DESY Seminar

Hamburg & Zeuthen · 13 & 14 November 2012

Recent BABAR Results Semileptonic/Leptonic B Decays: Impact on New Physics



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Outline

- BaBar dataset
- Quark Mixing Matrix and Unitarity Triangle
- Semileptonic Decays
 - Recent |V_{ub}| measurement
 - Results on $B \to D^{(\star)} \, \tau \, \nu_\tau$
- Results on leptonic $B \rightarrow \tau v_{\tau}$ decay
- Implications for the widely discussed 2HDM
- Summary

PEP-II storage rings and BaBar experiment



- Operation 1999-2008
- Linear accelerator injects in PEP-II
 - Asymmetric beams
 - 9.0 GeV electrons
 - 3.1 GeV positrons
 - CM energy of the Y(4S) = 10.58 GeV most of the time
 - Collected data at the Y(2S-3S) and above the Y(5S)



May 1999 · Apr 2008

- At Y(4S) center of mass energy, large production of B meson from Y decays
 - σ_{10.58GeV}(e⁺e⁻→bb) = 1.06nb
 - LHC: $\sigma_{7TeV}(pp \rightarrow bb) \sim 200 \cdot 10^{3} nb$





- PEPII/BaBar: Charm / Tau factory!
 - σ(cc)=1.30nb
 - σ(ττ)=0.91nb
- Study light quark and cc production using ISR and γγ events



Broad physics program

- BaBar still producing a lot of results
 - 500 published papers in total
 - 13 published + 14 submitted since Jan/2012
 - Diverse Physics
 - Strong competition with Belle





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BELLE

CKM Matrix Introduction

• The Standard Model quark flavor sector requires the knowledge of the quark masses and of the strength of the charge-current gauge interactions (**C**abibbo **K**obayashi **M**askawa matrix)



- Experimental evidence of a strongly hierarchical structure
- V[†]_{CKM} V_{CKM} = 1 ⇒ 4 independent parameters
 - 3 real + 1 complex phase (CPV)
 - A, λ~0.22, |ρ+iη|=O(1)

The Wolfenstein parameterization

V_{km}

$$\mathcal{I}_{\mathsf{CKM}} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

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CKM Matrix

- CKM cannot be predicted in the SM
- Most SM extensions contain new CP-violating phases and new quark-flavor changing interactions

 $\begin{array}{ccc} V_{ud} & V_{us} \\ \pi \rightarrow \ell \nu & {\mathsf{K}} \rightarrow \pi \ell \nu \\ \beta \text{-decay} & {\mathsf{K}} \rightarrow \ell \nu \end{array}$ $b \to u \ell \nu$ $B \rightarrow \pi \ell \nu$ β-decay $\begin{array}{ccc} V_{cd} & V_{cs} & V_{cb} \\ D \rightarrow \pi \ell \nu & D \rightarrow K \ell \nu & b \rightarrow c \ell \nu \\ \nu + d \rightarrow c + \ell & D_s \rightarrow \ell \nu & B \rightarrow D \ell \nu \end{array}$ \mathbf{V}_{tb} $t \rightarrow b\ell v$ $egin{array}{c} {\sf V}_{{
m ts}} \ \langle B_{
m s} | \overline{B}_{
m s}
angle \ {\sf b} o {\sf s} \gamma \end{array}$ v_{td}

∼ |V_{km}|² **q**_m

The CKM matrix extracted from tree-level processes

Unitarity Triangle

- V[†]_{CKM} V_{CKM} = 1 ⇒ 9 conditions on the CKM parameters (6 triangular relations):
 - 3rd x 1st is of great phenomenological interest
 - all sizes of the same order in λ : CP violation is "visible"



 $1 - \lambda^2 / 2$

-λ

 $A\lambda^3(1-\rho-i\eta)$

 $A\lambda^{3}(\rho-i\eta)$

 $A\lambda^2$

1

λ

 $1 - \lambda^2 / 2$

-Aλ²

UT Constraints







From loop mediated processes particularly sensitive to New Physics







• Redundant constraints on UT and test of CKM unitarity \rightarrow powerful test of the SM

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UT Status (Summer 2012)

Redundant and consistent determinations of various CKM elements



UT Status (Summer 2012)

Redundant and consistent determinations of various CKM elements



IV_{ub}

Phys. Rev. D 86, 092004 (2012)

Phys.Rev.Lett. 109 101802 (2012)

Semileptonic Decays



- QCD corrections to parton level decay rate
- Operator Product Expansion predicts the total rate Γ_{u}



Semileptonic Decays



- QCD correction parameterized in the Form Factors
- Lattice-QCD, LCSR

$$\frac{d\Gamma(B^0 \to \pi^- \ell^+ \nu)}{dq^2} = \frac{G_F^2}{192\pi^3 m_B^3} \left[(m_B^2 + m_\pi^2 - q^2)^2 - 4m_B^2 m_\pi^2 \right]^{3/2} |\mathbf{V_{ub}}|^2 |\mathbf{f_+}(\mathbf{q^2})|^2$$

In the $m_{\ell} \sim 0$ only f+ contributes

V_{ub} from inclusive decays

 $\frac{\Gamma(b \to c\ell\nu)}{\Gamma(b \to u\ell\nu)} \approx 50$



Experimental resolution leads to irreducible $b \rightarrow c \ell v$ contamination

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Kinematics to extract the signal: $m_{\mu} << m_{c}$

- Cut limited region of phase space (f_{μ})
- From partial BF $\rightarrow |V_{ub}|$

$$|V_{ub}| = \sqrt{\frac{\Delta \mathcal{B}(\overline{B} \to X_u \ell \bar{\nu})}{\tau_B \, \Delta \Gamma_{\text{theory}}}}.$$

 m_{χ}

Not to scale!

Inclusive decays: Shape Function

- Cut in limited region of phase space is theoretically challenge:
 - OPE breaks down: $\Delta \Gamma_u$ depends on $O(1/m_b)$ non-perturbative effects!
 - Increase dependence on b-quark mass
 - Need the **shape function**:
 - In principle directly from experimental data with $B \rightarrow X_s \gamma$





In practice: the SF is determined indirectly from fitting many kinematical variables related to $B \rightarrow X_s \gamma$ and $B \rightarrow X_c \ell \nu$ or only to $B \rightarrow X_c \ell \nu$ with m_c constrained

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Events Reconstruction

- Two B in the Y decay: $e^+e^- \rightarrow Y(4S) \rightarrow BB$
 - Fully reconstruct one B in hadronic decay modes (with a D or a D*)
 - The rest comes from the other B (B^{recoil})
- Tag efficiency ~ 0.2-0.4%
- Recoil momentum is know from P_{tag}

$$ec{p}_{B^{ ext{recoil}}} = -ec{p}_{B^{ ext{tag}}}$$

$$p_{\nu} = p_{B^{\text{recoil}}} - p_X - p_\ell$$
 (p_v²=m²_{mise}

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$$p_X = \sum_i p_i^{\text{track}} + \sum_i p_i^{\text{clust.}}$$

$$q^2 = (p_{B^{\text{recoil}}} - p_X)^2$$



Recoil kinematics boosted in the B^{recoil}

 ΔB in the B rest frame

Fit results in limited regions of phase space



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Best Measurements ($\sigma_{exp} \oplus \sigma_{theory}$)

• Fit the (M_x,q^2) distribution in the region defined by $p_{lepton} > 1$ GeV: 89% of Phase Space



Measurement	BLNP $ V_{ub} $ [10 ⁻³]	$GGOU V_{ub} [10^{-3}]$	DGE $ V_{ub} $ [10 ⁻³]
$(m_X, q^2); p_\ell^{B *} > 1.0 \text{ GeV}$	$4.28 \pm 0.23^{+0.18}_{-0.20}$	$4.35 \pm 0.24^{+0.09}_{-0.10}$	$4.40 \pm 0.24^{+0.12}_{-0.13}$
$ ho_\ell^B*>1.0~{ m GeV}$	$4.30 \pm 0.28 ^{+0.18}_{-0.20}$	$4.36 \pm 0.30^{+0.09}_{-0.10}$	$4.42 \pm 0.30^{+0.13}_{-0.13}$
Belle [Phys.Rev.Lett.104:021801]	$4.47 \pm 0.27 ^{+0.19}_{-0.21}$	$4.54 \pm 0.27 ^{+0.10}_{-0.11}$	$4.60 \pm 0.27^{+0.11}_{-0.13}$

Largest systematics: signal model

Comparison of the available calculations



Exclusive $B \rightarrow \pi/\rho \ell v$





(the ρ and other mesons are difficult on lattice because these are unstable and have a large $\Gamma/m)$

Exclusive $|V_{ub}|$ from $\mathbf{B} \rightarrow \pi \ell \mathbf{v}$ HPQCD 10 One FF (for massless leptons) FNAL/MILC 8 LCSR L-QCD (HPQCD, FNAL) Unquenched calculations 6 Light Cone Sum Rules 4 Reliable at low q² $f_{+.0}(0)=0.28\pm0.03$ (Khodjamirian et al (KMOW) 2011) 0 5 10 20 25 15 a² (GeV²) M.Rotondo

Techniques employed at the BFactories





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Exclusive $B \rightarrow \pi \ell \nu$

- Identity only π+e/μ
- Neutrino from the rest of the event
 - (B momentum magnitude is know)



$$m_{ES} = \sqrt{E_{beam}^{*2} - \mathbf{p}_{\pi\ell\nu}^{*2}}$$
$$\Delta E = E_{\pi\ell\nu}^* - E_{beam}^*$$

- 12K signal $B \rightarrow (\pi^+ + \pi^0) \ell v$
 - S/N~0.1
- Same technique used also to extract the BF and the FF shape of other resonances
 - $B \rightarrow \eta \ell \nu$
 - B→η'ℓν
 - $B \rightarrow \omega \ell v$

|V_{ub}| from exclusive decays



Simultaneous fit to data and L-QCD calculations



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- Use the hadronic B tag with 710 fb⁻¹
 - New algorithm: tag selection based on NeuroBayes NN (NIM A654 (2011))

 $B^+ \to \pi^0 \ell \nu$



World average

World average



Inclusive · Exclusive difference

- Long standing puzzle
 - Despite progresses from B-factories+Theory, the inclusive-exclusive discrepancy still present: Δ @ 2.5-3.0 σ



Inclusive · Exclusive difference

- Long standing puzzle
 - Despite progresses from B-factories+Theory, the inclusive-exclusive discrepancy still present: Δ @ 2.5-3.0 σ



... if New Physics, what kind?

Left Right model

F. Bernlochner @ ICHEP12

New physics observable via right-handed currents? $|V_{ub}| = |V_{ub}^L| f(\epsilon_R' = \epsilon_R \Re \frac{V_{ub}^R}{V_{ub}^L})$



$B \longrightarrow D^{(*)} \tau \nu_{\tau}$

Evidence for an excess over the SM prediction

Phys.Rev.Lett. 109 101802 (2012)



Heavy leptons: introduction

- Charged Higgs required in multiple New Physics scenarios
 - Coupling is proportional to the fermion mass: H⁻ ℓ coupling $\propto m_{\ell}$



Introduction to the event reconstruction

Weak signal signature

- Decay with missing momentum
 - Many neutrinos in the final state
- Lack of kinematics constraints in the final state

Use the hadronic B sample: reconstruct one B and look at the rest of the event

- Expected only tracks from signal
- No additional tracks
- Little activity in the Calorimeter



$B \rightarrow D^{(*)} \tau v_{\tau}$: measurement

• We measure directly the *R*(D) and *R*(D*) ratios

Several experimental and theoretical uncertainties cancel in ratio

- D reconstruction / Particle ID /tracking eff.
- |V_{cb}| & FFs (partially)

Very precise SM prediction

 $\begin{array}{l} \mathsf{R}(\mathsf{D}) = 0.297 \pm 0.017 \text{ and } \mathsf{R}(\mathsf{D}^*) = 0.252 \pm 0.003 \\ \sigma = 5.7\% \qquad \qquad \sigma = 1.2\% \end{array}$

Existing measurements

Previous measurements exceed SM by 1-2 σ



Update BaBar 2008 with 2x data and 2x efficiency - improved tag B and better muon ID

$B \rightarrow D^{(*)} \tau v_{\tau}$: Yields Extraction

- Simultaneous un-binned M.L. Fit
 - 4 signal samples $D^0\ell$, $D^{*0}\ell$, $D^+\ell$, $D^{*+}\ell$
 - 4 $D^{(*)}\pi^0 \ell \nu$ Control samples
 - 2 dimensional distributions:

$$m_{miss}^2 = (p_{e+e-} - p_{tag} - p_{D(*)} - p_{\ell})^2$$

 p_{ℓ}^* in the B_{sig} rest-frame

- PDFs from MC: approximated using KEYS function
- Fitted Yields
 - 4 $D^{(*)}\tau v$ + 4 $D^{(*)}\ell v$ + 4 $D^{**}\ell v$





Results of Fit $B \rightarrow D^* \tau v_{\tau}$

Isospin Constrained



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D*0+D*+

1.5

 $p_1^{\overline{2}}$ (Gev)

Free

yields

Fixed



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Results and Systematics Uncertainties

Decay	$N_{\rm sig}$	$N_{\rm norm}$	$R(D^{(*)})$	$\mathcal{B}(B \to D^{(*)} \tau \nu) (\%)$	$\Sigma_{\rm tot}(\sigma)$
$D\tau^-\overline{\nu}_{\tau}$	489 ± 63	2981 ± 65	$0.440 \pm 0.058 \pm 0.042$	$1.02 \pm 0.13 \pm 0.11$	6.8
$D^* \tau^- \overline{\nu}_{\tau}$	888 ± 63	11953 ± 122	$0.332 \pm 0.024 \pm 0.018$	$1.76 \pm 0.13 \pm 0.12$	13.2

	<i>R</i> (D)	<i>R</i> (D*)	ρ_{corr}
D** τ/l ν	5.8	3.7	0.62
MC statistics	5.0	2.5	-0.48
Continuum and BB bkg	4.9	2.7	-0.30
$\epsilon_{sig}^{\prime}/\epsilon_{norm}$	2.6	1.6	0.22
Syst. Uncertainty	9.5	5.3	0.05
Stat. Uncertainty	13.1	7.1	-0.45
Total Uncertainty	16.2	9.0	-0.27

Uncertainties due to FFs, PID, tracks, photons and soft pion reconstruction cancel in the ratio: contribution ~1%

SM Predictions of R(D) and R(D*)

• The new measurements are fully compatible with earlier results

Average does not include this measurement



[*] Kaminik Mescia 2008 SM Predictions of R(D) and $R(D^*)$

The new measurements are fully compatible with earlier results

Average does not include this measurement

Fajfer et al 2012



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2HDM calculation

Differential decay rate in the SM

$$B \rightarrow D^{*} FFs$$

$$\frac{d\Gamma(\overline{B} \rightarrow D^{*}\ell^{-}\overline{\nu}_{\ell})}{dq^{2}} = \frac{G_{F}^{2}|V_{cb}|^{2}|p|q^{2}}{96\pi^{3}m_{B}^{2}} \left(1 - \frac{m_{\ell}^{2}}{q^{2}}\right)^{2} \left[\left(|H_{++}|^{2} + |H_{--}|^{2} + |H_{00}|^{2}\right) \left(1 + \frac{m_{\ell}^{2}}{2q^{2}}\right) + \frac{3m_{\ell}^{2}}{2q^{2}}|H_{0t}|^{2}\right]$$
A charged Higgs (**Type II 2HDM**) of
spin 0 coupling with the τ will affect H_{0t}

$$H_{0t}^{2HDM} \approx H_{0t}^{SM} \times \left(1 - \frac{\tan^{2}\beta}{m_{H^{+}}^{2}} \frac{q^{2}}{1 \mp m_{c}/m_{b}}\right)$$

$$- \text{ for } B \rightarrow D\tau \nu_{\tau}$$

$$+ \text{ for } B \rightarrow D^{\tau} \nu_{\tau}$$

$$+ \text{ for } B \rightarrow D^{\tau} \nu_{\tau}$$
Effects both the signal efficiency and the
signal yields (m^{2}_{miss} p^{*}_{t} \text{ shapes}): simulated
reweighting the MC signal events
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Can we explain the excess events?



- The combination of R(D) and R(D*) excludes the Type II
 2HDM in the full tanβ-m_H parameter space (with m_H>10 GeV) with a probability >99.8%
 - Low m_H range (m_H <~300 GeV) already excluded by $B \rightarrow X_s \gamma$ data!

Some interesting following papers

- SM prediction is sensitive to the f₀ FF
 - Becirevic, Kosnik, Tayduganov, (1206.4977) proposal, using lattice data
 - MILC collaboration: 1206.4992, first SM lattice calculation unquenched (difference with SM reduced to 3.2 σ)
- 2HDM type II (alone) cannot accommodate the results on $B\to\tau\,\nu_{\tau}$ and $B\to D^{(*)}\,\tau\,\nu_{\tau}$
- New models have been studied so far
 - Crivellin et al 1206.2634: possibile explanation with Type III 2HDM
 - Fajfer et al. 1206.1872: 2HDM with leptoquarks

Some interesting following papers

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 - MILC collaboration: 1206.4992, first SM lattice calculation unquenched (difference with SM reduced to 3.2 σ)
- New Belle measurements with improved B_{tag} (NeuroBayes[®]) welcome!
- Confirmation? If yes, look for other observables:
 - q² distribution, τ polarization using $\tau \rightarrow \pi v_{\tau}$, D* polarization from D* decay angular analysis => Rich physics for future SuperB factories and... perhaps also for LHCb now!!!

$B \to \tau \, \nu_\tau$

arXiv:1207.0698





Analysis of B $\rightarrow \tau \, \nu_{\tau}$

Theoretically very clean

$$\mathcal{B}(B \to l\nu) = \frac{G_F^2 m_B}{8\pi} m_l^2 (1 - \frac{m_l^2}{m_B^2})^2 f_B^2 |V_{ub}|^2 \tau_B$$

 $B\{ \frac{b}{\overline{u}} \xrightarrow{\mathbf{W}^{-}} \underbrace{\tau^{-}}_{\overline{v}_{\tau}}$

Allow $|V_{ub}|$ extraction from $\rm f_B$ & BF

Experimentally difficult:

- helicity suppression
- BF(τ)~10⁻⁴ (and BF(μ)~10⁻⁷ out of reach of current Bfactories
- only the Branching Fraction is accessible)

Power probe of physics beyond the SM



$$\begin{split} \mathcal{B}_{2HDM}(B \to \tau \nu) = \\ \frac{G_F^2 m_B m_\tau^2}{8\pi} \left(1 - \frac{m_\tau^2}{m_B^2} \right)^2 f_B^2 |V_{ub}|^2 \tau_B & \text{B(τ\nu$)}\\ \times \left(1 - m_B^2 \frac{\tan^2 \beta}{m_H^2} \right)^2 & \text{In the SM} \\ \end{split}$$

Results: $B \rightarrow \tau v_{\tau}$

Fit to residual energy in the EMC simultaneously in 4 τ decay modes •



Comparison with 2HDM-II

Uncertainty in SM prediction is dominated by $|V_{ub}|$

$$\begin{pmatrix} \mathcal{B}_{2HDM}(B \to \tau\nu) = \\ \frac{G_F^2 m_B m_\tau^2}{8\pi} \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 f_B^2 |V_{ub}|^2 \tau_B \\ \times \left(1 - m_B^2 \frac{\tan^2 \beta}{m_H^2}\right)^2 \end{pmatrix}$$



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Experimental Results on $B \rightarrow \tau v_{\tau}$



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Exclusion region for 2HDM-II

Using both D · D* results: Type II 2HDM excluded at 99.8% (3.1s)



Exclusion region for 2HDM-II

Using both D · D* results: Type II 2HDM excluded at 99.8% (3.1s)



Summary

- New (legacy) BaBar results on $|V_{ub}|$
 - Puzzle about the Inclusive-Exclusive difference and bad compatibility with indirect extractions
 - Will stay with us for long
- $B \rightarrow D^{(*)} \tau \nu_{\tau}$: not in agreement with SM prediction
 - Wait for a confirmation from Belle (LHCb ?)
- $B \to \tau \, \nu_\tau$: reached the B-factory limits
 - Status is cloudy
 - But will be explored with high precision at SupeB-Factorys.

BACK UP



|V_{ub}| extraction at B-factories

Leptonic Decays



Semileptonic Decays e, μ , τ V_{ub} W⁻ ν_{e} , ν_{μ} , ν

Leptonic and hadronic currents factorize v_{e}, v_{μ}, v_{τ} $\mathcal{M}(B \to \pi \ell^{-} \overline{\nu}) = -i \frac{G_{F}}{\sqrt{2}} \cdot V_{ub} \cdot L^{\mu} H_{\mu}$

 $B \rightarrow \pi$ hadronic current



CKM Matrix measurements

- CKM cannot be predicted in the SM
- Most SM extensions contain new CP-violating phases and new quark-flavor changing interactions

$$\begin{array}{c|ccccc} V_{ud} & V_{us} & V_{ub} \\ \hline \pi \rightarrow \ell \nu & & \mathsf{K} \rightarrow \pi \ell \nu & \mathsf{b} \rightarrow u \ell \nu \\ \hline \beta \text{-decay} & & \mathsf{K} \rightarrow \ell \nu & & \mathsf{B} \rightarrow \pi \ell \nu \\ \hline \\ V_{cd} & V_{cs} & V_{cb} \\ \hline D \rightarrow \pi \ell \nu & D \rightarrow \mathsf{K} \ell \nu & & \mathsf{b} \rightarrow c \ell \nu \\ \nu \text{+} d \rightarrow c \text{+} \ell & \mathsf{D}_s \rightarrow \ell \nu & & \mathsf{B} \rightarrow D \ell \nu \\ \hline \\ V_{td} & V_{ts} & V_{tb} \\ \hline \\ \langle B_d | \overline{B}_d \rangle & \langle B_s | \overline{B}_s \rangle & t \rightarrow \mathsf{b} \ell \nu \\ \hline \\ \mathsf{b} \rightarrow \mathsf{d} \gamma & \mathsf{b} \rightarrow \mathsf{s} \gamma \end{array}$$

 $q_m \sim |V_{km}|^2$

The CKM matrix extracted from tree-level processes

CKM Matrix measurements

- CKM cannot be predicted in the SM
- Most SM extensions contain new CP-violating phases and new quark-flavor changing interactions



Only $V_{td},\,V_{ts}$ cannot be accessed by tree-level processes

(From G.Isidori)

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Event Reconstruction: Details

- B_{tag} reconstructed in B→D^(*)X, B→D_s^(*)X, B→J/ψX (X=π,K modes with n_X<6) and selected using
 - beam energy substituted mass
 - the energy difference

$$m_{ES} = \sqrt{(E^*_{beam})^2 - (\mathbf{p}^*_{tag})^2}$$
$$\Delta E = E^*_{tag} - E^*_{beam}$$

- Signal side D^(*) in D⁰, D^{*0}, D⁺, D^{*+} and require an identified lepton
- No additional charged particles
- Kinematic selection: $q^2 = (p_B p_{D(*)})^2 = q^2 > 4 \text{ GeV}^2$
- Boosted Decision Tree (BDT)
 - Reduce combinatorial and D** backgrounds
- Because the $B \rightarrow D^{**}(\ell, \tau)v$ have large uncertainties
 - We fit simultaneously also a sample of 4 $D^{(*)}\pi^0 \ell v$
 - same selection as signal but added π^0 : captures $D^{**} \rightarrow D^{(*)} \pi^0$
- Three control samples to validate and correct the simulation:
 - E_{extra} >0.5 GeV, q²<4 GeV, m_{ES}<5.26 GeV
 - + off-peak data to correct lepton spectrum of simulated continuum events

Energy extra in the EMC





Background estimation



Two Higgs Doublet Model



- SM - $\tan\beta/m_{H^+} = 0.3 \text{ GeV}^{-1}$ - $\tan\beta/m_{H^+} = 0.5 \text{ GeV}^{-1}$ - $\tan\beta/m_{H^+} = 1.0 \text{ GeV}^{-1}$