Kilogram, Planck Units, and Quantum Hall Effect

Alexander Penin

University of Alberta & TTP Karlsruhe

DESY Hamburg, May 2012

Topics Discussed

- Systems of units and fundamental constants
  - *Planck system of units* vs *traditional SI*
  - *New definition of ampere and the value of electron charge*
  - *New definition of kilogram and the value of Planck constant*

- New SI *vs* Quantum Electrodynamics
  - *The role of the fine structure constant*
  - *QED corrections to quantum Hall effect*

System of units and fundamental constants

- Historical approach
  - *define units by artefact*
  - *calibrate experimental tools*
  - *measure fundamental constants*

- Planck's idea (Ann. Physik 1 (1900) 69)

  " . . . with the help of fundamental constants we have the possibility of establishing units of length, time, mass, and temperature, which necessarily retain their significance for all cultures, even unearthly and nonhuman ones".

Planck's units

- Five fundamental constants G (gravity), k_C (electricity), c (relativity), $\hbar$ (quantization), k_B (temperature)

$$E = G\frac{m_1 m_2}{r}, \quad E = k_C\frac{q_1 q_2}{r}, \quad E = \hbar\omega, \quad E = k_B T, \qquad E = mc^2$$

- Combinations with the dimension of mass, length, time, temperature and charge ⇨ *new units*

$$l_P = \sqrt{\frac{\hbar G}{c^3}} \sim 10^{-35}\ \mathrm{m}, \quad t_P = \frac{l_P}{c} \sim 10^{-43}\ \mathrm{s}, \quad m_P = \sqrt{\frac{\hbar c}{G}} \sim 10^{-8}\ \mathrm{kg},$$

$$q_P = \sqrt{\frac{\hbar c}{k_C}} \sim 10^{-18}\ \mathrm{C}, \quad T_P = \frac{m_P \hbar c^2}{k_B} \sim 10^{32}\ \mathrm{K}$$

  - *in these units* $\hbar = c = k_C = k_B = G = 1$
  - *a device measuring one of the combinations is a new standard*

Planck's units

- Perfect theoretical concept!

Planck's units

- Perfect theoretical concept!

  ➥ *but as usual does not work in practice*

why? *the accuracy of all the fundamental constants should be equal or better than the accuracy of the "traditional" units definition based on artefact*

- Weak link in Planck’s system
  - *Gravitational constant from torsion balance:*

$\delta G/G \approx 10^{-4}$ *(2010 CODATA, compare to* 10^{-2} *by Cavendish 1798)*

Planck's units

- Never give up!

  ➥ *just follow the concept with*

  $$\hbar = c = k_C = k_B = 1$$

  or better use some predetermined numbers close to "traditional" values but with zero uncertainty

The first step: meter *vs* speed of light

- Old definition

- Current definition
  - *1 second: the duration of 9 192 631 770 periods of the radiation corresponding to the ground state HFS transition of the caesium-133 atom*
  - *1 meter:* $c = 299\,792\,458 \text{ m/s}$ *sharp*

    (in practice $1 \text{ m} = 1\,650\,763.73$ *wavelengths of the orange-red emission line of the krypton-86)*
  - *counting interference experiment*

The next step - ampere *vs* electron charge

- Traditional SI units
  - *electric charge* ⇨ *electric current* $I = q/t$
  - *1 A =1 C/1 s the constant current which will produce an attractive force of* $2 \cdot 10^7$ *newton per metre of length between two straight, parallel conductors of infinite length and negligible circular cross section placed one metre apart in a vacuum.*
  - *electron charge* $e = -1.602176487(40) \cdot 10^{-19}$ *C*
- "User friendly" Planck units
  - $e = -1/6.2415093 \cdot 10^{-18}$ *C sharp*

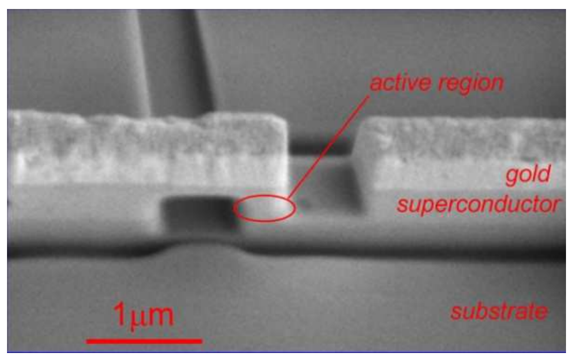
    to be formally proposed by the CIPM in 2015

The next step - ampere *vs* electron charge

- Practical realization: how to count so many electrons?
  - ➥ *make use of macroscopic quantum effects*

❶ Josephson effect

alternating current through S-I-S junction



Josephson frequency-voltage relation $\nu = K_J V$

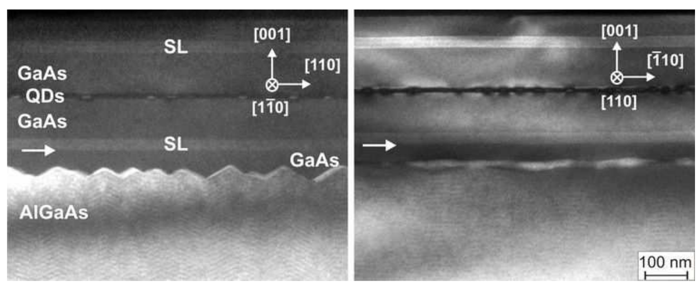
Josephson constant $K_J = \frac{2e}{h}$

The next step - ampere *vs* electron charge

- Practical realization: how to count so many electrons?
  - *make use of macroscopic quantum effects*

❷ Quantum Hall effect

2-dimesional electron gas conductivity in magnetic field



Quantum Hall current $I = nV/R_K$

von Klitzing constant $R_K = \frac{h}{e^2}$

The next step - ampere *vs* electron charge

- Practical realization: how to count so many electrons?
  - ↪ *make use of macroscopic quantum effects*

- *Quantum Hall effect* $+$ *Josephson effect*
  $=$ *fundamental current-frequency converter*

$$I = \frac{\nu}{K_J R_K} = \frac{e}{2}\nu$$

- 1 *ampere* ⇨ $2 \times 6.2415093 \cdot 10^{18}$ *Hz*

  Quantum Hall universality/precision: 10^{-11}

  F. Schopfer, W. Poirier (2007)

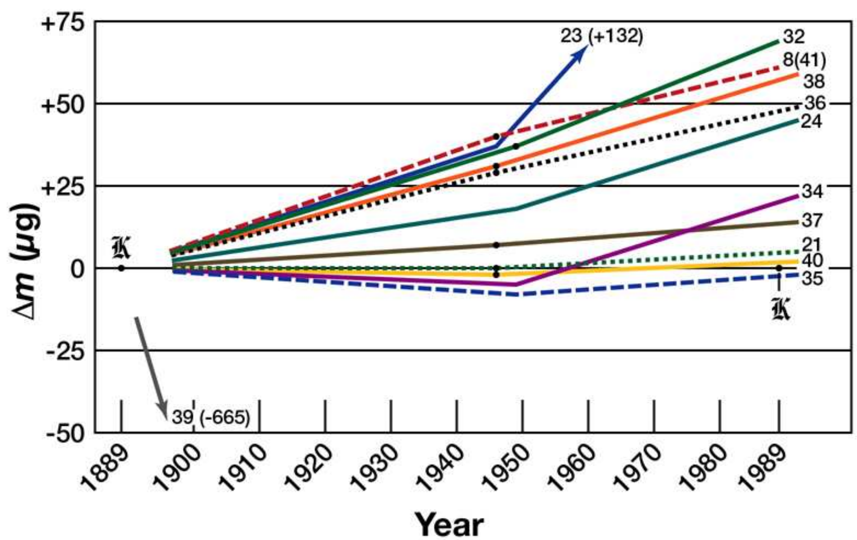
  Josephson universality/precision: 10^{-19}

  A.K. Jain, J.E. Lukens, J.-S. Tsai (1987)

The last step - kilogram *vs* Planck constant

- Current SI standard: *the last artefact*

- Prototype mass drift: *does kilogram get lighter?*



The last step - kilogram *vs* Planck constant

- Current SI:

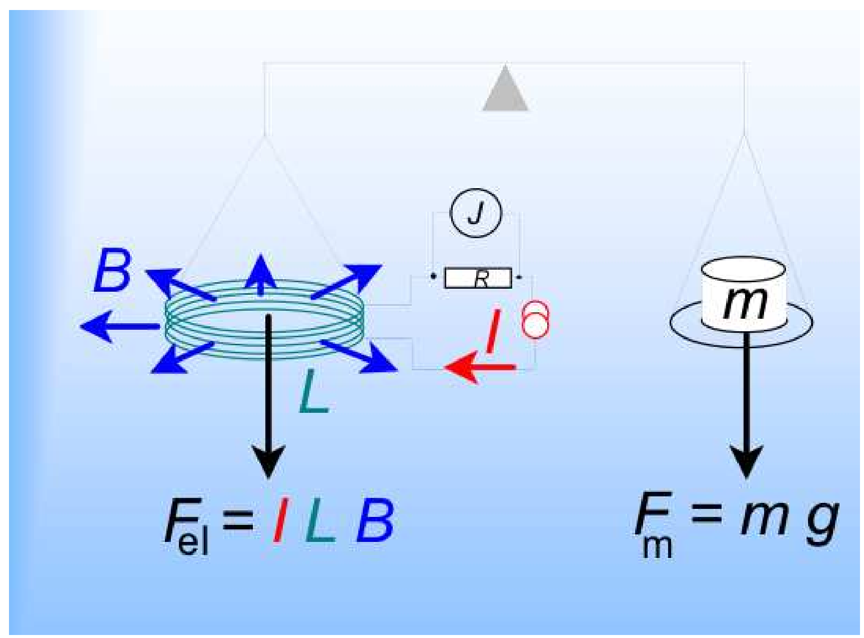
  Planck constant $h = 6.62606957(29) \cdot 10^{-34}\,\mathrm{kg\,m^2/s}$

- "User friendly" Plank units:

  define kilogram so that $h = 6.62606957 \cdot 10^{-34}\,\mathrm{kg\,m^2/s}$ *sharp*

- Practical realization

  Watt balance

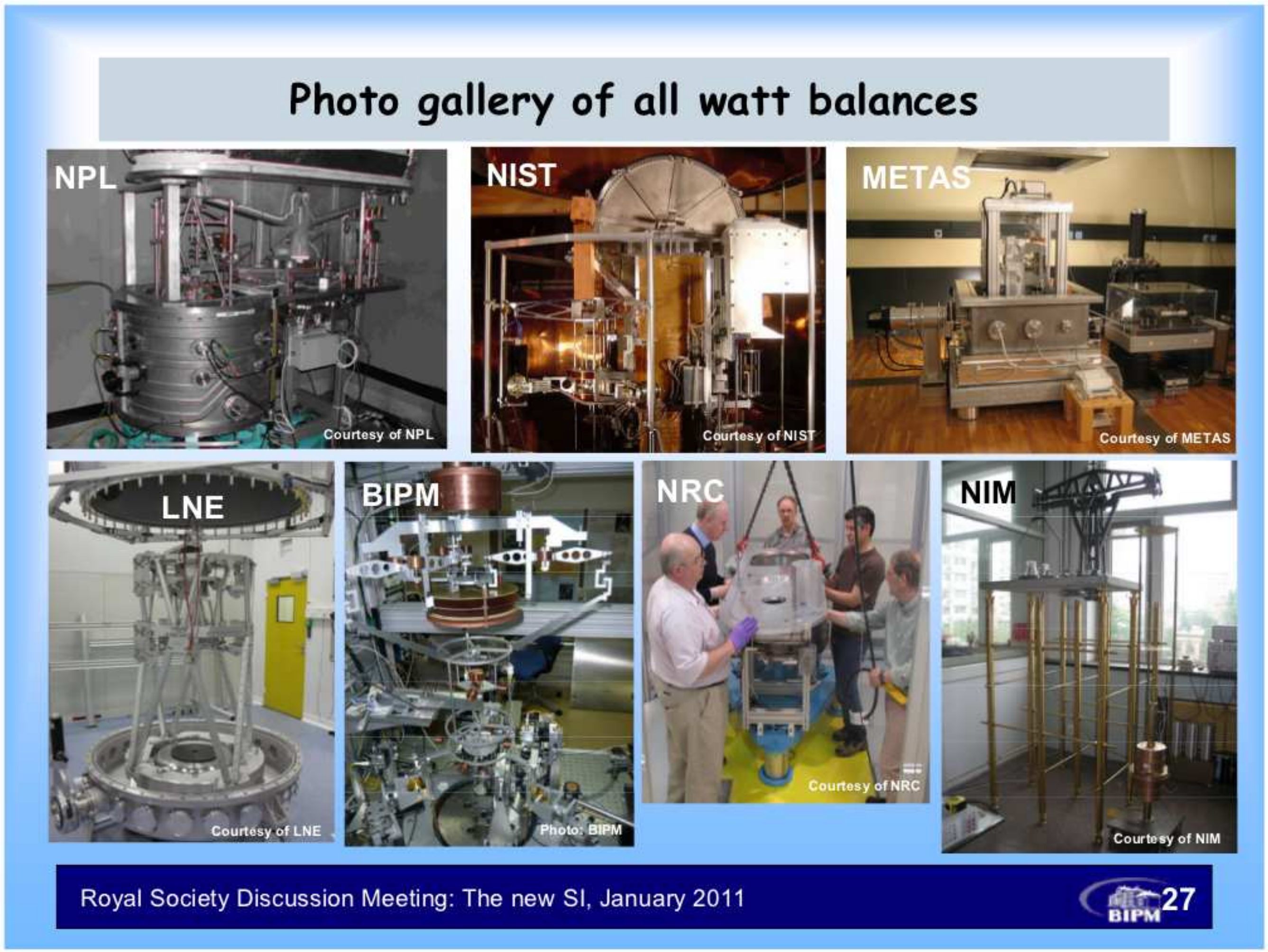


The last step - kilogram *vs* Planck constant

- Watt balance idea (simplified)
  - *electric power = mechanical power:* $$V\,I = m\,g\,v$$
  - *Josephson + quantum Hall effect:* $$V\,I = \frac{\nu}{K_J}\,\frac{\nu}{K_J R_K} = h\,\nu^2$$
  - *final equation:* $$m = h\,\frac{\nu^2}{gv}$$
  - *relative accuracy in g, ν, v better than* 10^{-10}
  - *relative accuracy of Watt balance: better than* 10^{-8}

Watt balances

(from talk by M. Stock)



The role of QED in new SI

Question $\mathcal{N}$1

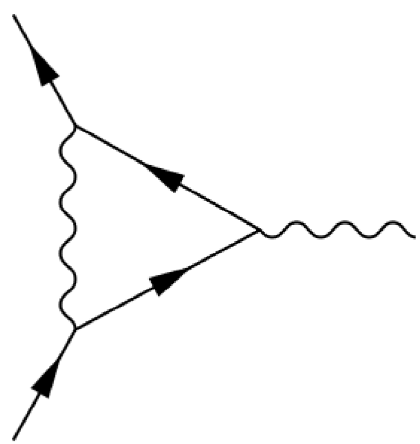
- How should the values of Planck constant and electron charge be choosen?

Question $\mathcal{N}$1

- How should the values of Planck constant and electron charge be choosen?
- The dimensionless fine structure constant
  - $\alpha = \frac{e^2}{h}\frac{c\mu_0}{2} \approx 1/137$
  - $\mu_0 = 4\pi \cdot 10^{-7}$ *predetermined value in SI*
- The predetermined values of h and e should leave $\mu_0 = 4\pi \cdot 10^{-7}$ unchanged

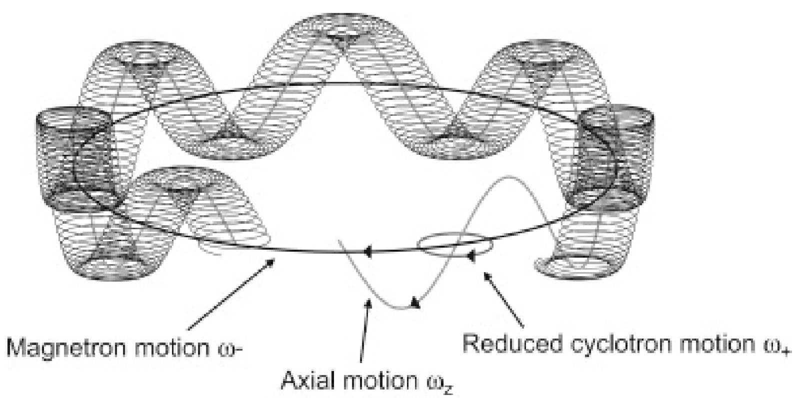
The fine structure constant

- Electron proper magnetic moment $\boldsymbol{\mu} = g\frac{e\hbar}{2m_e}\boldsymbol{s}$
  - *Dirac theory:* $g = 2$
  - *QED:* $g - 2 = \frac{\alpha}{\pi} + \ldots$ *"anomalous magnetic moment"*



The fine structure constant

- Determination from electron $g-2$



  - *experimental error:* 10^{-10}

G. Gabrielse et al. (2008)

  - *theoretical error:* 10^{-10} - *requires 4-loop calculation in QED*

T. Kinoshita et al. (2008)

Question $\mathcal{N}$2

- *Which constants should be taken as fundamental?*
- Experimental choice: Josephson and von Klitzing constants
  - $e = \frac{2}{K_J R_K}$
  - $h = \frac{4}{K_J^2 R_K}$
- Theoretical choice: Planck constant and electron charge
  - $R_K = \frac{h}{e^2}$
  - $K_J = \frac{2e}{h}$

$e,\ \hbar$ or R_K, K_J?

- Does not matter if the above relations are exact
  - *they are exact in quantum mechanics*
  - *is this true beyond quantum mechanics?*

$e,\ \hbar$ or R_K, K_J?

- Does not matter if the above relations are exact
  - *they are exact in quantum mechanics*
  - *is this true beyond quantum mechanics?*

- *Not in QED!*

A.Penin, *Phys.Rev.* **B79**, 113303 (2009)
Phys.Rev.Lett. **104**, 097003 (2010)

Gauge invariance and quantum phase

$\hbar = c = 1, \ \alpha = \frac{e^2}{4\pi}$

- ✔ Schrödinger equation $(i\partial_t - \mathcal{H})\,\Psi = 0$
- ✔ Wave function $\Psi = |\Psi| e^{i\theta}$
- ✔ Hamiltonian $\mathcal{H} = qA_0 + F(\boldsymbol{B}, \boldsymbol{E}, \boldsymbol{D})$

$$\boldsymbol{D} = \boldsymbol{\partial} - iq\boldsymbol{A}$$

↪ If $\boldsymbol{\nabla} \times \boldsymbol{A} = 0$ then

$$\theta(\boldsymbol{r}_1) - \theta(\boldsymbol{r}_2) = q \int_{\boldsymbol{r}_2}^{\boldsymbol{r}_1} \boldsymbol{A}(\boldsymbol{r}') \cdot d\boldsymbol{r}'$$

↪ Also $\phi(\boldsymbol{r}) = \theta(\boldsymbol{r}) - q \int^{\boldsymbol{r}} \boldsymbol{A}(\boldsymbol{r}') \cdot d\boldsymbol{r}'$ is gauge invariant

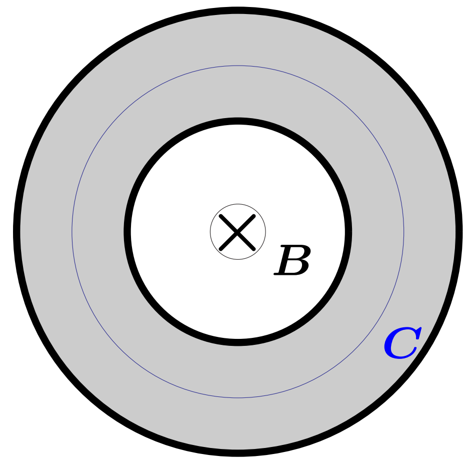
Flux quantization

F. London (1948)

Superconductivity ⇨ *coherent state of Cooper pairs* ⇨ $q = 2e$

Meissner effect ⇨ $\boldsymbol{B} = 0$ inside superconductor

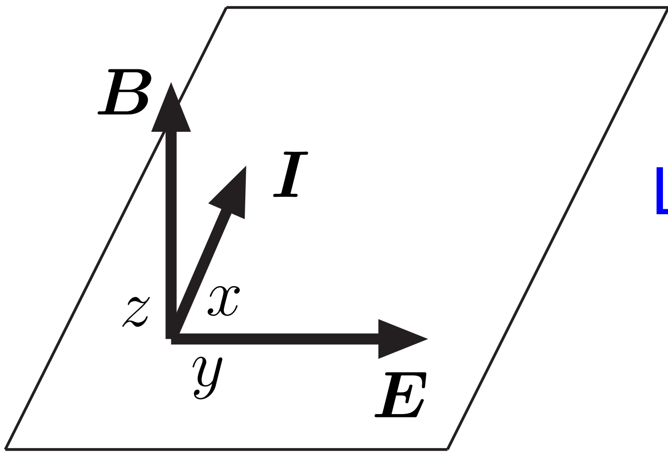
Superconducting ring:



Single-valued wave function ⇨ $\Delta\theta = 2e \oint_C \boldsymbol{A} \cdot d\boldsymbol{x} = 2\pi n$

Hall effect

E. Hall (1879)



Lorentz force *vs* electrostatic force

$$e\boldsymbol{E} = -e\boldsymbol{v} \times \boldsymbol{B}$$

current density

$$j = \rho e v = \frac{\rho e}{B} E$$

total current per length

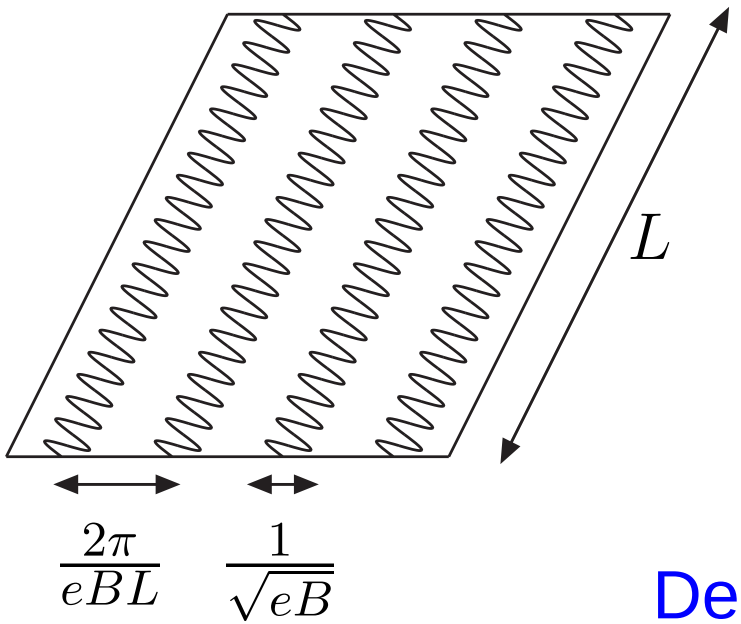
$$I = \frac{\rho e}{B} V$$

Hall conductivity

$$R^{-1} = \frac{\rho e}{B}$$

Quantum Hall effect

K. von Klitzing, G. Dorda, M. Pepper (1980)



Wave function:

$$\Psi(x,y) = e^{i2\pi m\frac{x}{L}}\ \psi(y-y_m)$$

$\psi(y-y_m)$ ⇨ *harmonic oscillator*

centered at $y_m = \frac{2\pi m}{eBL}$

Density of quantum states with

$\boldsymbol{n}$ Landau levels filled: $\rho = n\frac{eB}{2\pi}$

Quantum Hall conductivity:

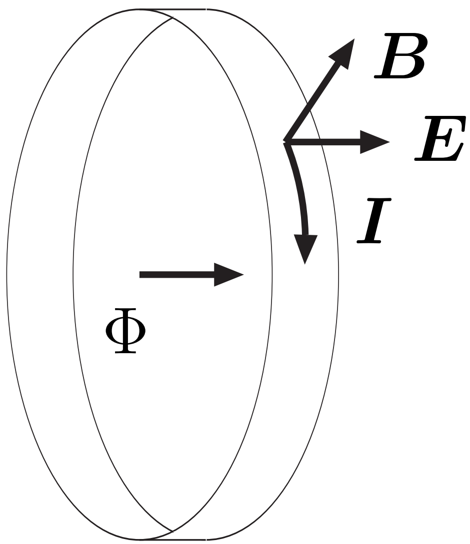
$$R^{-1} = 2n\alpha = n/R_K$$

von Klitzing constant:

$$R_K = \frac{h}{e^2}$$

Gauge invariance argument

R.B. Laughlin (1981)



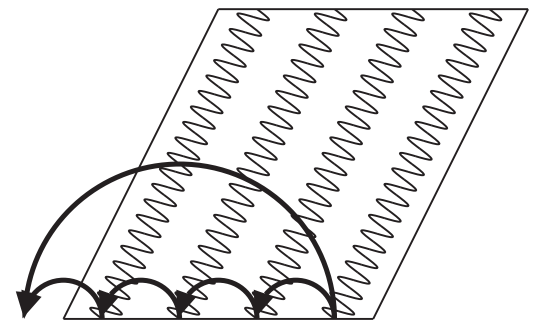
current density $\boldsymbol{j} \propto \dfrac{\delta\mathcal{H}}{\delta\boldsymbol{A}}$

total current $I = \dfrac{d\mathcal{E}}{d\Phi}$

Flux quantization $\Phi = 4\pi|\boldsymbol{A}|/L = n2\Phi_0 = n\,\frac{2\pi}{e}$

Flux dependence $y_m(\Phi + 2\Phi_0) = y_{m+1}(\Phi)$

for $d\Phi = 2\Phi_0$ ⇨ $d\mathcal{E} = neV$



↪ $I = \dfrac{neV}{2\Phi_0} = \dfrac{nV}{R_K}$

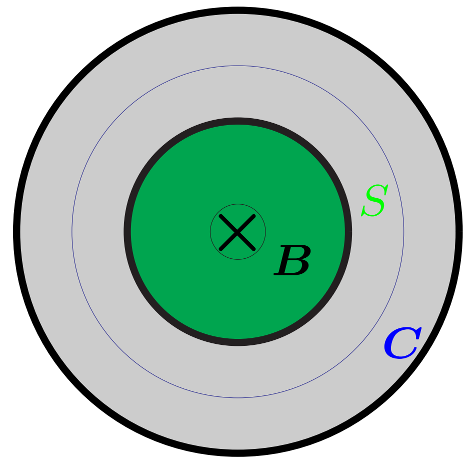
Flux quantization

F. London (1948)

Superconductivity ⇨ *coherent state of Cooper pairs* ⇨ $q = 2e$

Meissner effect ⇨ $\boldsymbol{B} = 0$ inside superconductor

Superconducting ring:



Single-valued wave function ⇨ $\Delta\theta = 2e \oint_C \boldsymbol{A} \cdot d\boldsymbol{x} = 2\pi n$

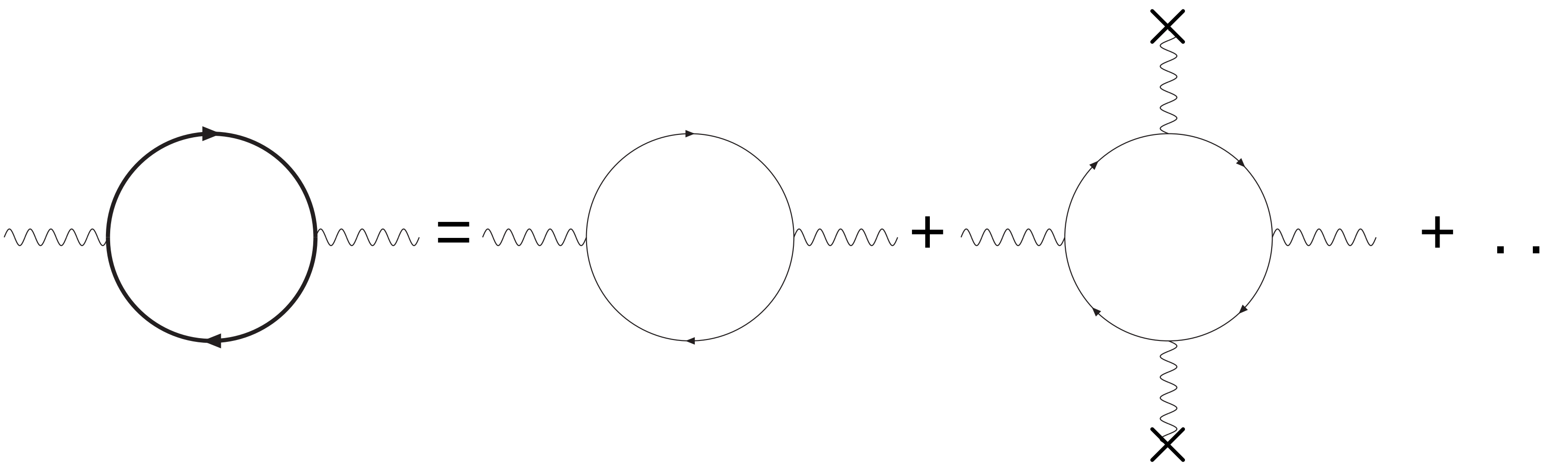
$\oint_C \boldsymbol{A}(\boldsymbol{r}) \cdot \boldsymbol{r} = \int_S \boldsymbol{B}(\boldsymbol{r}) \cdot d^2\boldsymbol{s} \equiv \Phi$ ⇨ $\Phi = \frac{\pi n}{e} \equiv n\Phi_0$

Flux quantum: $\Phi_0 = \frac{h}{2e}$

Nonlinear Electrodynamics

H. Euler, W. Heisenberg (1936)

J. Schwinger (1951)



Vacuum polarization in magnetic field

QED effects

- Correction to the photon dispersion:

$$v = \frac{k(\omega)}{\omega} \neq c$$

- Local charge renormalization:

$$e \to \left(1 + \frac{\alpha}{\pi}\left(\frac{eB}{m^2}\right)^2 \frac{1}{45}\right) e \equiv e'$$

QED corrections to R_K

Corrections to the filling factor $R_K^{-1} \propto e^2 \to ee'$

Result:

$$R_K^{-1} = \frac{e^2}{h}\left[1 + \frac{1}{45}\frac{\alpha}{\pi}\left(\frac{\hbar e B}{c^2 m^2}\right)^2\right] \approx \frac{e^2}{h}\left[1 + 10^{-20} \times \left(\frac{B}{10\,\mathrm{T}}\right)^2\right]$$

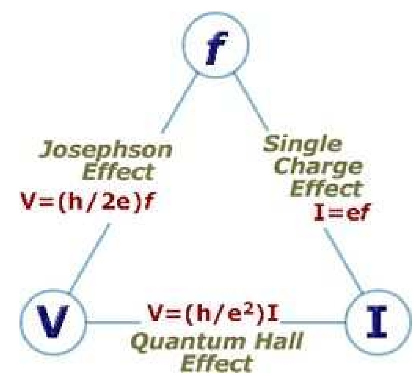
Testing the fundamental relations

- Fundamental relations
  - *electron charge* $e = \frac{2}{K_J R_K}$
  - *Planck constant* $h = \frac{4}{K_J^2 R_K}$

- Quantum metrology triangle

K.K. Likharev, A.B. Zorin (1985)

  - *quantum Hall effect* $V/I = R_K$
  - *Josephson effect* $V = \nu/K_J$
  - *Single electron tunneling* $I = e\nu$ **(?)**



Summary

- In forthcoming years the International System of units will finally transform from a set of artifacts into a “user friendly” Planck system.

Summary

- In forthcoming years the International System of units will finally transform from a set of artifacts into a "user friendly" Planck system.

- Two messages from *QED* to new SI:
  - *In the new "Planck inspired" SI the* electron charge *and* Plank constant *rather than* Josephson *and* von Klitzing constants *is the relevant choice of the fundamental constants*
  - *The predetermined values of the* electron charge *and* Plank constant *must be consistent with the measured value of the fine structure constant*