The top quark and its mass

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DESY

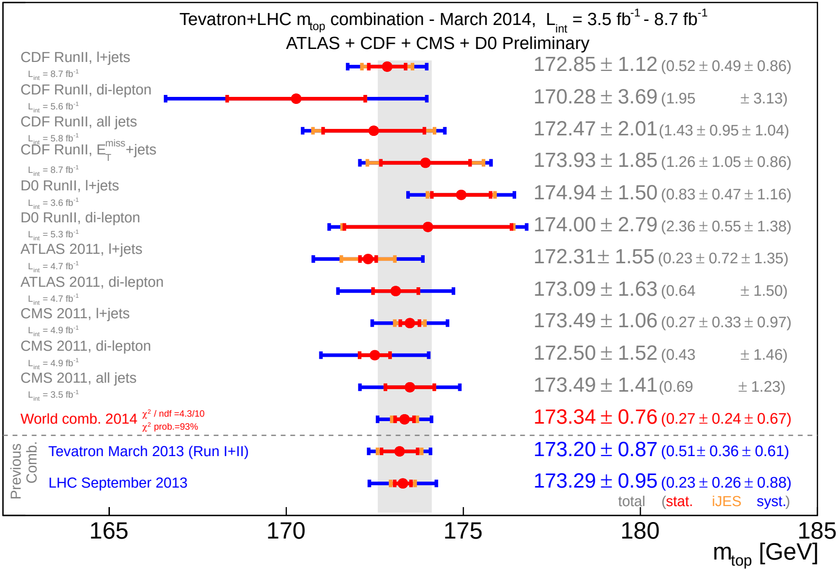
DESY, May 2016

Outline

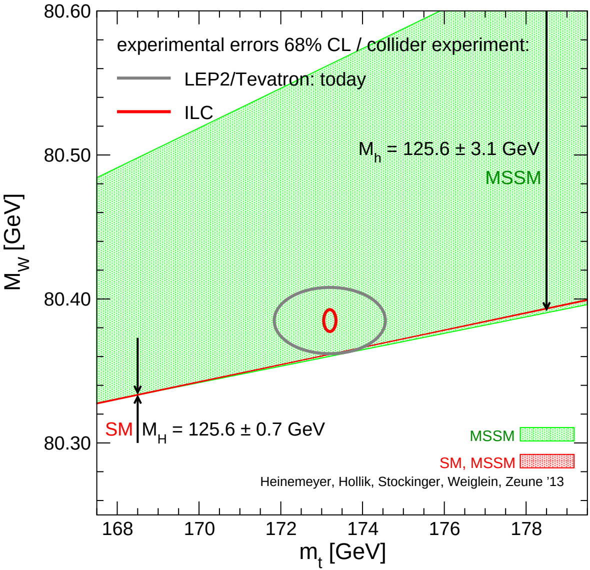
1. Introduction
2. Quark mass relations
3. Top pair production @ ILC

Outline

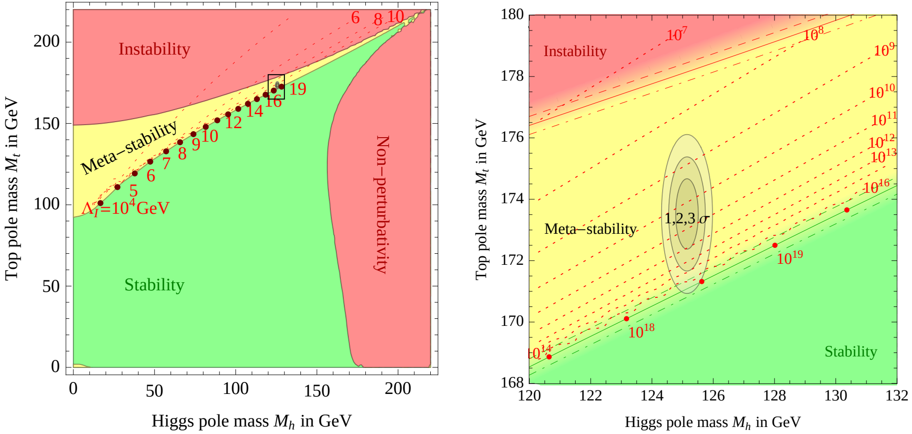
LHC results for m_t



Precision tests



Stability of the Higgs potential



[Buttazzo et al ’13]

QCD Lagrangian

The Standard Model is described by a quantum field theory with Lagrangian

$$\mathcal{L}_{\mathrm{QCD}} = \mathcal{L}_{\mathrm{gauge}} + \mathcal{L}_{\mathrm{int}} + m_0 \bar{\Psi}_0 \Psi_0 + \bar{\Psi}_0 \gamma^\mu \frac{\partial}{\partial x^\mu} \Psi_0$$

where Ψ_0 denotes the quark field.
The masses of the quarks m_0 (or the quark Yukawa coupling) are free parameter of the Standard Model.

To confront theory predictions with experimental measurements all input parameters including the masses have to be determined as precise as possible.

Renormalization

The calculation of the corresponding Feynman diagrams leads to singularities, which first have to be

regularized by using e.g. dimensional regularization

$$d = 4 \to d = 4 - 2\epsilon$$

and then the theory has to be

renormalized by systematically subtracting these singularities by redefinition of the parameters

$$m_0 = Z_m m_q \qquad \Psi_0 = Z_2^{\frac{1}{2}} \Psi$$

$$\mathcal{L}_{\mathrm{QCD}} = \mathcal{L}_{\mathrm{gauge}} + \mathcal{L}_{\mathrm{int}} + Z_m Z_2 m_q \bar{\Psi}\Psi + Z_2 \bar{\Psi}\gamma^\mu \frac{\partial}{\partial x^\mu}\Psi$$

Ren. schemes and mass definitions

To do the renormalization in a well defined way, we have to define renormalization schemes.
Two schemes are particularly important

- $\overline{\mathrm{MS}}$ scheme (running mass)
- on-shell scheme

The value of the mass depends on the choice of the renormalization scheme!
We must be able to translate between the schemes.

Scheme definitions

One considers the renormalized quark propagator

$$S_F(q) = \frac{-i\,Z_2}{\not{q} - Z_m m + \Sigma(q,m)}$$

with $\Sigma(q,m)$ the quark two-point function.
For the $\overline{\mathrm{MS}}$ scheme we require

$$S_F(q) \quad \text{finite}$$

and for the on-shell scheme we require a pole at the position of the mass

$$S_F(q) \overset{q^2 \to M^2}{\longrightarrow} \frac{-i}{\not{q} - M}$$

Even more quark mass definitions

- pole mass
- $\overline{\mathrm{MS}}$ mass
- PS mass [Beneke '98]
- 1S mass [Hoang,Smith,Stelzer,Willenbrock '99]
- kinetic mass [Bigi,Shifman,Uraltsev,Vainstein '97]
- ...

We need precise conversion formula for all these schemes!

Which mass do we measure at the LHC?

The mass measured at the LHC (and Tevatron) is an ”MC mass” and not directly related to any theoretically well defined mass but close to the pole mass.

$$M_{\mathrm{MC}} = M_{\mathrm{pole}} \pm \Delta$$

Recent studies indicate

$$\Delta = 500\,\mathrm{MeV} - 1\,\mathrm{GeV}$$

More work needed to better understand the problem.
Much better control when using a threshold scan at a linear collider.

Outline

Setup of the calculation

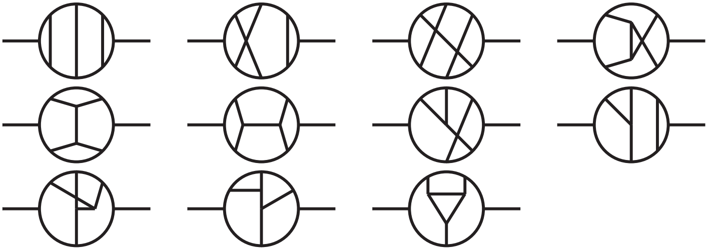
- Need to calculate mass renormalization constant Z_m^{OS} by calculating four-loop on-shell integrals
- Together with the renormalization constant in the $\overline{\mathrm{MS}}$-scheme $Z_m^{\overline{\mathrm{MS}}}$ [Chetyrkin '97; Larin, van Rittbergen, Vermaseren '97]
  we get

$$\left.\begin{aligned} m_{\mathrm{bare}} &= Z_m^{\mathrm{OS}} M \\ m_{\mathrm{bare}} &= Z_m^{\overline{\mathrm{MS}}} m \end{aligned}\right\} \Rightarrow m = M \frac{Z_m^{\mathrm{OS}}}{Z_m^{\overline{\mathrm{MS}}}}$$

  - 1-loop [Tarrach'81]
  - 2-loop [Gray,Broadhurst,Grafe,Schilcher'90]
  - 3-loop [Chetyrkin,Steinhauser'99; Melnikov, v. Ritbergen'00; Marquard,Mihaila,Piclum,Steinhauser'07]

Setup of the calculation

Need to calculate 4-loop on-shell diagrams of the form



where we have to consider all possible ways to route the external quark through the diagrams $\Rightarrow$ 100 integral families.

Setup of the calculation

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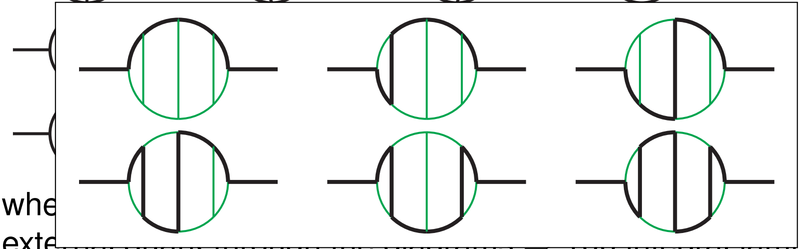

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external quark through the diagrams ⇒ 100 integral families.

Setup of the calculation

Follow the *standard* procedure for multi-loop calculations

- exploit that the appearing integrals are not linear independent but related through Integration-By-Parts identities [Chetyrkin, Tkachov '81]
- reduce all appearing integrals to a small set of basis integrals using `FIRE` [Smirnov] or `Crusher` [PM,Seidel]
- evaluate the remaining basis integrals ($\mathcal{O}(350)$) using analytic or numerical techniques (Mellin-Barnes or Sector Decomposition)

$\overline{\text{MS}}$–on-shell relation at four-loop order

$\overline{\text{MS}} \to$ on-shell

$$
\begin{aligned}
M_t &= m_t \Big(1 + 0.4244\,\alpha_s + 0.8345\,\alpha_s^2 + 2.375\,\alpha_s^3 \\
&\quad + (8.49 \pm 0.25)\,\alpha_s^4\Big) \\
&= 163.643 + 7.557 + 1.617 + 0.501 + 0.195 \pm 0.005 \text{ GeV}
\end{aligned}
$$

[PM, Steinhauser, Smirnov, Smirnov '15]

small remaining error of about 3% due to numerical integration of the master integrals using FIESTA [A. Smirnov] for the sector decomposition.

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$$
\begin{aligned}
M_b &= m_b \Big(1 + 0.4244\,\alpha_s + 0.9401\,\alpha_s^2 + 3.045\,\alpha_s^3 + (12.57 \pm 0.38)\,\alpha_s^4\Big) \\
&= 4.163 + 0.401 + 0.201 + 0.148 + 0.138 \pm 0.004 \text{ GeV}\,.
\end{aligned}
$$

Threshold mass schemes

- Potential-subtracted mass

$$m^{\mathrm{PS}} = M - \delta m(\mu_f)$$

$$\delta m(\mu_f) = -\frac{1}{2}\int_{|\vec{q}|<\mu_f} \frac{\mathrm{d}^3 q}{(2\pi)^3} V(\vec{q}) = \mu_f \frac{C_F \alpha_s}{\pi}(1 + \alpha_s \ldots)$$

need static potential @ three loops

[Smirnov,Smirnov,Steinhauser '09; Anzai,Kiyo,Sumino '09]

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- 1S mass

$$m^{1\mathrm{S}} = M + \frac{1}{2} E_1^{\mathrm{pt}}$$

$$E_1^{\mathrm{pt}} = -\frac{C_F^2 M \alpha_s^2}{8}(1 + \alpha_s \ldots)$$

need binding energy @ three loops [Beneke et al]

Convergence and error estimate

input #loops	$m^{\rm PS} =$ 171.792	$m^{\rm 1S} =$ 172.227	$m^{\rm RS} =$ 171.215	
1	165.097	165.045	164.847	
2	163.943	163.861	163.853	1-2 GeV

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4 ($\times$1.03)	163.637	163.637	163.637	6 MeV

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half the 4-loop contribution $\{22, 4, 10\}$
3% uncertainty $\equiv$ 6 MeV
$\Big\} \Rightarrow \{23, 7, 11\}$MeV error

Final result for PS and 1S mass scheme

$$\frac{\bar{m}_t(\bar{m}_t)}{\text{GeV}} = 163.643 \pm 0.023 + 0.074\Delta_{\alpha_s} - 0.095\Delta_{m_t}^{\mathrm{PS}}$$

$$\frac{m_t(m_t)}{\text{GeV}} = 163.643 \pm 0.007 + 0.069\Delta_{\alpha_s} - 0.096\Delta_{m_t}^{\mathrm{1S}}$$

$$\Delta_{m_t}^{\mathrm{PS}} = \frac{171.792\,\mathrm{GeV} - m_t^{\mathrm{PS}}}{0.1\,\mathrm{GeV}}$$

$$\Delta_{m_t}^{\mathrm{1S}} = \frac{172.227\,\mathrm{GeV} - m_t^{\mathrm{1S}}}{0.1\,\mathrm{GeV}}$$

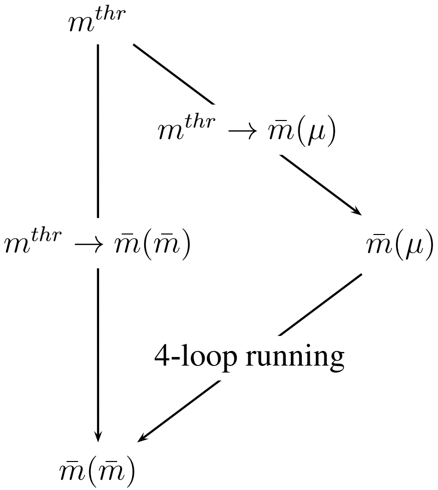
$$\Delta_{\alpha_s} = \frac{(0.1185 - \alpha_s(M_Z))}{0.001}$$

$m^{\text{thr}} \to \bar{m}(\bar{m})$

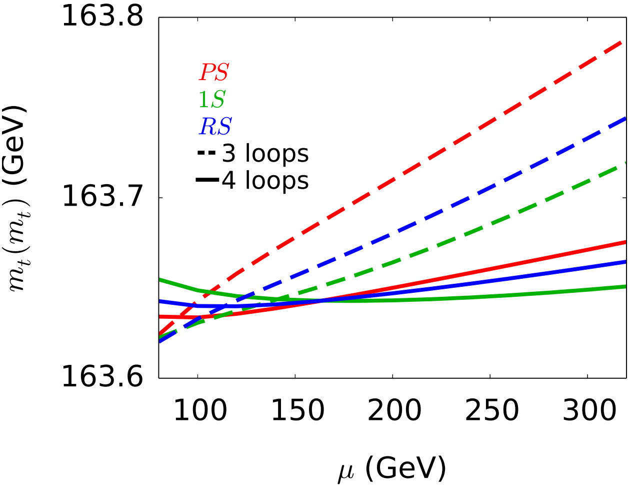
alternative error estimate by

- first calculating $\bar{m}(\mu)$
- and in a second step $\bar{m}(\bar{m})$

has to give the same result up to higher-order corrections



$m^{\text{thr}} \rightarrow \bar{m}(\bar{m})$



$\Rightarrow$ compatible error estimate

Beyond 4-loops

$$m_P = m(\mu)\left(1 + \sum_{n=1}^{\infty} c_n(\mu, \mu_m, m(\mu))\, \alpha_s^n(\mu)\right)$$

for large n

$$c_n(\mu, \mu_m, m(\mu_m)) \underset{n\to\infty}{\longrightarrow} N c_n^{(\mathrm{as})}(\mu, m(\mu_m)) \equiv N \frac{\mu}{m(\mu_m)}\, \tilde{c}_n^{(\mathrm{as})}\,,$$

[Beneke '94 '99]

where

$$\tilde{c}_{n+1}^{(\mathrm{as})} = (2b_0)^n \frac{\Gamma(n+1+b)}{\Gamma(1+b)} \left(1 + \frac{s_1}{n+b} + \frac{s_2}{(n+b)(n+b-1)} + \cdots\right)$$

b_0, b, s_1, s_2: Combinations of coefficients of the β-function.

Beyond 4-loops

Fit N to 4-loop term and take higher orders from asymptotic formula

j	$\tilde{c}_j^{(\mathrm{as})}$	$\tilde{c}_j^{(\mathrm{as})}\alpha_s^j$
5	0.985499×10^2	0.001484
6	0.641788×10^3	0.001049
7	0.495994×10^4	0.000880
8	0.443735×10^5	0.000854
9	0.451072×10^6	0.000942
10	0.513535×10^7	0.001164

$$\delta^{(5+)}m_P = 0.272^{+0.016}_{-0.041}\,(N) \pm 0.001\,(c_4) \pm 0.011\,(\alpha_s) \pm 0.066\;(\mathrm{amb})\;\mathrm{GeV}$$

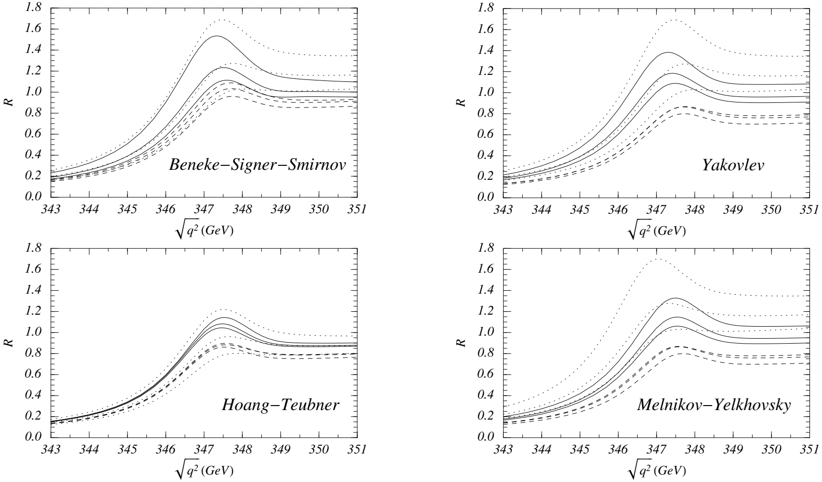
[Beneke, PM, Nason, Steinhauser '16]

Outline

Reminder

- Problem
  - Measurement of top-quark mass at hadron colliders limited
  - Measured mass not well defined $\rightarrow$ conversion to well-defined mass scheme problematic
- Solution: Threshold scan at a linear collider
- exp. stat. error [Seidel et al '13, Horiguchi et al '13]
  - $\Delta M_t = 20 - 30$ MeV
  - $\Delta\Gamma_t = 21$ MeV
- Goal: Measurement of top-quark mass with $\Delta M \approx 100\,\text{MeV}$

NNLO Results

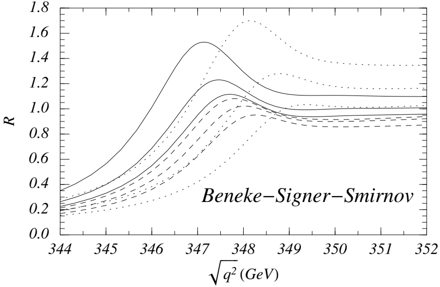


[Hoang,Teubner; Melnikov,Yelkhovsky; Yakovlev;

Beneke, Signer, Smirnov; Nagano, Ota, Sumino; Penin, Pivovarov '00]

Choosing an adequate mass scheme

pole mass



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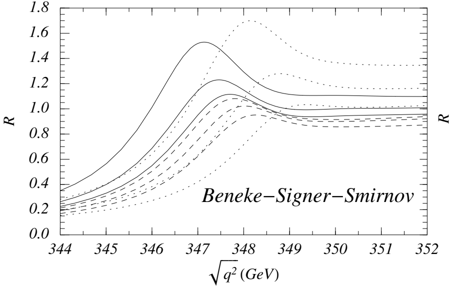
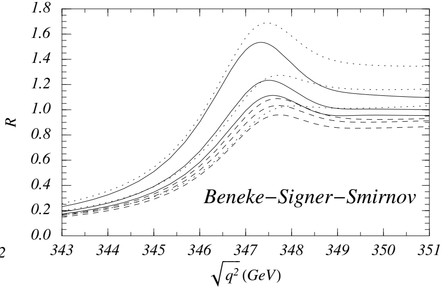


PS mass



Framework

Physics of bound states of heavy particles and threshold phenomena best described within an effective field theory:
Non-Relativistic QCD (NRQCD) [Caswell,Lepage'86; Bodwin,Braaten,Lepage'95]
and potential Non-Relativistic QCD (pNRQCD)

[Beneke,Smirnov'97; Pineda,Soto'98; Brambilla,Pineda,Soto,Vairo'00]

or vNRQCD [Luke,Manohar,Rothstein'00; Hoang,Stewart'03]

Prominent applications are

- production of $t\bar{t}$ pairs
- decays of $b\bar{b}$ bound states
- $b\bar{b}$ sum rules
- positronium spectra

Effective Field Theories

Chain of effective field theories:

QCD $\rightarrow$ NRQCD $\rightarrow$ p(otential)NRQCD

- QCD $\rightarrow$ NRQCD integrate out modes of order m_t
- NRQCD $\rightarrow$ pNRQCD integrate out everything besides
  - potential quarks $E_p \sim mv^2, |\vec{p}| \sim mv$
  - ultrasoft gluons $E_k \sim mv^2, |\vec{k}| \sim mv^2$

Master Formula

$$R = \frac{\sigma_{\text{tot}}(e^+e^- \to t\bar{t})}{\sigma_0}$$
$$= \frac{18\pi}{m_t^2}\text{Im}\left\{c_v\left[c_v - \frac{E}{m_t}\left(c_v + \frac{d_v}{3}\right)\right]G(E) + \cdots\right\}$$

Ingredients:

1. NRQCD matching coefficients: c_v, d_v

   [Kallen,Sabry'55;Czarnecki,Melnikov'97; Beneke,Signer,Smirnov'97;Marquard,Piclum,Seidel,Steinhauser'06'08'14]

2. pNRQCD Green function: $G(E)$

   [Beneke,Kiyo,Penin'07; Beneke,Kiyo'08; Beneke,Kiyo,Schuller'13]

QCD $\to$ NRQCD

integrate out heavy degrees of freedom

QCD vector current	NRQCD vector current
$j_v^\mu = \bar{Q}\gamma^\mu Q$	$\tilde{j}_v^k = \phi^\dagger \sigma^k \chi$

QCD $\rightarrow$ NRQCD

integrate out heavy degrees of freedom

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$j_v^\mu = \bar{Q}\gamma^\mu Q$	$\tilde{j}_v^k = \phi^\dagger \sigma^k \chi$

$$j_v^k = c_v \tilde{j}_v^k + \frac{d_v}{6M^2}\phi^\dagger \sigma^k D^2 \chi + \cdots$$

QCD → NRQCD

integrate out heavy degrees of freedom

QCD vector current	NRQCD vector current
$j_v^\mu = \bar{Q}\gamma^\mu Q$	$\tilde{j}_v^k = \phi^\dagger \sigma^k \chi$

$$j_v^k = c_v \tilde{j}_v^k + \mathcal{O}\left(\frac{1}{M^2}\right)$$

c_v can be extracted by calculating vertex corrections involving j_v and $\tilde{j}_v$

$$Z_2 \Gamma_v = c_v \tilde{Z}_2 \tilde{Z}_v^{-1} \tilde{\Gamma}_v + \cdots$$

Results

$$c_v \approx 1 - 2.667\frac{\alpha_s^{(n_l)}}{\pi} + \left(\frac{\alpha_s^{(n_l)}}{\pi}\right)^2 [-44.551 + 0.407 n_l]$$
$$+ \left(\frac{\alpha_s^{(n_l)}}{\pi}\right)^3 \left[-2091(2) + 120.66(0.01)\, n_l - 0.823\, n_l^2\right]$$
$$+\text{singlet terms}$$

[PM, Piclum, Seidel, Steinhauser '14]

- large NNNLO correction
- but, on its own not a physical quantity
- preliminary results confirm that singlet terms are small

Potential NRQCD

$$\mathcal{L}_{\text{PNRQCD}} = \psi^\dagger \left(i\partial_0 + g_s A_0(t,\mathbf{0}) + \frac{\partial^2}{2m} \right) \psi + \int d^3\mathbf{r} \left[\psi^\dagger \psi\right](x+\mathbf{r}) \left(-\frac{C_F \alpha_s}{r} + \delta V(r) \right) \left[\chi^\dagger \chi\right](x) - g_s \psi^\dagger \mathbf{x} \cdot \mathbf{E}(t,\mathbf{0}) \psi$$

[Gupta,Radford'81,. . . ,Manohar'97,. . . ,Kniehl,Penin,Smirnov,Steinhauser'02,. . . ,Beneke,Kiyo,Schuller'13]

ultrasoft contributions from $A_0(t,\mathbf{0})$ and $\mathbf{E}(t,\mathbf{0})$ start only at NNNLO [Beneke,Kiyo '08]

Potentials

$$\delta V = C_c(\alpha_s) V_C(|\vec{q}|) + C_{1/m}(\alpha_s) V_{1/m}(|\vec{q}|) + V_{1/m^2}(|\vec{q}|)$$

- static potential $V_C(|\vec{q}|) \propto \frac{\alpha_s}{\vec{q}^2}$: C_C @ 3 loops

[Smirnov,Smirnov,Steinhauser'08; Smirnov,Smirnov,Steinhauser'09; Anzai,Kiyo,Sumino'09]

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- 1/m potential: $V_{1/m}(|\vec{q}|) \propto \frac{\alpha_s^2}{m|\vec{q}|}$: $C_{1/m}$ @ 2 loops

  [Penin,Smirnov,Steinhauser'13; Beneke,Kiyo,Marquard,Penin,Seidel,Steinhauser'14]

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  [Penin,Smirnov,Steinhauser'13; Beneke,Kiyo,Marquard,Penin,Seidel,Steinhauser'14]
- $1/m^2$ potential: $V_{1/m^2}(|\vec{q}|)$: several potentials needed up to 1 loop

Potential insertions

- LO: Coulomb solution

$$\left(-\frac{\Delta}{m_t} - \frac{C_F\alpha_s}{r} - E\right) G_0(\vec{r},\vec{r}\,',E) = \delta(\vec{r}-\vec{r}\,')$$

- NLO,NNLO,... : treat by perturbation theory in momentum space

$$\begin{aligned} G(E) &= \int d\mathbf{p}d\mathbf{p}' \Big[G_0(\mathbf{p},\mathbf{p}',E) \\ &+ \int d\mathbf{p}_1 d\mathbf{p}_1' G_0(\mathbf{p},\mathbf{p}_1,E)\delta V(\mathbf{p}_1,\mathbf{p}_1')G_0(\mathbf{p}_1',\mathbf{p}',E) \\ &+ \cdots \Big] \end{aligned}$$

coulombic insertions up to third order [Beneke,Kiyo,Schuller '05]
non-coulombic insertions up to second order [Beneke,Kiyo,Schuller tbp]

Diagrammatic description

$\Pi(E) =$

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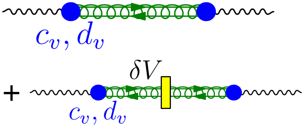

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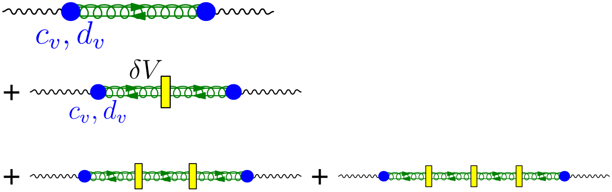

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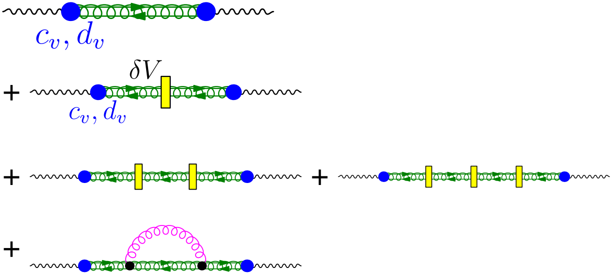
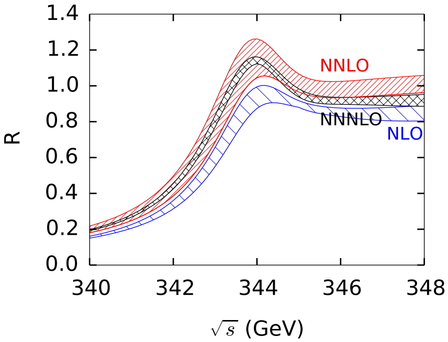


Total cross section

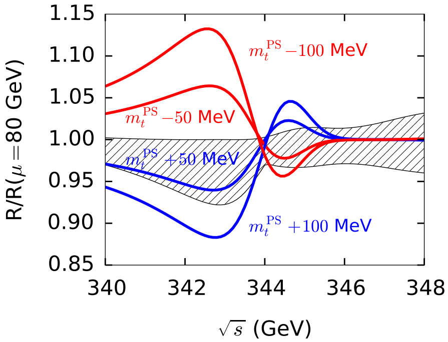


[Beneke,Kiyo,PM,Piclum,Steinhauser,Penin ’15]

$m_t = 171.5\,\mathrm{GeV}$, $\Gamma_t = 1.33\,\mathrm{GeV}$,
$50\,\mathrm{GeV} \leq \mu \leq 350\,\mathrm{GeV}$

- very good convergence of perturbative series below threshold
- above threshold -8% shift driven by large negative corrections to c_v

Scale uncertainty and mass sensitivity

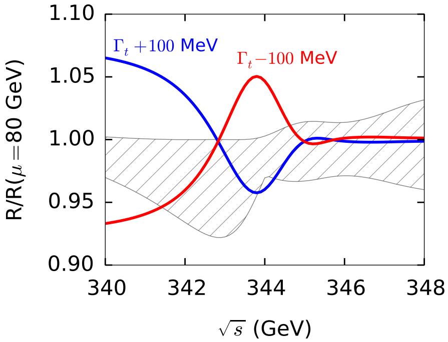


[Beneke,Kiyo,PM,Piclum,Steinhauser,Penin ’15]

$m_t = 171.5\,\mathrm{GeV}$, $\Gamma_t = 1.33\,\mathrm{GeV}$,
$50\,\mathrm{GeV} \leq \mu \leq 350\,\mathrm{GeV}$

- residual scale uncertainty of about 3% depending on center-of-mass energy
- extraction of top mass with $<$ 100 MeV uncertainty seems feasible $\Rightarrow$ need more detailed study

Width sensitivity

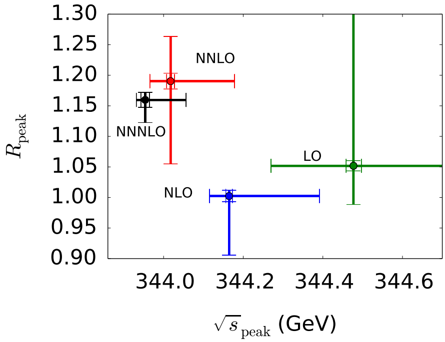


- shape sensitive to width of top quark
- smaller width $\rightarrow$ more pronounced peak

[Beneke,Kiyo,PM,Piclum,Steinhauser,Penin ’15]

$m_t = 171.5\,\mathrm{GeV}$, $\Gamma_t = 1.33\,\mathrm{GeV}$,
$50\,\mathrm{GeV} \leq \mu \leq 350\,\mathrm{GeV}$

Peak height vs position

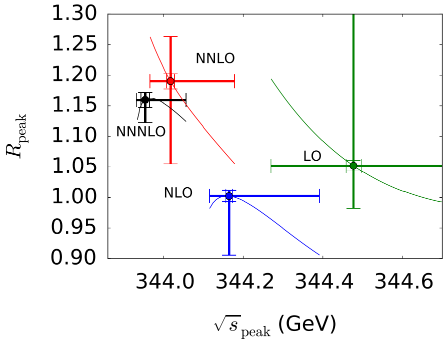


- inner error bars correspond to α_s error
- stabilization of peak position and height
- about 3% uncertainty on the peak height

[Beneke,Kiyo,PM,Piclum,Steinhauser,Penin '15]

$m_t = 171.5\,\mathrm{GeV}$, $\Gamma_t = 1.33\,\mathrm{GeV}$,
$50\,\mathrm{GeV} \leq \mu \leq 350\,\mathrm{GeV}$

Peak height vs position

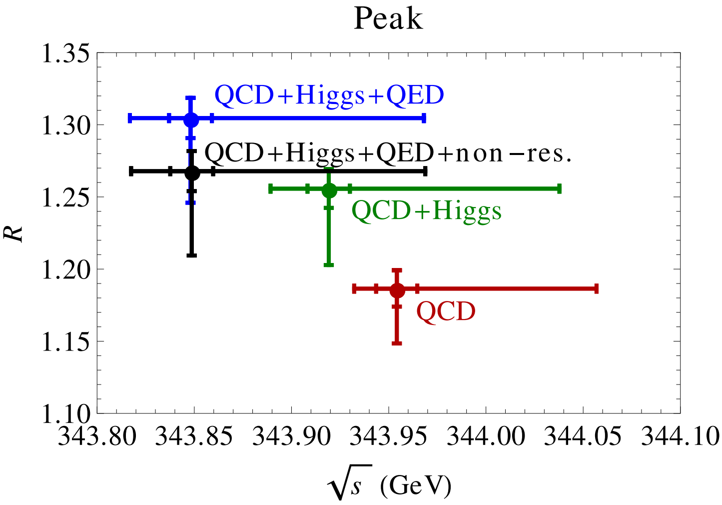


[Beneke,Kiyo,PM,Piclum,Steinhauser,Penin '15]

$m_t = 171.5\,\mathrm{GeV}$, $\Gamma_t = 1.33\,\mathrm{GeV}$,
$50\,\mathrm{GeV} \leq \mu \leq 350\,\mathrm{GeV}$

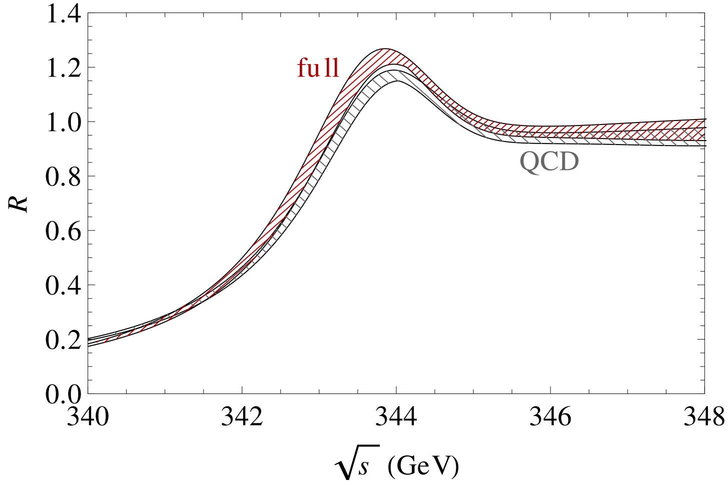
- Peak height and position are correlated under scale variation
- Correlation has to be taken into account in experimental analysis

Electro-weak effects



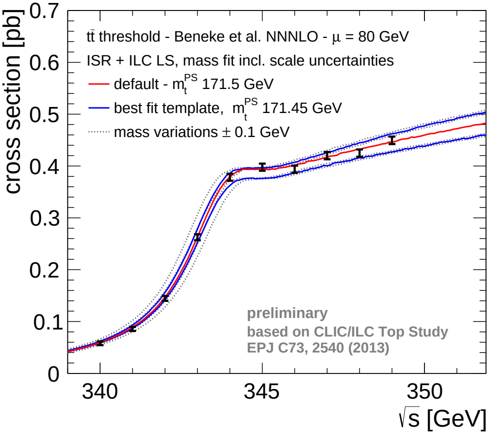
[Beneke,Maier,Piclum,Rauh '15]

Electro-weak effects



[Beneke,Maier,Piclum,Rauh '15]

Experimental study (prelim)



error budget:

- 32 MeV fit
- 45 MeV theory
- $\approx$ 10 MeV experimental systematics
- 32 MeV α_s

[Simon, LCWS '15]

Conclusions

- The mass of the top quark is an important input parameter in the context of precision observables
- presented the 4-loop (NNNLO) relations between the different mass renormalization schemes
- 4-loop contribution to the $\overline{\mathrm{MS}}$ – on-shell relation is $\mathcal{O}(200\,\mathrm{MeV})$, higher orders may result in $\mathcal{O}(250\,\mathrm{MeV})$ with an intrinsic uncertainty of $\mathcal{O}(60\,\mathrm{MeV})$
- presented the full NNNLO prediction for $t\bar{t}$ production at ILC
- prelim. exp. studies show $\Delta M_t < 100\,\mathrm{MeV}$ is possible