Heavy-flavor treatment at NNLO in CTEQ PDF analysis.

Marco Guzzi

Southern Methodist University

DESY Hamburg, April 24, 2012

In collaboration with P. Nadolsky, H.-L. Lai and C.P. Yuan
arXiv:1108.5112 (hep-ph)

CTEQ PDF analysis at NNLO

Work in progress on the CTEQ side!

- Careful analysis of PDF fits at NNLO;
- Benchmarking and validation to estimate PDF uncertainties;
- Validation of heavy-quark S-ACOT-χ scheme at $O(\alpha_s^2)$. (based on M.G., Nadolsky, Lai, Yuan, arXiv:1108.5112 (hep-ph), submitted to Phys. Rev. D).
  Also in Proceedings of the 2011 Workshop "New Trends in HERA Physics", Ringberg, Germany, 2011

CTEQ PDF at NNLO

Some new things in the NNLO analysis

- Include LHC W and Z rapidity data, ATLAS and CMS jet data, HERA 2011 F_L data
- Only inclusive p_T bins of D0 electron and muon charged asymmetry data
- Updated α_s, m_c, m_b values
- Flexible $\bar{d}/\bar{u}$ ratio at $x \to 1$, updated $(s+\bar{s})/(\bar{u}+\bar{d})$ at $x \lesssim 10^{-2}$

★ Constrained by the LHC W/Z rapidity distributions

♠ CT10W NNLO are posted on the CTEQ website
$http://hep.pa.msu.edu/cteq/public/ct10\_2012.html$

CTEQ PDF at NNLO

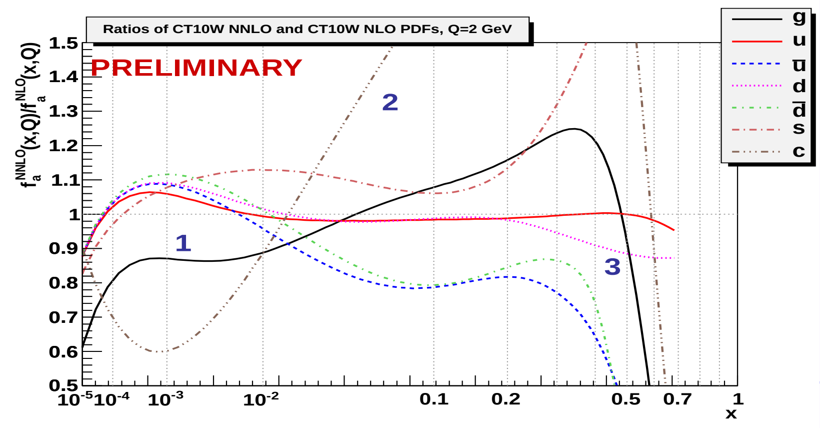
CTEQ PDFs at NNLO is a combined efforts:

M. G., J. Gao, P. Nadolsky, Z. Li, J. Huston, H.-L. Lai, Z. Liang, J. Pumplin, D. Soper, D. Stump, C.-P. Yuan

CTEQ PDF at NNLO

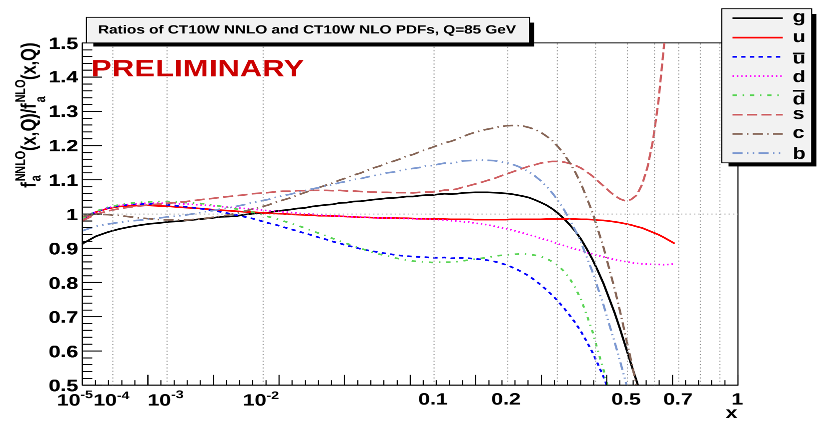
★ Shapes of the NNLO PDFs have noticeably evolved compared to NLO as a result of $O(\alpha_s^2)$ contributions, updated electroweak contributions, and revised statistical procedures.

CT10W NNLO central PDFs, as ratios to NLO, Q=2 GeV



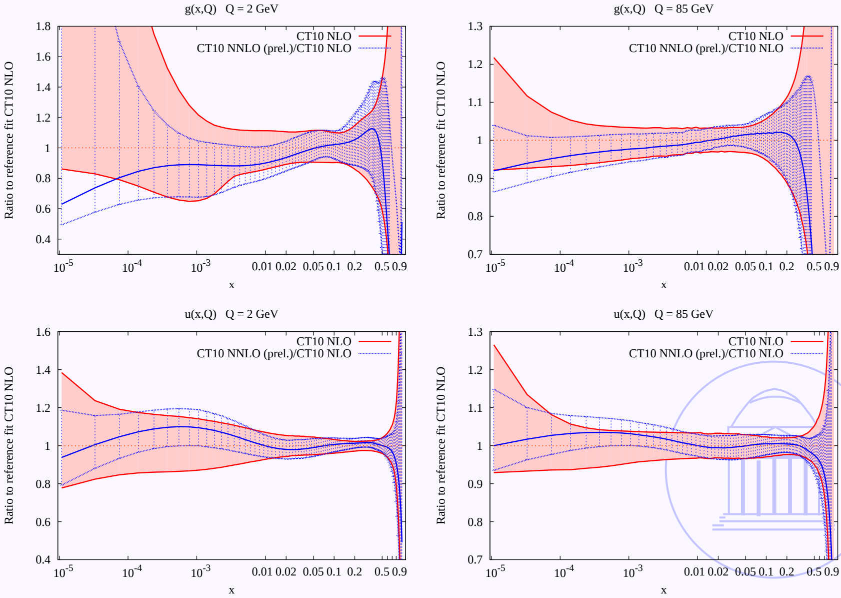
1. At $x < 10^{-2}$, $\mathcal{O}(\alpha_s^2)$ evolution suppresses $g(x,Q)$, increases $q(x,Q)$
2. $c(x,Q)$ and $b(x,Q)$ change as a result of the $\mathcal{O}(\alpha_s^2)$ GM VFN scheme
3. At $x > 0.1$, $g(x,Q)$ and $d(x,Q)$ are reduced by revised EW couplings, alternative treatment of correlated systematic errors, scale choices

CT10W NNLO central PDFs, as ratios to NLO, Q=85 GeV



(See also P. Nadolksy's Talk at DIS2012 in Bonn and J. Huston's Talk at Standard Model@LHC 2012 in Copenhagen)

CTEQ PDF uncertainties NNLO -PRELIMINARY-



The need of precise predictions

Do NNLO computations provide better estimates than NLO ones ?

★ IT´S NOT AUTOMATICALLY TRUE!

- We have differences among the PDF sets utilized
- Differences are compatible with the experimental errors
- Several uncertainties entering the computations compete with NNLO corrections even after the inclusion of the NNLO Wilson coeff.

★ One of the most important difference is the Heavy-flavor treatment

Massive quark contributions to neutral-current DIS

Several heavy-quark factorization schemes

FFN, ACOT, BMSN, CSN, FONLL, TR′...

Extensive recent work

Tung et al., hep-ph/0611254; Thorne, hep-ph/0601245; Tung, Thorne, arXiv:0809.0714; P.N., Tung, arXiv:0903.2667; Forte, Laenen, Nason, arXiv:1001.2312; J. Rojo et al., arXiv:1003.1241;Alekhin, Moch, arXiv:1011.5790;...

Do we have a consistent emerging picture?

Massive quark contributions to neutral-current DIS

★ We encorporate the best features of available schemes in S-ACOT-χ which is the default scheme for heavy-flavor treatment in CTEQ analyses.

★ We revisited the QCD factorization theorem for DIS with heavy quarks.

★ We provide algorithmic formulas for NNLO implementation

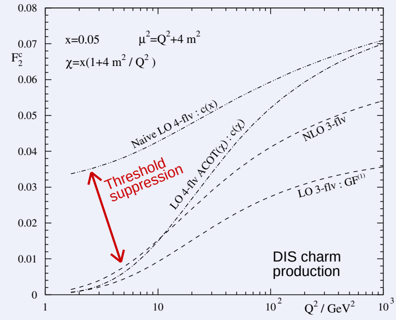
- A complementary calculation (a "hybrid mass scheme"; exact $O(\alpha_s)$ massive ACOT terms + approximate $O(\alpha_s^2)$ and $O(\alpha_s^3)$ massive terms), has been published by Stavreva, Olness, Schienbein, Jezo, Kusina, Kovarik, Yu, arXiv:1203.0282

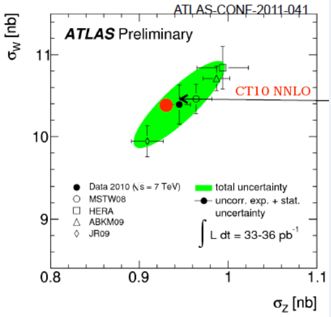
Heavy-quark DIS and LHC observables

Motivation:

General-mass (and not zero-mass of fixed-flavor number) treatment of c, b mass terms in DIS is essential for predicting precision W, Z cross sections at the LHC *(Tung et al., hep-ph/0611254)*

Several quark mass effects are comparable to NNLO radiative contributions, must be included in a consistent way

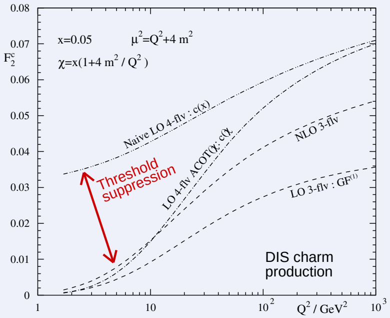


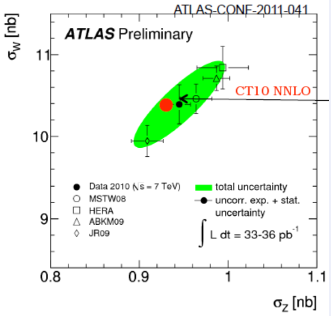


Heavy-quark DIS and LHC observables

We will discuss:

- an NNLO computation for heavy-quark DIS structure functions, $F_i^{c,b}(x,Q)$, in a general-mass scheme (S-ACOT-χ)
- a consistent treatment of all relevant factors in $F_i^{c,b}(x,Q)$ affecting CTEQ-TAO PDFs at NNLO accuracy

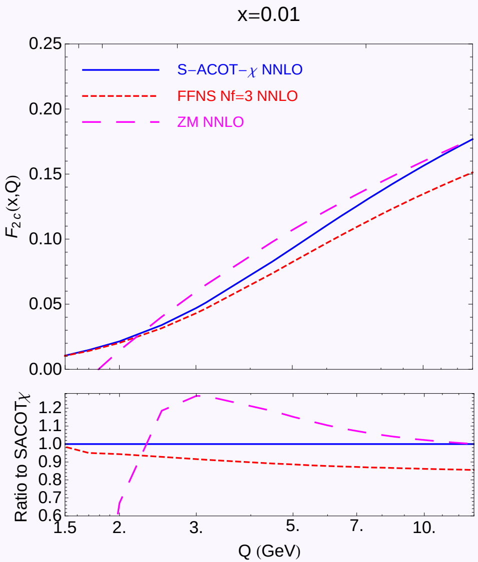




Main features of the S-ACOT-χ scheme

- It is proved to all orders by the QCD factorization theorem for DIS *(Collins, 1998)*
- It is relatively simple
  - One value of N_f (and one PDF set) in each Q range
  - sets $m_h = 0$ in ME with incoming $h = c$ or b
  - matching to FFN is **implemented as a part of the QCD factorization theorem**
- **Universal** PDFs
- It reduces to the ZM $\overline{MS}$ scheme at $Q^2 \gg m_Q^2$, without additional renormalization
- It reduces to the FFN scheme at $Q^2 \approx m_Q^2$
  - has reduced dependence on tunable parameters at NNLO

$F_2^c(x, Q^2)$ at NNLO



S-ACOT-χ reduces to FFNS at $Q \approx m_c$ and to ZM at $Q \gg m_c$

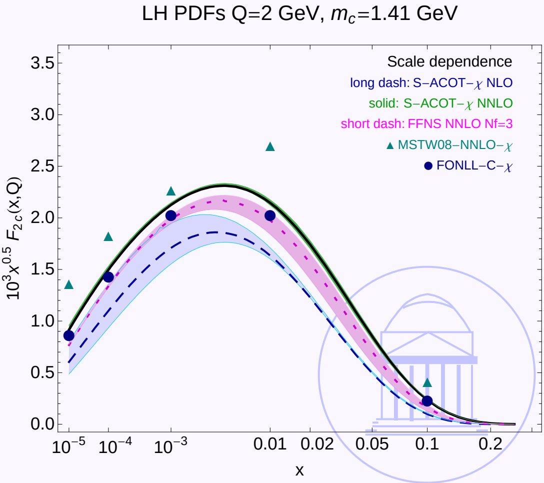
Les Houches toy PDFs, evolved at NNLO with threshold matching terms

NNLO predictions for F_L^c are in the backup slides

Results for $F_2^c(x, Q^2)$ at NLO/NNLO

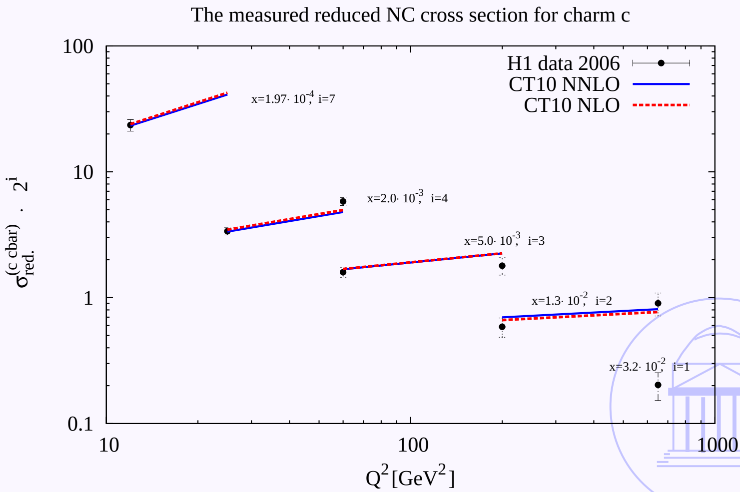
At NNLO and $Q \approx m_c$:

- S-ACOT-$\chi \approx$ FFN$(N_f = 3)$ without tuning
- It is close to other NNLO schemes
- S-ACOT-χ predictions are for a physically motivated rescaling variable $\zeta = x(1 + 4m_c^2/Q^2)$. Dependence on the form of ζ is also reduced



S-ACOT-χ predictions with CT10 NLO/NNLO -PRELIMINARY-

H1 Collaboration: Eur.Phys.J.C45:23-33,2006; Eur.Phys.J.C40:349-359,2005



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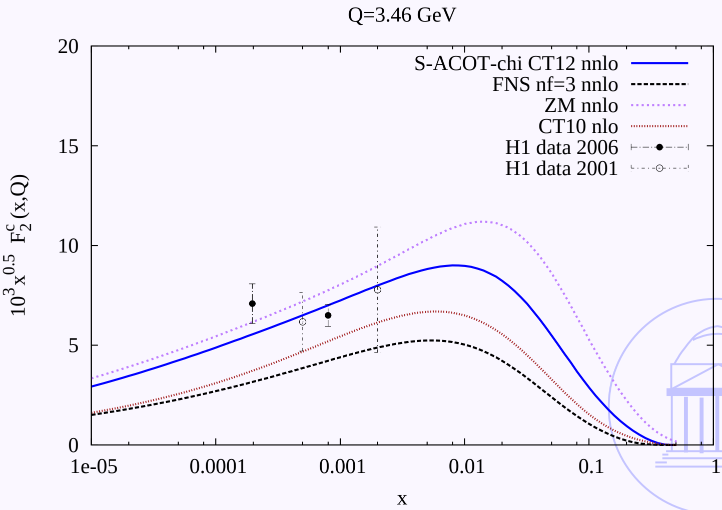

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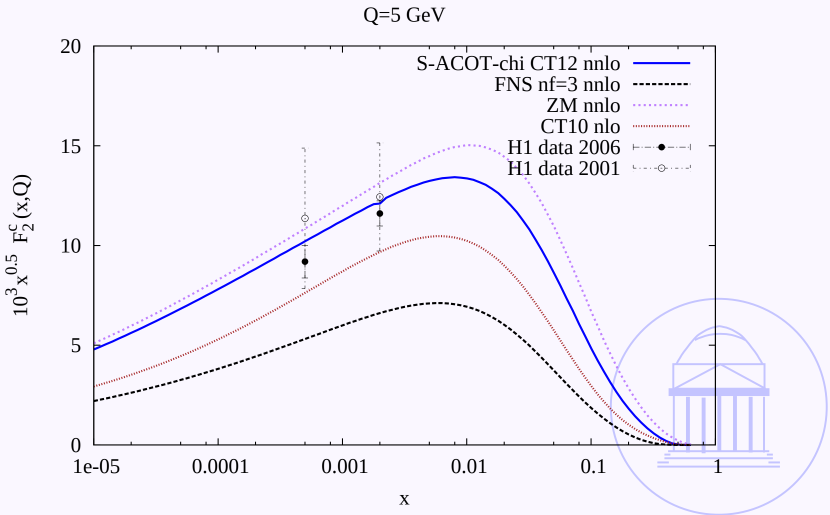

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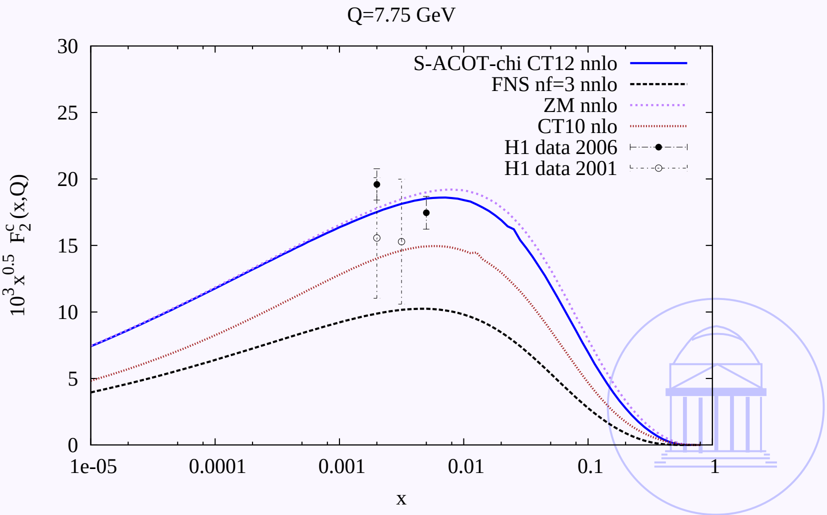
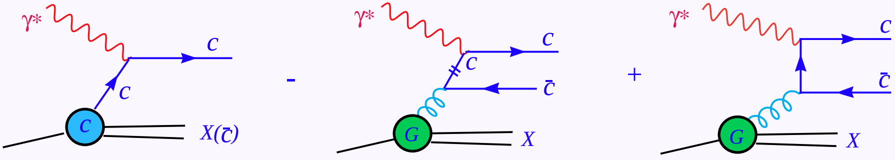


Pictorial description at lowest non-trivial order



- The first (LO 4-flv scheme) term is called flavor-excitation
- The middle (asymptotic/subtract) term represents the overlap between the LO 3-flv scheme and LO 4-flv scheme terms
- The third (LO 3-flv scheme) term is variously referred to as the flavor-creation, or gluon-fusion, or fixed-flavor-number term – since the charm quark never becomes an active parton flavor

$$c(\zeta,\mu)\,\omega^0 \;-\; \alpha_s(\mu)\ln\left(\frac{\mu}{m_H}\right)\int_\zeta^1 \frac{dz}{z} g(z,\mu) P_{g\to c}\left(\frac{\zeta}{z}\right)\omega^0 + \alpha_s(\mu)\int_\chi^1 \frac{dz}{z} g(z,\mu)\omega^1\left(\frac{\chi}{z},\frac{m_H}{Q}\right)$$

GM VFN schemes are Jazzy!

★ Jazz players always try to play solos by creating smooth connections over chord changes.

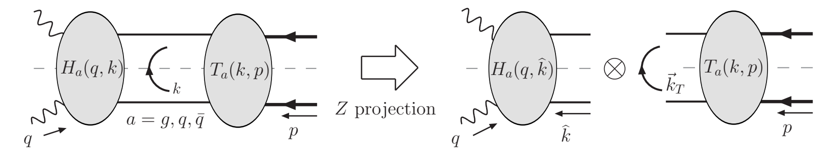
★ GM VFN smoothly interpolates through different flavor schemes!

Details of the computation at NNLO

- NNLO evolution for α_s and PDFs (HOPPET)
  - matching coefficients relating the PDFs in N_f and N_{f+1} schemes *(Smith, van Neerven, et al.)*
- NNLO Wilson coefficient functions for $F_2^c(x, Q)$, $F_L^c(x, Q)$
- One value of N_f and one PDF set in each Q-range
- $\Rightarrow$ S-ACOT: implementation as an algorithm that follows the proof of QCD factorization for DIS with massive quarks, J. Collins Phys.Rev.D 1998

★ **S-ACOT-χ implementation will be made available in HERAFITTER**

Rescaling to all orders of α_s in the QCD factorization theorem



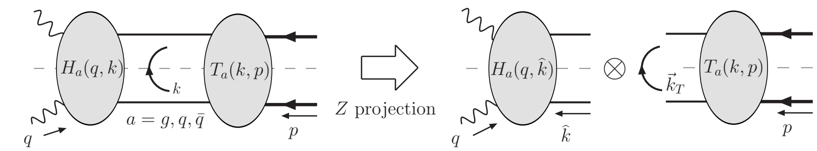
- is motivated on the same physics grounds as the proof of the S-ACOT scheme (without rescaling) by Collins (1998)

- only changes the parent's light-cone momentum p^+ in $H_c(q,\hat{k})$, the hard graph for $\gamma^*(q) + c(\hat{k}) \to X$; the light-parton hard graphs H_g, H_q and target graphs $T_a(k,p)$ are not affected

Approximate momentum $\hat{k}^\mu$ of the incoming c quark in $H_c(q,\hat{k})$

Scheme	Full ACOT	S-ACOT	S-ACOT-χ
$\left(\hat{k}^+, \hat{k}^-, \vec{0}_T\right)$ in H_c	$\left(\xi p^+, \frac{m_c^2}{2\xi p^+}, \vec{0}_T\right)$	$\left(\xi p^+, 0, \vec{0}_T\right)$	$\left(\xi \frac{p^+}{\kappa}, 0, \vec{0}_T\right)$
ξ range in $H_c \otimes T_c$	$\frac{x}{2}\ \left(1+\sqrt{\kappa}\right) \leq \xi \leq 1$	$x \leq \xi \leq 1$	$x\,\kappa \leq \xi \leq 1$
The ξ range is...	wrong	wrong	**OK** ($x\kappa = \chi$)

Rescaling to all orders of α_s in the QCD factorization theorem



$$F(x,Q) = \sum_{a=g,u,d,\ldots,c} \int \frac{d\xi}{\xi} C_a\left(\frac{x}{\xi}, \frac{Q}{\mu}, \frac{m_c}{Q}\right) f_{a/p}(\xi,\mu)$$

- **Wilson coefficients with initial heavy quarks** are

$$C_c\left(\frac{x}{\xi}, \frac{Q}{\mu}, \frac{m_c}{Q}\right) \approx C_c\left(\frac{\chi}{\xi}, \frac{Q}{\mu}, m_c = 0\right) \theta(\chi \le \xi \le 1)$$

where $\chi \equiv x\left(1 + \frac{4m_c^2}{Q^2}\right).$

- The target (PDF) subgraphs T_a are given by the same **universal** operator matrix elements in all ACOT schemes

Components of inclusive $F_{2,L}(x, Q)$.

Components of inclusive $F_{2,L}(x, Q^2)$ are classified according to the quark couplings to the photon

$$F = \sum_{l=1}^{N_l} F_l + F_h \quad (1)$$

$$F_l = e_l^2 \sum_a \left[C_{l,a} \otimes f_{a/p}\right](x, Q), \quad F_h = e_h^2 \sum_a \left[C_{h,a} \otimes f_{a/p}\right](x, Q). \quad (2)$$

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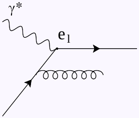

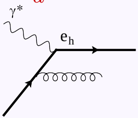

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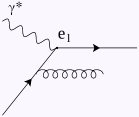

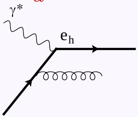

At $\mathcal{O}(\alpha_s^2)$:

$$F_h^{(2)} = e_h^2 \left\{ c_{h,h}^{NS,(2)} \otimes (f_{h/p} + f_{\bar{h}/p}) + C_{h,l}^{(2)} \otimes \Sigma + C_{h,g}^{(2)} \otimes f_{g/p} \right\}$$

$$F_l^{(2)} = e_l^2 \left\{ C_{l,l}^{NS,(2)} \otimes (f_{l/p} + f_{\bar{l}/p}) + c^{PS,(2)} \otimes \Sigma + c_{l,g}^{(2)} \otimes f_{g/p} \right\}. \qquad (3)$$

Components of inclusive $F_{2,L}(x, Q)$.

Components of inclusive $F_{2,L}(x, Q^2)$ are classified according to the quark couplings to the photon

The "light-quark" F_l contains some subgraphs with heavy-quark lines, denotd by "$G_{l,l,heavy}$". The "heavy-quark" $F_h \neq F_2^c$,

$$F_2^c = F_h + (G_{l,l,heavy})_{real}, \tag{1}$$

where $G_{i,j} = C_{i,j}^{(2)}, \ F_{i,j}^{(2)}$, and $A_{i,j}^{(2)}$

Notations

- Lower case $c^{(2)}_{a,b}$, $\hat{f}^{(k)}_{a,b}$ $\rightarrow$ ZM epxressions
  Zijlstra and Van Neerven PLB272 (1991), NPB383 (1992)
  S. Moch, J.A.M. Vermaseren and A. Vogt, NPB724 (2005)

- Upper case $C^{(2)}_{a,b}$, $F^{(k)}_{a,b}$ $A^{(k)}_{a,b}$ $\rightarrow$ coeff. functions, structure functions and subtractions with $m_c \neq 0$,
  Buza *et al.*, NPB 472 (1996); EPJC1 (1998);
  Riemersma, *et al.* PLB 347 (1995); Leanen *et al.* NPB392 (1993)

A parton level calculation of $C_{a,b}^{(k)}$

Coefficient functions $C_{a,b}^{(k)}$ are found from parton-level structure functions by considering $F(e+b \to e+X) \equiv \sum_{i=1}^{N_f} e_i^2 F_{i,b}$ for DIS on an initial-state parton b

$$F(e+b \to e+X) = \sum_{i=1}^{N_f} e_i^2 \sum_{a=-N_f}^{N_f} \left[C_{ia} \otimes f_{a/b}\right](x,Q), \tag{2}$$

$f_{a/p}$ are parton-level PDFs.

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$f_{a/p}$ are parton-level PDFs. Perturbative expansions as a series of $a_s \equiv \alpha_s(\mu, N_f)/(4\pi)$

$$\begin{aligned} f_{a/b}(x) &= \delta_{ab}\delta(1-x) + a_s\, A_{ab}^{(1)} + a_s^2 A_{ab}^{(2)} + \dots, \\ C_{i,a} &= C_{i,a}^{(0)} + a_s\, C_{i,a}^{(1)} + a_s^2 C_{i,a}^{(2)} + \dots, \\ F_{i,b} &= F_{i,b}^{(0)} + a_s\, F_{i,b}^{(1)} + a_s^2\, F_{i,b}^{(2)} + \dots, \end{aligned} \qquad (3)$$

$A_{ab}^{(m)}$ $(m = 0, 1, 2, \dots)$ are OME's defining the parton-level PDFs.

Operator matrix elements

★ A lot progress in the computation of OME's

- NNLO: Bierenbaum, Blümlein, Klein 2009 Nucl.Phys.B; $\rightarrow$ Mellin Moments
- Blümlein, Klein, Tödtli 2009 Phys. Rev. D
- Ablinger, Blümlein, Klein, Schneider, Wissbrock 2011 Nucl.Phys.B) contrib. $\propto n_f$ to F_2 (all N):

3-loop computations have been performed for

- $A_{qq,Q}^{PS}$, $A_{qg,Q}$ Complete.
- $A_{Qg}, A_{Qq}^{PS}, A_{qq}^{NS}, \ldots$: all terms of $O(n_f T_F^2 C_{A/F})$
- $A_{Qq}^{PS}, A_{qq,Q}^{NS}, A^{NS}, \ldots$: all terms of $O(T_F^2 C_{A/F})$

(See J. Blümlein's talk at DIS2012)

Operator matrix elements

It would be interesting to
explore features of S-ACOT-χ at $O(\alpha_s^3)$!

α_s^2-Coefficient functions in F_h

Perturbative contributions $C_{i,a}^{(k)}$ to the Wilson functions can be found as

$$
\begin{aligned}
C_{i,b}^{(0)} &= F_{i,b}^{(0)}, \\
C_{i,b}^{(1)} &= F_{i,b}^{(1)} - C_{i,a}^{(0)} \otimes A_{ab}^{(1)}, \\
C_{i,b}^{(2)} &= F_{i,b}^{(2)} - C_{i,a}^{(0)} \otimes A_{ab}^{(2)} - C_{i,a}^{(1)} \otimes A_{ab}^{(1)},
\end{aligned} \tag{4}
$$

S-ACOT prescription: use ZM expressions for $F_{a,b}^{(k)}$ and $C_{a,b}^{(k)}$ with incoming heavy-quark lines.

α_s^2-Coefficient functions in F_h

This leads to the following F_h

$$F_h^{(2)} = e_h^2 \left\{ c_{h,h}^{NS,(2)} \otimes (f_{h/p} + f_{\bar{h}/p}) + C_{h,l}^{(2)} \otimes \Sigma + C_{h,g}^{(2)} \otimes f_{g/p} \right\} \quad (5)$$

where

$$\begin{aligned} c_{h,h}^{NS,(2)} &= \widehat{f}_{h,h}^{NS,(2)}; \\ C_{h,l}^{(2)} &= \widehat{F}_{h,l}^{PS,(2)} - A_{hl}^{PS,(2)}; \\ C_{h,g}^{(2)} &= \widehat{F}_{h,g}^{(2)} - A_{hg}^{(2)} - c_{h,h}^{(1)} \otimes A_{hg}^{(1)}; \end{aligned} \quad (6)$$

$c_{h,h}^{(1)} = \widehat{f}_{h,h}^{(1)}$ and $\Sigma(x,\mu) = \sum_{i=1}^{N_f} \left[f_{i/p}(x,\mu) + \bar{f}_{i/p}(x,\mu) \right]$,
Available from literature!

α_s^2-Coefficient functions in F_h

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Available from literature!

Zijlstra, Van Neerven, PLB 272 (1991), NPB383 (1992), M.Buza *et al.*, NPB472 (1996), J.Smith *et al.* EPJC1 (1998)

...Similarly for F_l

while the function F_l is given by

$$F_l^{(2)} = e_l^2 \left\{ C_{l,l}^{NS,(2)} \otimes (f_{l/p} + f_{\bar{l}/p}) + c^{PS,(2)} \otimes \Sigma + c_{l,g}^{(2)} \otimes f_{g/p} \right\}. \qquad (7)$$

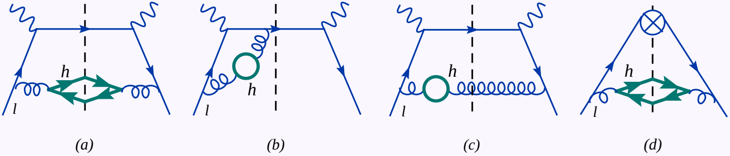
where

$$C_{l,l}^{NS,(2)} = \widehat{f}_{l,l,light}^{NS,(2)} + F_{l,l,heavy}^{NS,(2)} - A_{ll,heavy}^{NS,(2)}; \qquad (8)$$

$c^{PS,(2)}$ and $c_{l,g}^{(2)}$: ZM expressions by Zijlstra, Van Neerven, PLB 272 (1991), NPB383 (1992)

S. Moch, J.A.M. Vermaseren and A. Vogt, NPB724 (2005)

Light-quark component of $F(x,Q)$



Among all terms, only non-singlet contributions $F_{l,l}^{NS,(2)}$ and $A_{ll}^{NS,(2)}$ to $C_{l,l}^{(2)}$ include disconnected heavy-quark lines in (cut) fermion loops and should be evaluated with full dependence on m_h. These heavy-quark contributions can be explicitly identified inside the non-singlet functions as

$$G = g_{light}(\{m\}=0) + G_{heavy}(m_h) \qquad \text{for } G = F_{l,l}^{NS,(2)}, A_{ll}^{NS,(2)},$$

Experimentally defined HQ structure function

The heavy-quark component F_h of inclusive $F(x,Q)$:
Not directly measurable!
At Q accessible at HERA the observable semi-inclusive $F_{h,SI} \approx F_2^c$ is related to F_h by

$$F_{h,SI}(x,Q) = F_h(x,Q) + \sum_{l=1}^{N_l} e_l^2 (F^{NS,(2)}_{l,l,heavy})_{real} \otimes (f_{l/p} + f_{\bar{l}/p}), \quad (9)$$

$(F^{NS,(2)}_{i,i,heavy})_{real}$ is a part of the non-singlet contribution with the incoming light quark that is contributed by real emission diagrams.

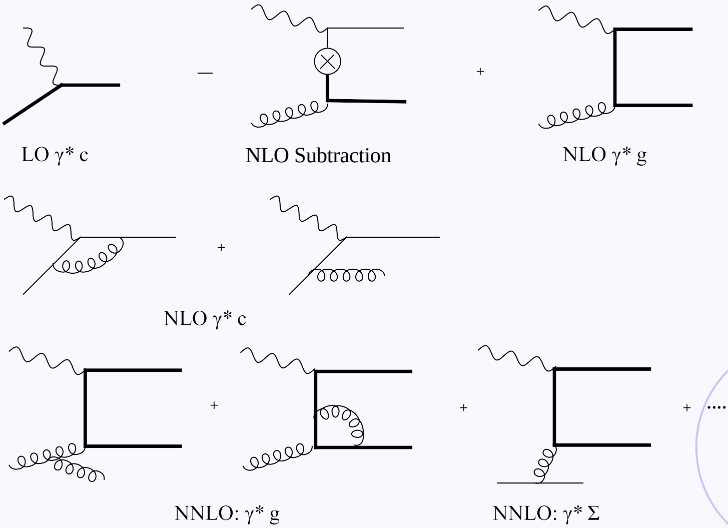
At higher Q the experimental $F_{h,SI}$ can be regularized in the collinear region as shown in Chuvakin *et al.* Phys. Rev. D 61, (2000).

Experimentally defined HQ structure function

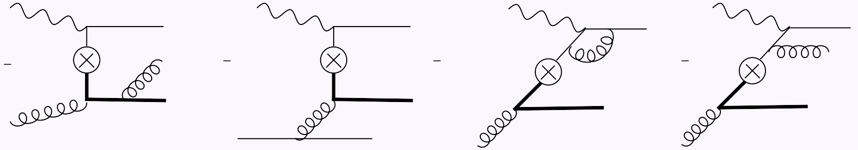
We use $F_{h,SI}$ to compute $F_{2,L}^{c}$ at moderate Q.

We use F_l and F_h to compute inclusive F_2 and F_L.

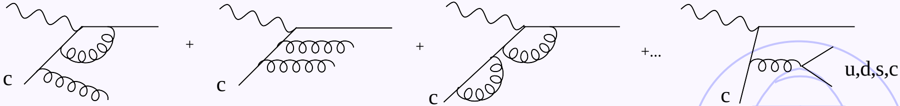
Classes of Feynman diagrams I



Classes of Feynman Diagrams II



NNLO Subtractions



NNLO γ^* c

Cancellations between Feynman diagrams

Validity of the S-ACOT calculation was verified by checking for certain cancelations at $Q \approx m_c$ and $Q \gg m_c$

- $Q \approx m_c$:

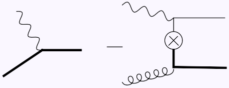
$$D^{(2)}_{C1} \ll D^{(2)}_{C0} \ll D^{(1)}_{C0} \leq F_2^c(x, Q)$$

- $Q \gg m_c$:

$$D^{(2)}_{g} \ll D^{(1)}_{g} < F_2^c(x, Q)$$

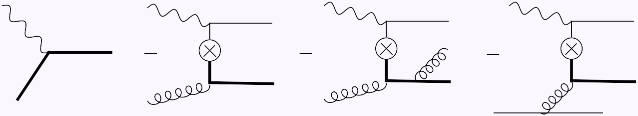
These cancellations are indeed observed in our results

NNLO: Cancellations at $Q^2 \approx m_c^2$



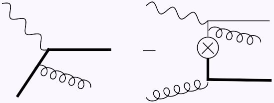
$D^{(1)}_{C0}$

$$D^{(1)}_{C0} = c^{(0)}_{h,h} \otimes c - a_s\, c^{(0)}_{h,h} \otimes A^{(1)}_{hg} \otimes g; \qquad a_s = \frac{\alpha_s}{(4\pi)} \tag{10}$$



$D^{(2)}_{C0}$

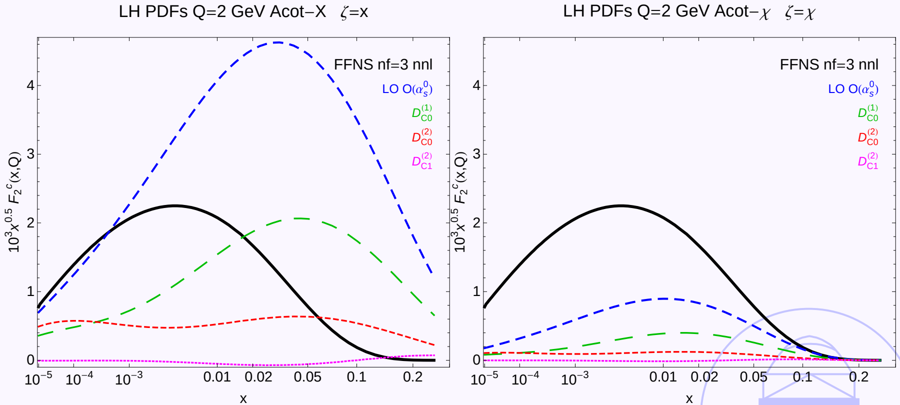
$$D^{(2)}_{C0} = D^{(1)}_{C0} - a_s^2\, c^{(0)}_{h,h} \otimes A^{(2)}_{hg} \otimes g - a_s^2\, c^{(0)}_{h,h} \otimes A^{PS,(2)}_{hl} \otimes \Sigma \tag{11}$$



$D^{(2)}_{C1}$

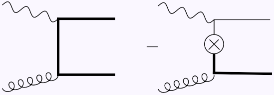
$$D^{(2)}_{C1} = a_s\, c^{(1)}_{h,h} \otimes c - a_s^2\, c^{(1)}_{h,h} \otimes A^{(1)}_{hg} \otimes g \tag{12}$$

NNLO: Cancellations at $Q^2 \approx m_c^2$



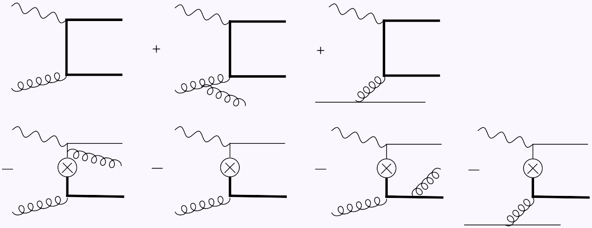
$D_{C1}^{(2)} << D_{C0}^{(2)} << D_{C0}^{(1)} \leq$ FFN at NNLO both for $\zeta = x$ and $\zeta = \chi$.

NNLO: Cancellations at $Q >> m_c$



$D_g^{(1)}$

$$D_g^{(1)} \equiv C_{h,g}^{(1)} = a_s \left(F_{h,g}^{(1)} \otimes g - c_{h,h}^{(0)} \otimes A_{hg}^{(1)} \otimes g \right) \quad (13)$$

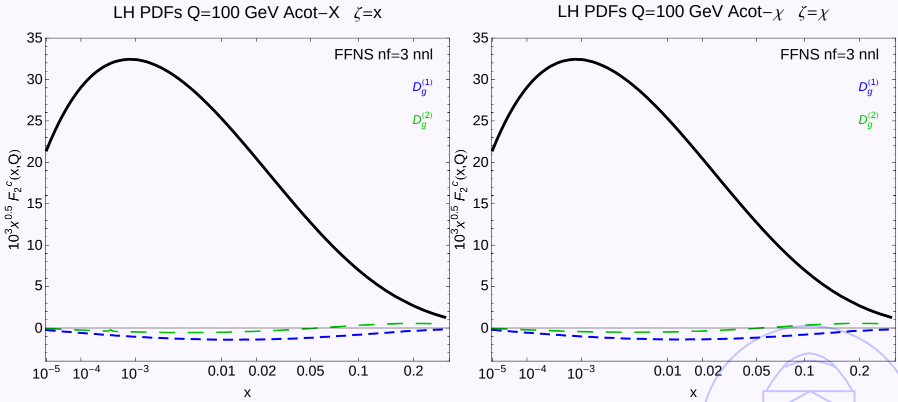


$D_g^{(2)}$

$$D_g^{(2)} = D_g^{(1)} + a_s^2 \left[\widehat{F}_{h,g}^{(2)} \otimes g + \widehat{F}_{h,l}^{PS,(2)} \otimes \Sigma - c_{h,h}^{(1)} \otimes A_{hg}^{(1)} \otimes g \right.$$
$$\left. - c_{h,h}^{(0)} \otimes A_{hg}^{(2)} \otimes g - c_{hh}^{(0)} \otimes A_{hl}^{PS,(2)} \otimes \Sigma \right] \quad (14)$$

$D_g^{(1)}$ is of order of α_s^2 while $D_g^{(2)}$ is of order of α_s^3.

F_2^c at NNLO: Cancellations at $Q = 100$ GeV



$D_g^{(2)} << D_g^{(1)} <$ FFN at NNLO $<$ ACOT

$\log \frac{Q^2}{m_c^2}$ terms in FFN are canceled well by subtractions.

Conclusions

- A lot of work in progress to deliver CTEQ at NNLO the sooner
- An NNLO calculation for $F_2^{c,b}$ and $F_L^{c,b}$ in the S-ACOT-χ scheme is proven to be viable
- This is the most challenging component of the NNLO CTEQ PDF analysis, which will be made available soon.
- NNLO predictions are stable and show a remarkable reduction in the dependence on free parameters, compared to NLO.
- They will help us to reduce tuning of m_c and scale parameters, to achieve excellent agreement with the HERA DIS data

BACK UP SLIDES

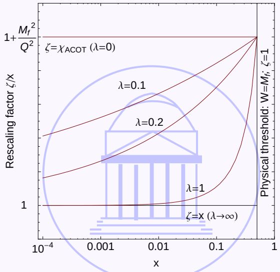
S-ACOT-χ input parameters

At $Q \approx m_c$, F_2^c depends significantly on

1. **Charm mass:** $m_c = 1.3$ GeV in CT10
2. **Factorization scale:** $\mu = \sqrt{Q^2 + \kappa m_c^2}$; $\kappa = 1$ in CT10
3. **Rescaling variable** $\zeta(\lambda)$ for matching in $\gamma^* c$ channels
   (*Tung et al., hep-ph/0110247; Nadolsky, Tung, PRD79, 113014 (2009)*)

$$F_i(x, Q^2) = \sum_{a,b} \int_\zeta^1 \frac{d\xi}{\xi}\, f_a(\xi, \mu)\, C^a_{b,\lambda}\left(\frac{\zeta}{\xi}, \frac{Q}{\mu}, \frac{m_i}{\mu}\right)$$

$$x = \zeta \Big/ \left(1 + \zeta^\lambda \cdot (4m_c^2)/Q^2\right)$$, with $0 \leq \lambda \lesssim 1$

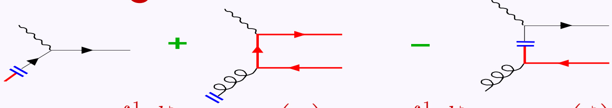


CT10 uses

$$\zeta(0) \equiv \chi \equiv x\left(1 + 4m_c^2/Q^2\right),$$

motivated by momentum conservation

Rescaling at the lowest order



$$c(\zeta) + \frac{\alpha_s}{4\pi}\int_\chi^1 \frac{d\xi}{\xi} g(\xi) C_{h,g}^{(1)}\left(\frac{\chi}{\xi}\right) - \frac{\alpha_s}{4\pi}\int_\zeta^1 \frac{d\xi}{\xi} g(\xi) A_{h,g}^{(1)}\left(\frac{\zeta}{\xi}\right)$$

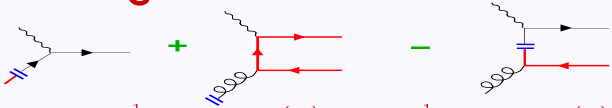
ζ takes place of x in terms 1 and 3

- Term 2 ($\gamma^* g$ fusion) is unambiguous
- Terms 1 and 3 are essentially

$$\int_0^1 \frac{d\xi}{\xi}\delta\left(1-\frac{\zeta}{\xi}\right)c(\xi) - \frac{\alpha_s}{4\pi}\int_0^1 \frac{d\xi}{\xi} g(\xi) A_{h,g}^{(1)}\left(\frac{\zeta}{\xi}\right)\theta\left(1-\frac{\zeta}{\xi}\right).$$

Rescaling $\zeta \to \kappa\zeta$, $\xi \to \kappa^{-1}\xi$ changes the ξ range in the **Wilson** coefficients $\delta(1-\frac{\zeta}{\xi})$ and $A_{h,g}^{(1)}\left(\frac{\zeta}{\xi}\right)\theta\left(1-\frac{\zeta}{\xi}\right)$, but does not change their magnitude

Rescaling at the lowest order

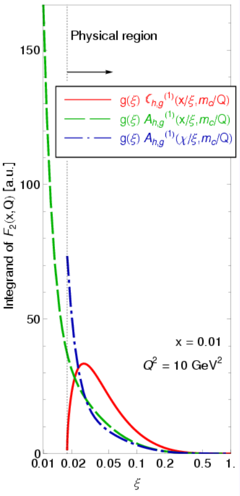


ζ takes place of x in terms 1 and 3

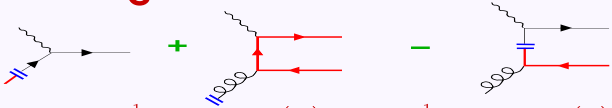
$$c(\zeta) + \frac{\alpha_s}{4\pi}\int_{\chi}^{1}\frac{d\xi}{\xi}g(\xi)C^{(1)}_{h,g}\left(\frac{\chi}{\xi}\right) - \frac{\alpha_s}{4\pi}\int_{\zeta}^{1}\frac{d\xi}{\xi}g(\xi)A^{(1)}_{h,g}\left(\frac{\zeta}{\xi}\right)$$

$Q^2 = 10$ GeV^2

- Red curve: $g(\xi)C^{(1)}_{h,g}(\chi/\xi)$ at $\chi \leq \xi \leq 1$
- Green: $\zeta = \mathbf{x}$; $\kappa = 1$
  - $g(\xi)A^{(1)}_{h,g}(x/\xi) \neq 0$ at $\xi < \chi$
  - its integral cancels poorly with $c(x)$
- Blue: $\zeta = \chi$; $\kappa = 1 + 4m_c^2/Q^2$
  - $g(\xi)A^{(1)}_{h,g}(\chi/\xi) = 0$ at $\xi < \chi$
  - its integral cancels better with $c(\chi)$

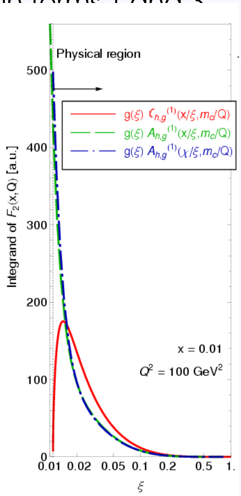


Rescaling at the lowest order



$$c(\zeta) + \frac{\alpha_s}{4\pi}\int_{\chi}^{1}\frac{d\xi}{\xi}g(\xi)C_{h,g}^{(1)}\left(\frac{\chi}{\xi}\right) - \frac{\alpha_s}{4\pi}\int_{\zeta}^{1}\frac{d\xi}{\xi}g(\xi)A_{h,g}^{(1)}\left(\frac{\zeta}{\xi}\right)$$

ζ takes place of x in terms 1 and 3

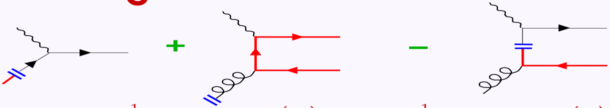


$Q^2 = 100$ GeV^2

$\chi \approx x$

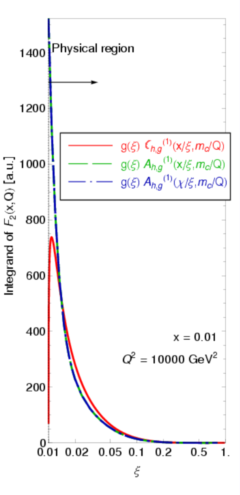
$g(\xi)A_{h,g}^{(1)}(\zeta/\xi)$ approximates the logarithmic growth in $g(\xi)C_{h,g}^{(1)}(\chi/\xi)$

Rescaling at the lowest order



$$c(\zeta) + \frac{\alpha_s}{4\pi}\int_\chi^1 \frac{d\xi}{\xi} g(\xi) C_{h,g}^{(1)}\left(\frac{\chi}{\xi}\right) - \frac{\alpha_s}{4\pi}\int_\zeta^1 \frac{d\xi}{\xi} g(\xi) A_{h,g}^{(1)}\left(\frac{\zeta}{\xi}\right)$$

ζ takes place of x

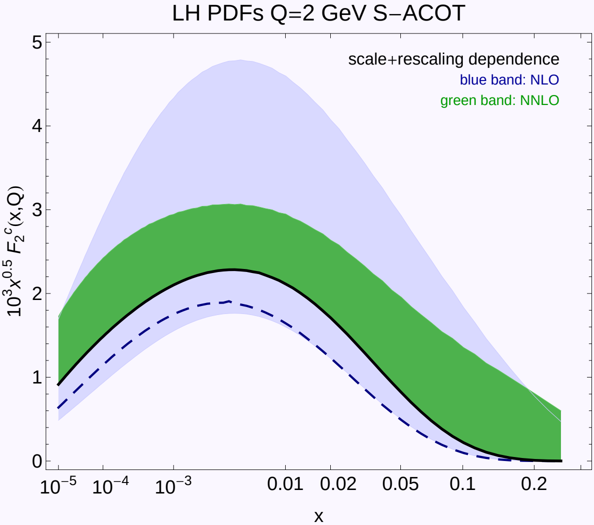


$Q^2 = 10000$ **GeV**^{2}

$\chi \approx x$

$g(\xi)A_{h,g}^{(1)}(\zeta/\xi)$ approximates the logarithmic growth in $g(\xi)C_{h,g}^{(1)}(\chi/\xi)$

Results for $F_2^c(x, Q^2)$ at NLO/NNLO



Plots for $Q^2 = 10\ GeV^2$ and $Q^2 = 100\ GeV^2$ are in the backup.

PS and NS structure of $F(x,Q)$

Using PS and NS decomposition we write

$$F(x,Q) = \sum_{i,a} e_i^2 c_{i,a} \otimes f_{a/p} = \sum_i e_i^2 \left\{ \sum_j \left[c^{PS} + \delta_{ij} c^{NS} \right] \otimes f_{j/p} + c_g \otimes f_{g/p} \right\}$$

$$= \sum_i e_i^2 c^{NS} \otimes f_{i/p} + \left(\sum_i e_i^2 \right) \left\{ c^{PS} \otimes f_{\Sigma/p} + c_g \otimes f_{g/p} \right\}$$

$$= c^{NS} \otimes \Sigma^{+,NS} + \frac{\left(\sum_i e_i^2\right)}{N_f} \left\{ c^S \otimes \Sigma + N_f c_g \otimes f_{g/p} \right\}. \qquad (15)$$

Here $\Sigma^{+,NS}$ and Σ are the non-singlet and singlet sums of (anti-)quark PDFs,

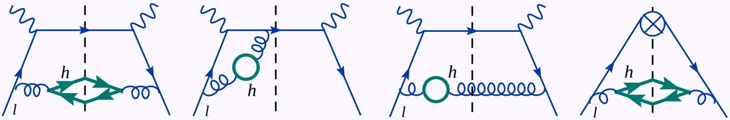
$$\Sigma(x,\mu) = \sum_{i=1}^{N_f} \left[f_{i/p}(x,\mu) + \bar{f}_{i/p}(x,\mu) \right],$$

$$\Sigma^{+,NS}(x,\mu) = \sum_{i=1}^{N_f} e_i^2 \left(f_{i/p}(x,\mu) + \bar{f}_{i/p}(x,\mu) - \frac{1}{N_f} \Sigma(x,\mu) \right),$$

and

$$c^S = c^{NS} + N_f c^{PS}.$$

Light-quark component of $F(x, Q)$



(a) (b) (c) (d)

The non-singlet heavy-quark coefficient function,

$$F^{NS,(2)}_{l,l,heavy}(x, Q^2/m_h^2) = \left(L^{NS,(2)}_{I,q}(x, Q^2/m_h^2)\right)_+ + \frac{2}{3}\ln\left(\frac{Q^2}{m_h^2}\right) c^{(1)}_{l,l}(x), \tag{16}$$

is composed of contributions of several classes shown in Figs (a)-(c). The real emission of heavy-quark pair as in Fig. (a) are accounted from $L^{NS,(2)}_{I,q}(x, Q^2/m_h^2)$, Buza *et al.* Nucl.Phys.B472 1996, for which Adler's sum rule holds, so that

$$\int_0^1 F^{NS,(2)}_{l,l,heavy}(x, Q^2/m_h^2)\, dx = 0. \tag{17}$$

Light-quark component of $F(x,Q)$

In the $Q^2 \gg m_h^2$ limit for the inclusive F_2

- $F^{NS,(2)}_{l,l,heavy} \Rightarrow$ large terms $\propto \ln(Q^2/m_h^2)$ + finite,
- Those coincide with $A^{NS,(2)}_{l,l,heavy}$ to the heavy-quark PDF from light quark flavors.

After we subtract from the coeff. function as shown before

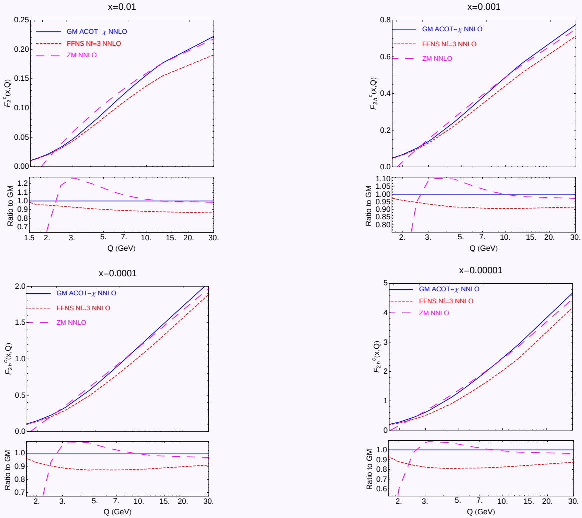
$$C^{(2),NS}_{l,l} = \widehat{f}^{NS,(2)}_{l,l,light} + F^{NS,(2)}_{l,l,heavy} - A^{NS,(2)}_{ll,heavy}, \quad (18)$$

- $C^{(2),NS}_{l,l}$ does not contain heavy quark lines,
- for $m_h^2/Q^2 \to 0$, $C^{(2),NS}_{l,l} \approx$ the ZM expression computed by Zijlstra, Van Neerven, PLB272 1991

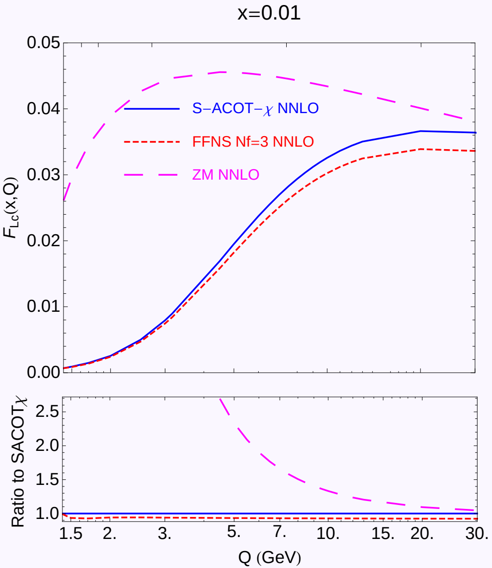
Finally we obtain

$$F_l^{(2)} = e_l^2 \left\{ C^{NS,(2)}_{l,l} \otimes (f_{l/p} + f_{\bar{l}/p}) + c^{PS,(2)} \otimes \Sigma + c^{(2)}_{l,g} \otimes f_{g/p} \right\}. \quad (19)$$

$F_2^c(x, Q^2)$ at NNLO, other x bins - Preliminary

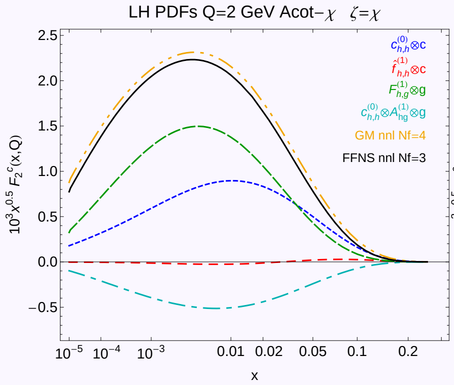


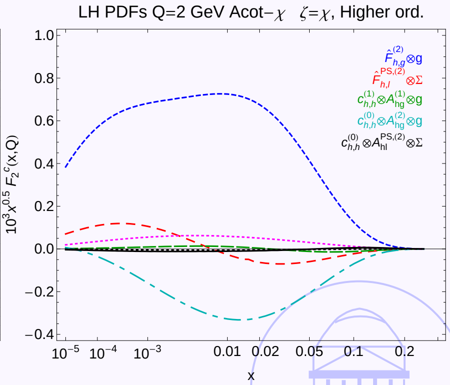
$F_L^c(x, Q^2)$ at NNLO - Preliminary



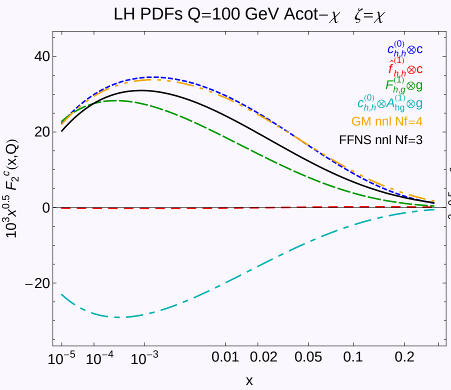
S-ACOT is close to FFNS at all Q. ZM over-estimates S-ACOT everywhere

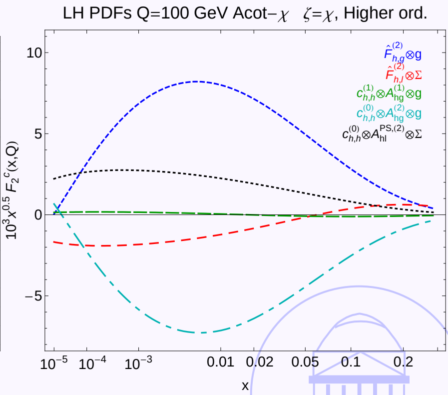
F_2^c: Anatomy of the contributions $Q = 2$ GeV





F_2^c: Anatomy of the contributions $Q = 100$ GeV





FFNS expression for $F^c_{2,L}(x,Q)$

- Riemersma, Smith, Van Neerven, Phys. Lett. B347 (1995) 143-151- The structure functions are given by

$$\begin{aligned} F_k(x,Q) &= \frac{Q^2\alpha_s}{4\pi^2 m^2}\int_x^{z_{max}} \frac{dz}{z}\left[e_H^2\, g\left(\frac{x}{z},\mu^2\right) c^{(0)}_{k,g}\right] \\ &+ \frac{Q^2\alpha_s^2}{\pi m^2}\int_x^{z_{max}} \frac{dz}{z}\left\{e_H^2\, g\left(\frac{x}{z},\mu^2\right)\left(c^{(1)}_{k,g} + \bar{c}^{(1)}_{k,g}\ \ln\frac{\mu^2}{m^2}\right)\right. \\ &+ \sum_{i=q,\bar{q}}\left[e_H^2\, f_i\left(\frac{x}{z},\mu^2\right)\left(c^{(1)}_{k,i} + \bar{c}^{(1)}_{k,i}\ \ln\frac{\mu^2}{m^2}\right)\right. \\ &+ \left.\left. e^2_{L,i}\, f_i\left(\frac{x}{z},\mu^2\right)\left(d^{(1)}_{k,i} + \bar{d}^{(1)}_{k,i}\ \ln\frac{\mu^2}{m^2}\right)\right]\right\}, \end{aligned} \quad (20)$$

Here e_H is the charge of the heavy quark while e_L refers to the light quark. Furthermore $k = 2, L$, $z_{max} = Q^2/(Q^2 + 4m^2)$ and $i = g, q, \bar{q}$.