

Quantum simulations of high-energy physics models

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MPQ
Max-Planck-Institut
für Quantenoptik



Physics Colloquium,
Zeuthen, January 19th, 2016

QUANTUM PHYSICS

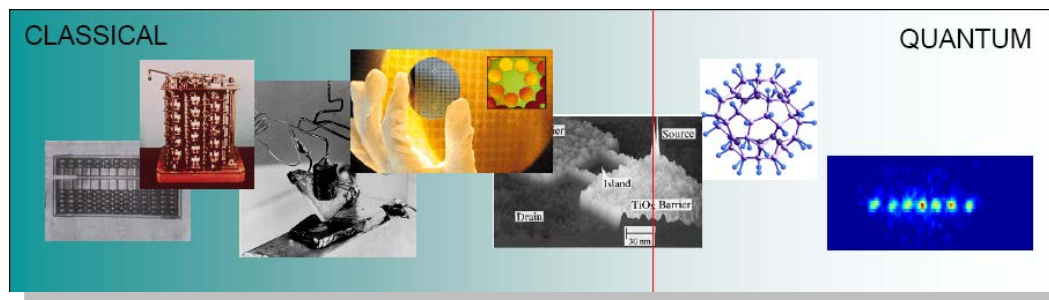


„There is plenty of room at the bottom“

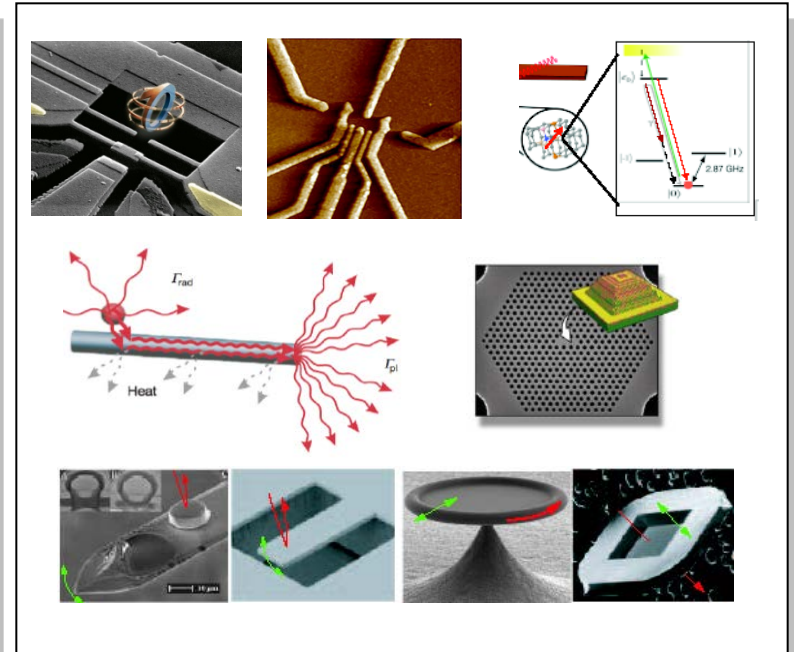
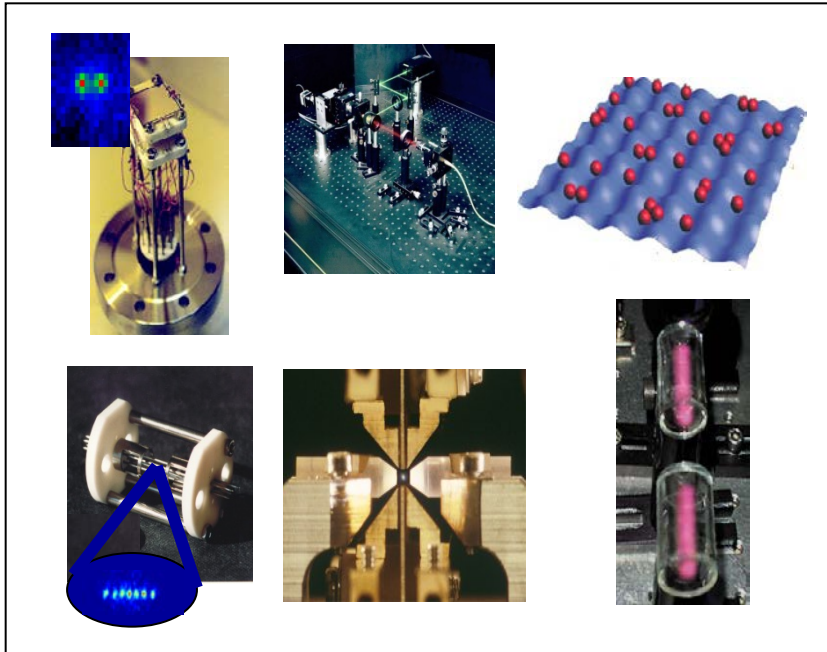
Ricard Feynman, 29.12.1959. Annual meeting of the American Physical Society, CALTECH



- When we get to the very, very small world we have a lot of new things that would happen that represent completely new opportunities for design.
- Atoms on a small scale behave like nothing on a large scale, for they satisfy the laws of quantum mechanics.
- We are working with different laws, and we can expect to do different things.



QUANTUM PHYSICS PROGRESS



Simulating Physics with Computers

Richard P. Feynman

Department of Physics, California Institute of Technology, Pasadena, California 91107

Received May 7, 1981



1. INTRODUCTION

On the program it says this is a keynote speech—and I don't know what a keynote speech is. I do not intend in any way to suggest what should be in this meeting as a keynote of the subjects or anything like that. I have

be understood very well in analyzing the situation. And I'm not happy with all the analyses that go with just the classical theory, because nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy. Thank you.



OUTLINE



- Simulating many-body systems
- Analog quantum simulation
- Cold atoms in optical lattices
- Quantum simulation of HEP models
- Tensor Networks and HEP models

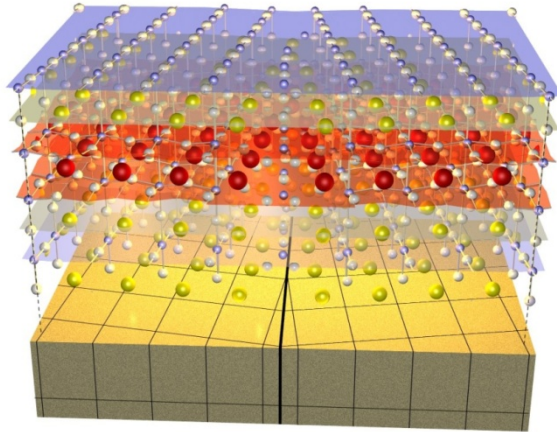
SIMULATING MANY-BODY SYSTEMS



MANY-BODY SYSTEM MODELS

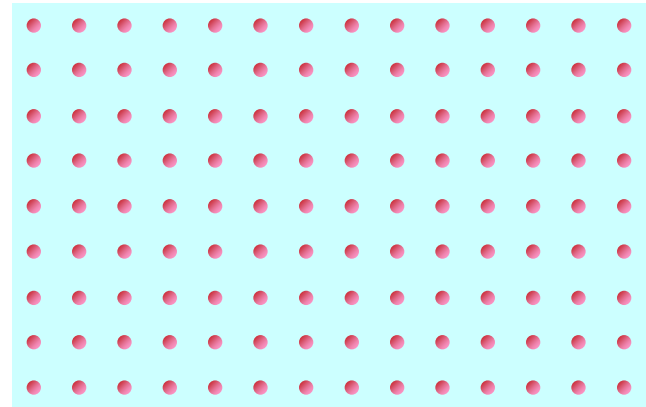


PHYSICAL SYSTEM



- Dynamics
- Thermal equilibrium (T)

MODEL



Model Hamiltonian

$$H = \dots$$

Computation time/memory scales exponentially with the number of constituents



MANY-BODY SYSTEM

CLASSICAL MODELS



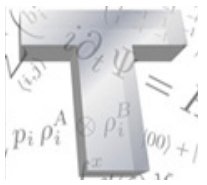
- EXAMPLE: classical spins

Which is the spin configuration at T=0?

	Configuration	Energy
	000...000	0.274
	000...001	0.298
	000...010	0.345
	000...011	-0.177 ●

$E = E(1,2) + E(2,3) + \dots$	111...111	-0.122

- There are 2^N different configurations
- Impossible if $N > 100$



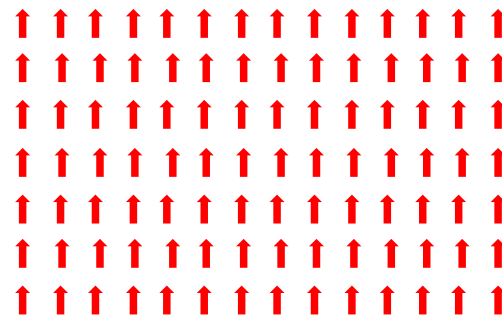
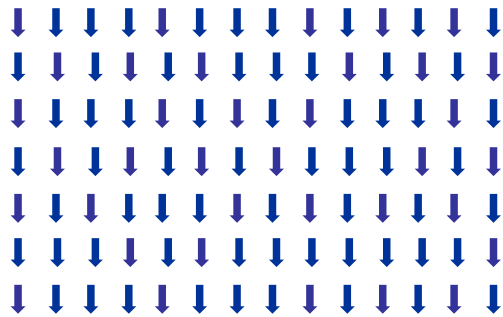
MANY-BODY SYSTEM

CLASSICAL MODELS



- EXAMPLE: classical spins + SYMMETRIES

Which is the spin configuration at $T=0$?



- Ferromagnetic
- Symmetries: simple

The problem is difficult in general (NP-Hard)
In practice, it may turn to be simple

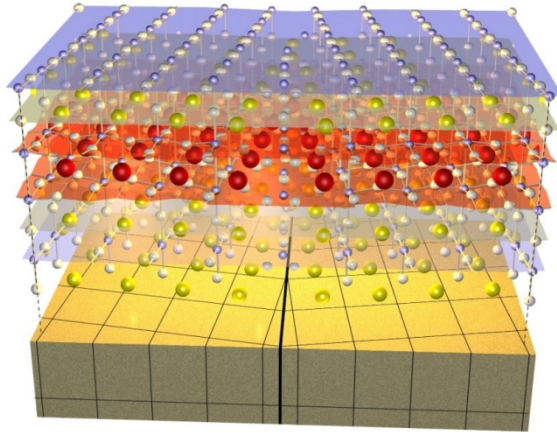


MANY-BODY SYSTEM

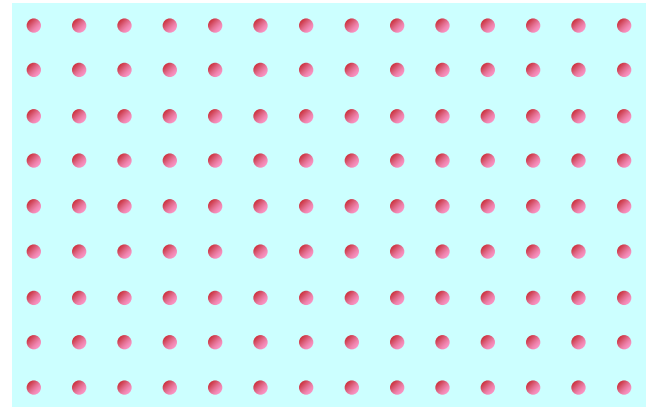
QUANTUM MODELS



PHYSICAL SYSTEM



MODEL

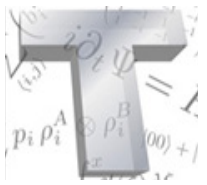


Model Hamiltonian

$$H = \dots$$

Which is the spin configuration at $T=0$?

$$|\Psi\rangle = c_1 |000\dots 0\rangle + c_2 |000\dots 1\rangle + \dots + c_{2^N} |111\dots 1\rangle$$

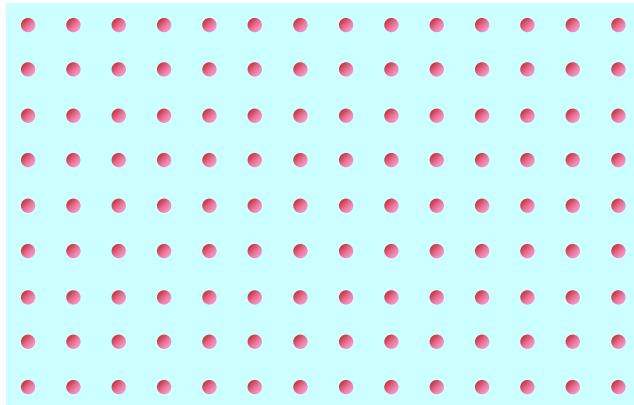


QUANTUM SIMULATIONS

QUANTUM SYSTEMS

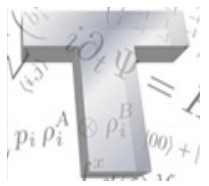


- Quantum spins: $|\Psi\rangle = c_1 |000\dots 0\rangle + c_2 |000\dots 1\rangle + \dots + c_{2^N} |111\dots 1\rangle$



N. spins	Memory
10	128 bytes
50	150 Tbytes
100	10^{29} bytes
500	10^{150} bytes

- Symmetries do not help much



QUANTUM SIMULATIONS

QUANTUM SYSTEMS



- Quantum superpositions are responsible for many physical phenomena:
 - Superconductivity
 - Superfluidity
 - Giant magnetoresistance
 - Insulators (Mott, Anderson, topological)
 - Nuclear reactions
 - Chemical reactions
 - Quark confinement
- There are many theoretical techniques
 - In some relevant problems they fail

ANALOG QUANTUM SIMULATION



Simulating Physics with Computers

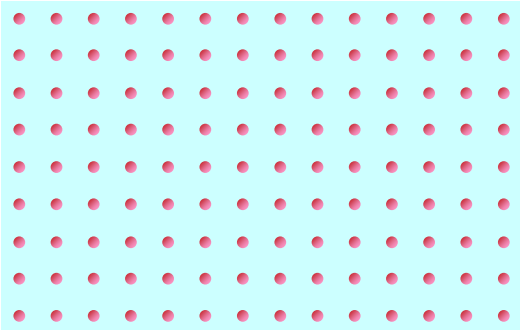
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$$c_1 |000\dots 0\rangle + c_2 |000\dots 1\rangle + \dots + c_{2^N} |111\dots 1\rangle$$

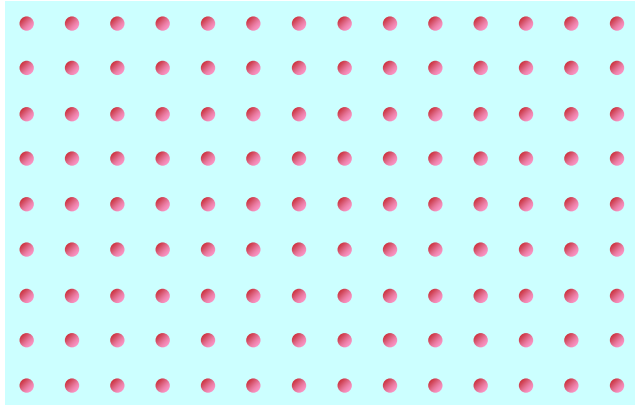


QUANTUM SIMULATORS

ANALOG



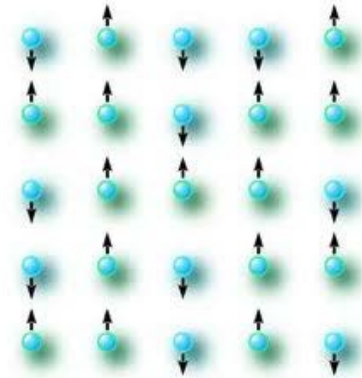
MODEL



Model Hamiltonian

$$H = \dots$$

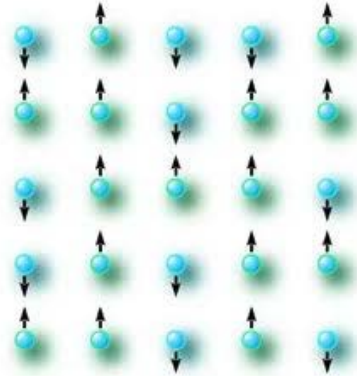
QUANTUM SIMULATOR



Model Hamiltonian

$$H = \dots$$

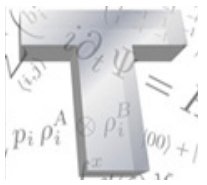
QUANTUM SIMULATORS



$$H = \dots$$

■ Questions:

- Ground state: $H |\Psi_0\rangle = E_0 |\Psi_0\rangle$
- Thermal state: $\rho \propto e^{-H/T}$
- Dynamics: $|\Psi(t)\rangle = e^{-iHt} |\Psi(0)\rangle$
- Physical properties: $\langle \sigma_n \rangle, \langle \sigma_n \sigma_m \rangle, \dots$

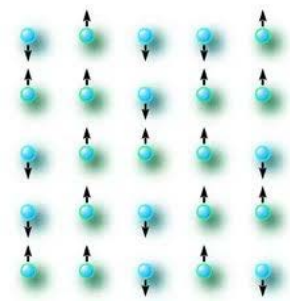


QUANTUM SIMULATORS

ANALOG

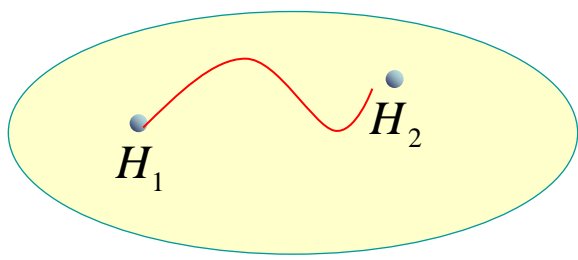


How does it work?

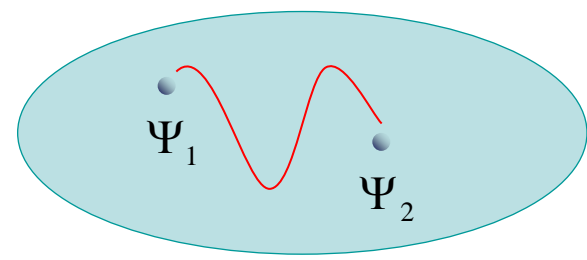


- Dynamics:
- Ground state:

Hamiltonians H



States $|\Psi\rangle$

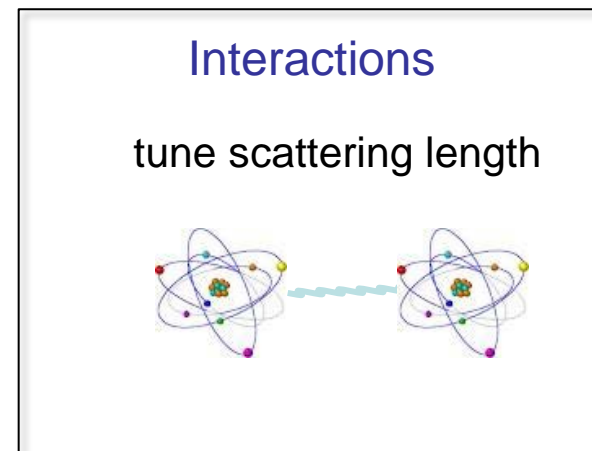
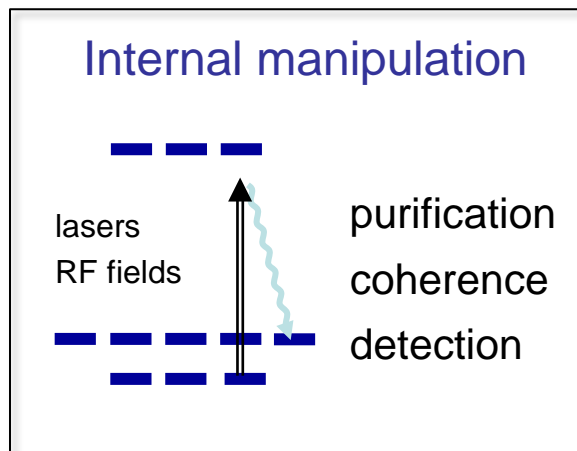
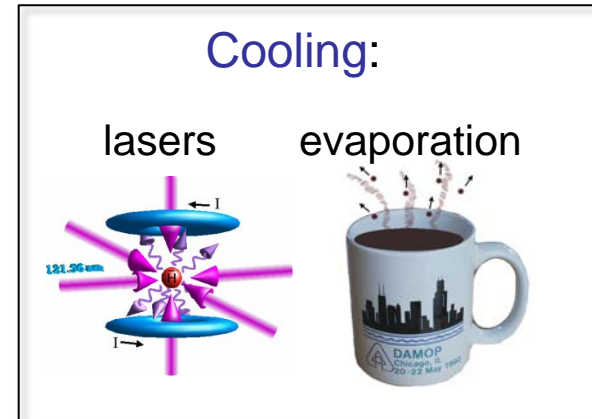
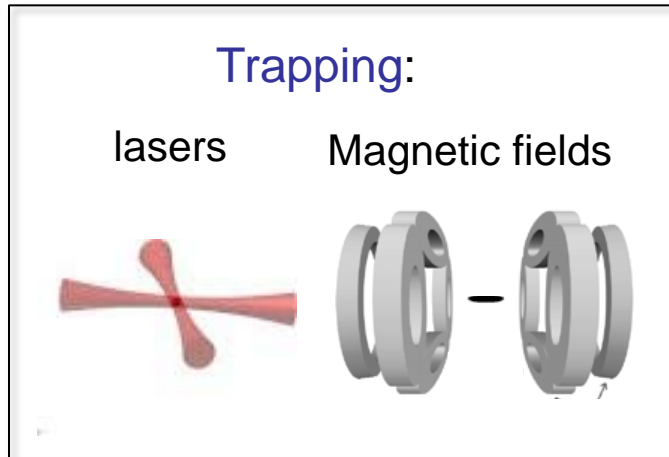


COLD ATOMS IN OPTICAL LATTICES

COLD ATOMS



- Control: External fields



COLD ATOMS

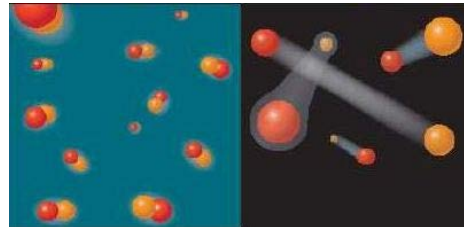
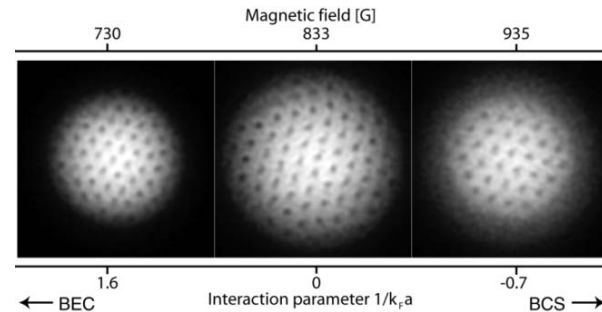
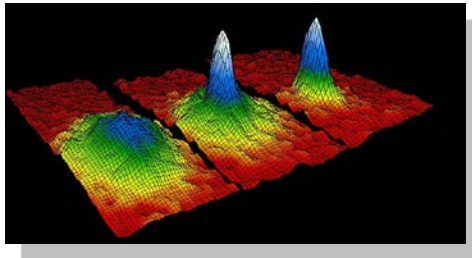
ACHIEVEMENTS



■ Many-body phenomena

- Degeneracy: bosons and fermions (BE/FD statistics)
- Coherence: interference, atom lasers, four-wave mixing, ...
- Superfluidity: vortices
- Disorder: Anderson localization
- Fermions: BCS-BEC

+ many other phenomena



- Cold atoms are described by simple quantum field theories:

$$H = \int \Psi_{\sigma}^{\dagger} \left(-\nabla^2 + V(r) \right) \Psi_{\sigma} + u_{\sigma_i} \int \Psi_{\sigma_1}^{\dagger} \Psi_{\sigma_2}^{\dagger} \Psi_{\sigma_3} \Psi_{\sigma_4}$$

- We can have bosons or fermions (or both).
- We can have different internal states (spin).
- The external potential, V , and interaction coefficients, u , can be engineered using lasers, and electric and magnetic fields.
- In certain limits, one obtains effective theories that are interesting in other fields of Physics.



Quantum Simulations



COLD ATOMS

OPTICAL LATTICES



- Laser standing waves: dipole-trapping

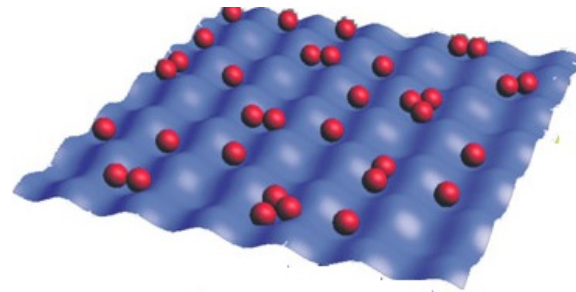
VOLUME 81, NUMBER 15

PHYSICAL REVIEW LETTERS

12 OCTOBER 1998

Cold Bosonic Atoms in Optical Lattices

D. Jaksch,^{1,2} C. Bruder,^{1,3} J. I. Cirac,^{1,2} C. W. Gardiner,^{1,4} and P. Zoller^{1,2}



COLD ATOMS

OPTICAL LATTICES



- Laser standing waves: dipole-trapping

$$H = \int \Psi_{\sigma}^{\dagger} \left(-\nabla^2 + V(r) \right) \Psi_{\sigma} + u_{\sigma_i} \int \Psi_{\sigma_1}^{\dagger} \Psi_{\sigma_2}^{\dagger} \Psi_{\sigma_3} \Psi_{\sigma_4}$$

Lattice theory: Bose/Fermi-Hubbard model

$$H = -t \sum_n \left(a_n^{\dagger} a_{n+1} + h.c \right) + U \sum_n a_n^{\dagger 2} a_n^2$$

COLD ATOMS

OPTICAL LATTICES



- Laser standing waves: dipole-trapping

$$H = \int \Psi_{\sigma}^{\dagger} \left(-\nabla^2 + V(\mathbf{r}) \right) \Psi_{\sigma} + u_{\sigma_i} \int \Psi_{\sigma_1}^{\dagger} \Psi_{\sigma_2}^{\dagger} \Psi_{\sigma_3} \Psi_{\sigma_4}$$

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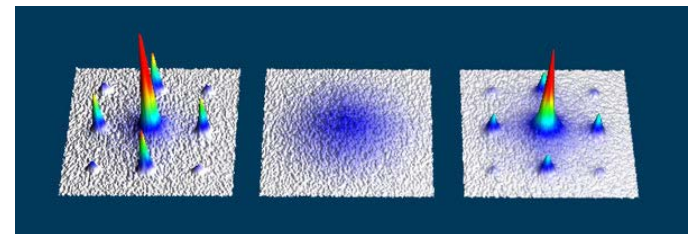
articles

Quantum phase transition from a superfluid to a Mott insulator in a gas of ultracold atoms

Markus Greiner*, Olaf Mandel*, Tilman Esslinger†, Theodor W. Hänsch* & Immanuel Bloch*

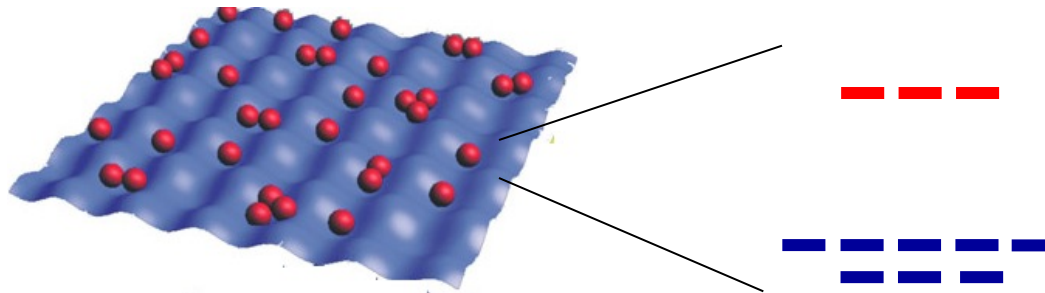
* Sektion Physik, Ludwig-Maximilians-Universität, Schellingstrasse 4/III, D-80799 Munich, Germany, and Max-Planck-Institut für Quantenoptik, D-85748 Garching, Germany

† Quantenelektronik, ETH Zürich, 8093 Zurich, Switzerland



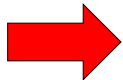
COLD ATOMS

QUANTUM SIMULATION



▪ Bosons/Fermions:
$$H = - \sum_{\substack{\langle n,m \rangle \\ \sigma, \sigma'}} (t_{\sigma, \sigma'} a_{n, \sigma}^\dagger a_{m, \sigma'} + h.c.) + \sum_{\substack{n \\ \sigma, \sigma'}} U_{\sigma, \sigma'} a_{n, \sigma}^\dagger a_{n, \sigma'}^\dagger a_{n, \sigma} a_{n, \sigma}$$

▪ Spins:
$$H = - \sum_{\substack{\langle n,m \rangle \\ \sigma, \sigma'}} (J_x S_n^x S_m^x + J_y S_n^y S_m^y + J_z S_n^z S_m^z) + \sum_{\substack{n \\ \sigma, \sigma'}} B_n S_n^z$$

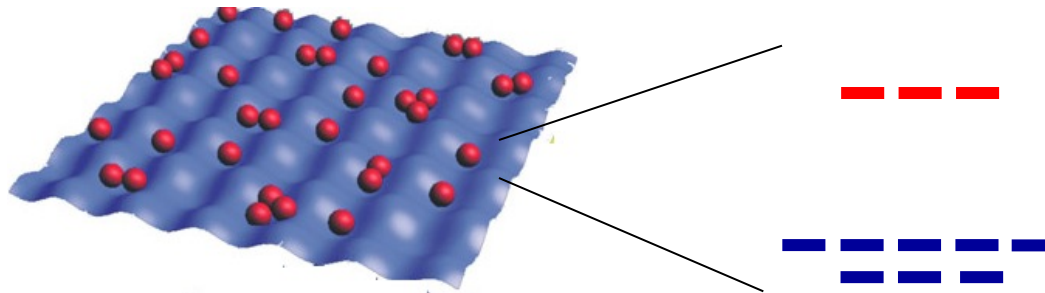


CONDENSED MATTER PHYSICS

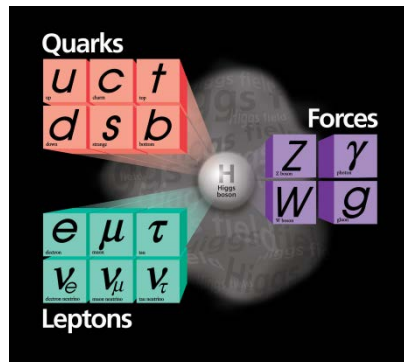


COLD ATOMS

QUANTUM SIMULATION



HIGH ENERGY PHYSICS?



QUANTUM SIMULATIONS OF HEP MODELS

QUANTUM SIMULATION HEP MODELS

INGREDIENTS



$$S = \int \bar{\Psi}(i\gamma^\mu \partial_\mu - m)\Psi - Q \int A_\mu \bar{\Psi}\gamma^\mu\Psi - \frac{1}{4} \int F_{\mu\nu}F^{\mu\nu} + \dots$$

- Matter + Gauge Fields
- Relativistic theory
- Gauge invariant

▪ Hamiltonian formulation: $i\partial_t |\Psi\rangle = H |\Psi\rangle$

- Gauss law

$$G(x) |\Psi\rangle = 0$$

$$[H, G(x)] = 0$$

QUANTUM SIMULATION HEP MODELS

INGREDIENTS



■ Problem:

$$H = \int \Psi_\sigma^\dagger \left(-\nabla^2 + V(r) \right) \Psi_\sigma + u \int \Phi_\mu^\dagger \Phi_\sigma \Psi_\sigma^\dagger \Psi_\sigma + v \int \Phi_\sigma^\dagger \Phi_\sigma^\dagger \Phi_\sigma \Phi_\sigma + \dots$$

choose $V(r)$, u , v , etc such that (in some limit), we have

$$\begin{aligned} i\partial_t |\Psi\rangle &= H |\Psi\rangle \\ G(x) |\Psi\rangle &= 0 \end{aligned} \quad [H, G(x)] = 0$$

corresponding to

$$S = \int \bar{\Psi} (i\gamma^\mu \partial_\mu - m) \Psi - Q \int A_\mu \bar{\Psi} \gamma^\mu \Psi - \frac{1}{4} \int F_{\mu\nu} F^{\mu\nu} + \dots$$

QUANTUM SIMULATION HEP MODELS

INGREDIENTS



■ Matter + Gauge Fields

$$S = \int \bar{\Psi}(i\gamma^\mu \partial_\mu - m)\Psi - Q \int A_\mu \bar{\Psi} \gamma^\mu \Psi - \frac{1}{4} \int F_{\mu\nu} F^{\mu\nu} + \dots$$



We need **bosonic** and **fermionic** atoms

We need **interactions** among themselves

$$H = \int \Psi_\sigma^\dagger (-\nabla^2 + V(r)) \Psi_\sigma + u \int \Phi_\mu^\dagger \Phi_{\sigma'} \Psi_\sigma^\dagger \Psi_{\sigma'} + v \int \Phi_\sigma^\dagger \Phi_{\sigma'}^\dagger \Phi_{\sigma'} \Phi_\sigma + \dots$$

QUANTUM SIMULATION HEP MODELS

INGREDIENTS

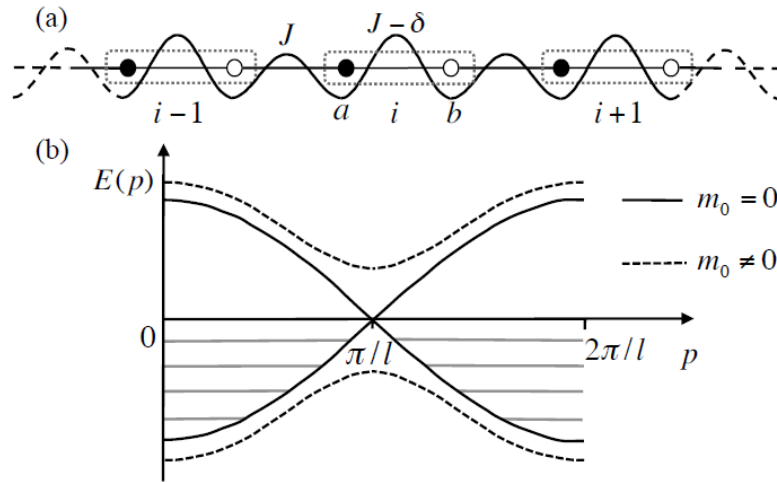


Relativistic

$$S = \int \bar{\Psi}(i\gamma^\mu \partial_\mu - m)\Psi - Q \int A_\mu \bar{\Psi}\gamma^\mu\Psi - \frac{1}{4} \int F_{\mu\nu}F^{\mu\nu} + \dots$$

$$H = \int \Psi_\sigma^\dagger (-\nabla^2 + V(r))\Psi_\sigma + u \int \Phi_\mu^\dagger \Phi_{\sigma'} \Psi_\sigma^\dagger \Psi_{\sigma'} + v \int \Phi_\sigma^\dagger \Phi_{\sigma'}^\dagger \Phi_{\sigma'} \Phi_\sigma + \dots$$

➔ Use a superlattice: it possesses the right limit in the continuum



(staggered fermions)



QUANTUM SIMULATION HEP MODELS

INGREDIENTS



$$H = \int \Psi_\sigma^\dagger (-\nabla^2 + V(r)) \Psi_\sigma + u \int \Phi_\mu^\dagger \Phi_\sigma \Psi_\sigma^\dagger \Psi_\sigma + v \int \Phi_\sigma^\dagger \Phi_\sigma^\dagger \Phi_\sigma \Phi_\sigma + \dots$$

Lattice

Fermion-gauge field
coupling

Gauge field
dynamics

▪ Matter + Gauge Fields

▪ Relativistic theory

▪ Hamiltonian formulation:

- Gauge invariance: abelian, non-abelian
- Gauss law

➔ Bosonic and Fermionic atoms
Low energy sector

➔ Lattices

➔ Angular momentum

➔ Interactions / Initial conditions

+ perturbation theory

J. Kogut and L. Susskind, Phys. Rev. D **11**, 395 (1975).
 J. B. Kogut, Rev. Mod. Phys. **51**, 659 (1979).
 J. B. Kogut, Rev. Mod. Phys. **55**, 775 (1983).

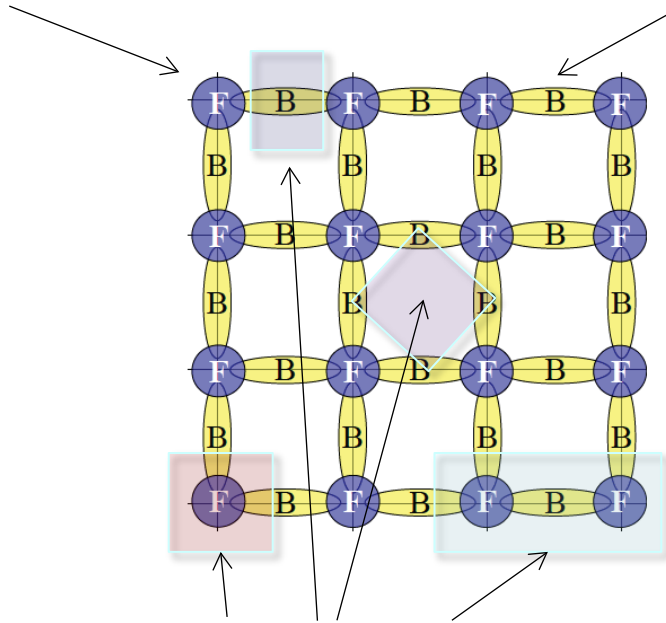
HEP LATTICE MODELS

HAMILTONIAN FORMULATION



Matter (Fermions): can move

Gauge fields (Bosons): Static



- Hamiltonian: $H = H_M + H_{KS} + H_{\text{int}}$

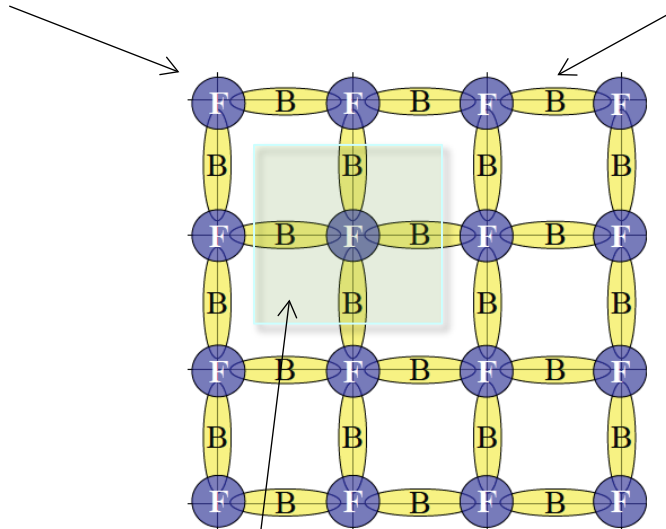
HEP LATTICE MODELS

HAMILTONIAN FORMULATION



Matter (Fermions): can move

Gauge fields (Bosons): Static



- **Hamiltonian:** $H = H_M + H_{KS} + H_{\text{int}}$
- **Gauge invariance:** Gauge group: $U(1)$, Z_N , $SU(N)$, etc

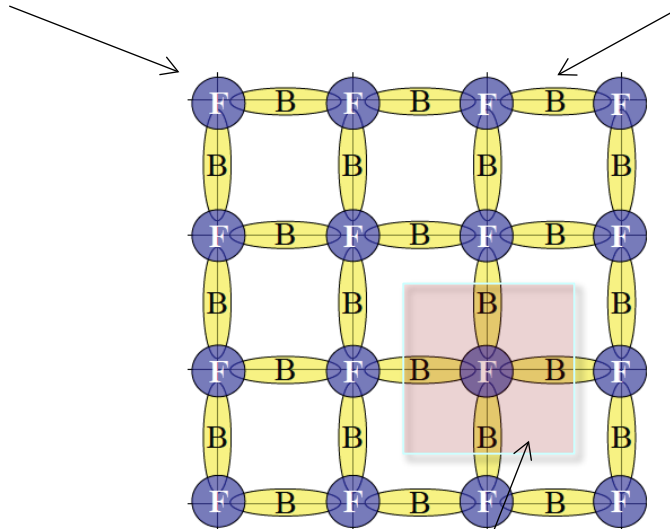
HEP LATTICE MODELS

HAMILTONIAN FORMULATION



Matter (Fermions): can move

Gauge fields (Bosons): Static



- **Hamiltonian:** $H = H_M + H_{KS} + H_{\text{int}}$
- **Gauge invariance:** Gauge group: $U(1)$, Z_N , $SU(N)$, etc
- **Gauss law:** $G_{\text{plaquette}} | \text{phys} \rangle = 0$

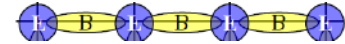
HEP LATTICE MODELS

HAMILTONIAN FORMULATION



Example: compact-QED in 1D

- Hamiltonians:



$$H_M = \sum_{\mathbf{n}} M_{\mathbf{n}} \psi_{\mathbf{n}}^{\dagger} \psi_{\mathbf{n}}$$

$$H_{int} = \epsilon \sum_{\mathbf{n}, \mathbf{k}} \left(\psi_{\mathbf{n}}^{\dagger} e^{i\phi_{\mathbf{n}, \mathbf{k}}} \psi_{\mathbf{n}+\mathbf{k}} + \psi_{\mathbf{n}+\mathbf{k}}^{\dagger} e^{-i\phi_{\mathbf{n}, \mathbf{k}}} \psi_{\mathbf{n}} \right)$$

$$H_{KS} = H_E = \frac{g^2}{2} \sum_{\mathbf{n}, \mathbf{k}} E_{\mathbf{n}}^2$$

$$[E_{\mathbf{n}, \mathbf{k}}, \phi_{\mathbf{m}, \mathbf{l}}] = -i\delta_{\mathbf{n}\mathbf{m}}\delta_{\mathbf{k}\mathbf{l}} \quad (\text{ie, compact})$$

- Gauss law: $G_n |phys\rangle = 0$
 - Gauge invariance: $e^{-i\theta G_n} H e^{i\theta G_n} = H$
- $$G_n = E_{n+1} - E_n - \psi_n^{\dagger} \psi_n$$

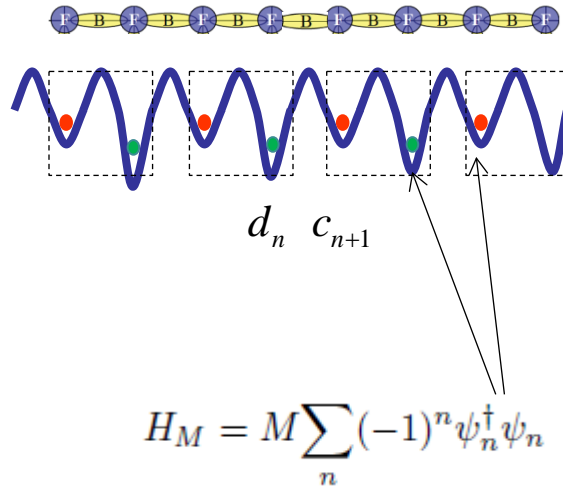
SCHWINGER MODEL

QUANTUM SIMULATION

SCHWINGER MODEL 1+1



Fermions:



internal states



$$\{c_n, c_n^\dagger\} = \{d_n, d_n^\dagger\} = 1$$

$$H = \int \Psi_\sigma^\dagger (-\nabla^2 + V(r)) \Psi_\sigma + u \int \Phi_\mu^\dagger \Phi_\sigma \Psi_\sigma^\dagger \Psi_\sigma + v \int \Phi_\sigma^\dagger \Phi_\sigma^\dagger \Phi_\sigma \Phi_\sigma + \dots$$

- Even sites: hole = particle
- Odd sites: fermion = antiparticle

Staggered Fermions:

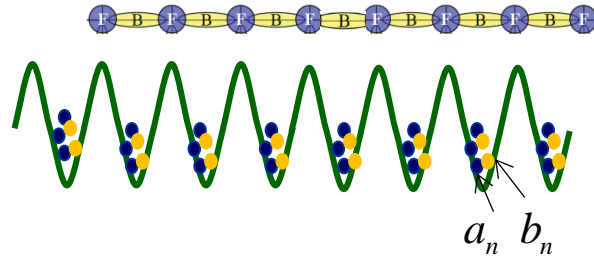
- L. Susskind, Phys. Rev. D **16**, 3031 (1977).
- G. 't Hooft, Nucl. Phys. B **75**, 461 (1974)

QUANTUM SIMULATION

SCHWINGER MODEL 1+1



■ Bosons:



internal states

$$[a_n, a_n^\dagger] = [b_n, b_n^\dagger] = 1$$

● Schwinger rep:

$$\begin{aligned}
 L_+ &= a^\dagger b \\
 L_z &= \frac{1}{2} (N_a - N_b) \\
 \ell &= \frac{1}{2} (N_a + N_b)
 \end{aligned}
 \quad \xrightarrow{\ell \gg 1} \quad
 \begin{aligned}
 L_+ &\approx \ell e^{i\phi} \\
 L_z &\approx i\partial_\phi
 \end{aligned}$$

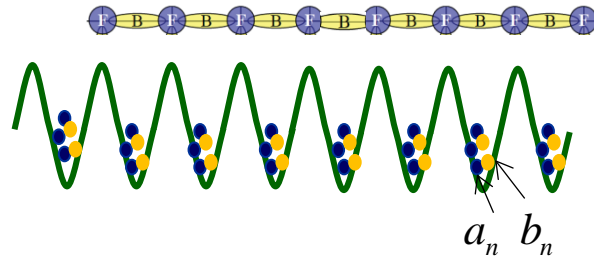
- If ℓ is small (eg 2 atoms), we obtain a truncated version
- One can also use a single atom with few internal levels (Z_M is the gauge group)

QUANTUM SIMULATION

SCHWINGER MODEL 1+1



□ Bosons:



internal states



$$[a_n, a_n^\dagger] = [b_n, b_n^\dagger] = 1$$

Schwinger rep:

$$L_+ = a^\dagger b$$

$$L_z = \frac{1}{2} (N_a - N_b)$$

$$\ell = \frac{1}{2} (N_a + N_b)$$

$$H_E = \frac{g^2}{2} \sum_n L_{z,n}^2$$

$$= \frac{g^2}{8} \sum_n (N_{a,n}^2 + N_{b,n}^2 - 2N_{a,n}N_{b,n})$$

$$H = \int \Psi_\sigma^\dagger (-\nabla^2 + V(r)) \Psi_\sigma + u \int \Phi_\mu^\dagger \Phi_\sigma \Psi_\sigma^\dagger \Psi_\sigma + v \int \Phi_\sigma^\dagger \Phi_\sigma^\dagger \Phi_\sigma \Phi_\sigma + \dots$$

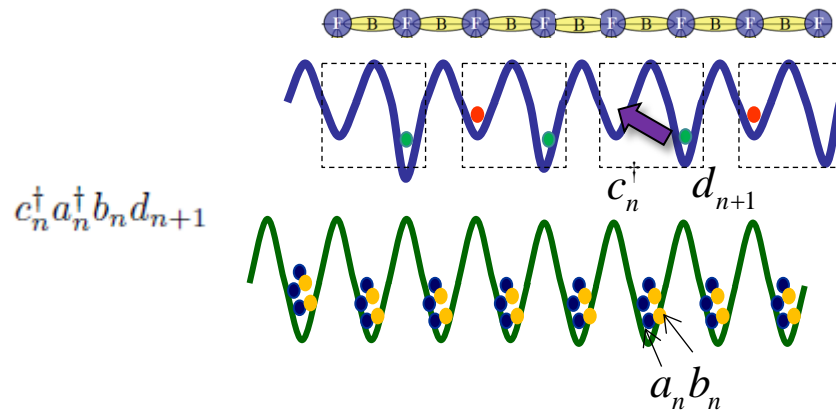


QUANTUM SIMULATION

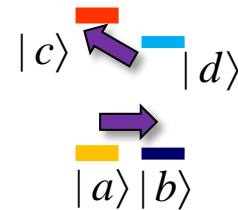
SCHWINGER MODEL 1+1



Interactions:



internal states



↕ 2M

$$H = \int \Psi_\sigma^\dagger (-\nabla^2 + V(r)) \Psi_\sigma + u \int \Phi_\mu^\dagger \Phi_{\sigma'} \Psi_\sigma^\dagger \Psi_{\sigma'} + v \int \Phi_\sigma^\dagger \Phi_{\sigma'}^\dagger \Phi_{\sigma'} \Phi_\sigma + \dots$$

conserves angular momentum locally

→ Gauge invariance

$$H_{int} \frac{1}{\sqrt{\ell(\ell+1)}} \psi_n^\dagger a_n^\dagger b_n \psi_{n+1} \approx \psi_n^\dagger e^{i\phi_n} \psi_{n+1}$$

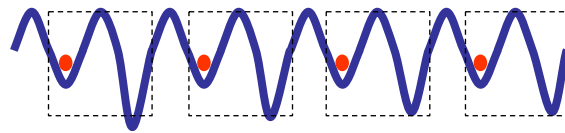
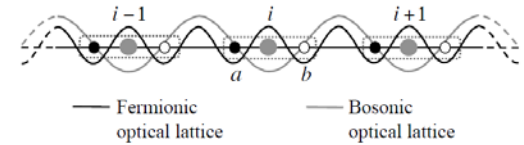


QUANTUM SIMULATION

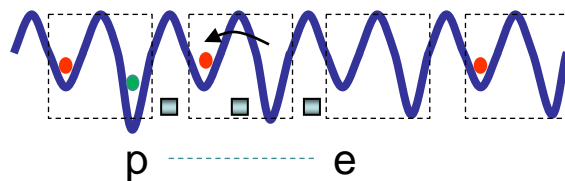
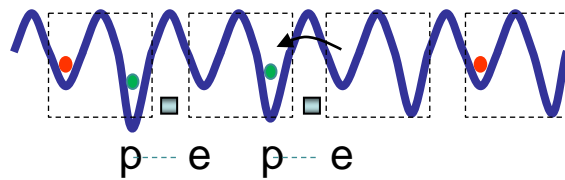
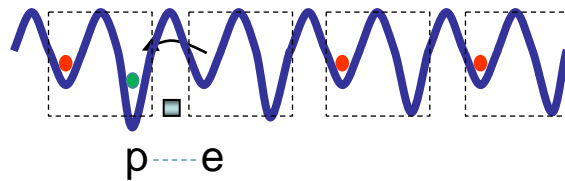
SCHWINGER MODEL 1+1



Physical processes:



non-interacting vacuum



TABLE

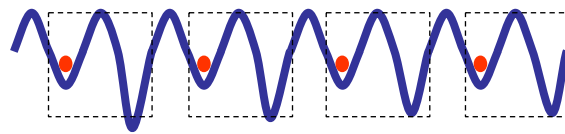
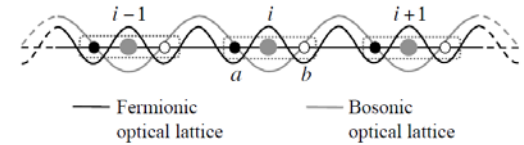
$ 0\rangle_e 0\rangle_p$	
$ 1\rangle_e 0\rangle_p$	
$ 1\rangle_e 1\rangle_p$	
$ 0\rangle_e 1\rangle_p$	

QUANTUM SIMULATION

SCHWINGER MODEL 1+1



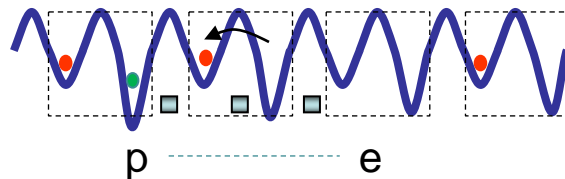
Preparation:



non-interacting vacuum



switch on interactions

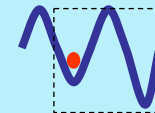


interacting vacuum

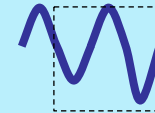
- Confinement
- Excitations: vector + scalar
- Time-dependent phenomena
- First experiments: few bosonic atoms

TABLE

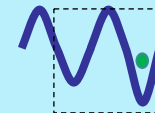
$$|0\rangle_e |0\rangle_p$$



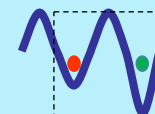
$$|1\rangle_e |0\rangle_p$$



$$|1\rangle_e |1\rangle_p$$



$$|0\rangle_e |1\rangle_p$$



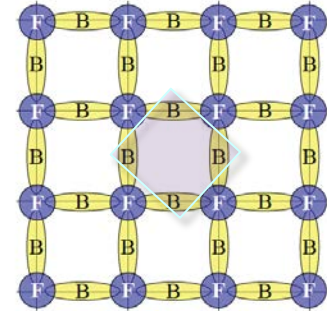
QUANTUM SIMULATION

HIGHER DIMENSIONS, NON-ABELIAN



- Plaque interactions:

$$H_B = -\frac{2\epsilon^4}{\lambda^3} \sum_{\mathbf{n}} \left(U_{\mathbf{n},1} U_{\mathbf{n}+\hat{1},2} U_{\mathbf{n}+\hat{2},1}^\dagger U_{\mathbf{n},2}^\dagger + h.c. \right) = -\frac{4\epsilon^4}{\lambda^3} \sum_{\mathbf{n}} \cos \left(\phi_{\mathbf{n},1} + \phi_{\mathbf{n}+\hat{1},2} - \phi_{\mathbf{n}+\hat{2},1} - \phi_{\mathbf{n},2} \right)$$

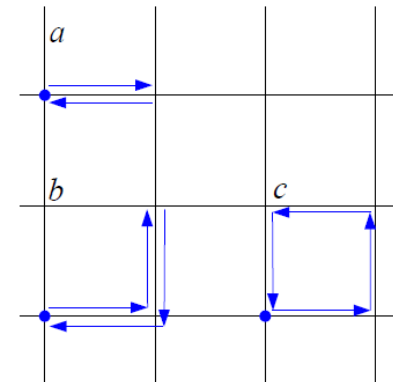


- Non-abelian gauge theories:

$$H_E = \frac{g^2}{2} \sum_{\mathbf{n},k,a} (E_{\mathbf{n},k})_a (E_{\mathbf{n},k})_a$$

$$H_B = -\frac{1}{g^2} \sum_{\text{plaquettes}} \left(\text{Tr} \left(U_1 U_2 U_3^\dagger U_4^\dagger \right) + h.c. \right)$$

$$H_{int} = \epsilon \sum_{\mathbf{n},k} \left(\psi_{\mathbf{n}}^\dagger U_{\mathbf{n},k}^r \psi_{\mathbf{n}+\hat{k}} + h.c. \right)$$



Link



L R

{a₁, a₂} {b₁, b₂}

bosonic modes



COLD ATOMS

EXPERIMENTAL CONSIDERATIONS



- Cold bosons in optical lattices
 - Mott insulator – superfluid transition
 - Exchange interaction (2nd order perturbation theory)
 - Dynamics
 - Anderson-Higgs mechanism in 2D

- Cold fermions in optical lattices
 - Mott insulator in 2D

- Cold fermions and bosons in optical lattices
 - Mean-field dynamics

- Techniques
 - Tuning of interactions: Magnetic/optical Feschbach resonances
 - Lattice geometry
 - Time of flight measurements
 - Single-site addressing: initializaton
 - Single-site measurement

- Challenges: temperature, decoherence, control ...



QUANTUM SIMULATION HIGH ENERGY MODELS



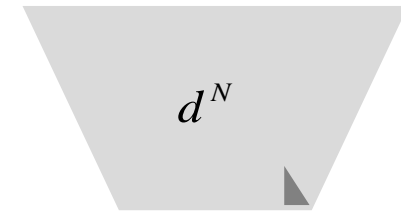
NEW METHODS: TENSOR NETWORKS

QUANTUM STATES

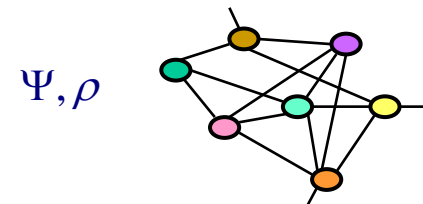


Hilbert space

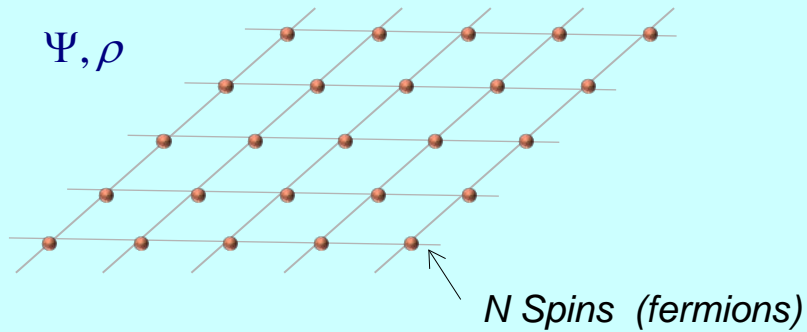
$$|\Psi\rangle = \sum c^{i_1, i_2, \dots, i_N} |i_1, i_2, \dots, i_N\rangle$$



Tensor networks



MANY-BODY QUANTUM SYSTEM



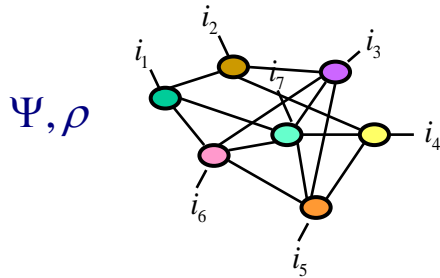


TENSORS NETWORKS

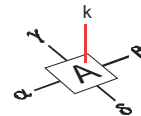
EFFICIENT DESCRIPTIONS



TENSOR NETWORK STATES:

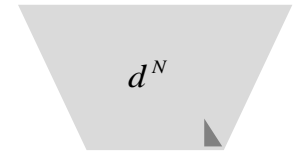


N tensors:



parameters: Nd^z

compare



- Provide a local description
- Represent a wide range of physical behavior
- Classification of (gapped) phases (in 1D)
- Bulk-boundary correspondence
- Algorithms



TENSORS NETWORKS

APPLICATIONS



1+1 dimensions:

- Schwinger models
- Non-abelian models
- $T=0$, finite temperature
- Dynamics

Collaboration: M. Banuls, S. Kühn (MPQ)
K. Jansen, H. Saito (Desy)
K. Cichy (Frankfurt)

2+1 dimensions:

- (non-relativistic) abelian
- (non-relativistic) non-abelian models
- $T=0$

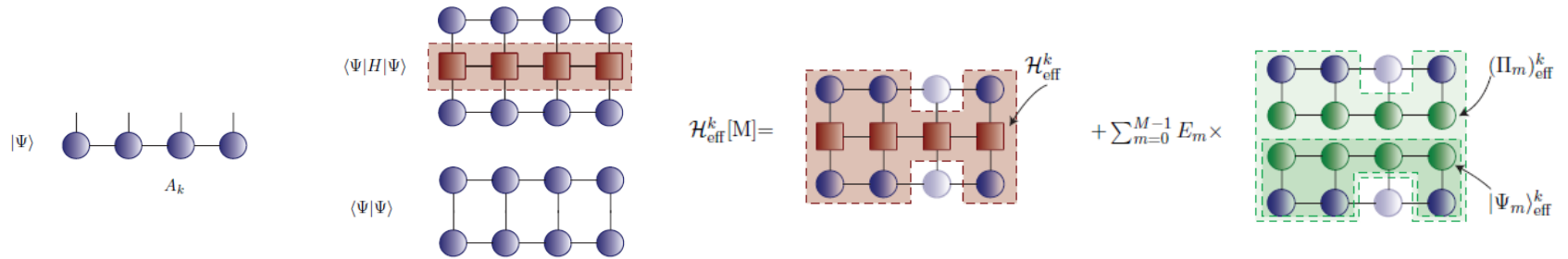
Collaboration: E. Zohar, M. Burrello (MPQ)
T. Wahl (Cambridge)
J. Haegeman, F. Verstraete (Ghent)

NUMERICAL METHODS

SCHWINGER MODEL 1+1



Method: Tensor networks



Results

	Scalar binding energy		
m/g	MPS with OBC	SCE result [35]	exact
0	1.1283(10)	1.11(3)	1.12838
0.125	1.221(2)	1.22(2)	-
0.25	1.239(6)	1.24(3)	-
0.5	1.213(5)	1.20(3)	-

NUMERICAL METHODS

SCHWINGER MODEL 1+1



Truncation + adiabatic evolution

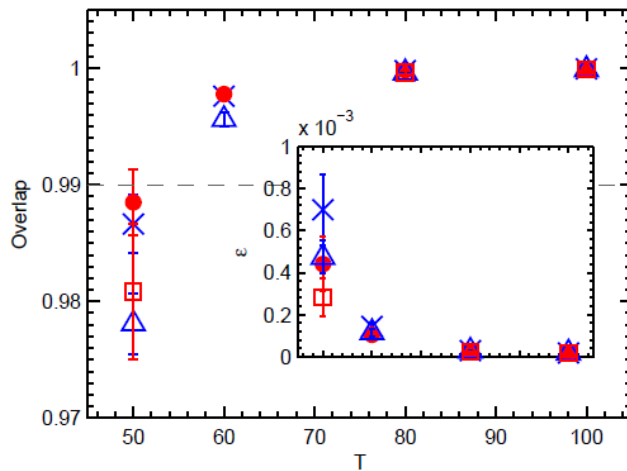


FIG. 3. Truncated cQED model with $D = 50$. Final overlap with the exact ground state at the end of the adiabatic preparation as a function of the total evolution time. The (blue) \times 's represent the data for $N = 50$, $d = 3$; (blue) triangles for $N = 100$, $d = 3$; (red) circles, for $N = 50$, $d = 9$; and (red) squares for $N = 100$, $d = 9$. Error bars were obtained from the difference in results with bond dimension $D = 50$ vs $D = 30$. Inset: Relative error of the energy with respect to the exact ground state.

Broken gauge invariance:

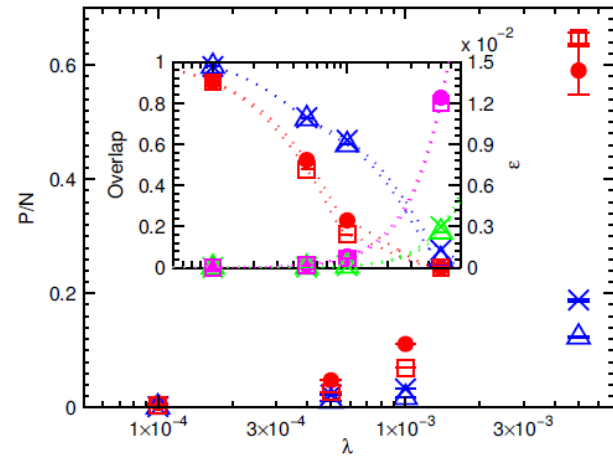


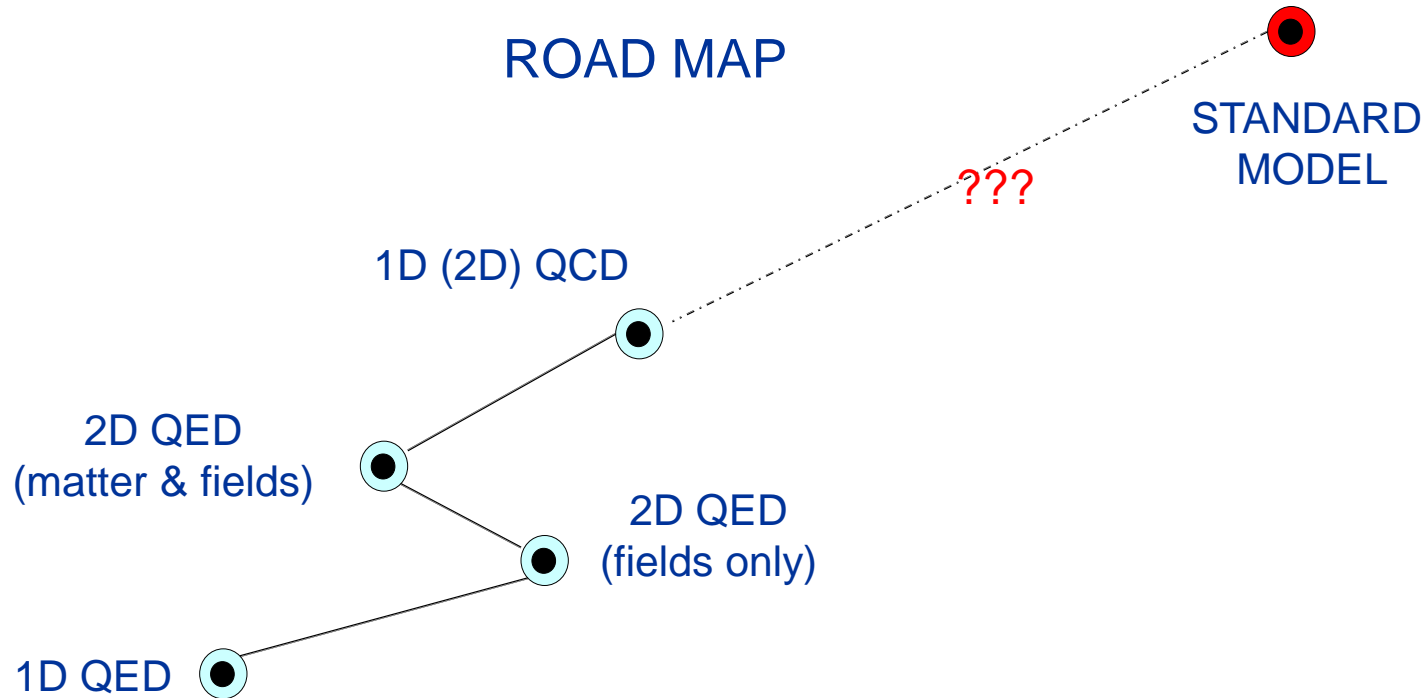
FIG. 5. Truncated cQED model. Penalty energy per site at the end of the noisy adiabatic preparation as a function of the noise strength. The [blue (green)] \times 's represent the values for $N = 50$, $d = 3$; the [blue (green)] triangles, the $N = 100$, $d = 3$ case; the [red (magenta)] circles, the $N = 50$, $d = 5$ case; and the [red (magenta)] squares, the $N = 100$, $d = 5$ case. Error bars were computed the same way as in the noiseless case. Inset: Overlap (blue and red symbols) and relative error in energy (green and magenta symbols) with respect to the noise-free exact ground state. As a guide for the eye, data points are connected.



QUANTUM SIMULATION HIGH ENERGY MODELS



ROAD MAP



IC, Maraner, Pachos, PRL **105**, 19403 (2010)
Zohar, IC, Reznik, PRL **107**, 275301 (2011)
Zohar, IC, Reznik, PRL **109**, 125302 (2012)
Zohar, IC, Reznik, PRL **110**, 125304 (2013)
[Zohar, IC, Reznik, PRA **88**, 023617 \(2013\)](#)
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Saito, Banuls, Cichy, IC, Jasnen
Kühn, IC, Banuls, PRA **90**, 042305 (2014)
Banuls, Cichy, IC, Jansen, Saito, arXiv:1505:00279
Kühn, Zohar, IC, Banuls, arXiv:1505.04441

See also:

Kapit, Mueller, PRA **83**, 033625 (2011)
Banerjee, ..., Wiese, Zoller, PRL **109**, 175302 (2013)
Banerjee, ..., Wiese, Zoller, PRL **110**, 125303 (2013)
Gauge fields: Lewenstein et al