# From Rutherford to HERA

Robin Devenish
Oxford

- Early days
- The nuclear atom
- Models and quantum ideas
- 1940s to now

# Rutherford

- Born 1871 Brightwater, New Zealand;
- BA, MA, BSc – Canterbury College NZ
- 1895-1898 Postgraduate study at Cambridge (UK) – '1851 Scholarship'
- 1898-1907 Macdonald Professor of Physics at McGill University (Montreal)
  - research on radioactivity – introduced concept of the 'half-life'
- Nobel Prize 1908 for Chemistry
  - *For investigations into the disintegration of the elements, and the chemistry of radioactive substances*
- 1907-1919 Langsworthy Professor of Physics, Manchester University
  - his most notable work on nuclear physics was done in Manchester
- 1919-1937 Director of the Cavendish Laboratory at Cambridge
  - he attracted talented colleagues and students from around the World
  - at this time the Cavendish was the leading centre in the World for nuclear physics
- died 1937 Cambridge, England

# Introduction

## Atomic Physics at the start of the 20th Century

# Hydrogen spectra

- Spectral analysis of elements developed rapidly during 2nd half of 19th Century, particularly the work of Bunsen and Kirchoff

- Kit was simple but accurate: bunsen burner; platinum wire 'ringlet' to hold the sample; prism; telescope; scale.

- Spectral analysis developed rapidly becoming an important tool and having a huge impact on astrophysics

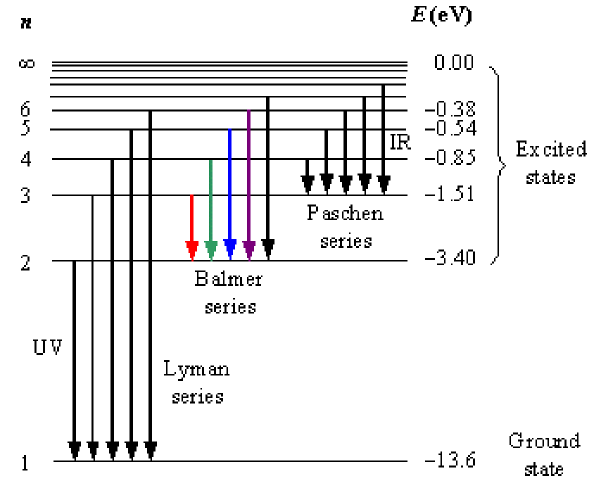
- Frequency of lines: Ritz 'combination principle'

$$E = h\nu = E_1 - E_2$$

- Balmer formula (1885)

$$\nu = \mathrm{R}\left(\frac{1}{n^2} - \frac{1}{m^2}\right) \quad n = 1, 2, \ldots, \; m > n$$

R is the Rydberg-Ritz constant which Balmer calculated as $109721\ \mathrm{cm}^{-1}$ (accuracy of about one part in ten thousand)




Energy levels of the hydrogen atom with some of the transitions between them that give rise to the spectral lines indicated.

*Übereinstimmung die im höchsten Grade überraschen muss!*

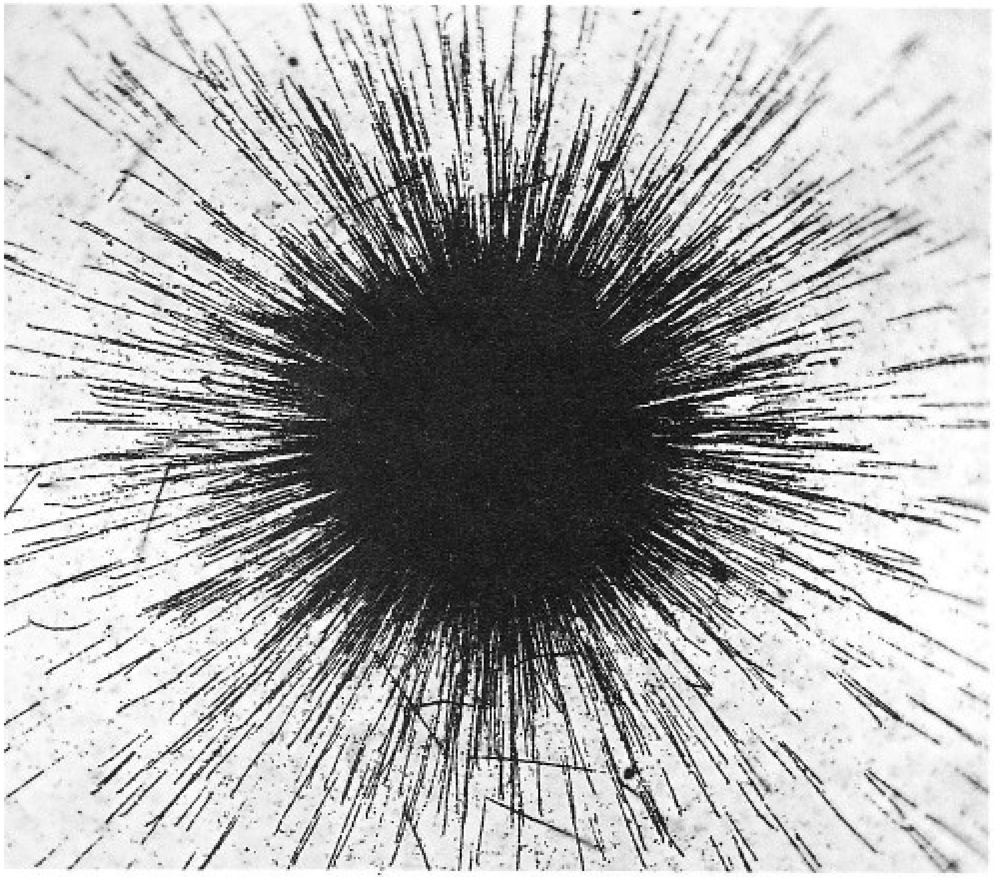
(Balmer on comparing his formula for H-lines with $n = 2$, $m = 5$ - 16.)

# Radioactivity 1896 - 1908

$\alpha$, $\beta$, $\gamma$ decays of heavy elements (Uranium, Thorium) discovered and classified

- Roentgen (1895): X-rays, conduction of electricity through rarified gases
- Becquerel (1896): 'On radiation emitted in phosphorescence' (potassium uranyl disulfate)
- J J Thomson (1897): Charge to mass ratio for cathode rays – discovery of the electron
- Marie and Pierre Curie (1898): Isolated polonium and radium from pitchblende
- Kaufmann (1902): Established that $\beta$-rays are electrons

# Radium



speck of Radium (about 0.1 mm across) emitting $\alpha$-particles in all directions captured in an emulsion photograph

you can almost feel the aggressive power of the source

# Rutherford on Bequerel Rays

- (1899) '...uranium radiation is complex, and there are at present at least two distinct types of radiation – one that is readily absorbed, which will be termed for convenience the α radiation, and the other of a more penetrative character, will be termed the β radiation.'

- (1903-1905) α particles:: positive charge from charge/mass measurements –
  if charge on the α is same as that of H ion, then the mass is twice that of H

- 1908 (Proc. Roy. Soc. with Geiger): '...evidence strongly in favour of the view that the charge of the α particle is *2e*, where *e* is proton charge'
  'We may conclude... that the α particle is a helium atom,...
  '

- 1909 (Phil.Mag with Royds): 'We can conclude with certainty... that the α particle is a helium atom' - they had demonstrated that the He spectrum was produced when an electrical discharge was applied to α particles collected from radium decay

# Nature of the α-Particle

α particles collected here, the number counted electrically
total charge measured by an electrometer

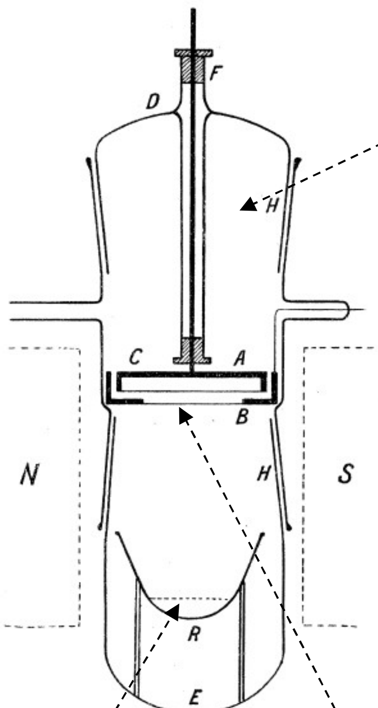


Fig. 1.

Source
Radium C ($^{214}$Bi)

Aluminium foil
window

*Nature of the α-Particle.*

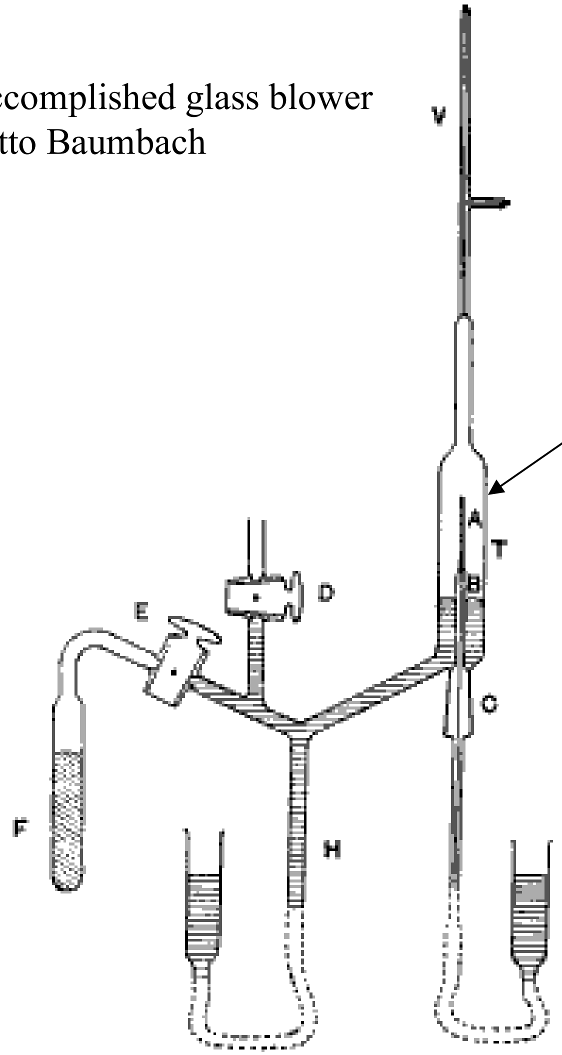
The value of E/M—the ratio of the charge on the α-particle to its mass—has been measured by observing the deflection of the α-particle in a magnetic and in an electric field, and is equal to $5{\cdot}07 \times 10^3$ on the electromagnetic system.* The corresponding value of $e/m$ for the hydrogen atom set free in the electrolysis of water is $9{\cdot}63 \times 10^3$. We have already seen that the evidence is strongly in favour of the view that $E = 2e$. Consequently $M = 3{\cdot}84m$, *i.e.*, the atomic weight of an α-particle is 3·84. The atomic weight of the helium atom is 3·96. Taking into account probable experimental errors in the estimates of the value of E/M for the α-particle, we may conclude that *an α-particle is a helium atom*, or, to be more precise, *the α-particle, after it has lost its positive charge, is a helium atom.*

Rutherford and Geiger, *Proc. Roy. Soc.* **A81,** 162, 1908

# Rutherford & Royds

Phil. Mag. **17,** 281, 1909

The accomplished glass blower was Otto Baumbach



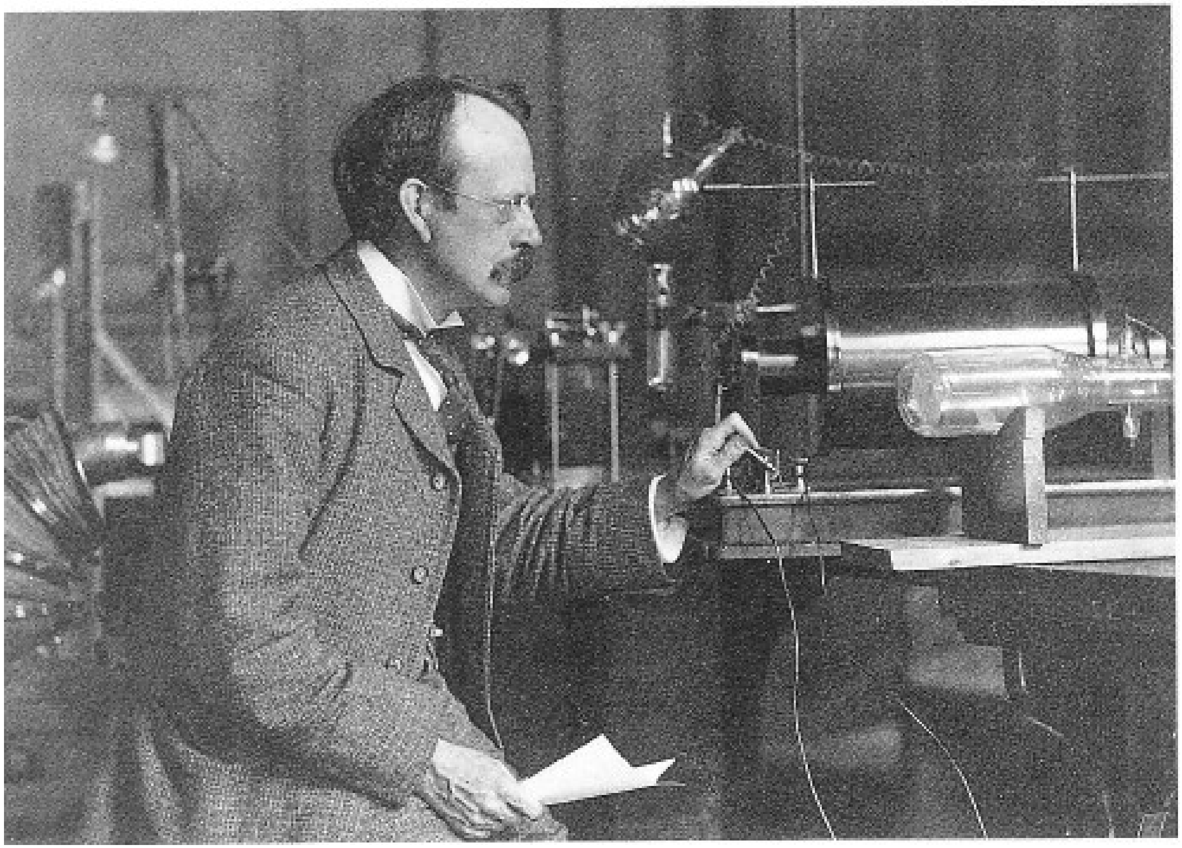

Radium emanation ($Rn^{222}$) from 140 mg radium at A, inside thin glass tube

Tube A – wall thickness < 1/100 mm thin enough to allow an $\alpha$ to escape but impervious to passage of helium or other radioactive products

Ranges of alphas ~ 4 -7 cm, so majority escape and range determined by use of zinc-sulphide screen

Enough alphas were retained in the outer tube (after 6 days) to give clear spectroscopic evidence for Helium

# J J Thomson – discovered the electron 1897



A posed photo!

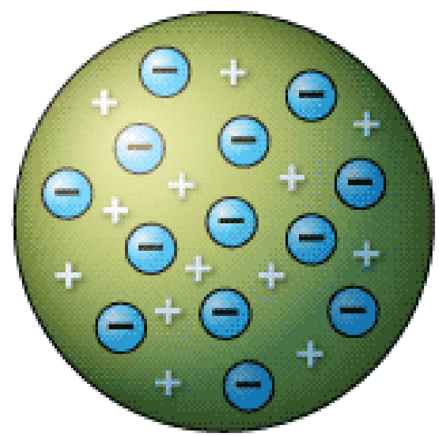
Thomson with equipment he used to measure electron charge to mass ratio e/m

Thomson also put forward the ‘plum pudding’ model of the atom

# Plum Pudding(s)

The one you eat –
particularly at Christmas

Thomson's
'plum pudding model'



# J J Thomson – atomic models (1903-1906)

- Started with a planar model - electrons in rings about a central nuclear charge - ignored radiation
  - found stable structures for well defined numbers per ring - hard to generalise to a sphere

- 3-D model - electrons in a sphere of positive charge radius $R$ - the 'plum pudding' model

- He derived results for:

  (a) the refractive index on monoatomic gases;

  (b) the fractional energy loss of an X-ray beam per unit length: $\sigma Nn$

  where $\sigma = \frac{8\pi}{3} r_0^2, \quad r_0 = e^2 \big/ \left(4\pi\varepsilon_0 mc^2\right)$ is the classical radius of the electron

  ($N$ atoms per unit volume; $n$ electrons of mass $m$)

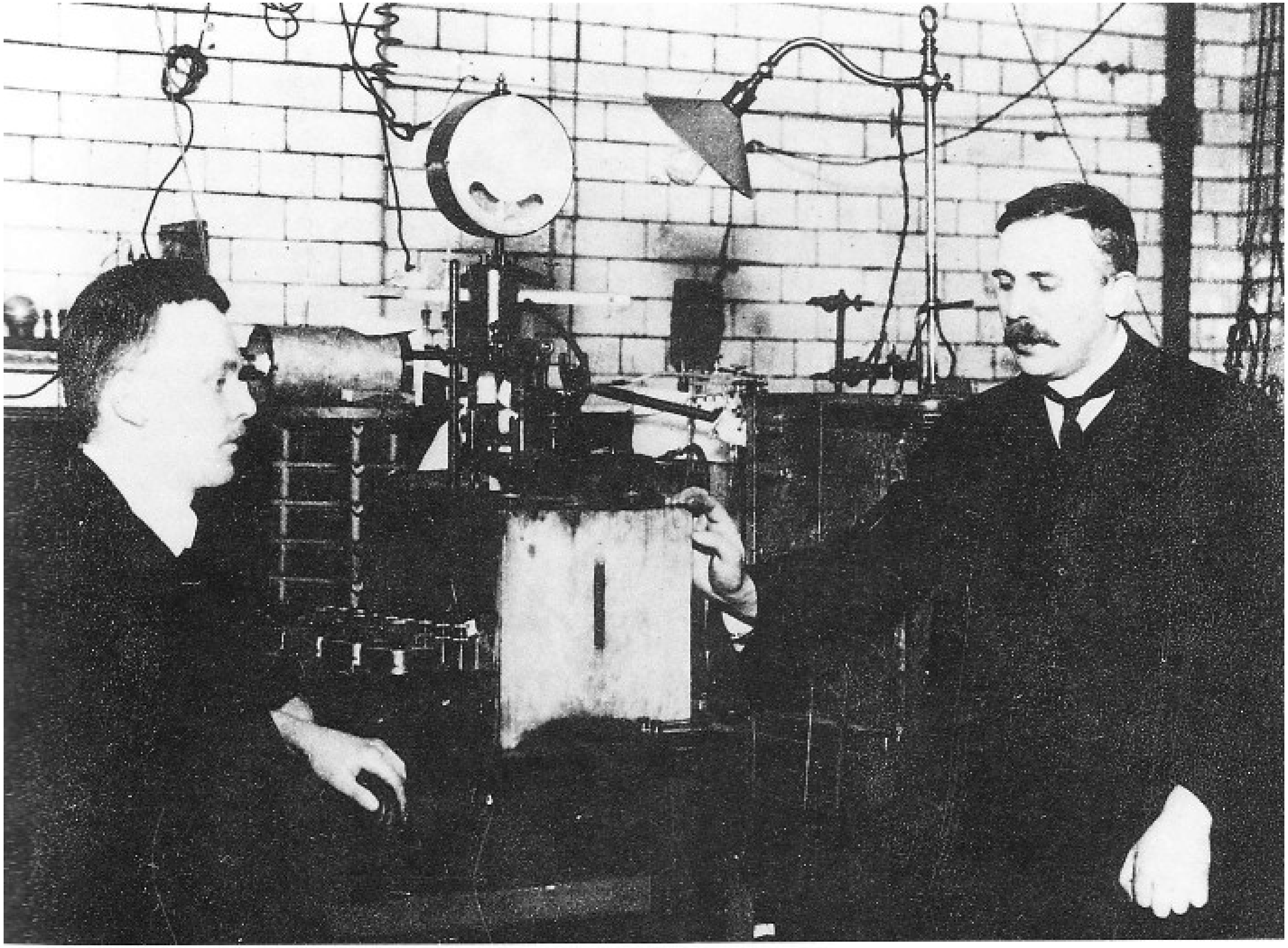
  (c) the absorption of $\beta$-rays in matter by scattering off atomic electrons

- Compared results, favourably, with experimental data from Kettler, Barkla and Rutherford

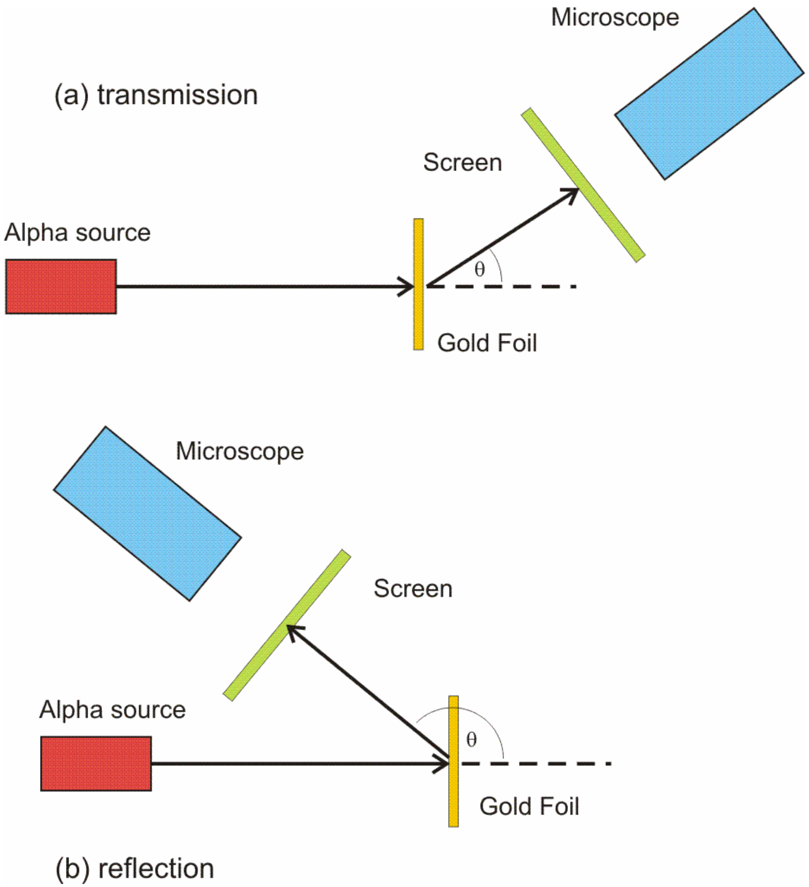
- Thomson concluded that the number of electrons per atom is
  between 0.2 and 2 times $Z$ (atomic number)

# The nuclear atom

# Rutherford & Geiger



# The nuclear atom




"See if you can get some effect of α-particles directly reflected from the surface" – fig (b)

**Geiger & Marsden 1909**
*On a Diffuse Reflection of the α-Particles*

"If the high velocity and mass of the α-particle be taken into account, it seems surprising that some of the α-particles, as the experiment shows, can be turned within a layer of $6 \times 10^{-5}$ cm. of gold through an angle of $90^{o}$, and even more*. To produce a similar effect by a magnetic field, the enormous field of $10^{9}$ absolute units would be required."

* About 1 in 8000 reflected i.e. $\theta > 90^{o}$.

This work was done in Manchester about 18 months after Rutherford had been appointed as Langsworthy Professorship of Physics

The experiments required very good eyesight and a lot of patience – you were the readout!

# Reaction to the G-M experiment

- Rutherford’s reaction was one of astonishment
- The ‘plum pudding’ model was totally inadequate
- an $\alpha$-particle with speed ~10000 km/sec hitting point-like electrons in the positive jelley and coming back at you??
- ‘It was quite the most incredible event that has ever happened in my life. It was almost as incredible as if you had fired a 15-inch shell at a piece of tissue paper and it came back and hit you.’

(Robin Marshall, based on studies of archive material in Manchester, mentioned in a seminar recently that Rutherford may not have said this – however it is well established in the ‘folk lore’!)

# Rutherford's paper of 1911

## *The Scattering of $\alpha$ and $\beta$ Particles by Matter and the Structure of the Atom*

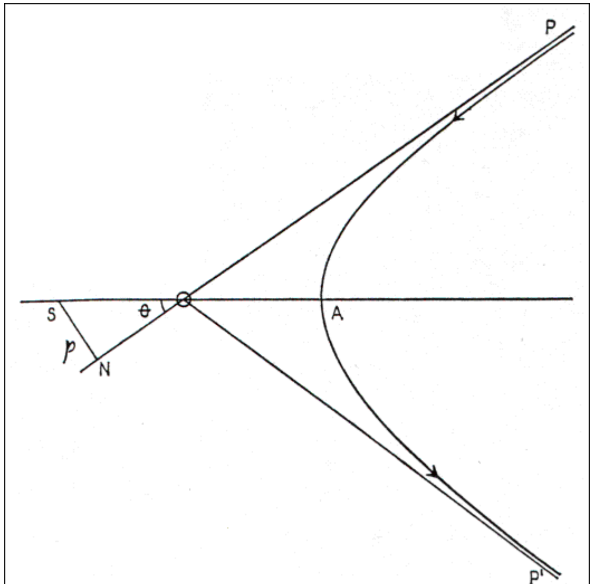


Fig. 1

$\varphi = \pi - 2\theta$ is the angle of deflection

- $1/r^2$ force; $\alpha$ incident PO; exits along OP′
  $\alpha$-particle trajectory - hyperbola
  external focus S at centre of the atom.

- for an $\alpha$ shot directly at the centre of the atom, 'distance of closest approach', $b$, given by:

$$\frac{mu^2}{2} = NeE\left(\frac{1}{b} - \frac{3}{2R} + \frac{b^2}{2R^3}\right) \approx \frac{NeE}{b} \quad \text{for } b \ll R$$

$Ne$: total positive charge of atom, uniform in sphere radius $R$

$E$, $m$, $u$: charge, mass, velocity of $\alpha$-particle

e.g. for $N = 100$ and $u = 2.09 \times 10^9$ cm/sec, $b \approx 3.4 \times 10^{-12}$ cm. Atomic radius $R \sim 10^{-8}$ cm, ... the $\alpha$ penetrates so close to the central charge that the effect of the negative electricity (electrons) may be neglected.

# Rutherford's paper of 1911 – 'cross-section'

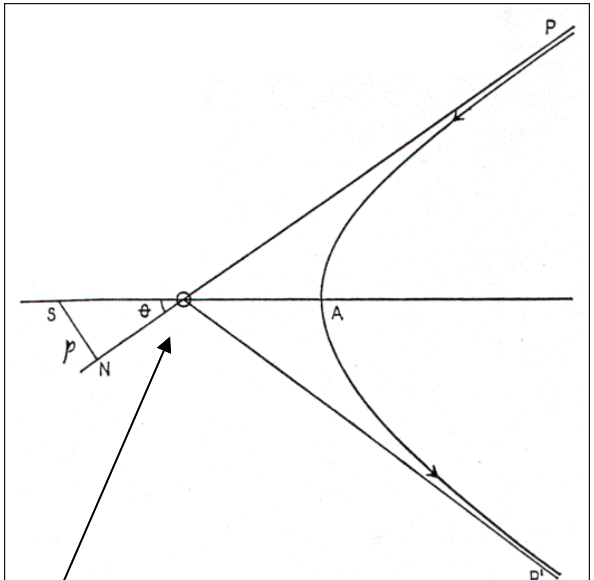


Fig. 1

$\varphi = \pi - 2\theta$ is the angle of deflection between the incoming and outgoing asymptotic directions of the $\alpha$

the number $y$ of particles on unit area of zinc sulphide detector screen that have been deflected through angle $\varphi$:

$$y = (ntb^2.Q.\operatorname{cosec}^4\varphi/2)/16r^2$$

where:

$n$: number of atoms per unit volume

$Q$: total of $\alpha$-particles incident

$r$: distance from point of incidence of $\alpha$-beam on target to screen

$t$: thickness of scattering material

$b$: distance of closest 'head on' approach, $b = 2NeE/mu^2$

$Ne$: total positive charge of atom, uniform in sphere radius $R$

$E$, $m$, $u$: charge, mass, velocity of $\alpha$-particle

He notes that the expression for $y$ is proportional to:

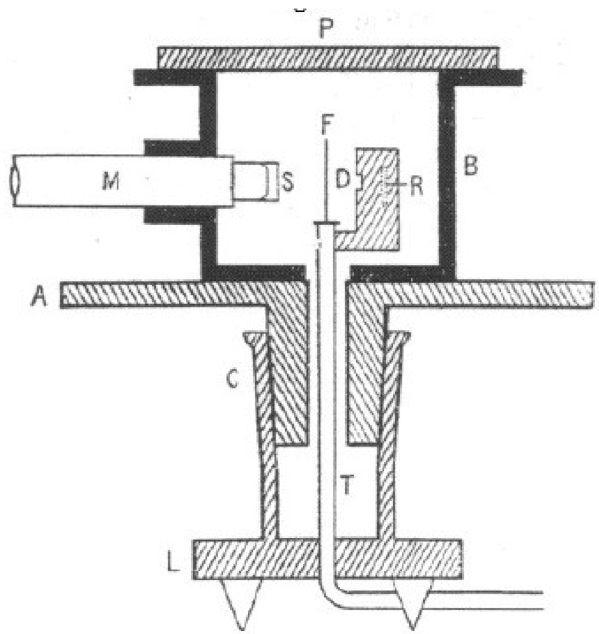
(1) $\operatorname{cosec}^4 \varphi/2$

(2) thickness $t$ of scattering material (assumed small)

(3) magnitude of central charge $Ne$

(4) the inverse of $(mu^2)^2$ (i.e. square of initial K.E. of the $\alpha$)

# Geiger & Marsden 1913 paper




Predictions of Rutherford's formula tested:

(1) Variation with angle.
(2) Variation with thickness of scattering material.
(3) Variation with atomic weight of scattering material.
(4) Variation with velocity of incident particles.
(5) The fraction of particles scattered through a definite angle.

Rn[222] (emanation) source at R, diaphragm at D directed a pencil of $\alpha$-particles normally onto screen F, viewed thru' telescope M

| I. | II. | III. | IV. | V. | VI. |
|---|---|---|---|---|---|
| | | SILVER. | | GOLD. | |
| Angle of deflexion, f | $\frac{1}{\sin^4 a / 2}$ | Number of scintil-lations, N | $\frac{N}{\sin^4 f / 2}$ | Number of scintil-lations, N | $\frac{N}{\sin^4 f / 2}$ |
| 150 | 1.15 | 22.2 | 19.3 | 33.1 | 28.8 |
| 135 | 1.38 | 27.4 | 19.8 | 43.0 | 31.2 |
| 120 | 1.79 | 33.0 | 18.4 | 51.9 | 29.0 |
| 105 | 2.53 | 47.3 | 18.7 | 69.5 | 27.5 |
| 75 | 7.25 | 136 | 18.8 | 211 | 29.1 |
| 60 | 16.0 | 320 | 20.0 | 477 | 29.8 |
| 45 | 46.6 | 989 | 21.2 | 1435 | 30.8 |
| 37.5 | 93.7 | 1760 | 18.8 | 3300 | 35.3 |
| 30 | 223 | 5260 | 23.6 | 7800 | 35.0 |
| 22.5 | 690 | 20300 | 29.4 | 27300 | 39.6 |
| 15 | 3445 | 105400 | 30.6 | 132000 | 38.4 |

'…it can be calculated that the number of elementary charges composing the centre of the atom is equal to half the atomic weight.'

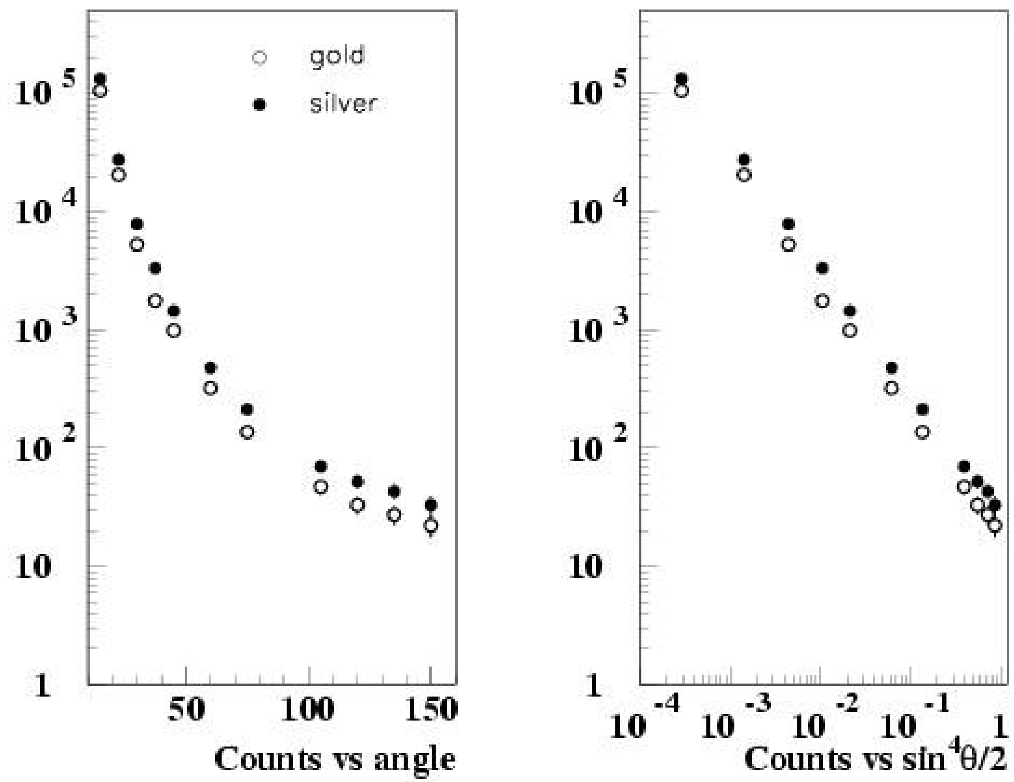
# Test of Rutherford's formula

Plots of the data from the 1913 G-M paper

$\alpha$-particle scattering from gold and silver foils

LH plot shows the angular distributions

RH plot shows the same data plotted against $\sin^4(\theta/2)$

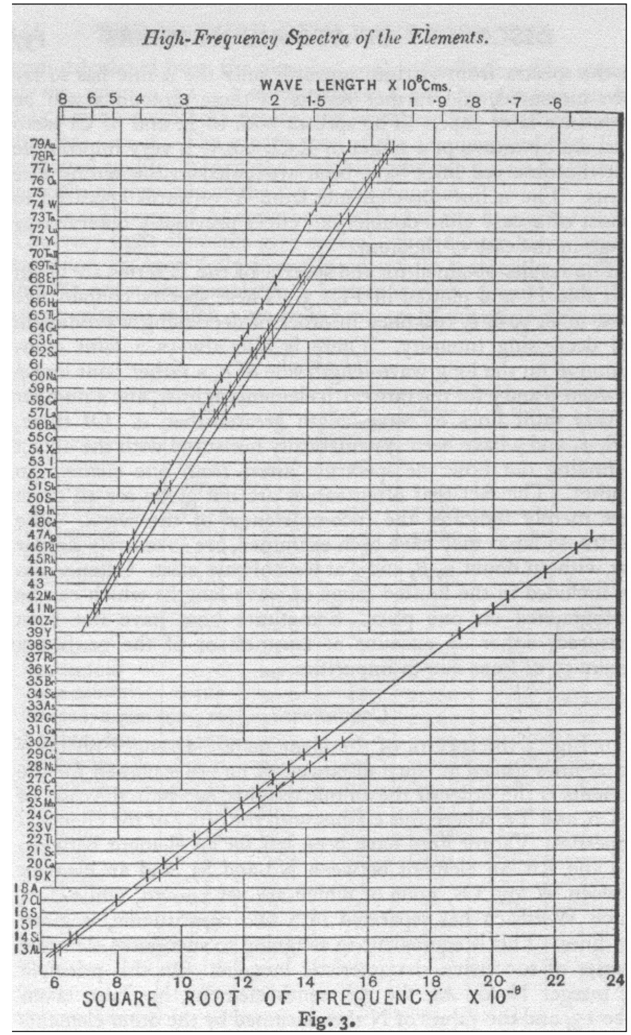


# Moseley and X-ray spectra – ordering the elements

- Measured precision  X-ray spectra
  - 1913  also in Manchester
- 'Balmer' type formula for frequency

$$\nu(\mathrm{Z}) = \mathrm{R}(\mathrm{Z}\text{-}\sigma)^2\left(\frac{1}{1^2}-\frac{1}{2^2}\right)$$

(Moseley died in World War I, aged 27)




$\sqrt{\nu}$ vs Z

## Consequences

## Bohr model and early quantum physics

## Bohr model for the H-atom - timeline

- First decade 20th C - it still wasn't known how many electrons an atom contained – even for Hydrogen (the simplest atom)

- Bohr visited Cambridge in 1911 hoping to work with J J Thomson
  - it didn't work out, but Bohr did hear Rutherford talk about his nuclear atom
  - he was also familiar with Thomson's atomic model

- Bohr decided he must visit Manchester – which he did 1911-12
  - Bohr realised that an additional constraint was needed to determine the radius of electron orbits in a 'planetary' model'

- Back in Copenhagen (1913) Bohr published his 'trilogy' of papers

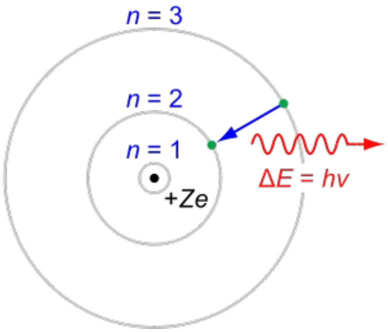
  *On the Constitution of Atoms and Molecules*

  - his key insight was to quantize the angular momentum of the electron

## Bohr model for the H-atom

- a single electron in orbit around the central nucleus, just one proton for H (*Z=1*)
- Coulomb $1/r^2$ force between positive charge *e* (proton) and the negative charge of the electron
- The radius of the orbit fixed by quantisation of angular momentum
- Energies in 'allowed' orbits were then fixed and electrons did not radiate continuously
- Energy of photon emitted or absorbed given by difference in energy levels

A stunning success!



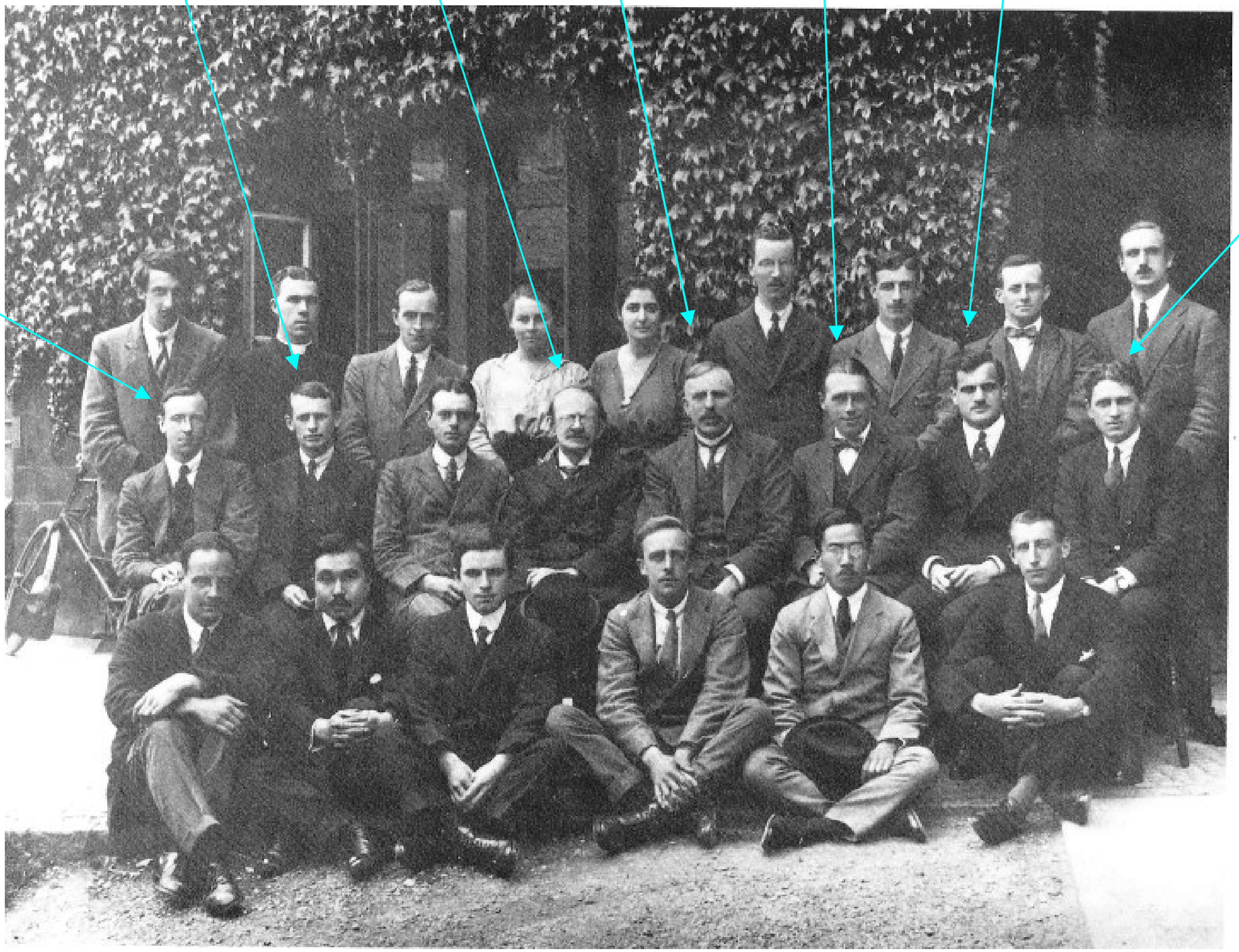

- Quantisation condition:

$$mvr = n\,h/2\pi \equiv n\hbar$$

- Photon energy: $h\nu = E_n - E_m$
- $$E_n = -\frac{1}{2}\left(\frac{Ze^2}{4\pi\varepsilon_0}\right)^2 \frac{m}{n^2\hbar^2}$$
- For H, $E_1$ = -13.6 eV
  (Z=1; the nuclear charge)

# 1930s

- Dirac proposes the positron (1931)
- Discovery of neutron (1932, Chadwick, Joliot-Curie..)
- First nuclear process initiated by an accelerator (1932)
- Fermi theory of beta decay (1933)




Rutherford, Thomson and research students; Cavendish Laboratory 1920

# The need to get higher energies

"It would be of great scientific interest if it were possible to have a supply of electrons… which the individual energy of motion is greater even than that of of the alpha particle." [Ernest Rutherford, PRS, 30 Nov 1927]

- Why the need for higher energy?

- Firstly: to overcome the Coulomb barrier.

  - for an $\alpha$-particle incident on a nucleus of charge $Z$, radius $R_A$

$$\mathrm{E}^{\alpha}_{\mathrm{min}} = \frac{2Ze^2}{4\pi\varepsilon_0 R_A}$$

e.g. for gold, Z=79, gives 32.6 MeV

- Secondly: Quantum mechanics $\Delta(pc)\Delta x \geq \hbar c$

  - to probe $10^{-15}m$ or less requires momenta (pc) of 200 MeV or more

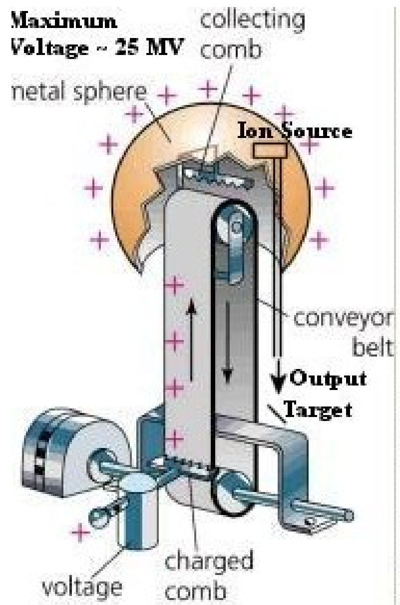
# First accelerators: Van de Graaff 1929

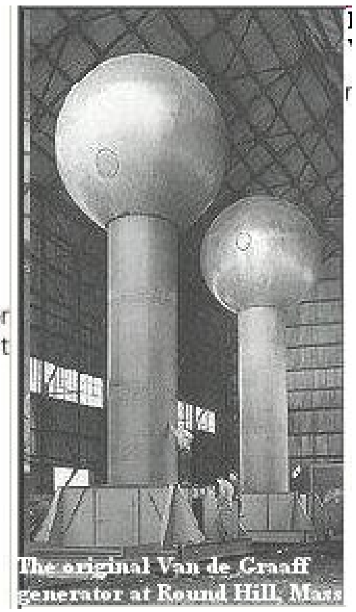
- Ttransfer charge from a voltage source via a continuous belt to the spherical isolated upper terminal

- Belt: a flexible dielectric material, (e.g. silk), for small machines. For large devices the ‘belt’: isolated metal plates capable of holding larger charge on a flexible backing

- Maximum energy given by upper terminal voltage (Typical $V_{max}$ of 25 MV); also limited by height above ground (avoid ‘breakdown’)

- Very stable energies, so many nuclear physics laboratories had them





The original Van de Graaff generator at Round Hill, Mass

The Van de Graaff at Oxford

# First accelerators: Cockcroft-Walton 1932

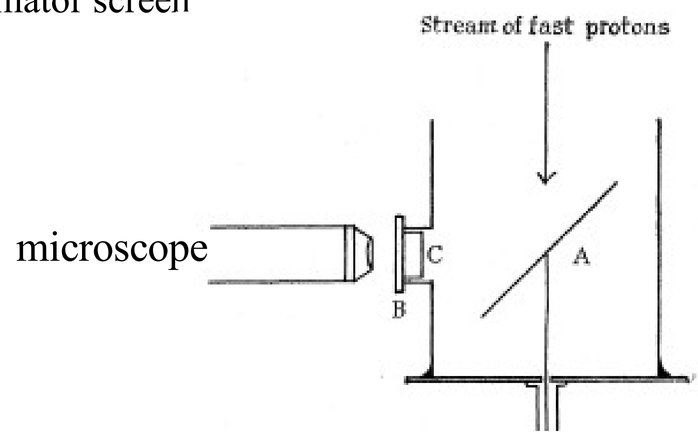
- Cockcroft-Walton 'voltage multiplier' - built in 1937 by Philips (Eindhoven) - now in the Science Museum

- Cockcroft and Walton used their device to 'split the atom':

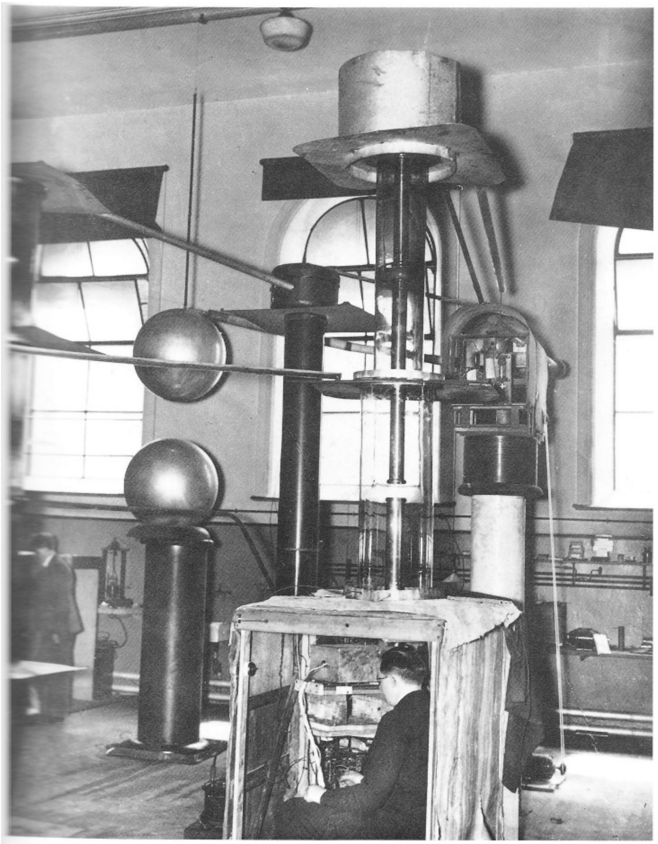
$$p + Li^7 \rightarrow 2\alpha$$

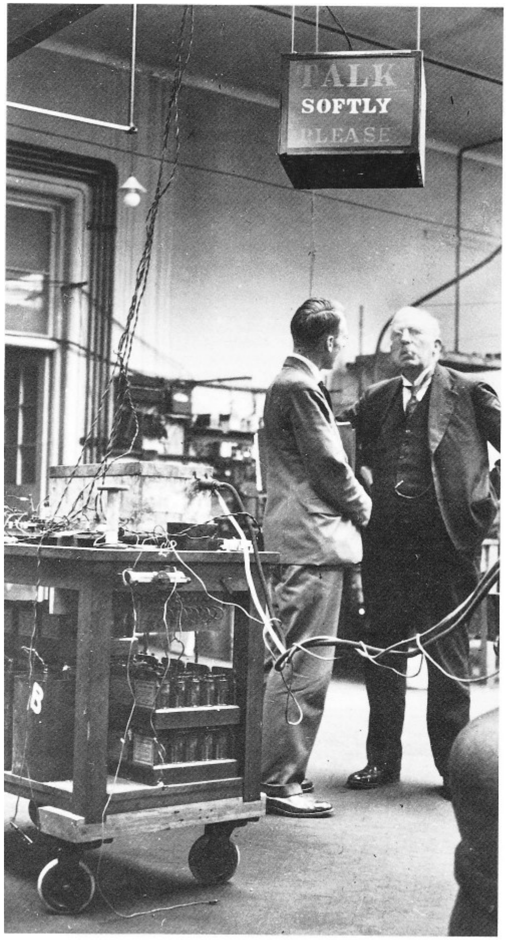
protons, energy 0.12 MeV, incident on the $Li^7$ target at A

- Detector was a zinc-sulphide scintillator screen, B, viewed through the microscope
- The $\alpha$'s were identified by inserting mica screens, C, of known stopping power between the exit window and scintillator screen




# An early Cockcroft-Walton



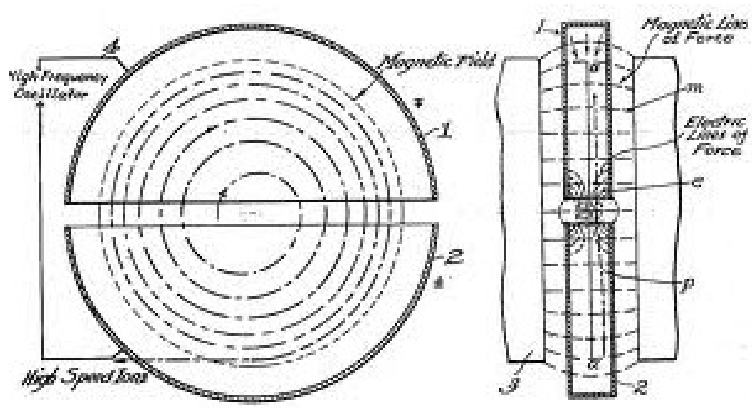


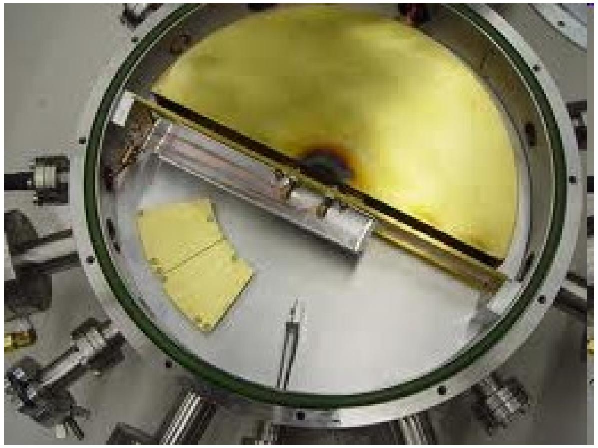

Left: Walton taking data (1931);  Right: Rutherford & John Ratcliffe (1932)

# First accelerators: cyclotron (E Lawrence) 1932

- Key idea – circular path allows fixed ‘acceleration kick’ to be used many times
- High frequency alternating voltage applied across the D-shaped electrodes
- Inject at the centre - acceleration when crossing the gap between the ‘dees’
- Maximum energy at periphery – beam extracted onto target and detector
- Largest had 60 inch diameter giving protons and deuterons of 8 and 16 MeV

- BUT could not accommodate relativistic effects and energy loss by radiation



Lawrence had a talent for raising money to support his projects

# Late 1940s to present day

**Brief mention of cosmic ray physics**

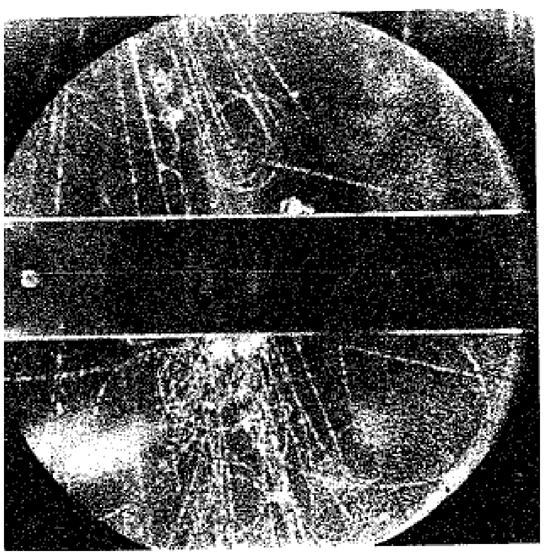
**Circular vs linear accelerators**

**Inside the nucleus and nucleon**

**Partons, quarks and gluons**

# Cosmic rays (mountain observatories or balloons)

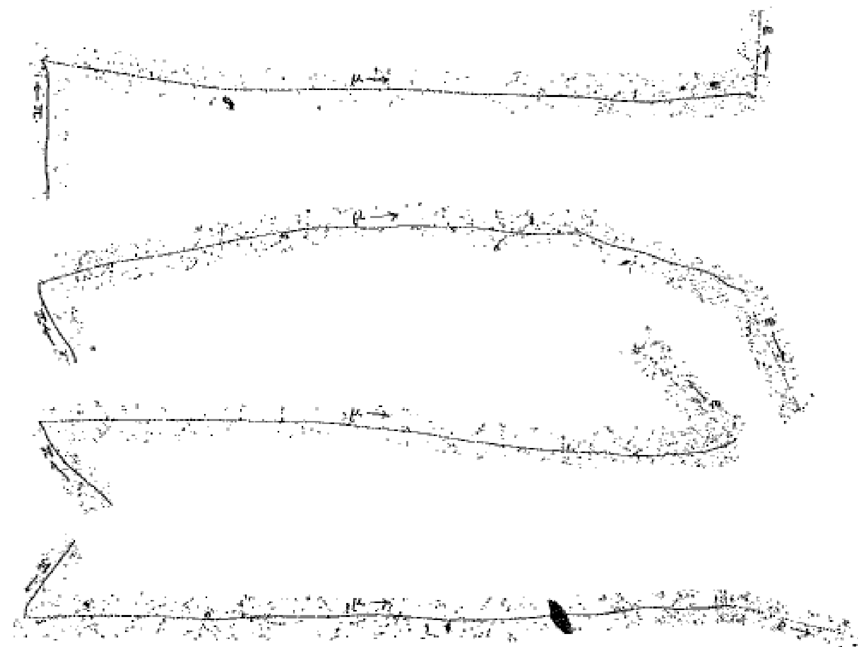
Pic du Midi Observatory (Pyrenees)



Before accelerators, cosmic rays provided a source of very high energy protons and new particle discoveries:

bottom left: '$V^0$' in a cloud chamber photo (the $K^0$)

below: $\pi \to \mu\nu \to e\nu\nu$, captured in photographic emulsion



# The synchrotron

- Mark Oliphant (Australian), Vladimir Veksler (Russian), Edwin McMillan (USA) were the key players (working independently)
- Circular machines – fixed radius beam pipe within a ring of bending magnets – no huge electromagnet which limited the cyclotron
- Synchronize accelerating voltage frequency and magnetic field strength with particle beam rotation – ‘phase stability’
- Oliphant built one of the first proton synchrotrons at Birmingham University (UK) – started operating in 1953 – energy 1 GeV
- McMillan - first electron synchrotron (300 MeV),

  then the Bevatron (1.8 BeV LBL)
- Rapidly adopted worldwide for particle physics, many are still in use
  - both electrons (DESY) and protons (CERN PS)

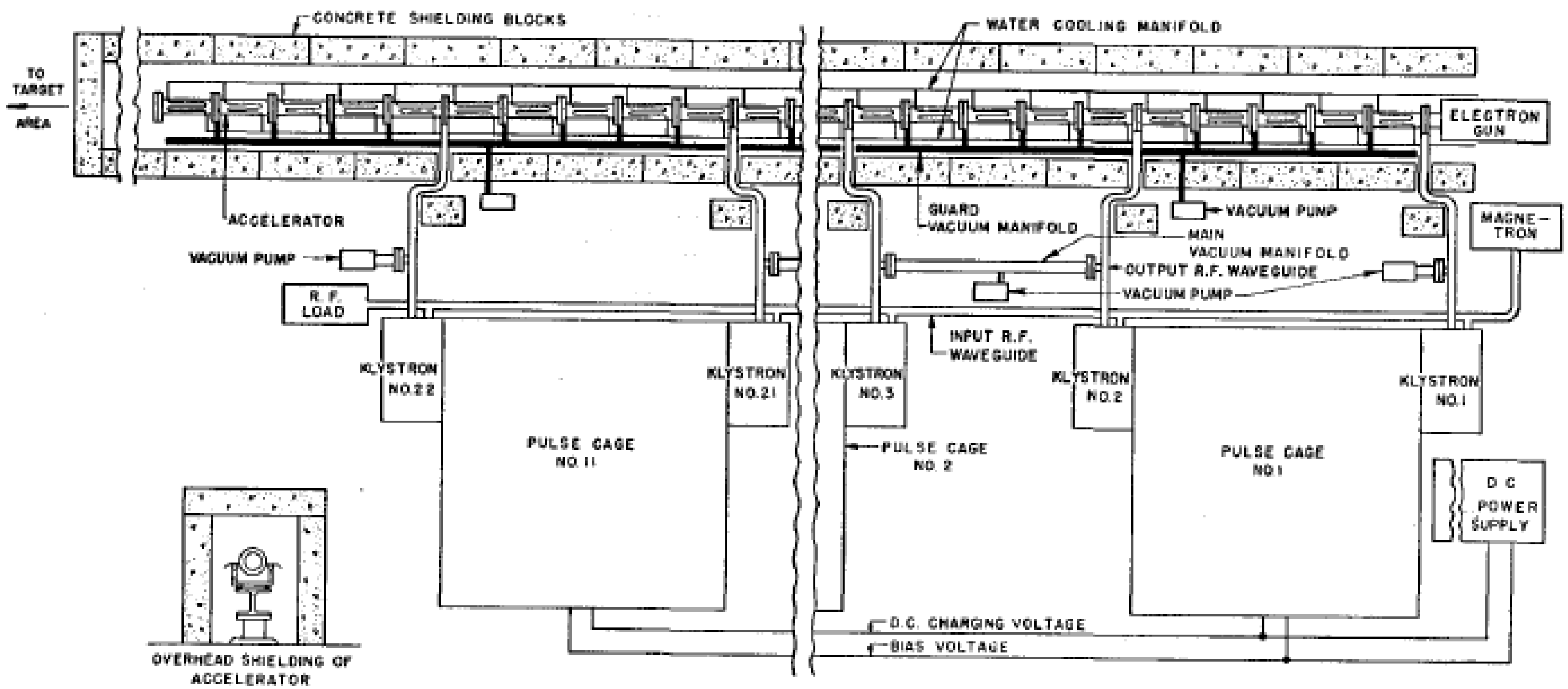
# A little nostalgia – DESY - 7.5 GeV electron synchrotron

February 24 1964: first beam at DESY! Professor Willibald Jentschke toasts
Drs Degele, Kumpfert and team in the accelerator control room

# Protons or electrons?

- Acceleration of protons gives the highest beam energies (Tevatron and LHC)
- The electron (or positron) is a point-like lepton without structure
- Many examples of electron synchrotrons (CEA, DESY, NINA…) and
  $e^+e^-$ colliders (Frascati (AdA), CEA, DORIS, PETRA, PEP, LEP
- BUT, synchrotron radiation energy loss limits maximum energy
- This is  avoided if the accelerator is linear
- Stanford linac then SLAC   (nuclear then particle physics, respectively)

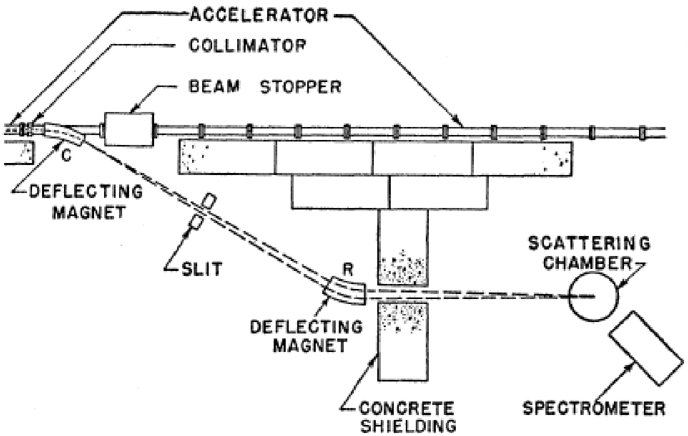
# Stanford Linac Mk III




Details in Rev. Sci. Inst. **26** 134 (1955)

Initial maximum energy with 21 klystrons was 630 MeV

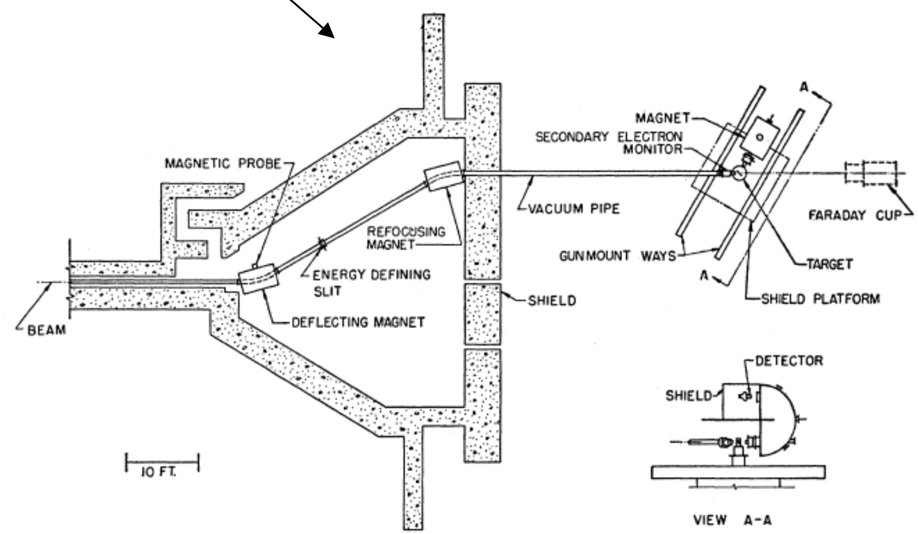
# Hofstadter and Nuclear Physics




**Halfway station** – 190 MeV spectrometer

Spectrometer is mounted on an obsolete gun mount turntable on loan from the US Navy

**End station** – 550 MeV spectrometer




Energy $E'$ and scattering angle

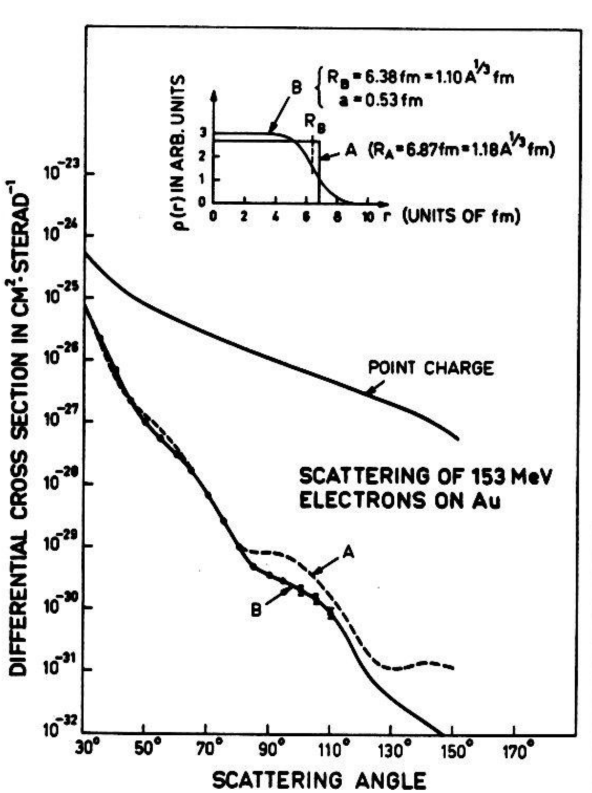
$\theta$ measured in both cases

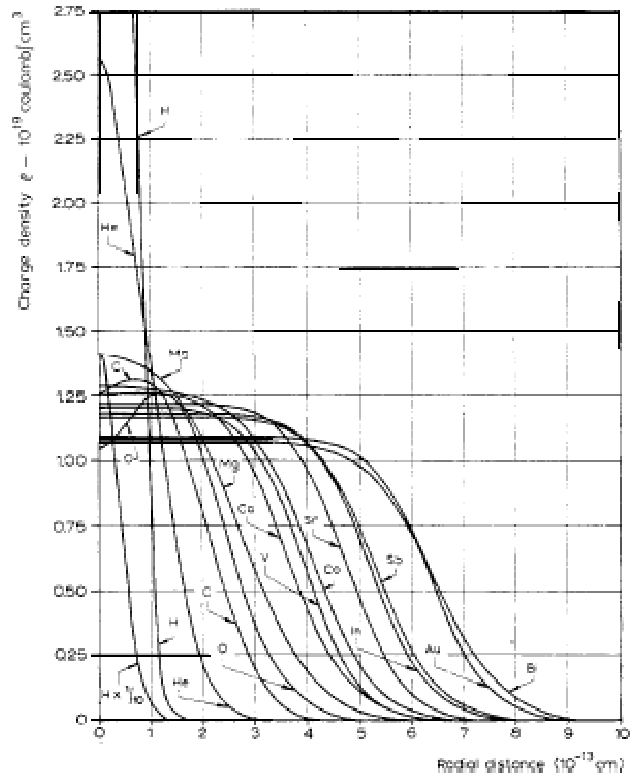
Elastic scatter: $E' = E$

3-momentum transfer q given by

$$q^2 = 4E^2 \sin^2(\theta/2)$$

# Elastic scattering and nuclear charge density

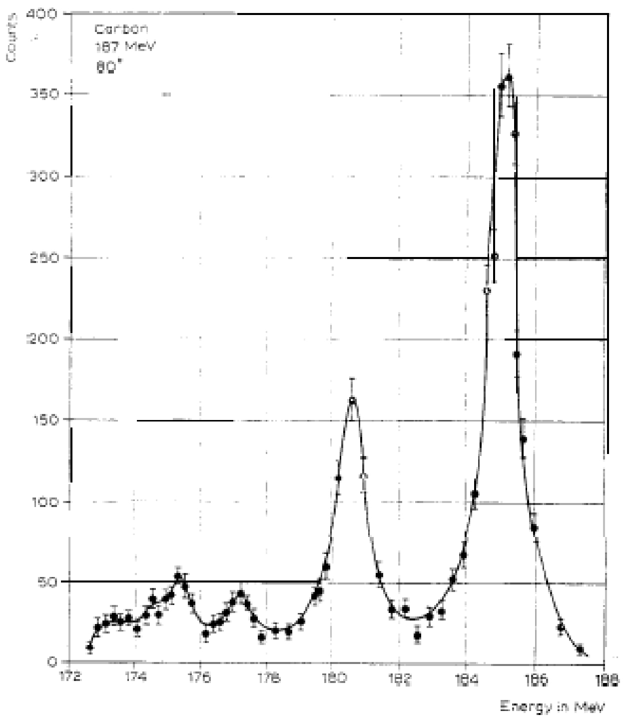



- Typical angular distribution for eA elastic scattering
- Note the much faster decrease compared to a point charge

- nuclear radius: $R_A = (1.07 \pm 0.02) \times A^{1/3}$ fm
- nuclear 'skin thickness': $2.4 \pm 0.3$ fm (constant)

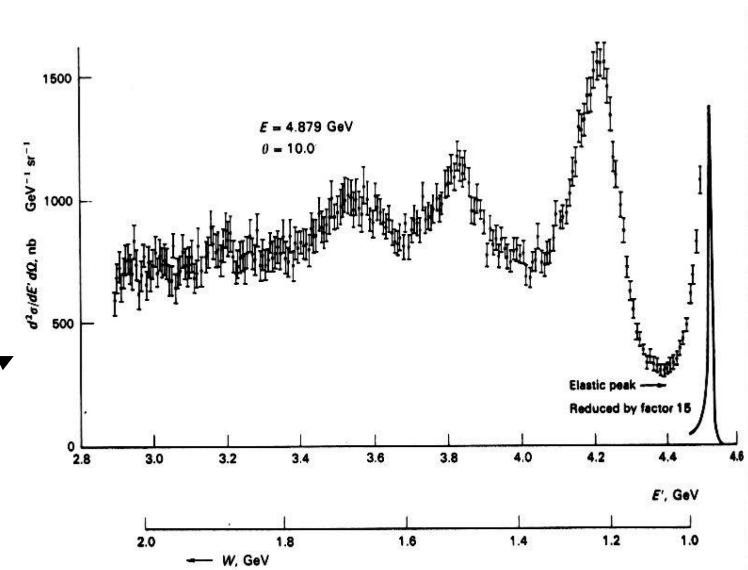
# Inelastic eA scattering

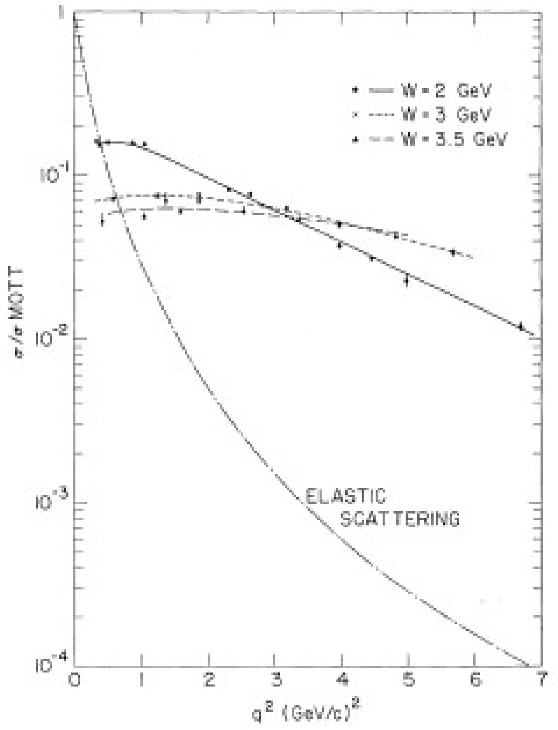



Left plot shows the scattered electron energy for a 187 MeV incident beam on $^{12}C$ measured at $80^0$ at Stanford

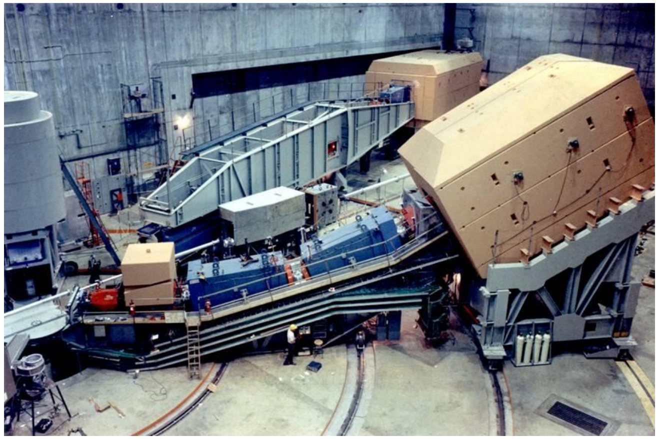
- elastic peak
- inelastic peaks from excited states the first near 180 MeV relates to the 4.43 MeV $^{12}C$ level

Right: $ep \rightarrow e'X$; $E_e = 4.878$ GeV; from DESY; $W \equiv m_X$ moving left the peaks are at the positions of the nucleon resonances starting with the $\Delta(1232)$ spin 3/2, isospin 3/2 state






# SLAC - 1969



20 GeV linear accelerator, plot shows inelastic $ep \to e'X$ scattering $\sigma_{ep\to e'X} / \sigma_{elastic}$ vs $q^2$ ($q$ is 4-momentum transfer) for fixed values of $M_X \equiv W$; the almost constant behaviour cannot be described by a uniform mass distribution within a sphere of radius $\sim 10^{-15} m$

Evidence for 'hard' (point-like) constituents

# Neutrinos - CDHS

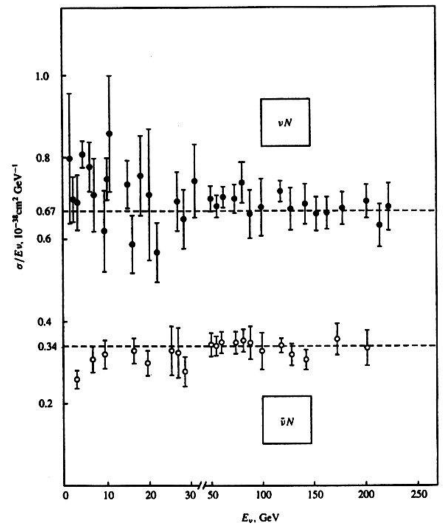
- CDHS experiment neutrino

deep inelastic scattering:

$$\nu(\bar{\nu})\mathrm{N} \rightarrow \mu^{-}(\mu^{+})\mathrm{X}$$

- $E_{\nu}^{max} \sim$ 200 GeV
- Detector: 19 modules

  toroidally magnetised iron plates




$$\sigma_{tot}(\nu N) = \frac{G_F^2 s}{2\pi}\left[\langle Q\rangle + \frac{1}{3}\langle\bar{Q}\rangle\right]$$

$$\sigma_{tot}(\bar{\nu} N) = \frac{G_F^2 s}{2\pi}\left[\langle\bar{Q}\rangle + \frac{1}{3}\langle Q\rangle\right]$$

- Plot shows
  $\sigma_{tot}(\nu N)/E_{\nu}^{LAB}$, $\sigma_{tot}(\bar{\nu} N)/E_{\nu}^{LAB}$ vs $E_{\nu}^{LAB}$

  data show $\boxed{\sigma_{tot}(\nu N),\ \sigma_{tot}(\bar{\nu} N) \propto E_{\nu}^{LAB}}$

- Magnitude of anti-quark content (from plot) :

  $$\frac{\sigma_{tot}(\nu N)}{\sigma_{tot}(\bar{\nu} N)} = \frac{3+r}{1+3r} \approx 2$$

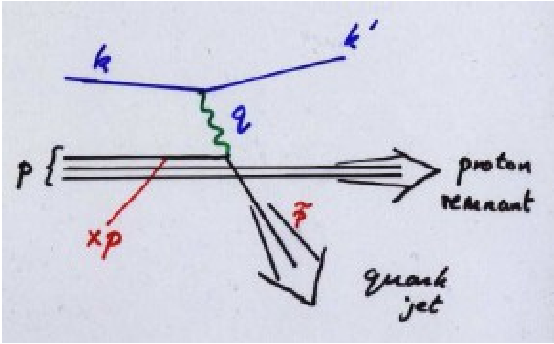
  gives $r \equiv \dfrac{\langle\bar{Q}\rangle}{\langle Q\rangle} \approx 0.2$

# Quark parton model for ‘deep’ inelastic scattering

The key assumption of the QPM is that the cross-section factorises

$$d\sigma(eN \to e'X) = \sum_i d\hat{\sigma}(eq_i \to eq_i) \otimes f_i(x),$$

$d\hat{\sigma}$, elastic $eq_i$ scattering; $f_i(x)$, the probability that quark $q_i$ has a fraction $x$ of the nucleon's momentum



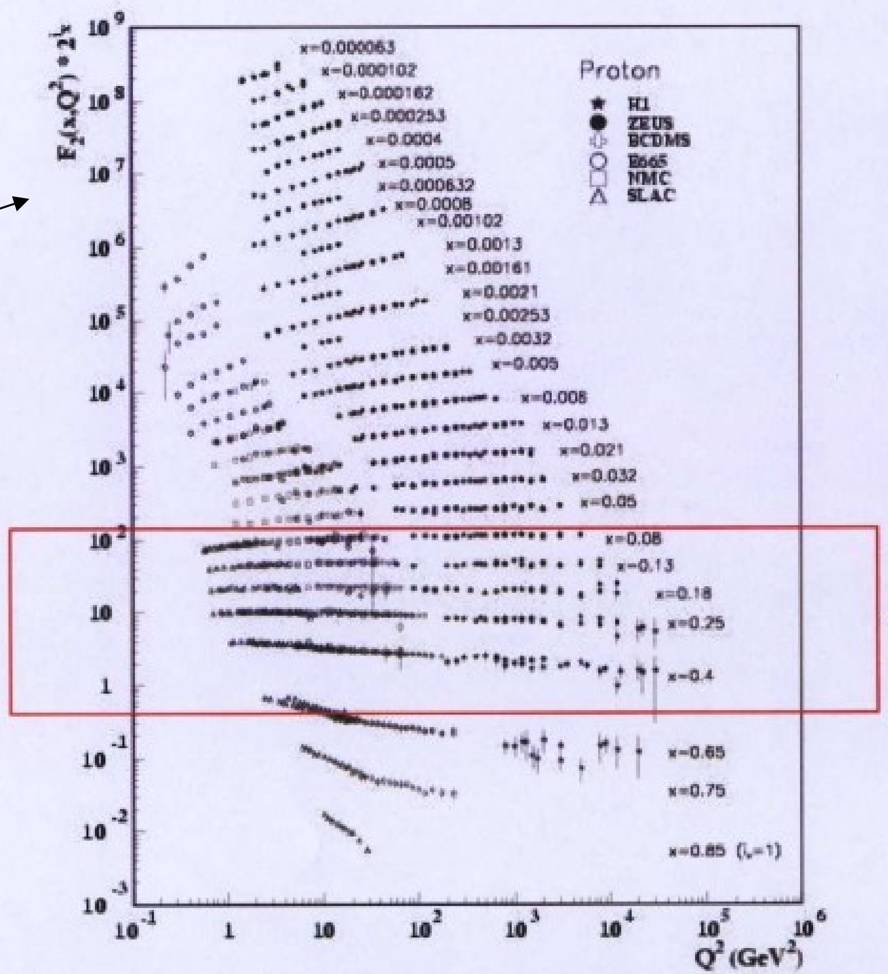

$$\frac{d^2\sigma}{dxdQ^2} = \frac{2\pi\alpha^2}{xQ^4}\left[(1+(1-y)^2)F_2(x,Q^2) - y^2F_L(x,Q^2)\right]$$

QPM predictions for the two structure functions (for spin-$\frac{1}{2}$ partons)

$$\begin{aligned} F_2(x,Q^2) &= \sum_i e_i^2 x f_i(x) \\ F_L(x,Q^2) &= 0 \end{aligned}$$

# $F_2^p$ data from fixed targets and HERA

BUT what is happening at small x values?




$F_2^p(x,Q^2)$

plotted at fixed x as a fcn of $Q^2$

Region of early data from SLAC showing approximate 'scaling' – ie no dependence on $Q^2$ at fixed x

# QPM – problems

- Momentum sum rule

$$\int_0^1 x(u + \bar{u} + d + \bar{d} + s + \bar{s})dx \approx 0.5 \quad \text{not } 1!$$

Implies other partons without electro-weak couplings $\longrightarrow$ gluons

- No free quarks, however hard the proton is hit: HERA $ep$ $E_{CM} \sim 320\,\text{GeV}$; Tevatron $p\bar{p}$ $E_{CM} \sim 1900\,\text{GeV}$
- Hadrons are bound states of quarks – very tightly bound
- BUT, QPM assumes essentially free quarks in deep-inelastic scattering
- The strong force has a short range of the order of a femtometre
- Surely these requirements are incompatible?
- Amazingly Quantum Chromodynamics (QCD) has the answers.

# QCD enhanced parton model

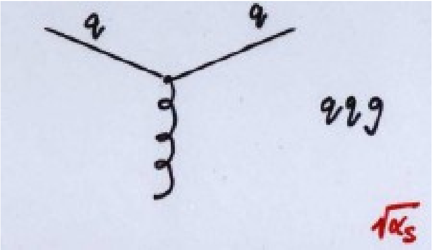
- gluon – massless, spin 1
- gluons carry the colour charge of QCD
- self-interact as well as coupling to quarks

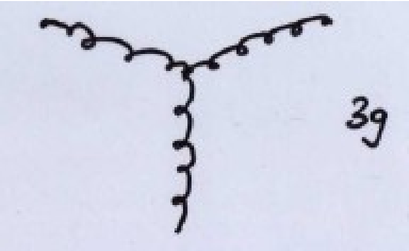
- the strong coupling $\alpha_S$ (in analogy to
  a (fine structure constant)
  decreases as the scale, $Q^2$, increases

- 'Asymptotic freedom'

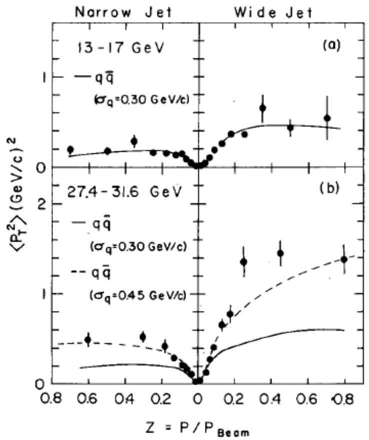
- Perturbative QCD

- Evidence for the gluon?

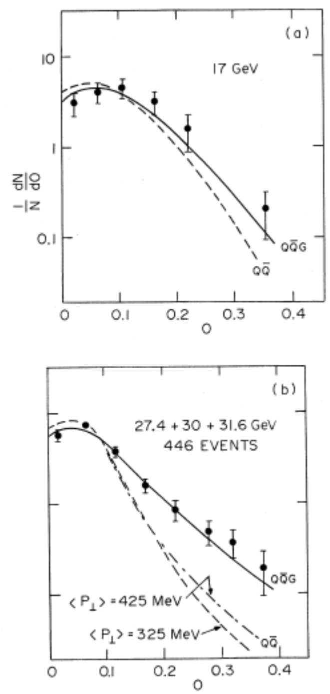


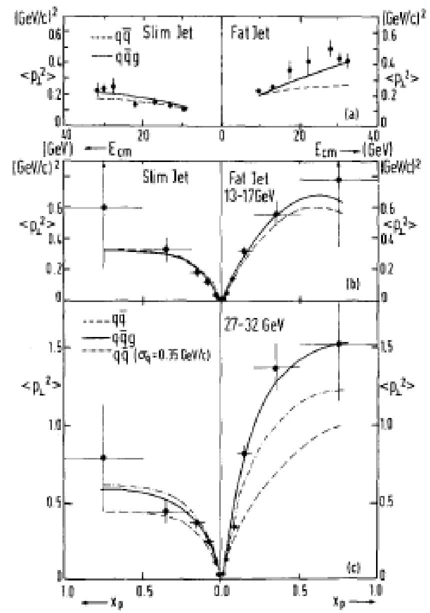
# PETRA 1979 – discovery of the gluon



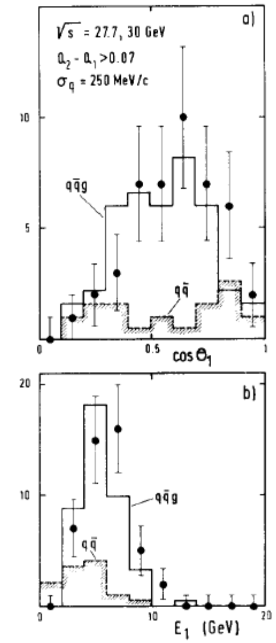

TASSO




Mark-J




PLUTO




JADE

$$e^+e^- \rightarrow \text{jets}$$

The four experiments on PETRA, using different measures of 'jetiness' showed clearly that three jet events and jet broadening agreed quantitatively with the predictions of QCD (quantum chromodynamics).

# Conclusion

- Since 1911 ‘Rutherford scattering’ has been an indispensible tool
  for exploring the physics of the nucleus, then the nucleons
- New ideas have been required – the most radical not yet completely understood was the development of quantum mechanics
- At times it appeared that the task was hopeless, but then a new order appears out of the chaos

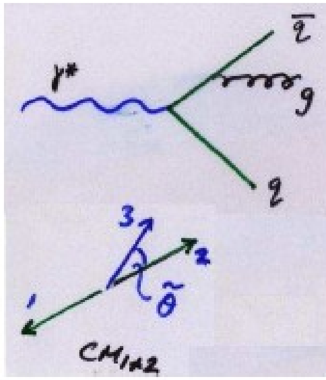
- Improvements in instrumentation and experiments have driven the subject

- It has been quite a century of achievement!
- We look forward to even greater surprises from the LHC.

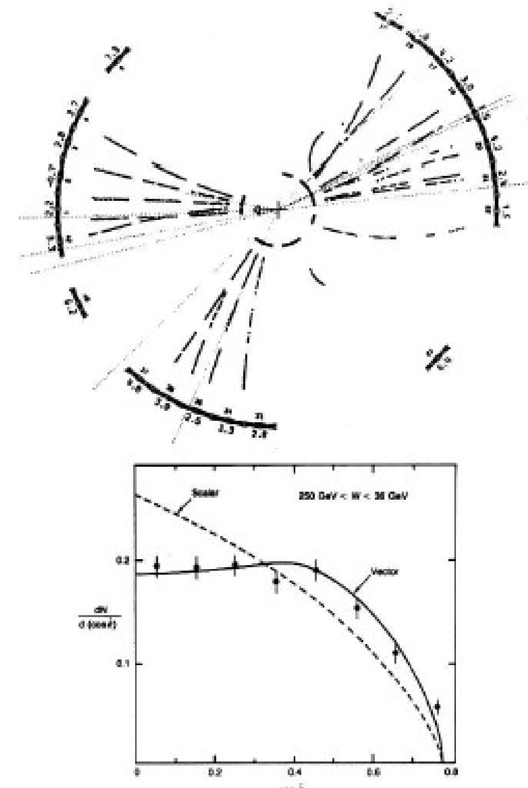
Apologies to all those on experiments that I have not mentioned.

# EXTRAS

# $e^+e^-$ annihilation at PETRA – 3-jet events




- The 3-jet final-state is explained by hard gluon radiation - 'hard' to ensure well-separated jets - the three jets should lie in a plane (no missing energy)
- Using pQCD both rate ($\propto \alpha_S$) and angular behavior can be calculated
- Direct evidence for the gluon and that it has spin 1
- Order jets by energy $E^1 > E^2 > E^3$, boost to CM frame of jets 1+2, the angle $\tilde{\theta}$ is between jets 2 and 3, as shown above




Top: is a 3-jet event from JADE
Bottom: $\cos\tilde{\theta}$ from TASSO

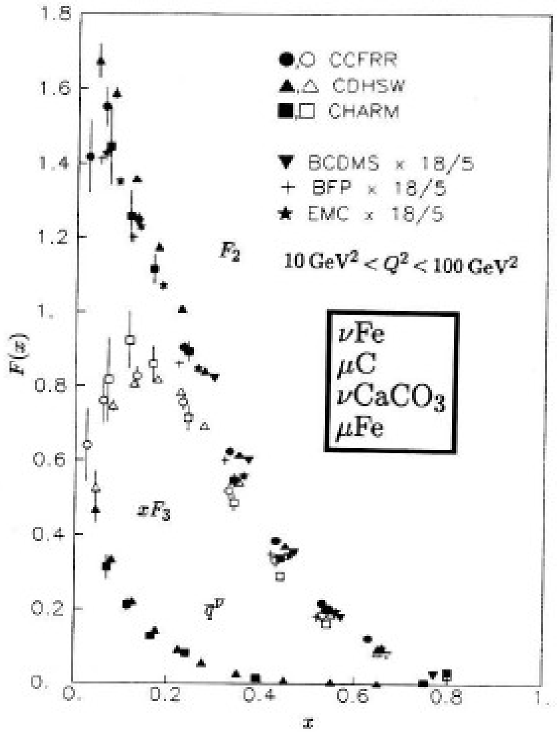
# Mean quark (charges)$^2$ in the nucleon

- $(n+p)/2 = (uud + ddu)/2$, so,

  $\langle Q_i^2 \rangle = \frac{1}{6}\left(\frac{4}{9}+\frac{4}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{4}{9}\right) = \frac{5}{18}$
- $F_2^{\nu N} = x(u+\bar{u}+d+\bar{d})$
- $F_2^{eN}$ for an isoscalar target

$$\begin{aligned} F_2^{eN} &= \frac{1}{2}\left(F_2^{ep}+F_2^{en}\right) \\ &= \frac{1}{2}x\left[\frac{4}{9}(u+\bar{u})+\frac{1}{9}(d+\bar{d})\right. \\ &\quad \left.+(\frac{1}{9}(u+\bar{u})+\frac{4}{9}(d+\bar{d})\right] \\ F_2^{eN} &= \frac{5}{18}x\left[u+\bar{u}+d+\bar{d}\right] \end{aligned}$$

so $F_2^{\nu N} \approx \frac{18}{5}F_2^{eN}$ (not exact because of small corrections)



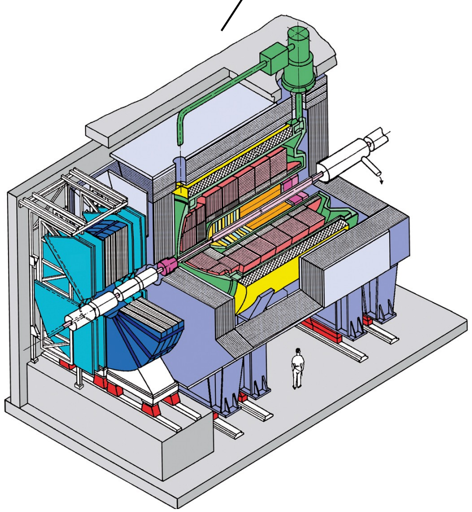

PDG 2008

# HERA – the microscope

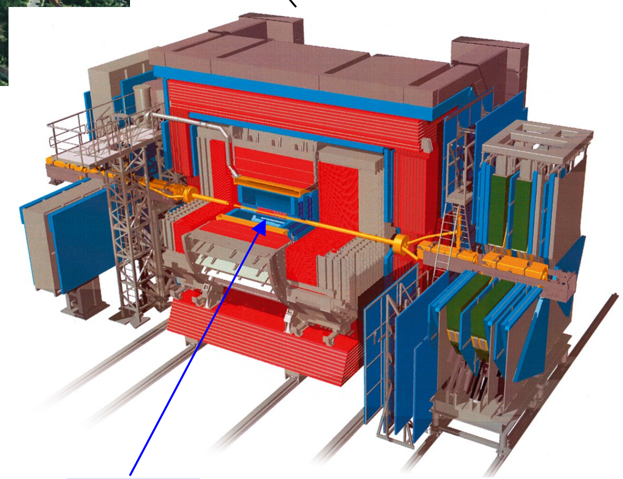
e-p collider
$e^{+/-}$ 27.5 GeV
p 920 GeV
$E_{cm}$= 318 GeV

Circumference
~ 11 km


H1 detector



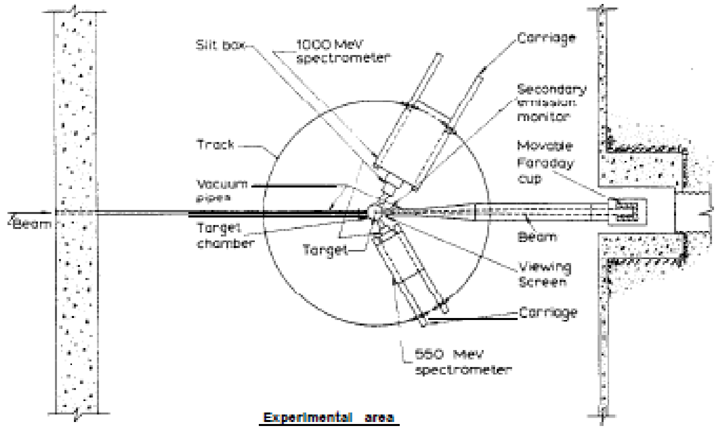
ZEUS detector



CTD

# Hofstadter – electron spectrometer

End station – spectrometer moved
On a circular track about the target
- It weighed about 30 tonnes



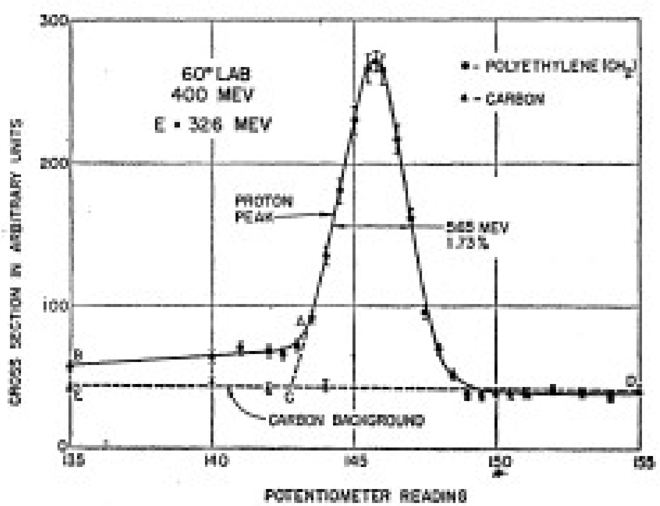
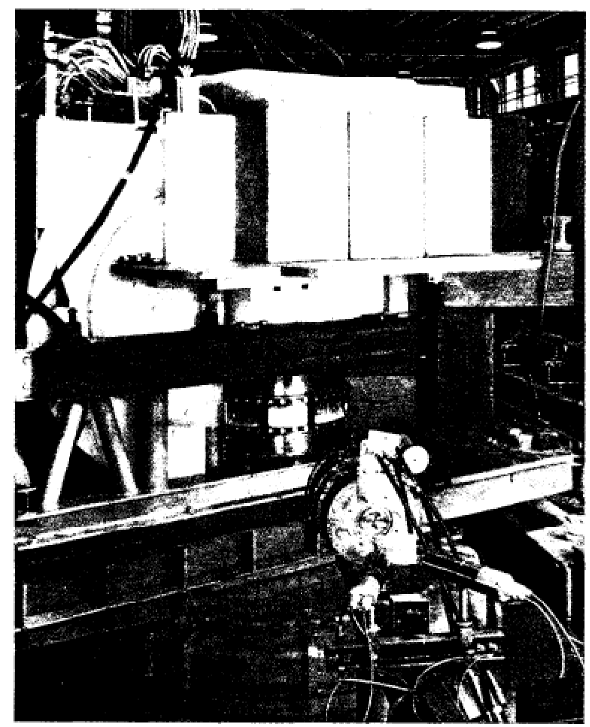




FIG. 25. Elastic scattering of 400-Mev electrons from protons in polyethylene at a laboratory angle of 60°.



# Bohr Model – more detail

- nucleus with positive charge Ze
- electrons in stable orbit of radius r, given by

$$\frac{mv^2}{r} = \frac{Ze^2}{4\pi\varepsilon_0}\frac{1}{r^2} \quad \text{with energy } E_{tot} = \frac{1}{2}mv^2 - \frac{Ze^2}{4\pi\varepsilon_0}\frac{1}{r} = -\frac{1}{2}\frac{Ze^2}{4\pi\varepsilon_0}\frac{1}{r}$$

- BUT - what determines r?

Bohr postulated the quantisation condition for 'stable' orbits:

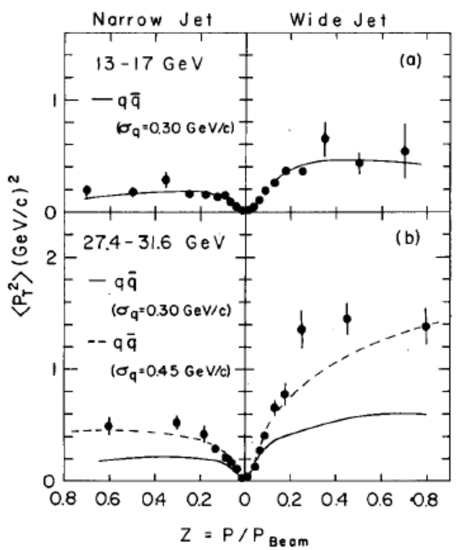
angular momentum $mvr = n\hbar \quad (n \geq 1)$

- so $$\frac{n^2\hbar^2}{mr} = \frac{Ze^2}{4\pi\varepsilon_0} \Rightarrow \frac{1}{r} = \frac{Ze^2}{4\pi\varepsilon_0}\frac{m}{n^2\hbar^2}$$

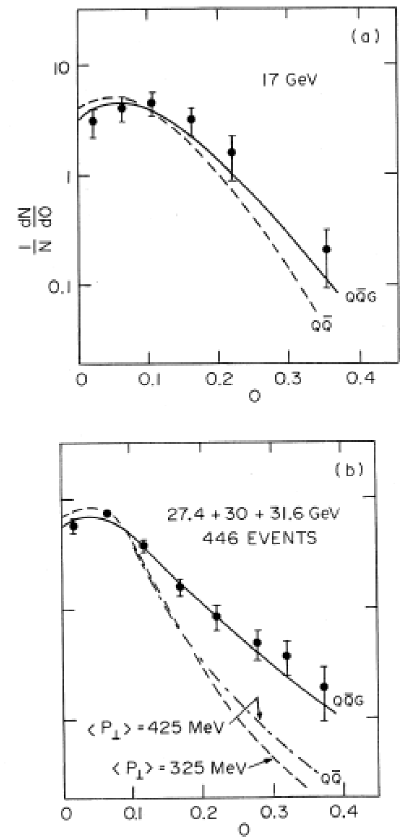
- $$E_n = -\frac{1}{2}\left(\frac{Ze^2}{4\pi\varepsilon_0}\right)^2\frac{m}{n^2\hbar^2} = -\frac{1}{2}\left(\frac{Ze^2}{4\pi\varepsilon_0\hbar c}\right)^2\frac{mc^2}{n^2} = -\frac{1}{2}\alpha^2\frac{mc^2}{n^2}$$

for hydrogen gives $E_1 = -13.6$ eV

# PETRA 1979 – discovery of the gluon II




TASSO



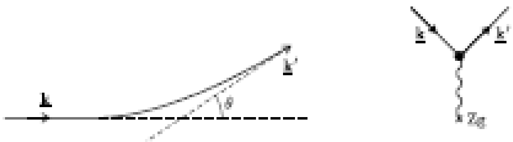

Mark-J

# Rutherford scattering

## Point-like charge

Elastic scattering from a fixed centre of force

$$V(r) = Zg^2 \frac{e^{-\mu r}}{r} \qquad \left(\mu = \frac{Mc}{\hbar}\right)$$



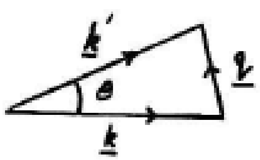
(a) (b)

the matrix element ( $(\text{coupling})^2 \times$ propagator)

$$\begin{aligned} \langle \mathbf{k}' | V(r) | \mathbf{k} \rangle &= \int e^{-i\mathbf{k}'\cdot\mathbf{r}} V(r) e^{i\mathbf{k}\cdot\mathbf{r}} d^3\mathbf{r} \\ &= \frac{4\pi Z g^2}{q^2 + \mu^2} \end{aligned}$$

$$\mathbf{q} = \mathbf{k} - \mathbf{k}'$$

$$|\mathbf{k}| = |\mathbf{k}'|$$




for electromagnetic $\mu^2 \to 0$ and $g^2 \to \frac{e^2}{4\pi\epsilon_0 \hbar c}$

$$\frac{d\sigma}{dq^2} = \frac{4\pi\alpha^2}{\beta^2}\frac{Z^2}{q^4} \quad \text{or} \quad \frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2 Z^2}{4|\mathbf{k}|^2 \sin^4(\theta/2)}$$

$$q^2 = 4|\mathbf{k}|^2 \sin^2(\theta/2)$$

POINT-LIKE Scattering (Rutherford scattering)

# Form-factor

Scattering from an extended charge distribution

$$V(r) = Zg^2 \int \frac{\rho(r')}{|\mathbf{r}-\mathbf{r}'|} d^3\mathbf{r}' \qquad \int \rho(r) d^3\mathbf{r}' = 1$$

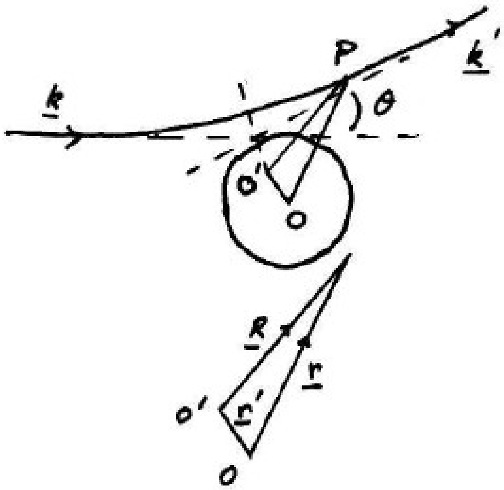
$$\begin{aligned} \langle \mathbf{k}'|V(r)|\mathbf{k}\rangle &= Zg^2 \int\int \frac{\rho(r')}{|\mathbf{r}-\mathbf{r}'|} e^{i\mathbf{q}\cdot\mathbf{r}} d^3\mathbf{r}' d^3\mathbf{r} \\ &= \frac{4\pi Z g^2}{q^2} F(q^2) \end{aligned}$$

where $$F(q^2) = \int \rho(r') e^{i\mathbf{q}\cdot\mathbf{r}'} d^3\mathbf{r}'$$

The FORM-FACTOR of the charge distribution, and

$$\frac{d\sigma}{dq^2} = \frac{4\pi\alpha^2}{\beta^2} \frac{Z^2}{q^4} |F(q^2)|^2$$

To evaluate the m.e. change variable $\mathbf{R} = \mathbf{r} - \mathbf{r}'$ - introduce a factor $e^{-\mu R}$ to integrate, then $\mu \to 0$.




Elastic scatter from charge element at $O'$ within an extended source centered on $O$.

$\mathbf{R}$ is vector from $O'$ to point $P$ on electron trajectory

# Sum rules F2n

- PDFs ($u(x)$ etc) cannot be calculated at present – they have to be determined from data
- Constraints – proton has $uud$ valence quark structure (i.e. quarks needed to give quantum numbers of proton) – so

$$\int_0^1 (u(x) - \bar{u}(x))dx = 2; \quad \int_0^1 (d(x) - \bar{d}(x))dx = 1; \quad \int_0^1 (s(x) - \bar{s}(x))dx = 0$$

- PDFs are intrinsic to a given hadron and independent of the process under study

$F_2^n$ (neutron has $ddu$ valence structure, measured using $ed$ scattering)

$$F_2^n(x) = \frac{1}{9}x[d^n(x) + \bar{d}^n(x)] + \frac{4}{9}x[u^n(x) + \bar{u}^n(x)] + \frac{1}{9}x[s^n(x) + \bar{s}^n(x)]$$

Appeal to strong interaction isospin symmetry – $d$ in neutron has same wave-fcn as $u$ in proton so

$$d^n(x) = u^p(x); \quad u^n(x) = d^p(x); \quad s^n(x) = s^p(x); \quad \text{giving}$$

$$F_2^n(x) = \frac{1}{9}x[u(x) + \bar{u}(x)] + \frac{4}{9}x[d(x) + \bar{d}(x)] + \frac{1}{9}x[s(x) + \bar{s}(x)]$$

## n/p

$$\frac{F_2^n}{F_2^p} = \frac{(u+\bar{u}) + 4(d+\bar{d}) + (s+\bar{s})}{4(u+\bar{u}) + (d+\bar{d}) + (s+\bar{s})}$$

- Large $x$ – "valence region" – ignore $q\bar{q}$ sea and using $u_V = u - \bar{u}$ etc

$$\text{for } x \sim 0.5, \quad \frac{F_2^n}{F_2^p} \approx \frac{u_V + 4d_V}{4u_V + d_V} \sim 0.5$$

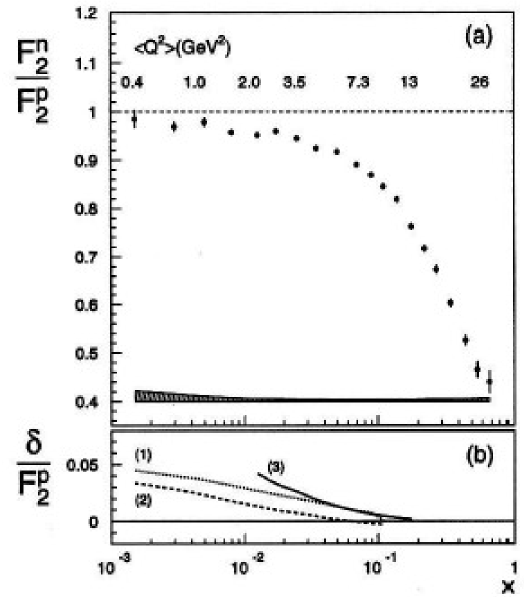
$F_2^n/F_2^p \approx 0.5 \Rightarrow u_V/d_V \approx 3.5$

- Small $x$

$$\frac{F_2^n}{F_2^p} \to 1$$

Appears that valence structure is irrelevant and only number of $q\bar{q}$ pairs matter

- More on the determination of PDFs in lecture V




NMC Nucl. Phys. B487 3 (1997)

NB $x$ and $\langle Q^2 \rangle$ are correlated

The ratio can be measured accurately

because many systematic effects cancel.

# QPM – problem?

| | | | | |
|---|---|---|---|---|
| • | $e(\mu)N$ DIS | $\longrightarrow$ | charged point-like spin-$\frac{1}{2}$ partons | $\surd$ |
| | | $\longrightarrow$ | scaling of $F_2(x,Q^2)$; $F_L(x,Q^2)=0$ | $\surd$ |
| • | $\nu(\bar{\nu})N$ DIS | $\longrightarrow$ | $\sigma_{tot}^{\nu(\bar{\nu})N} \propto s$ (or $E_\nu^{Lab}$) | $\surd$ |
| | | $\longrightarrow$ | $\sigma^{\nu N}/\sigma^{\bar{\nu}N} \rightarrow \bar{Q}/Q \neq 0$ | $\surd$ |
| • | $F_2^{\nu N}/F_2^{eN}$ | $\longrightarrow$ | "5/18" quark charges | $\surd$ |
| • | Sum rules | | | $\surd$ |

BUT

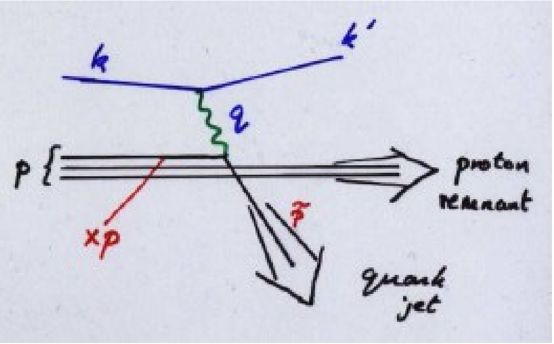
- $q\bar{q}$ sea – especially dominance at low $x$
- $F_2(x,Q^2)$ at low $x$ and scaling violations
- Momentum sum rule

$$\int_0^1 x(u+\bar{u}+d+\bar{d}+s+\bar{s})dx \approx 0.5 \quad \text{not } 1!$$

Implies other partons without electro-weak couplings $\longrightarrow$ gluons

- XX
  - Ss
    - aa

# Kinematics for DIS




Lorentz invariant variables:

$$
\begin{aligned}
s &= (k+p)^2, \quad \text{(CM energy)}^2, \\
Q^2 &= -(k-k')^2, \quad \text{photon virtuality}, \\
x &= Q^2/(2p\cdot q), \quad \text{Bjorken x}, \\
y &= (p\cdot q)/(p\cdot k), \quad \text{inelasticity}.
\end{aligned}
$$

Variable are not independent, at high energies, ignore masses

$$Q^2 = sxy$$