Theory of double parton scattering: basics and open questions

M. Diehl

Deutsches Elektronen-Synchroton DESY

DESY Forum on Double Parton Scattering
19 and 20 May 2015

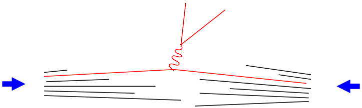
Hadron-hadron collisions

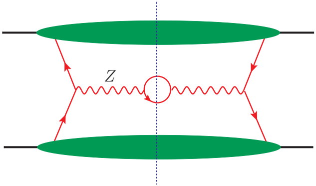
- standard description based on factorization formulae

cross sect = parton distributions × parton-level cross sect

example: Z production

$$pp \to Z + X \to \ell^+\ell^- + X$$





- factorization formulae are for inclusive cross sections $pp \to Y + X$
  where $Y =$ produced in parton-level scattering, specified in detail
  $X =$ summed over, no details

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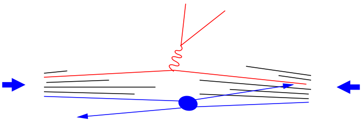
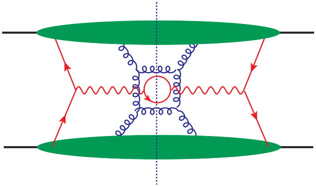
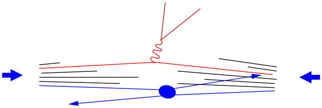
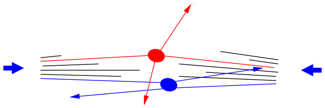




- factorization formulae are for inclusive cross sections $pp \to Y + X$
where $Y =$ produced in parton-level scattering, specified in detail
$X =$ summed over, no details
- also have interactions between "spectator" partons
their effects cancel in inclusive cross sections thanks to unitarity
but they affect the final state X

Multiparton interactions





- secondary, tertiary etc. interactions generically take place in hadron-hadron collisions
- predominantly low-p_T scattering $\rightsquigarrow$ underlying event
- at high collision energy can be hard $\rightsquigarrow$ multiple hard scattering
- many studies:

  theory: phenomenology, theory foundations (1980s, recent activity)

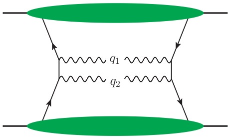
  experiment: ISR, SPS, HERA (photoproduction), Tevatron, LHC

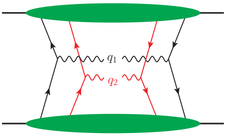
  Monte Carlo generators: Pythia, Herwig++, Sherpa, ...

  and ongoing activity: see e.g. the MPI@LHC workshop series `http://indico.cern.ch/event/305160`
- this forum: concentrate on double hard scattering (DPS)

Single vs. double hard scattering

- example: prod'n of two gauge bosons, transverse momenta $\boldsymbol{q}_{T1}$ and $\boldsymbol{q}_{T2}$





single scattering:

$|\boldsymbol{q}_{T1}|$ and $|\boldsymbol{q}_{T1}| \sim$ hard scale Q^2

$|\boldsymbol{q}_{T1} + \boldsymbol{q}_{T2}| \ll Q^2$

double scattering:

both $|\boldsymbol{q}_{T1}|$ and $|\boldsymbol{q}_{T1}| \ll Q^2$

- for transv. mom. $\sim \Lambda \ll Q$:

$$\frac{d\sigma_{\text{single}}}{d^2\boldsymbol{q}_{T1}\, d^2\boldsymbol{q}_{T2}} \sim \frac{d\sigma_{\text{double}}}{d^2\boldsymbol{q}_{T1}\, d^2\boldsymbol{q}_{T2}} \sim \frac{1}{Q^4\,\Lambda^2}$$

but single scattering populates larger phase space :

$$\sigma_{\text{single}} \sim \frac{1}{Q^2} \gg \sigma_{\text{double}} \sim \frac{\Lambda^2}{Q^4}$$

Single vs. double hard scattering

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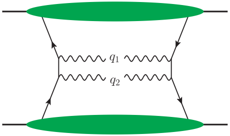
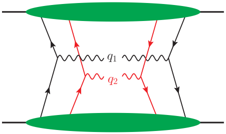




single scattering:

$|\boldsymbol{q}_{T1}|$ and $|\boldsymbol{q}_{T1}| \sim$ hard scale Q^2

$|\boldsymbol{q}_{T1} + \boldsymbol{q}_{T2}| \ll Q^2$

double scattering:

both $|\boldsymbol{q}_{T1}|$ and $|\boldsymbol{q}_{T1}| \ll Q^2$

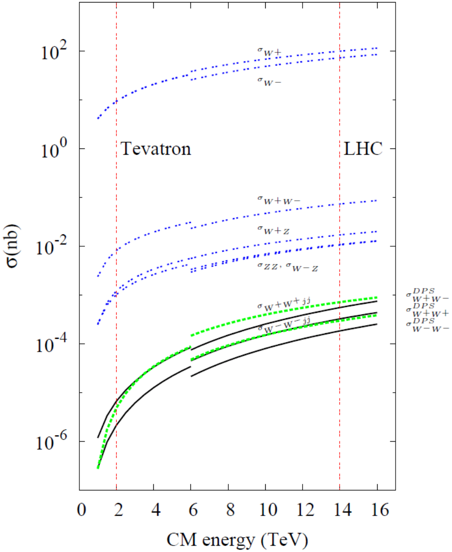
- for small parton mom. fractions x
  double scattering enhanced by parton luminosity
- process dependent:
  enhancement or suppression by parton type (quarks vs. gluons), coupling constants, etc.

A numerical estimate

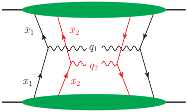
gauge boson pair production

single scattering:
$qq \to qq + W^+W^+$
suppressed by α_s^2

J Gaunt et al, arXiv:1003.3953
based on pocket formula to be discussed shortly



Cross section formula



$$\frac{d\sigma_{\text{double}}}{dx_1\, d\bar{x}_1\, dx_2\, d\bar{x}_2} = \frac{1}{C}\, \hat{\sigma}_1\, \hat{\sigma}_2 \int d^2\boldsymbol{y}\; F(x_1, x_2, \boldsymbol{y})\, F(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$$

C = combinatorial factor
$\hat{\sigma}_i$ = parton-level cross sections
$F(x_1, x_2, \boldsymbol{y})$ = double parton distribution (DPD)
$\boldsymbol{y}$ = transv. distance between partons

- follows from Feynman graphs and hard-scattering approximation
  no semi-classical approximation required
- can make $\hat{\sigma}_i$ differential in further variables (e.g. for jet pairs)
- can extend $\hat{\sigma}_i$ to higher orders in α_s
  get usual convolution integrals over x_i in $\hat{\sigma}_i$ and F

Pocket formula

- make simplest possible assumptions
- if two-parton density factorizes as

$$F(x_1, x_2, \boldsymbol{y}) = f(x_1)\, f(x_2)\, G(\boldsymbol{y})$$

  where $f(x_i) =$ usual PDF

- if assume same $G(\boldsymbol{y})$ for all parton types
  then cross sect. formula turns into

$$\frac{d\sigma_{\text{double}}}{dx_1\, d\bar{x}_1\ dx_2\, d\bar{x}_2} = \frac{1}{C}\, \frac{d\sigma_1}{dx_1\, d\bar{x}_1}\ \frac{d\sigma_2}{dx_2\, d\bar{x}_2}\ \frac{1}{\sigma_{\text{eff}}}$$

  with $1/\sigma_{\text{eff}} = \int d^2\boldsymbol{y}\ G(\boldsymbol{y})^2$

  $\rightsquigarrow$ scatters are completely independent

- underlies bulk of phenomenological estimates
- fails if any of the above assumptions is invalid
  or if original cross sect. formula misses important contributions
  (will encounter examples later)

  cf. Calucci, Treleani 1999; Frankfurt, Strikman, Weiss 2003-04; Blok et al 2013

Pocket formula

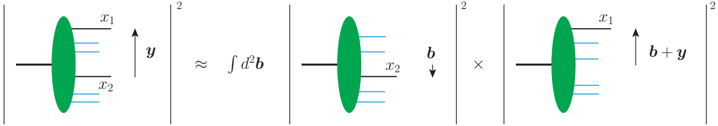
- make simplest possible assumptions
- if two-parton density factorizes as

$$F(x_1, x_2, \boldsymbol{y}) = f(x_1)\, f(x_2)\, G(\boldsymbol{y})$$

- if neglect correlations between two partons

$$G(\boldsymbol{y}) = \int d^2\boldsymbol{b}\; F(\boldsymbol{b})\, F(\boldsymbol{b}+\boldsymbol{y})$$

where $F(\boldsymbol{b})$ = impact parameter distrib. of single parton



- for Gaussian $F(\boldsymbol{b})$ with average $\langle \boldsymbol{b}^2 \rangle$

$$\sigma_{\text{eff}} = 4\pi \langle \boldsymbol{b}^2 \rangle = 41\,\text{mb} \;\times \langle \boldsymbol{b}^2 \rangle / (0.57\,\text{fm})^2$$

phenomen. determinations of $\langle \boldsymbol{b}^2 \rangle$ give $(0.57\,\text{fm} - 0.67\,\text{fm})^2$

is $\gg$ $\sigma_{\text{eff}} \sim 5$ to $20\,\text{mb}$ from experimental extractions ($\rightsquigarrow$ next talks)

same conclusions for alternatives to Gaussian $F(\boldsymbol{b})$

Parton correlations

at certain level of accuracy expect correlations between

- x_1 and x_2 of partons
  - most obvious: energy conservation $\Rightarrow\ x_1 + x_2 \leq 1$
  - significant x_1 – x_2 correlations found in constituent quark model

    Rinaldi, Scopetta, Vento 2013

- x_i and $\boldsymbol{y}$

  even for single partons see correlations between x and $\boldsymbol{b}$ distribution
  - HERA results on $\gamma p \to J/\Psi\, p$ give

    $$\langle \boldsymbol{b}^2 \rangle \propto \text{const} + 4\alpha' \log(1/x) \quad \text{with} \quad 4\alpha' \approx (0.16\,\text{fm})^2$$

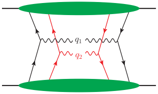
    for gluons with $x \sim 10^{-3}$
  - lattice simulations $\to$ strong decrease of $\langle \boldsymbol{b}^2 \rangle$ with x above ~ 0.1

  plausible to expect similar correlations in double parton distributions

  even if two partons not uncorrelated

  impact on observables: R Corke, T Sjöstrand 2011; B Blok, P Gunnellini 2015

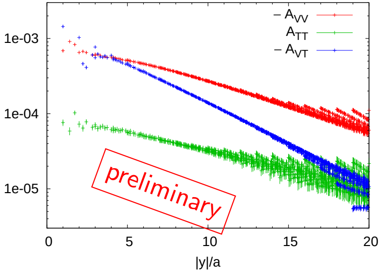
Spin correlations



- polarizations of two partons can be correlated even in unpolarized proton
  - quarks: longitudinal and transverse pol.
  - gluons: longitudinal and linear pol.
- can be included in factorization formula
  $\rightsquigarrow$ extra terms with polarized DPDs and partonic cross sections
- if spin correlations are large → large effects for
  rate and final state distributions of double hard scattering
  A. Manohar, W. Waalewijn 2011; T. Kasemets, MD 2012
  M. Echevarria, T. Kasemets, P. Mulders, C. Pisano 2015
- large spin correlations found in MIT bag model
  Chang, Manohar, Waalewijn 2012
- for x_1, x_2 small: size of correlations unknown
  known: evolution to higher scales tends to wash out polarization
  unpol. densities evolve faster than polarized ones
  MD, T. Kasemets 2014

Spin correlations

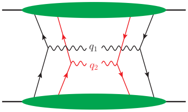
- can (almost) compute x_1, x_2 moments of DPDs in lattice QCD
- pilot study for the pion G Bali, L Castagnini, S Collins, MD, M Engelhardt, J Gaunt, B Gläßle, A Sternbeck, A Schäfer, Ch Zimmermann



lattice spacing $a \approx 0.07\,\mathrm{fm}$
pion mass $280\,\mathrm{MeV}$

- VV: spin averaged
- TT: transverse spin corr. $\propto \boldsymbol{s}_u \cdot \boldsymbol{s}_{\bar{d}}$
  find very small $A_{TT} \sim -0.1 \times A_{VV}$
- AA: longitudinal spin corr. even smaller (not shown)
- VT: correlation $\propto \boldsymbol{y} \cdot \boldsymbol{s}_{\bar{d}}$
  maximal at small $|\boldsymbol{y}|$, then decreases

Color correlations



- color of two quarks and gluons can be correlated
- suppressed by Sudakov logarithms

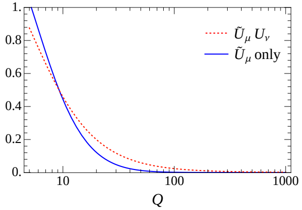
Mekhfi 1988

... but not necessarily negligible
for moderately hard scales

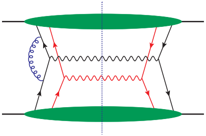
Manohar, Waalewijn arXiv:1202:3794

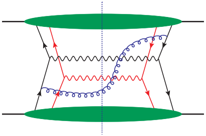
U = Sudakov factor for quarks

Q = hard scale



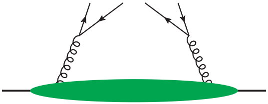
from incomplete cancellation between graphs with real/virtual soft gluons





Behavior at small interparton distance

- for $\boldsymbol{y} \ll 1/\Lambda$ in perturbative region $F(x_1, x_2, \boldsymbol{y})$ dominated by graphs with splitting of single parton



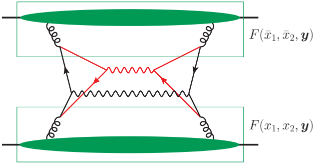
- gives strong correlations in x_1, x_2, spin and color between two partons

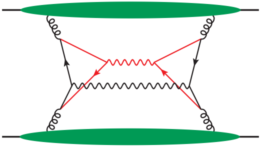
  e.g. -100% correlation for longitudinal pol. of q and $\bar{q}$

- can compute short-distance behavior:

$$F(x_1, x_2, \boldsymbol{y}) \sim \frac{1}{\boldsymbol{y}^2} \text{ splitting fct} \otimes \text{usual PDF}$$

Problems with the splitting graphs



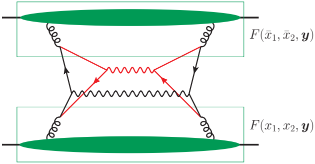


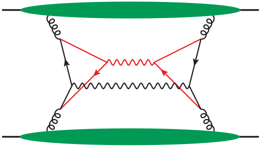
- contribution from splitting graphs in cross section gives UV divergent integrals $\int d^2\boldsymbol{y}\, F(x_1, x_2, \boldsymbol{y})\, F(\bar{x}_1, \bar{x}_2, \boldsymbol{y}) \sim \int d\boldsymbol{y}^2/\boldsymbol{y}^4$
- double counting problem between double scattering with splitting and single scattering at loop level

MD, Ostermeier, Schäfer 2011; Gaunt, Stirling 2011; Gaunt 2012
Blok, Dokshitzer, Frankfurt, Strikman 2011; Ryskin, Snigirev 2011, 2012
already noted by Cacciari, Salam, Sapeta 2009

- possible solution:
subtract splitting contribution from two-parton dist's when $\boldsymbol{y}$ is small
will also modify their scale evolution; remains to be worked out

Problems with the splitting graphs





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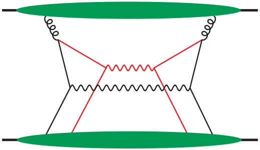
- also have graphs with single PDF for one and double PDF for other proton

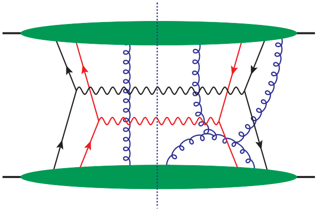
$\sim \int d\boldsymbol{y}^2/\boldsymbol{y}^2 \times F_{\textit{no split}}(x_1, x_2, \boldsymbol{y})$

B Blok et al 2011-13
J Gaunt 2012
B Blok, P Gunnellini 2015



Does the DPS cross section factorize at all?

- problem already for single hard scattering:
  exchange of soft gluons in specific kinematics (Glauber region)
  - physics: soft rescattering between partons in the two protons
  - must show that effects cancel by unitarity



- can generalize proof of soft-gluon-cancellation
  for single to double Drell-Yan process

  MD, J. Gaunt, D. Ostermeier, D. Plößl, A. Schäfer: in progress

Conclusions

- multiple hard scattering is often suppressed, but not necessarily
  - for multi-differential cross sections, high-multiplicity final states
  - in specific kinematics
  - if single scattering disfavored by coupling constants, PDFs etc.
- most phenomenology relies on strong simplifications
  some improvements are being explored
- have more and more elements for a formulation of factorization
  but important open questions still unsolved
  - crosstalk with single hard scattering at small distances
- double hard scattering depends on detailed hadron structure
  including correlation and interference effects, largely unknown
- subject remains of high interest for
  - understanding final states at LHC
  - study of hadron structure in its own right

Backup

Scale evolution for distributions without color correlation

- if define two-parton distributions as operator matrix elements in analogy with usual PDFs

$$F(x_1, x_2, \boldsymbol{y}; \mu) \sim \langle p|\mathcal{O}_1(\boldsymbol{0}; \mu)\, \mathcal{O}_2(\boldsymbol{y}; \mu)|p\rangle \qquad f(x; \mu) \sim \langle p|\mathcal{O}(\boldsymbol{0}; \mu)|p\rangle$$

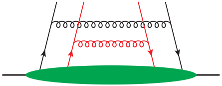
where $\mathcal{O}(\boldsymbol{y}; \mu)$ = twist-two operator renormalized at scale μ

- $F(x_i, \boldsymbol{y})$ for $\boldsymbol{y} \neq \boldsymbol{0}$:

separate DGLAP evolution for partons 1 and 2

$$\frac{d}{d \log \mu} F(x_i, \boldsymbol{y}) = P \otimes_{x_1} F + P \otimes_{x_2} F$$

two independent parton cascades

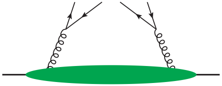


- $\int d^2\boldsymbol{y}\, F(x_i, \boldsymbol{y})$:

extra term from $2 \to 4$ parton transition

since $F(x_i, \boldsymbol{y}) \sim 1/\boldsymbol{y}^2$

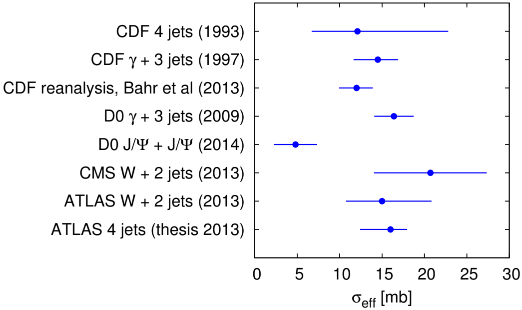
Kirschner 1979; Shelest, Snigirev, Zinovev 1982
Gaunt, Stirling 2009; Ceccopieri 2011



Phenomenological estimates of double parton scattering

- pocket formula used in most estimate for DPS contribution
- some recent studies (apologies for omissions):
  - double dijets — Domdey, Pirner, Wiedemann 2009; Berger, Jackson, Shaughnessy 2009
  - W/Z + jets — Maina 2009, 2011
  - $\gamma\gamma$ + jets — Tao et al, 2015
  - like-sign W pairs — Kulesza, Stirling 2009; Gaunt et al 2010; Berger et al 2011
  - double Drell-Yan — Kom, Kulesza, Stirling 2011
  - double charmonium — Kom, Kulesza, Stirling 2011; Baranov et al. 2011, 2012; Novoselov 2011
  - double charm — Berezhnoy et al 2012; Luszczak et al 2011; Cazaroto et al 2013; Maciula, Szczurek 2012, 2013; van Hameren, Maciula, Szczurek 2014, 2015
- also several studies for proton-nucleus collisions

Experimental investigations (very incomplete)



- other channels:
  - double charm production $(c\bar{c}c\bar{c})$ — LHCb 2011, 2012; CMS 2014
    $J/\Psi + J/\Psi$, $J/\Psi + C$, $C + C$ with $C = D^0, D^+, D_s^+, \Lambda_c^+$
  - $W + J/\Psi$ — ATLAS 2014