Bulava, D. Chu, P. Gerhold, J. Kallarackal, B. Knippschild, D. Lin, K. Nagai, A. Nagy, K.J.)



- Higgs boson mass right in the funnel of mass bounds
- validity scale of the standard model

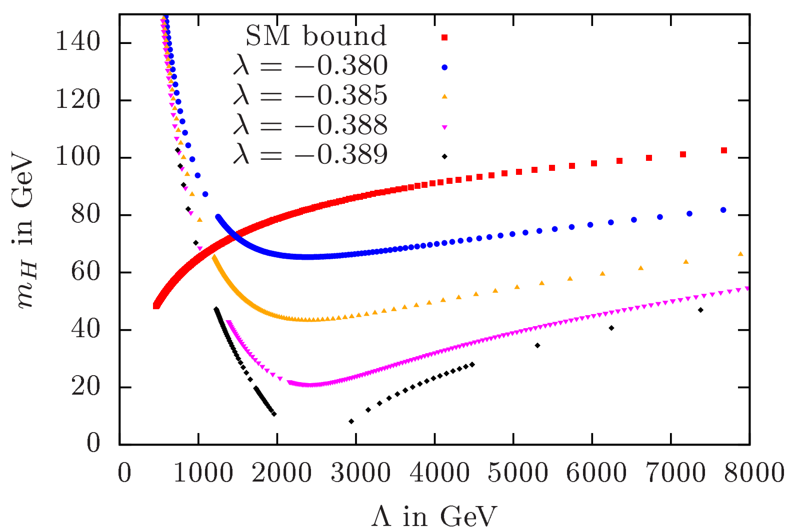
Lattice: Fermion mass dependence of Higgs boson mass bounds



- strong dependence on fermion mass
- exclude 4th generation quarks

Effect of adding a $\lambda_6\Phi^6$ term

- Higher dimensional operators
  - are allowed since theory is only defined with cutoff
  - can mimic extensions of standard model, e.g. a second scalar particle
  - with $\lambda_6 > 0$, quartic coupling can be negative



- already a simple $\lambda_6\Phi^6$ term can change bound

Further lattice QCD activities

- obtain precision results
  (e.g. strong coupling, quark masses, decay constants)

- better understanding of QCD
  (e.g. chiral symmetry breaking, topology)

- understanding hadrons
  (e.g. hadronic form factors, proton spin puzzle, PDFs)

- help to uncover BSM physics
  (e.g. B-system, electroweak observables, anomalous magnetic lepton moments)

- conceptual and algorithmic developmens

$\Rightarrow$ grand challenges for the future


DB
ICE

→

Exercise after coffee

- programme action of quantum mechanical oscillator
- evaluate path integral by Metropolis algorithm
- compute observables $\langle x \rangle$, $\langle x^2 \rangle$ with error
- compare $\langle x^2 \rangle$ to theoretically known value

$\Rightarrow$ have fun (we, J. Volmer and K.J. will help)