Parton Distribution Functions

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SINGLE PARTON DISTRIBUTIONS

- introduction and overview
- LHC benchmark cross sections
- issues and outlook

DOUBLE PARTON DISTRIBUTIONS

- introduction and overview

in collaboration with Alan Martin, Robert Thorne, Graeme Watt, Jo Gaunt, Steve Kom and Anna Kulesza

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introduction and overview

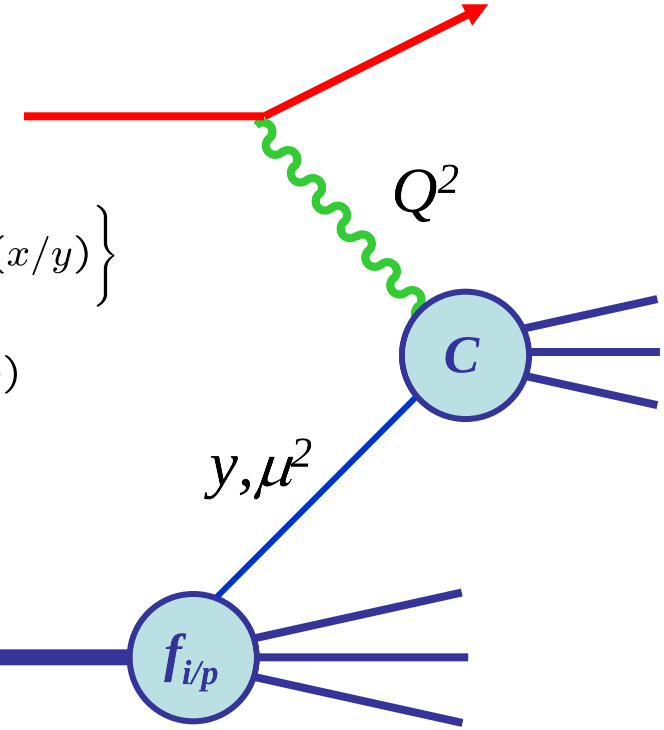
(single) parton distribution functions

$$f_{i/A}(x, Q^2)$$

- introduced by Feynman (1969) in the *parton model*, to explain Bjorken scaling in deep inelastic scattering data; interpretation as probability distributions

- according to the QCD *factorisation theorem* for inclusive hard scattering processes, universal distributions containing long-distance structure of hadrons; related to parton model distributions at leading order, but with logarithmic scaling violations (DGLAP)

- key ingredients for Tevatron and LHC phenomenology for example, in Deep Inelastic Scattering



$$\frac{1}{x}F_2^{lp}(x,Q^2) = x\sum_q e_q^2 \int_x^1 \frac{dy}{y} q(y,Q^2)\left\{\delta(1-\frac{x}{y}) + \frac{\alpha_s(Q^2)}{2\pi}C_q(x/y)\right\}$$
$$+ \; x\sum_q e_q^2 \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dy}{y} g(y,Q^2) C_g(x/y) + \mathcal{O}(\alpha_S^2)$$
$$+ \; \mathcal{O}(1/Q^2) \;\; \text{(higher twist, mass corrections)}$$

where the scale dependence of the parton distributions is calculable in QCD perturbation theory

$$\mu^2 \frac{\partial}{\partial \mu^2} f_i(x,\mu^2) = \frac{\alpha_S(\mu^2)}{2\pi}\sum_j \int_x^1 \frac{dy}{y} f_j(y,\mu^2) P_{ij}(x/y, \alpha_S(\mu^2))$$

Dokshitzer
Gribov
Lipatov
Altarelli
Parisi

… and $f_i(x,\mu_0^2)$ determined from

- lattice QCD (in principle)
- fits to data (in practice)

how pdfs are obtained*

- choose a factorisation scheme (e.g. $\overline{\text{MS}}$), an order in perturbation theory (LO, NLO, NNLO) and a 'starting scale' Q_0 where pQCD applies (e.g. 1-2 GeV)
- parametrise the quark and gluon distributions at Q_0, e.g.

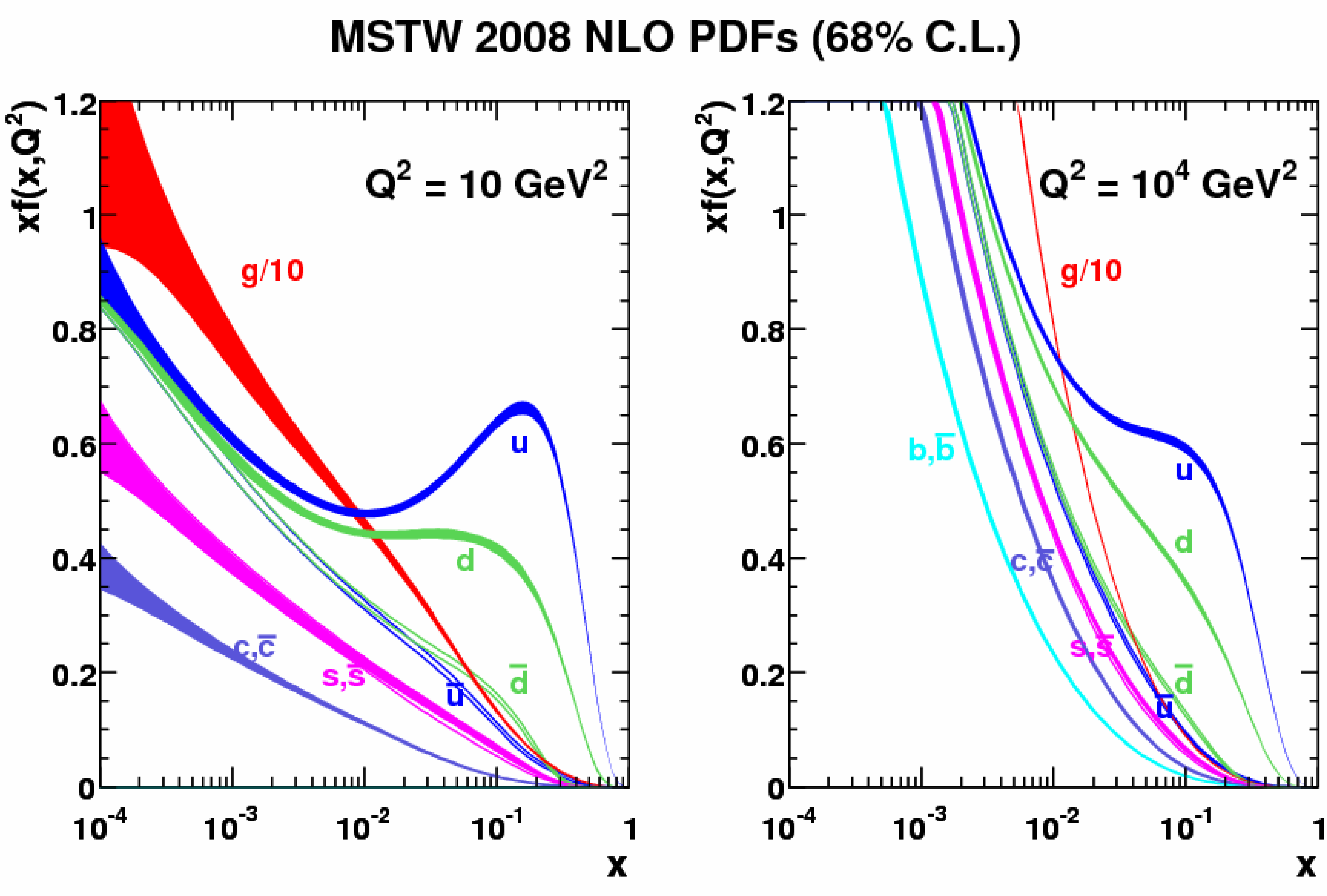
$$f_i(x,Q_0^2) = A_i x^{a_i}[1 + b_i\sqrt{x} + c_i x](1-x)^{d_i}$$

- solve DGLAP equations to obtain the pdfs at any x and scale $Q > Q_0$; fit data for parameters $\{A_i, a_i, \ldots \alpha_S\}$
- approximate the exact solutions (e.g. interpolation grids, expansions in polynomials etc) for ease of use; thus the output 'global fits' are available 'off the shelf", e.g.

```
SUBROUTINE PDF(X,Q,U,UBAR,D,DBAR,…,BBAR,GLU)
```

input | output

*traditional method


MSTW 2008 NLO PDFs (68% C.L.)
xf(x,Q^2)
Q^2 = 10 GeV^2
g/10
u
d
$\bar{d}$
$\bar{u}$
s,$\bar{s}$
c,$\bar{c}$
x
xf(x,Q^2)
Q^2 = 10^4 GeV^2
g/10
u
d
$\bar{d}$
$\bar{u}$
s,$\bar{s}$
c,$\bar{c}$
b,$\bar{b}$
x

the pdf industry

- many groups now extracting pdfs from 'global' data analyses (MSTW, CTEQ, NNPDF, …)

- broad agreement, but differences due to
  - choice of data sets (including cuts and corrections)
  - treatment of data errors
  - treatment of heavy quarks (s,c,b)
  - order of perturbation theory
  - parameterisation at Q_0
  - theoretical assumptions (if any) about:
    - flavour symmetries
    - $x \rightarrow 0,1$ behaviour
    - …

HERA-DIS
FT-DIS
Drell-Yan
Tevatron jets
Tevatron W,Z
other

examples of data sets used in fits*

Data set	$N_{\text{pts.}}$
H1 MB 99 e^+p NC	8
H1 MB 97 e^+p NC	64
H1 low Q^2 96–97 e^+p NC	80
H1 high Q^2 98–99 e^-p NC	126
H1 high Q^2 99–00 e^+p NC	147
ZEUS SVX 95 e^+p NC	30
ZEUS 96–97 e^+p NC	144
ZEUS 98–99 e^-p NC	92
ZEUS 99–00 e^+p NC	90
H1 99–00 e^+p CC	28
ZEUS 99–00 e^+p CC	30
H1/ZEUS $e^{\pm}p$ F_2^{charm}	83
H1 99–00 e^+p incl. jets	24
ZEUS 96–97 e^+p incl. jets	30
ZEUS 98–00 $e^{\pm}p$ incl. jets	30
DØ II $p\bar{p}$ incl. jets	110
CDF II $p\bar{p}$ incl. jets	76
CDF II $W \to l\nu$ asym.	22
DØ II $W \to l\nu$ asym.	10
DØ II Z rap.	28
CDF II Z rap.	29

Data set	$N_{\text{pts.}}$
BCDMS μp F_2	163
BCDMS μd F_2	151
NMC μp F_2	123
NMC μd F_2	123
NMC $\mu n/\mu p$	148
E665 μp F_2	53
E665 μd F_2	53
SLAC ep F_2	37
SLAC ed F_2	38
NMC/BCDMS/SLAC F_L	31
E866/NuSea pp DY	184
E866/NuSea pd/pp DY	15
NuTeV νN F_2	53
CHORUS νN F_2	42
NuTeV νN xF_3	45
CHORUS νN xF_3	33
CCFR $\nu N \to \mu\mu X$	86
NuTeV $\nu N \to \mu\mu X$	84
All data sets	**2743**

red font = new wrt MRST2006 fit

*MSTW2008

recent global or quasi-global pdf fits

pdfs	authors	arXiv
ABKM	S. Alekhin, J. Blümlein, S. Klein, S. Moch, and others	0908.3128, 0908.2766, …
CTEQ	H.-L. Lai, M. Guzzi, J. Huston, Z. Li, P. Nadolsky, J. Pumplin, C.-P. Yuan, and others	1007.2241, 1004.4624, 0910.4183, 0904.2424, 0802.0007, …
GJR	M. Glück, P. Jimenez-Delgado, E. Reya, and others	0909.1711, 0810.4274, …
HERAPDF	H1 and ZEUS collaborations	1006.4471, 0906.1108, …
MSTW	A.D. Martin, W.J. Stirling, R.S. Thorne, G. Watt	1006.2753, 0905.3531, 0901.0002, …
NNPDF	R. Ball, L. Del Debbio, S. Forte, A. Guffanti, J. Latorre, J. Rojo, M. Ubiali, and others	1005.0397, 1002.4407, 0912.2276, 0906.1958, …

	MSTW08	CTEQ6.6[x]	NNPDF2.0	HERAPDF1.0	ABKM09	GJR08
HERA DIS	✓	✓	✓*	✓*	✓	✓
F-T DIS	✓	✓	✓	✘	✓	✓
F-T DY	✓	✓	✓	✘	✓	✓
TEV W,Z	✓	✓+	✓	✘	✘	✘
TEV jets	✓	✓+	✓	✘	✘	✓
GM-VFNS	✓	✓	✘	✓	✘	✘
NNLO	✓	✘	✘	✘	✓	✓

+ Run 1 only

* includes new combined H1-ZEUS data → 1 – 2.5% increase in quarks at low x (depending on procedure), similar effect on $\alpha_S(M_Z^2)$ if free and somewhat less on gluon; more stable at NNLO (MSTW prelim.)

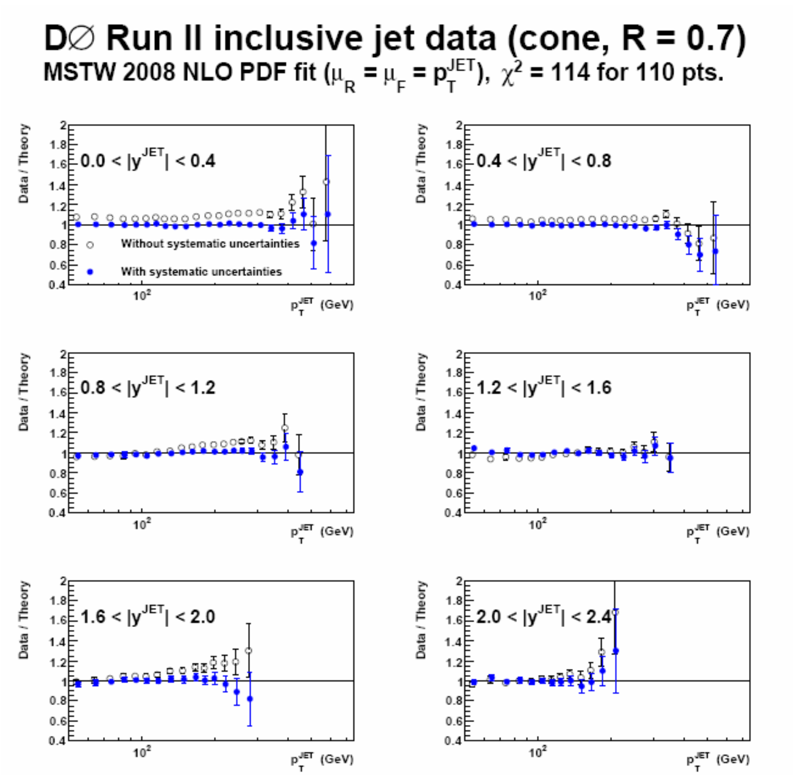
x New (July 2010) CT10 includes new combined H1-ZEUS data + Run 2 jet data + extended gluon parametrisation + … → more like MSTW08

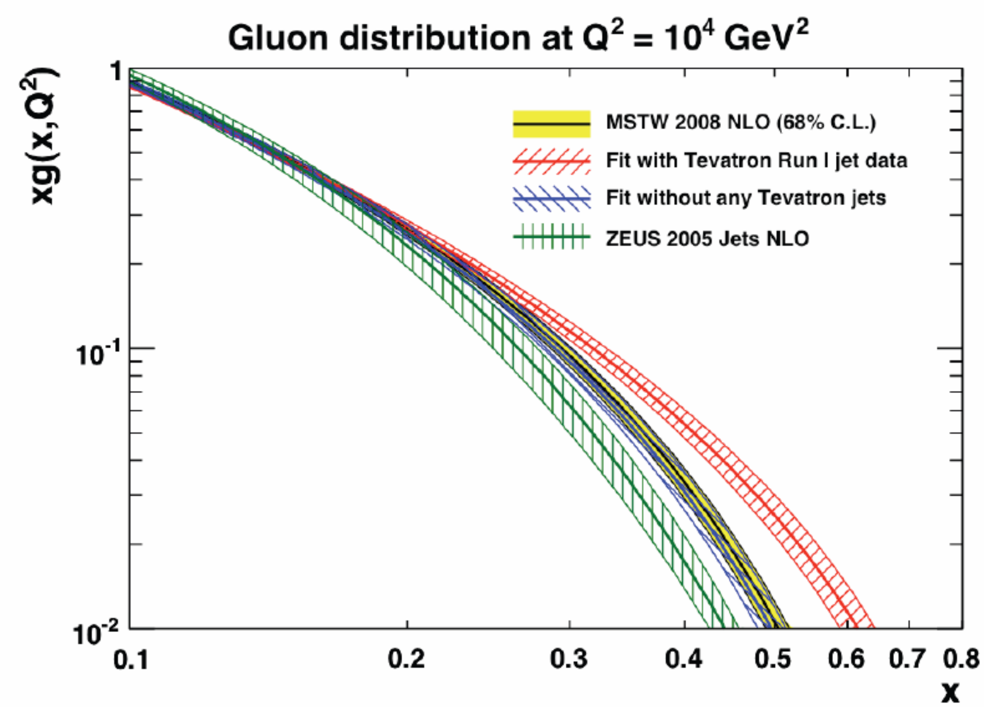
impact of Tevatron jet data on fits

- a distinguishing feature of pdf sets is whether they use (MRST/MSTW, CTEQ, NNPDF, GJR,…) or do not use (HERAPDF, ABKM, …) Tevatron jet data in the fit: the impact is on the *high-x gluon*

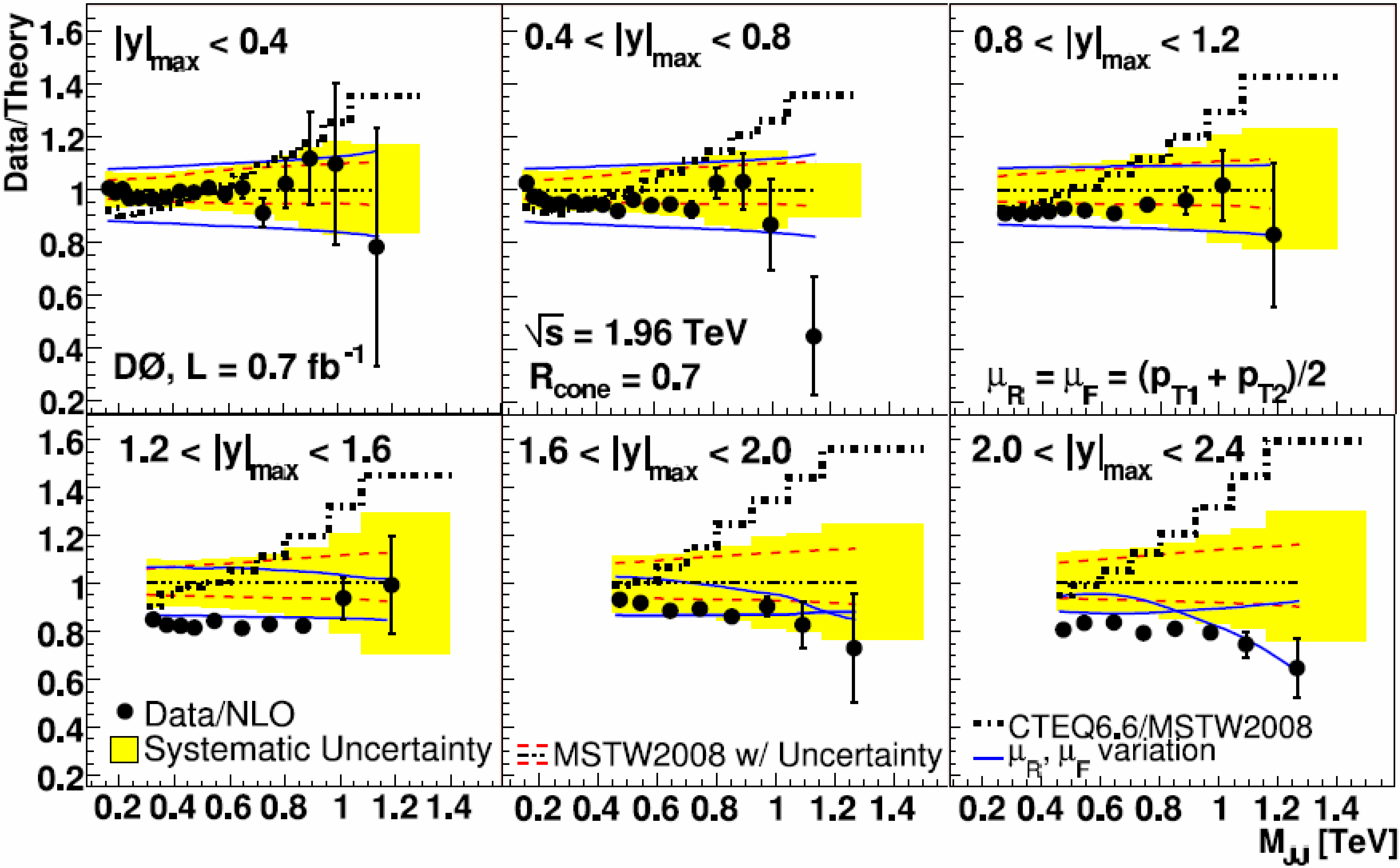
  (Note: Run II data requires slightly softer gluon than Run I data)

- the (still) missing ingredient is the full NNLO pQCD correction to the cross section, but not expected to have much impact in practice [Kidonakis, Owens (2001)]





dijet mass distribution from D0



D0 collaboration: arXiv:1002.4594

LO *vs* NLO *vs* NNLO?

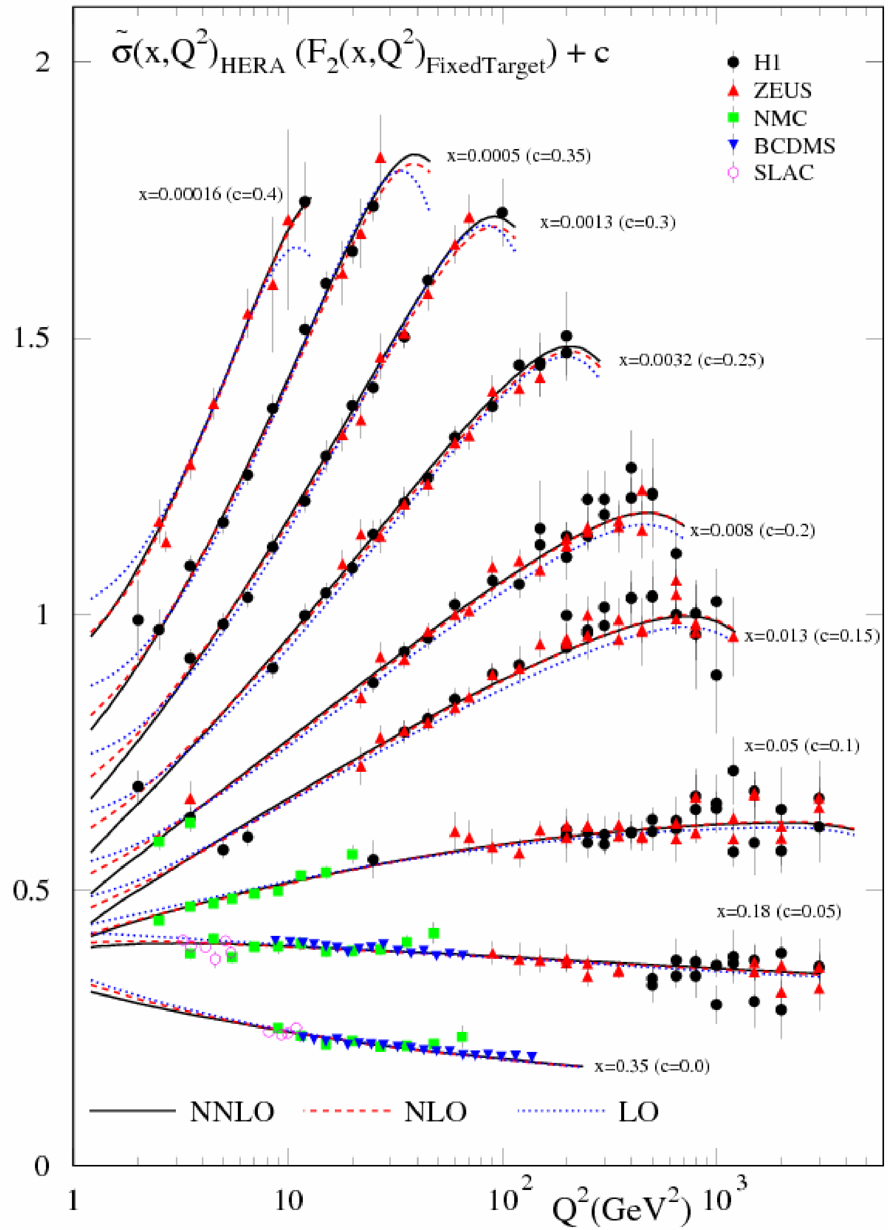
in the MSTW2008 fit

χ^2_{global} /dof = 3066/2598 (LO)
2543/2699 (NLO)
2480/2615 (NNLO)

LO evolution too slow at small x; NNLO fit marginally better than NLO

Note:
- an important ingredient missing in the full NNLO global pdf fit is the NNLO correction to the Tevatron high E_T jet cross section
- LO can be improved (e.g. LO*) for MCs by adding K-factors, relaxing momentum conservation, etc.

MSTW 2008



pdf uncertainties

- all groups produce 'pdfs with errors'
- typically, 20-40 'error' sets based on a 'best fit' set to reflect $\pm 1\sigma$ variation of all the parameters* $\{A_i, a_i, \ldots, \alpha_S\}$ inherent in the fit
- these reflect the uncertainties on the data used in the global fit (e.g. $\delta F_2 \approx \pm 3\% \rightarrow \delta u \approx \pm 3\%$)
- however, there are also systematic pdf uncertainties reflecting theoretical assumptions/prejudices in the way the global fit is set up and performed (see earlier slide)

* e.g. $f_i(x, Q_0^2) = A_i x^{a_i} [1 + b_i \sqrt{x} + c_i x] (1-x)^{d_i}$

pdf uncertainties (contd.)

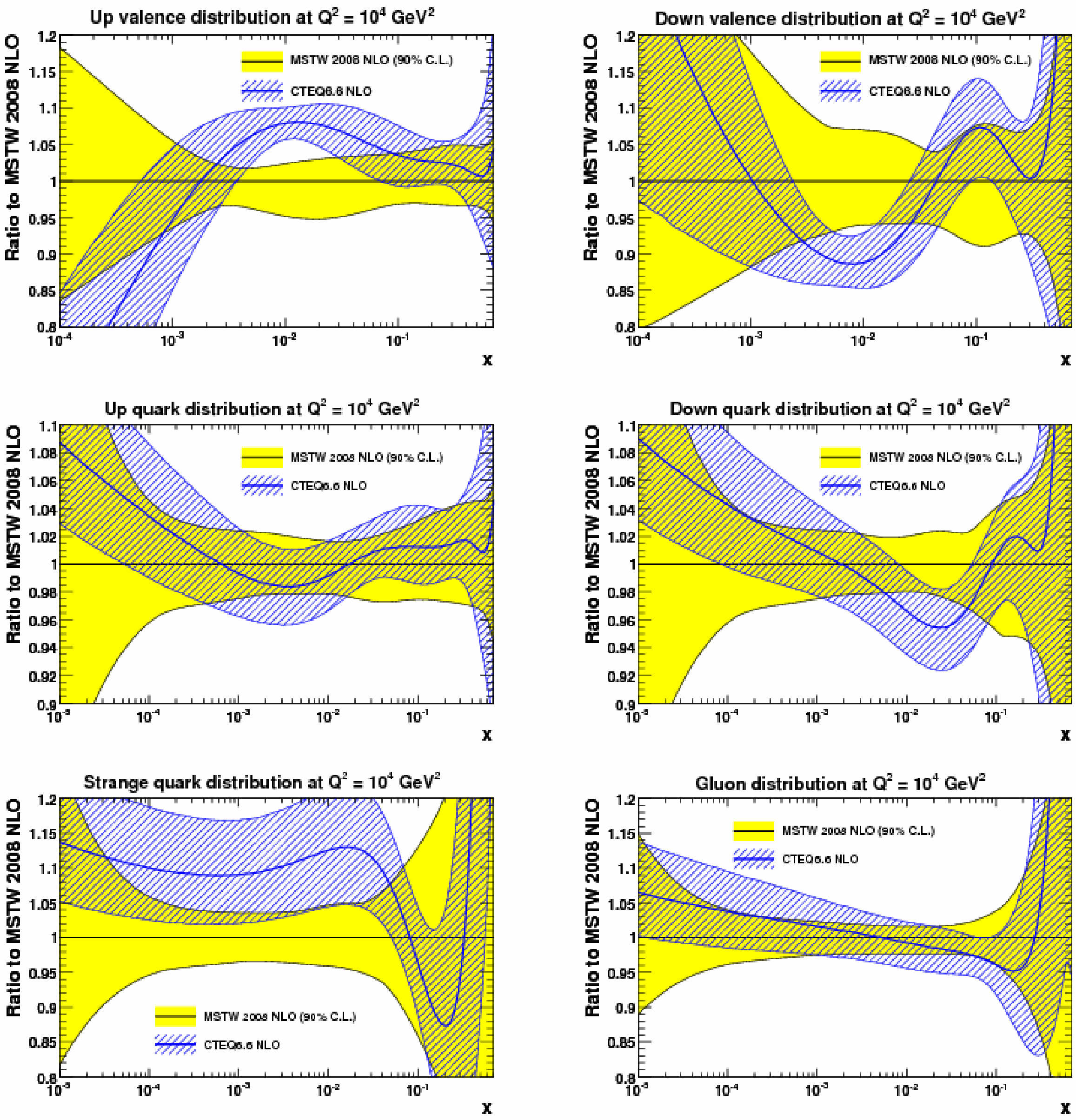
- NNPDF create many replicas of data and obtain PDF replicas in each case by fitting to training set and comparing to validation set → uncertainty determined by spread of replicas. Direct relationship to χ^2 in global fit not trivial.

- NNPDF and MSTW (due to extra parameters) have more complicated shape for gluon at smaller x and bigger small-x uncertainty, ditto for CTEQ at large x

- different theory assumptions in strange quark pdf leads to vastly different uncertainties — e.g. MSTW small, NNPDF large; feeds into other ‘light’ quarks

- perhaps surprisingly, all get rather similar uncertainties for pdfs and predicted cross sections — see later

example: MSTW2008(NLO) vs. CTEQ6.6



Note:

CTEQ error bands comparable with MSTW 90%cl set (different definition of tolerance)

CTEQ light quarks and gluons slightly larger at small x because of imposition of positivity on gluon at Q_0^2

CTEQ gluons slightly larger at large x - only Run 1 jet data in fit

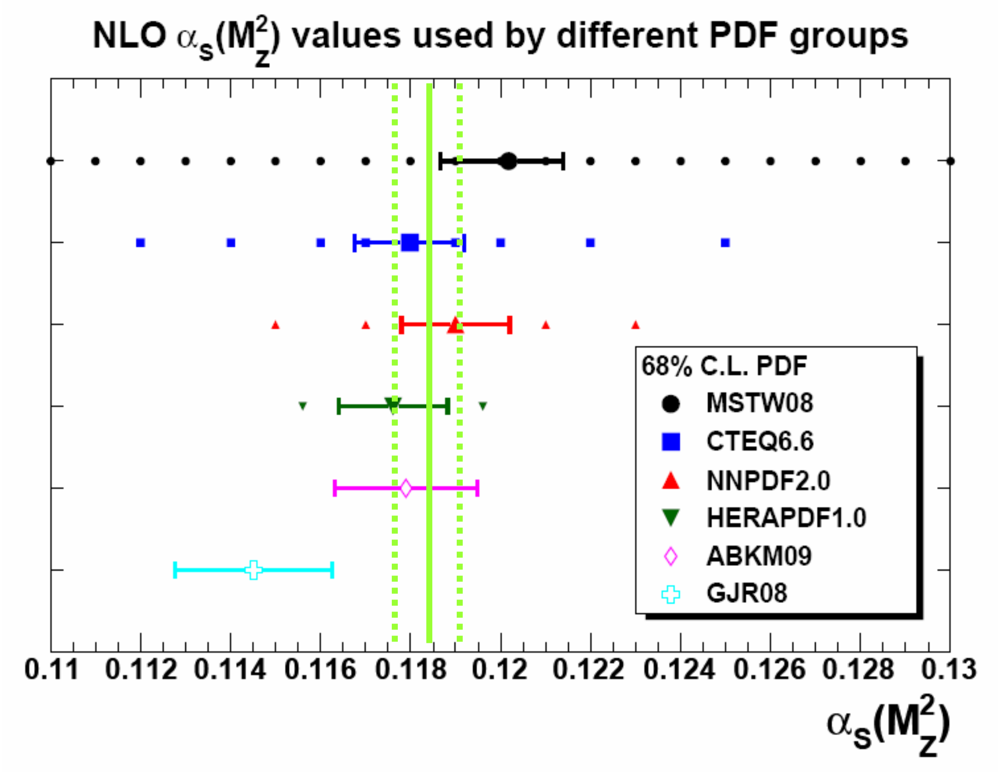
→ implications for 'precision' LHC cross sections (later)

pdfs and $\alpha_S(M_Z^2)$

- MSTW08, ABKM09 and GJR08: $\alpha_S(M_Z^2)$ values and uncertainty determined by global fit
- NNLO value about 0.003 – 0.004 lower than NLO value, e.g. for MSTW08

$$\alpha_S^{\overline{MS},NLO}(M_Z^2) = 0.1202 {}^{+0.012}_{-0.015}$$

$$\alpha_S^{\overline{MS},NNLO}(M_Z^2) = 0.1171 {}^{+0.014}_{-0.014}$$



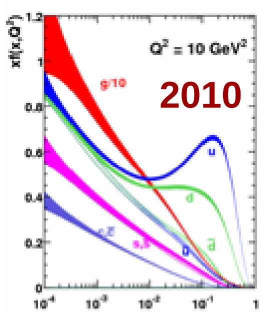
- CTEQ, NNPDF, HERAPDF choose standard values and uncertainties
- world average (PDG 2009)

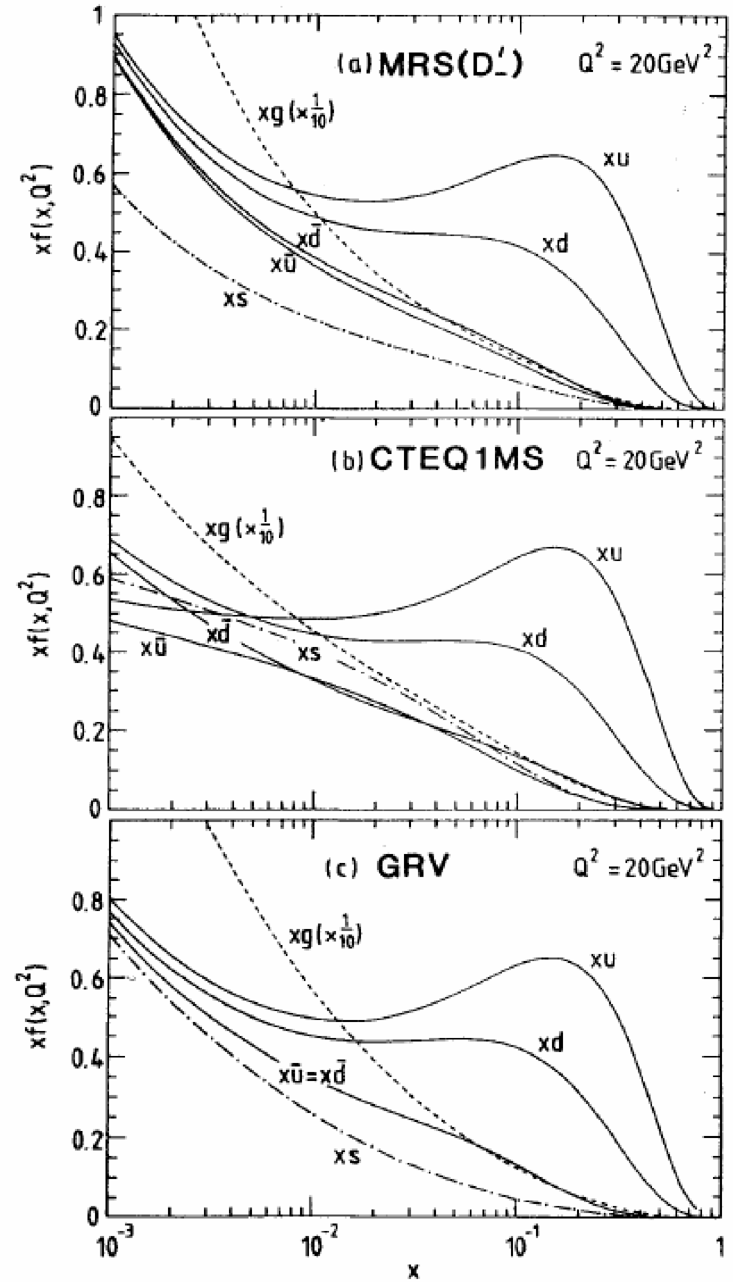
$$\alpha_S^{\overline{MS}}(M_Z^2) = 0.1184 \pm 0.0007$$

- note that the pdfs and α_S are correlated!
- e.g. gluon – α_S anticorrelation at small x and quark – α_S anticorrelation at large x

cf. pdfs in 1993

	μ-DIS	ν-DIS	Prompt γ	D-Yan	W, Z
MRS '88	EMC + ..	CDHSW	AFS($+J/\psi$)	–	–
DFLM '88	(EMC + ..)	CHARM + ..	–	E288 + ..	–
ABFOW '89	BCDMS	–	WA70	–	–
HMRS '90	EMC BCDMS NMC(n/p)	CDHSW	WA70	E605	–
MT '90	EMC BCDMS	CDHSW	–	E288 E605	–
KMRS '90 (sets B_0,B_-)	BCDMS NMC(n/p)	CDHSW	WA70	E605	–
MRS (Apr '92) (sets D_0,D_-)	BCDMS NMC(p,n)†	CDHSW CCFR†	WA70	E605	(UA2, CDF
MRS (Nov '92) (sets D'_0,D'_-)	BCDMS NMC(p,n)	CCFR	WA70	E605	(UA2, CDF)
CTEQ ('93)	BCDMS NMC(p,n)	CCFR	WA70 E706,UA6	E605	





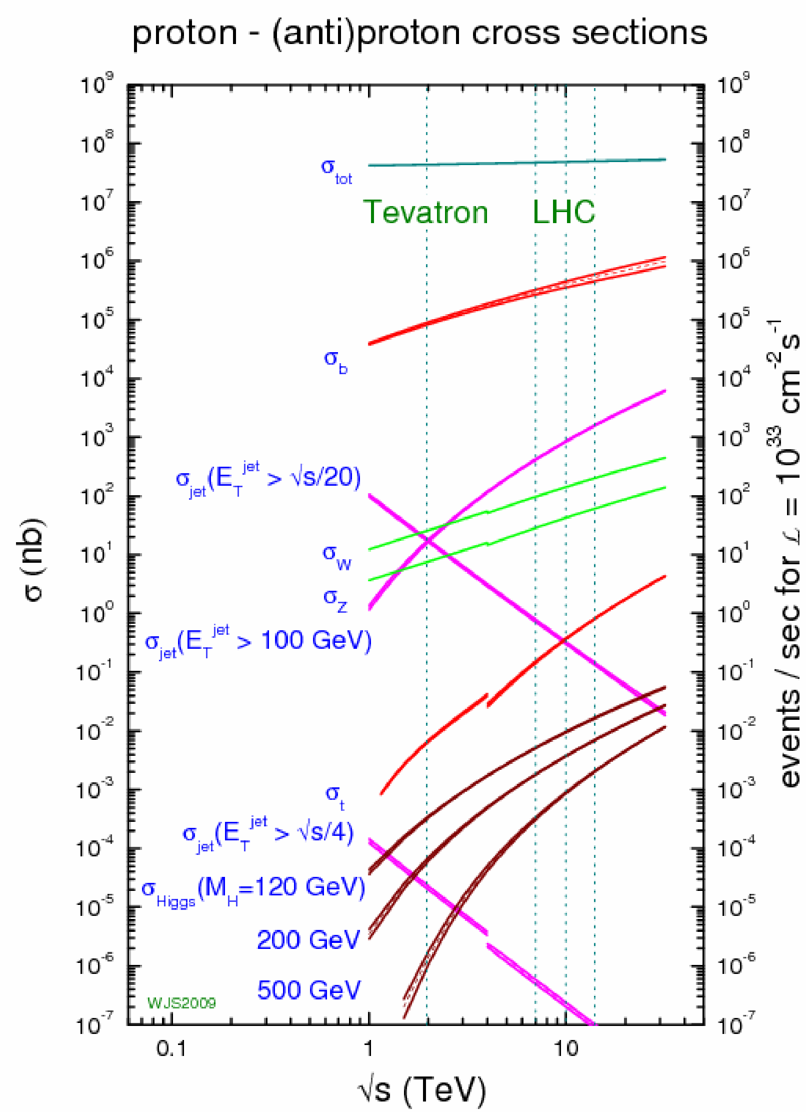
2

LHC benchmark cross sections

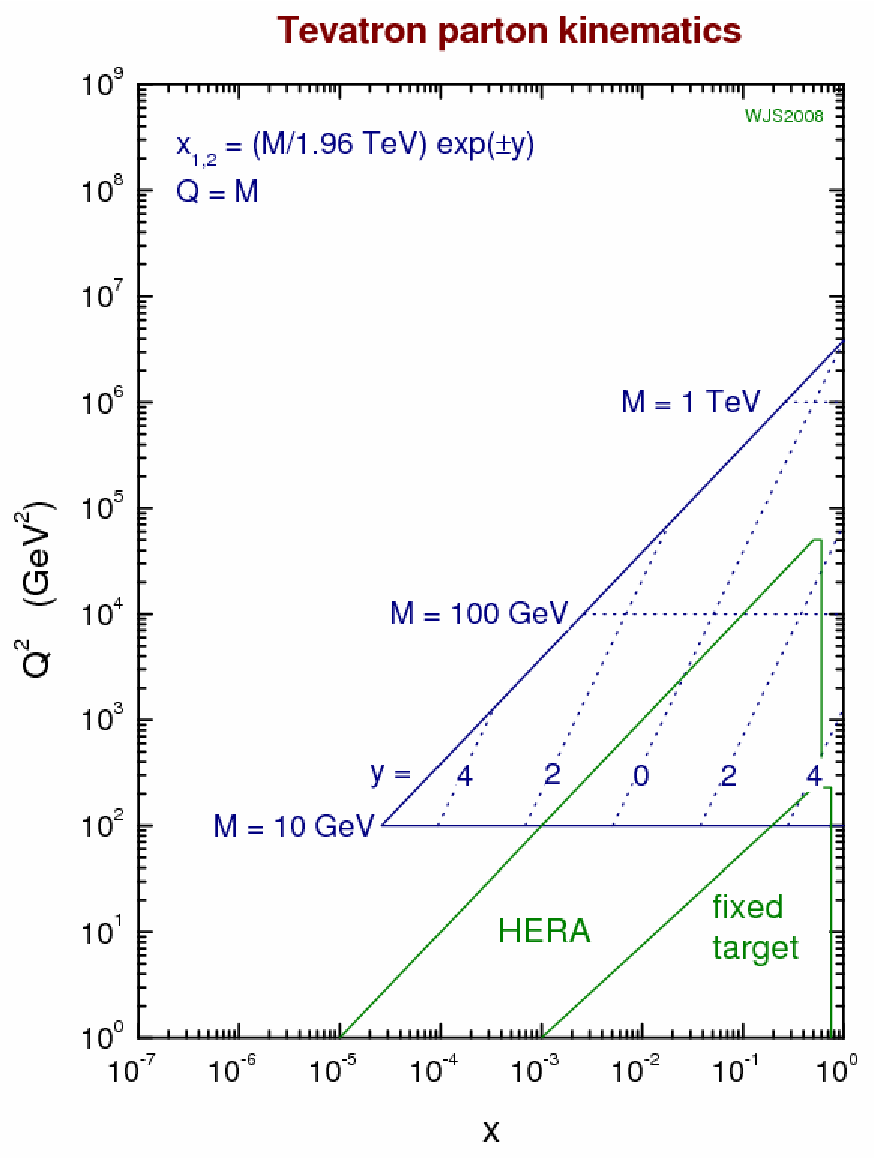
precision phenomenology at LHC

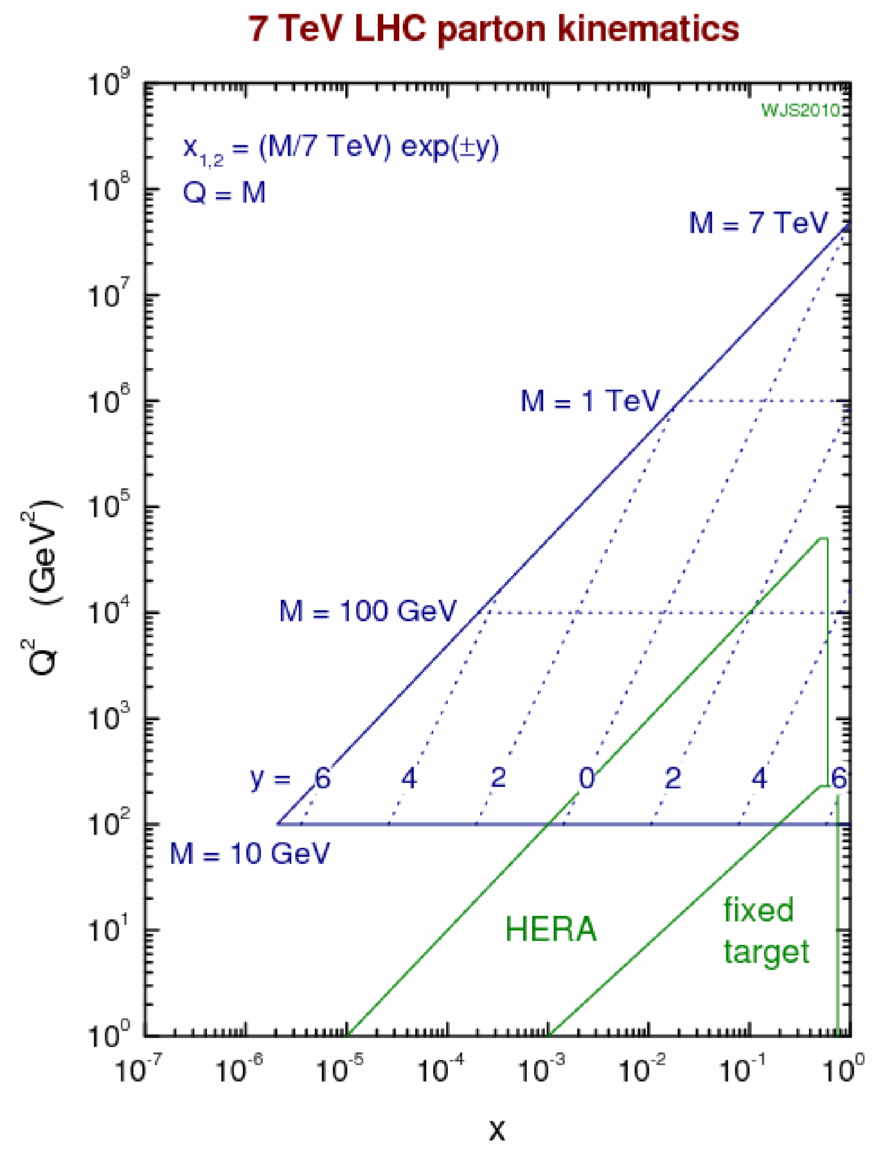
- LO for generic PS Monte Carlos
- NLO for NLO-MCs and many parton-level signal and background processes
- NNLO for a limited number of 'precision observables' (W, Z, DY, H, …)

\+ E/W corrections, resummed HO terms etc…



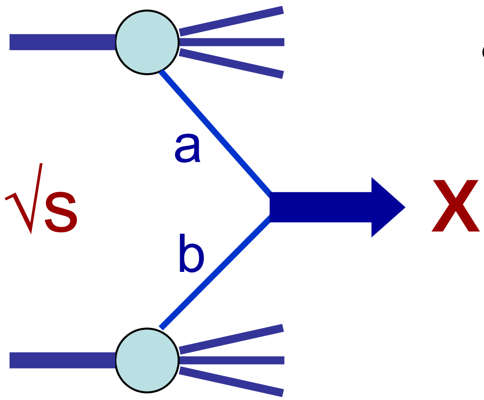
$$\delta\sigma_{th} = \delta\sigma_{pdf} \oplus \delta\sigma_{HO} \oplus \delta\sigma_{param} \oplus \cdots$$


Tevatron parton kinematics
$x_{1,2}$ = (M/1.96 TeV) exp(±y)
Q = M
WJS2008
M = 1 TeV
M = 100 GeV
M = 10 GeV
y = 4 2 0 2 4
HERA
fixed target
Q^2 (GeV^2)
x


7 TeV LHC parton kinematics
$x_{1,2}$ = (M/7 TeV) exp(±y)
Q = M
WJS2010
M = 7 TeV
M = 1 TeV
M = 100 GeV
M = 10 GeV
y = 6 4 2 0 2 4 6
HERA
fixed target
Q^2 (GeV^2)
x

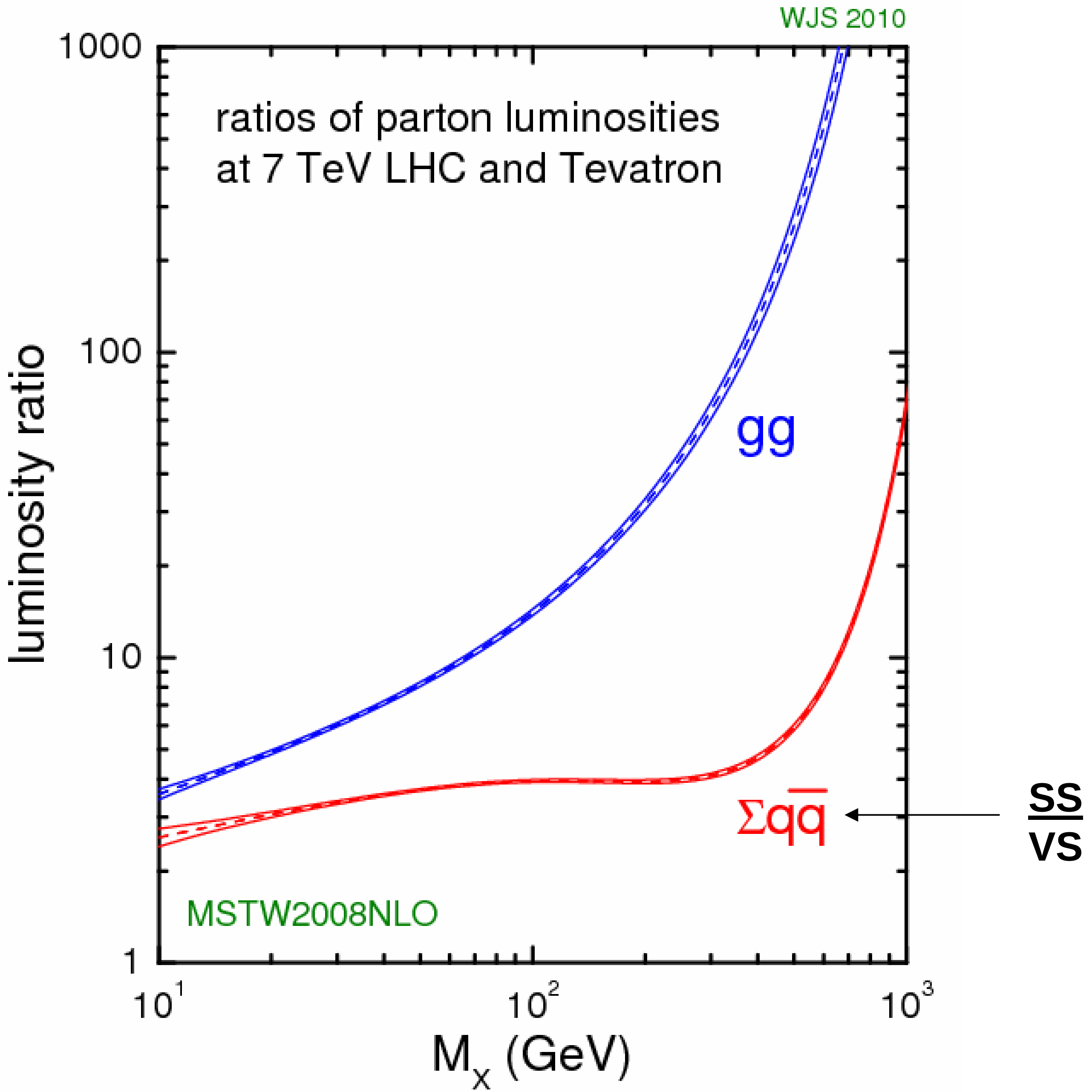
parton luminosity functions

- a quick and easy way to assess the mass, collider energy and pdf dependence of production cross sections

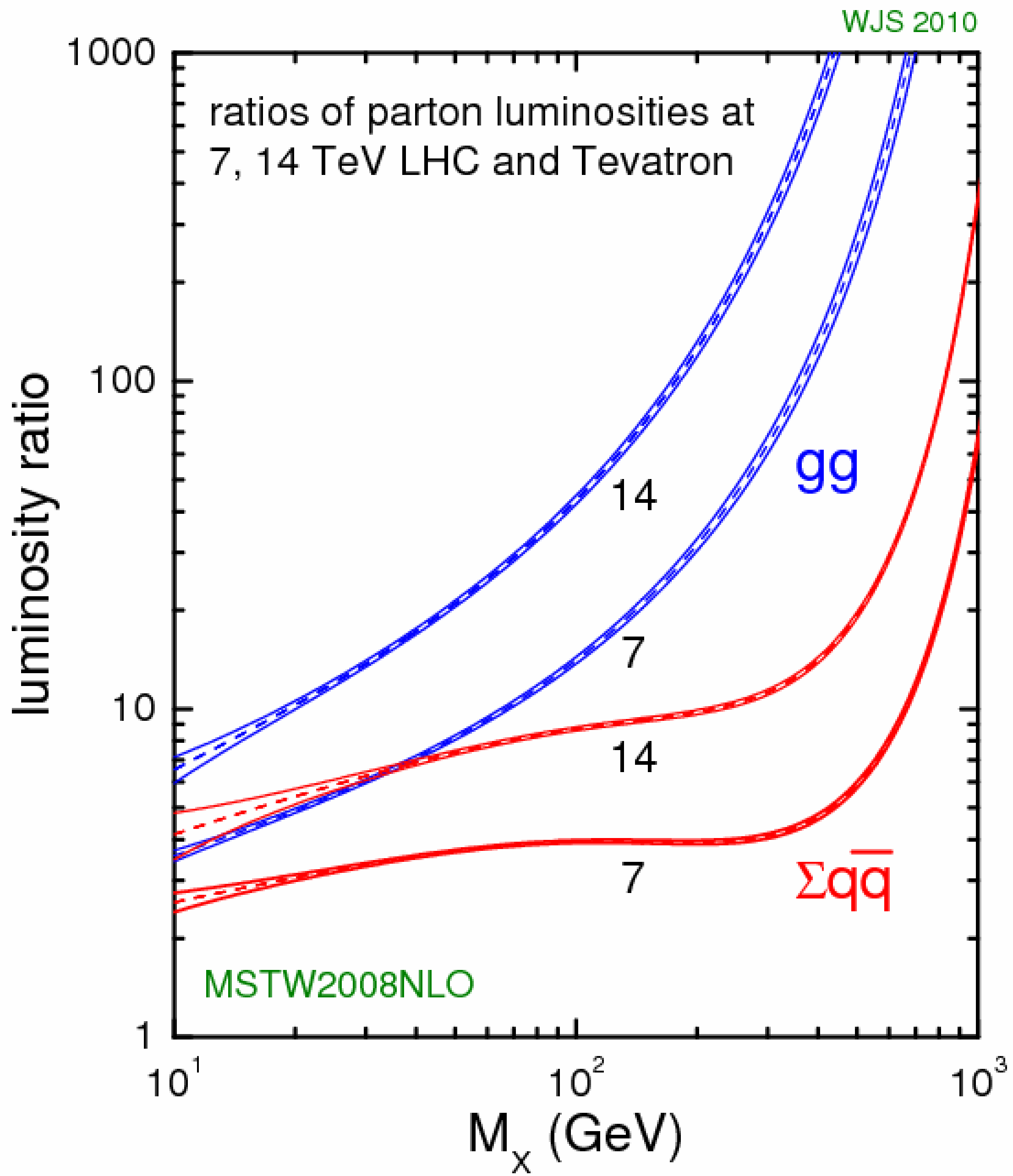


$$\begin{aligned}
\hat{\sigma}_{ab\to X} &= C_X \delta(\hat{s} - M_X^2) \\
\sigma_X &= \int_0^1 dx_a dx_b \, f_a(x_a, M_X^2) f_b(x_b, M_X^2) \, C_X \, \delta(x_a x_b - \tau) \\
&\equiv C_X \left[\frac{1}{s} \frac{\partial \mathcal{L}_{ab}}{\partial \tau} \right] \qquad\qquad (\tau = M_X^2/s) \\
\frac{\partial \mathcal{L}_{ab}}{\partial \tau} &= \int_0^1 dx_a dx_b \, f_a(x_a, M_X^2) f_b(x_b, M_X^2) \, \delta(x_a x_b - \tau)
\end{aligned}$$

- i.e. all the mass and energy dependence is contained in the X-independent parton luminosity function in [ ]
- useful combinations are $ab = gg, \sum_q q\bar{q}, \ldots$
- and also useful for assessing the uncertainty on cross sections due to uncertainties in the pdfs



more such luminosity plots available at www.hep.phy.cam.ac.uk/~wjs/plots/plots.html


WJS 2010
ratios of parton luminosities at
7, 14 TeV LHC and Tevatron
luminosity ratio
1000
100
10
1
14
7
14
7
gg
Σqq̄
MSTW2008NLO
10^1
10^2
10^3
M_X (GeV)

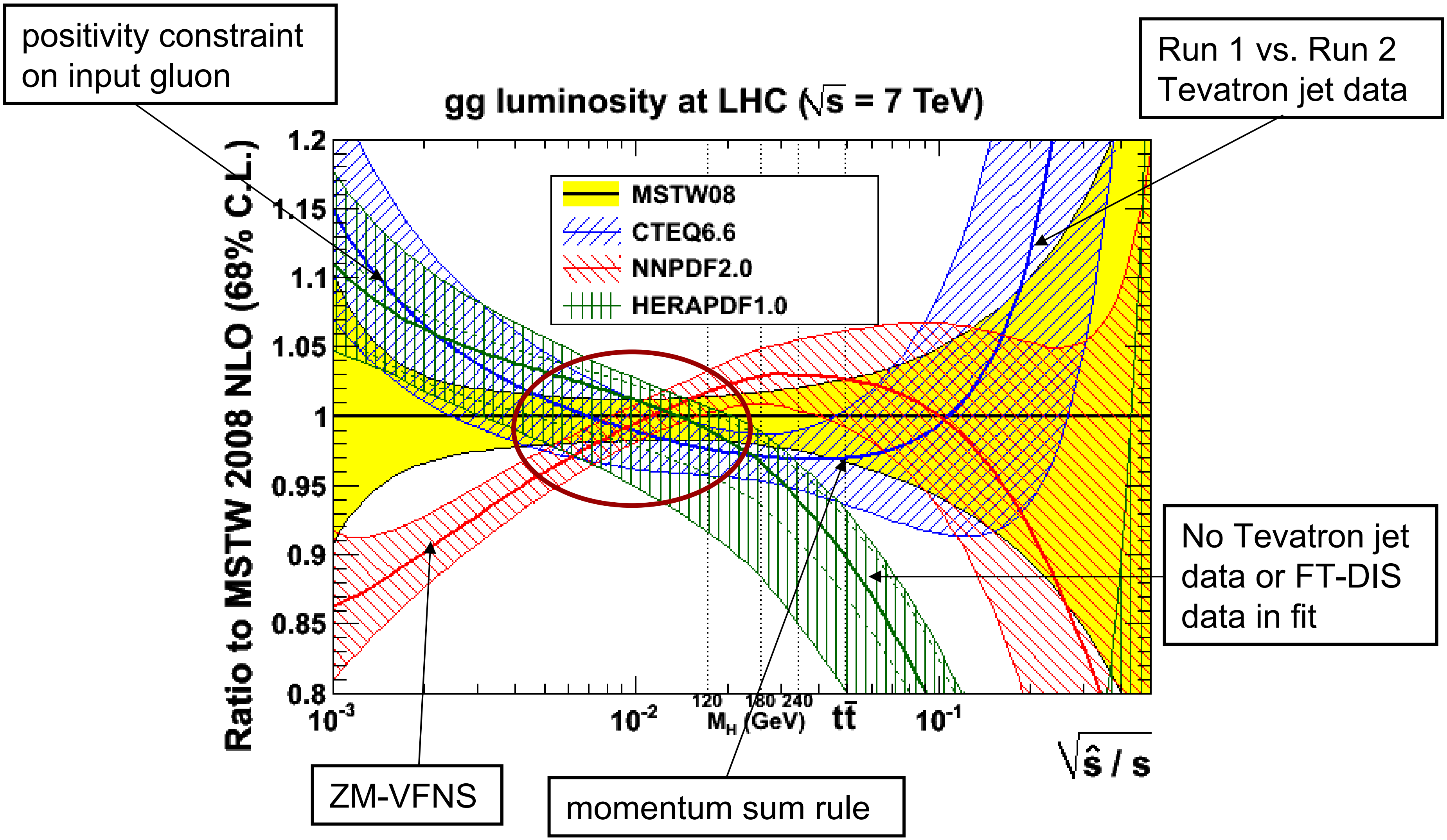
pdfs at LHC – the issues

- high precision cross section predictions require accurate knowledge of pdfs: $\delta\sigma_{th} = \delta\sigma_{pdf} + \ldots$

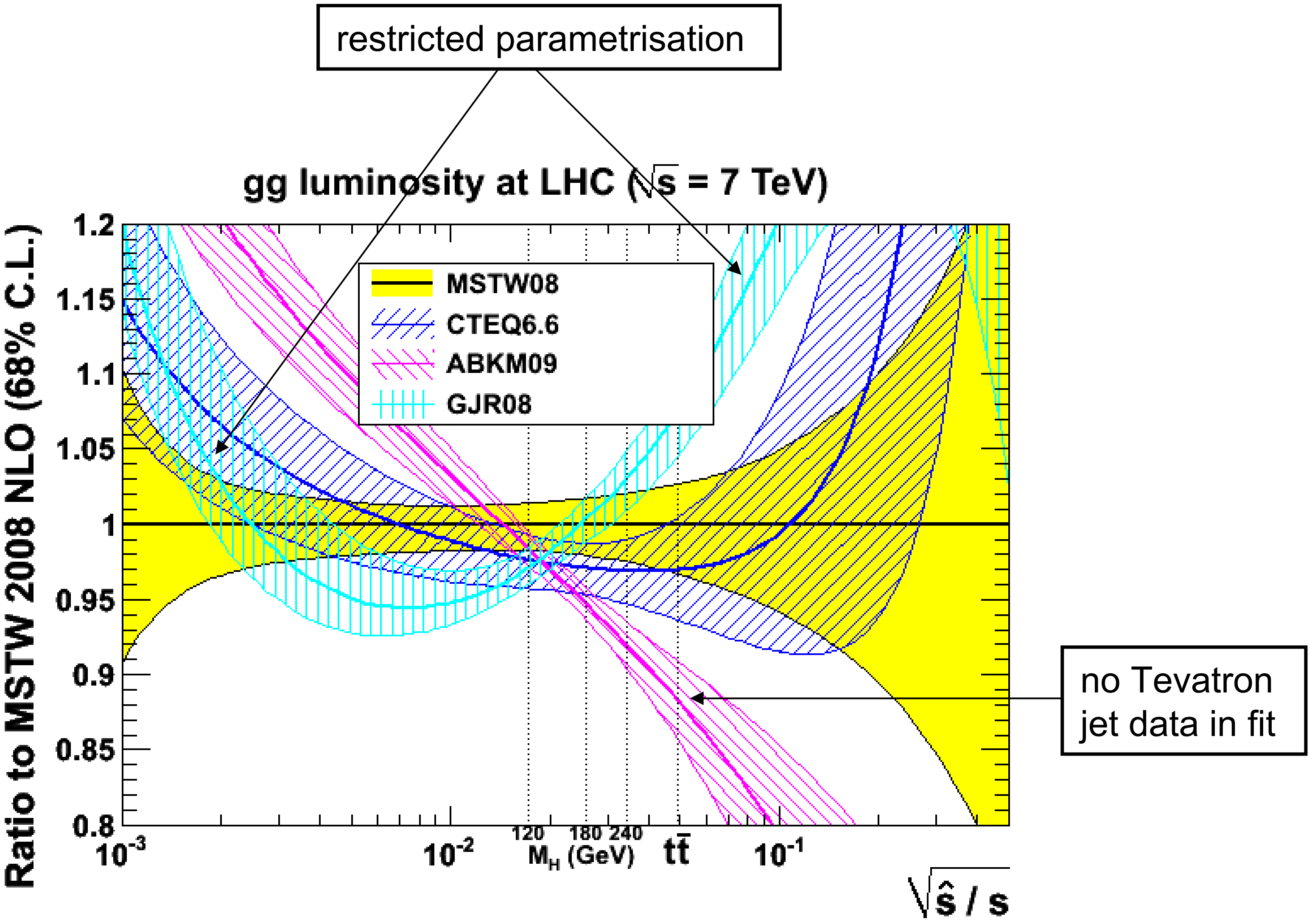
  → how do the different pdf sets compare?

- can we learn more about pdfs from LHC measurements, e.g.
  - high-E_T jets → gluon?
  - W^+, W^-, Z^0 → quarks?
  - very forward Drell-Yan (e.g. LHCb) → small x?
  - …

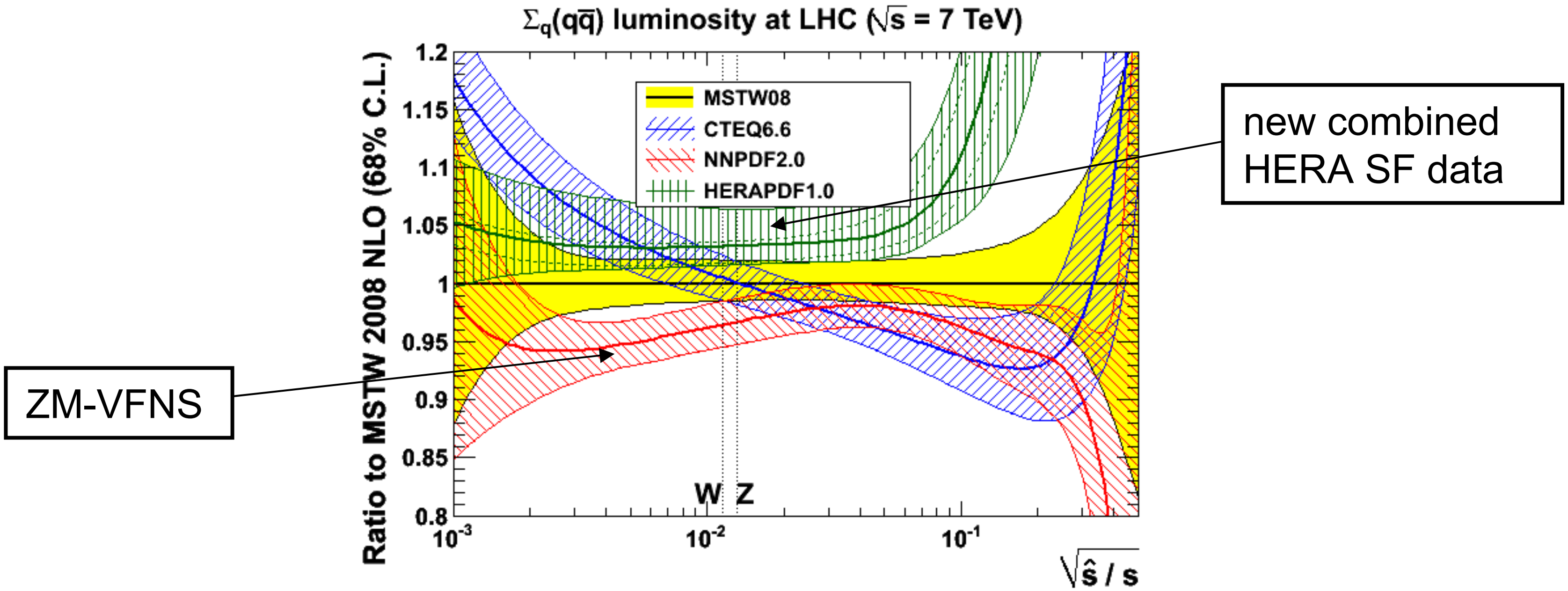
parton luminosity comparisons

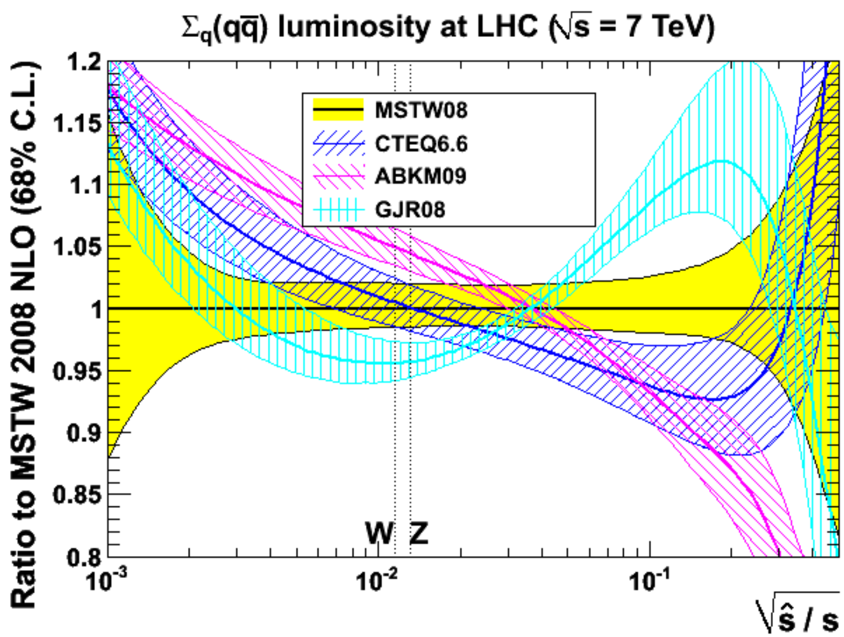


Luminosity and cross section plots from Graeme Watt (MSTW, in preparation), available at projects.hepforge.org/mstwpdf/pdf4lhc

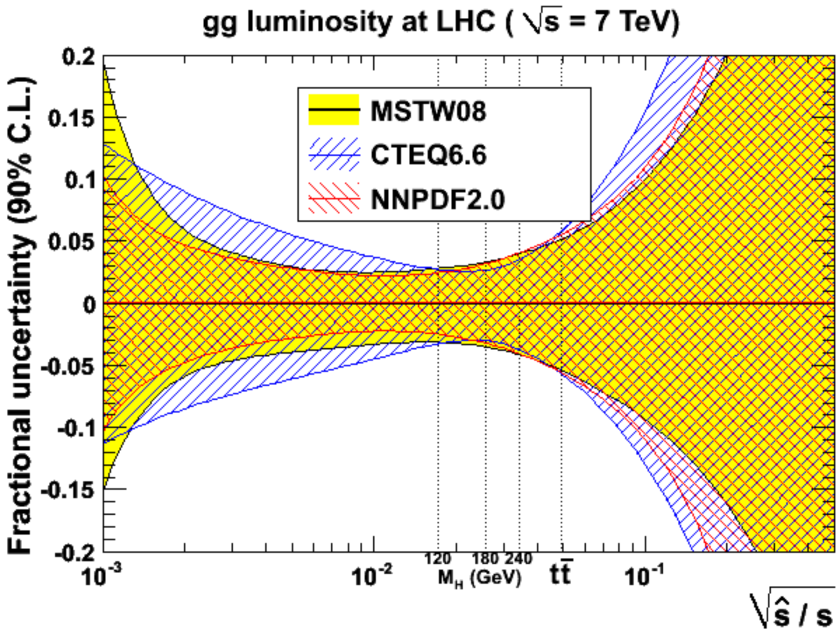


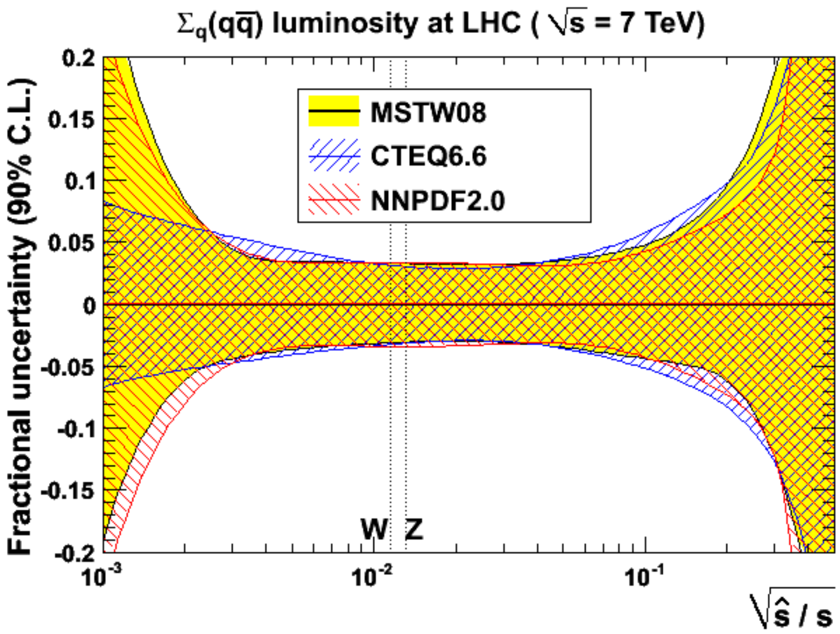
restricted parametrisation
gg luminosity at LHC (√s = 7 TeV)
MSTW08
CTEQ6.6
ABKM09
GJR08
Ratio to MSTW 2008 NLO (68% C.L.)
1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
0.8
10^-3
10^-2
120
180
240
M_H (GeV)
tt̄
10^-1
√(ŝ / s)
no Tevatron jet data in fit


Σq(qq̄) luminosity at LHC (√s = 7 TeV)
Ratio to MSTW 2008 NLO (68% C.L.)
1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
0.8
10-3
10-2
10-1
√ŝ / s
MSTW08
CTEQ6.6
NNPDF2.0
HERAPDF1.0
W
Z
new combined
HERA SF data
ZM-VFNS


Σq(qq̄) luminosity at LHC (√s = 7 TeV)
Ratio to MSTW 2008 NLO (68% C.L.)
1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
0.8
10-3
10-2
10-1
√ŝ / s
MSTW08
CTEQ6.6
ABKM09
GJR08
W
Z

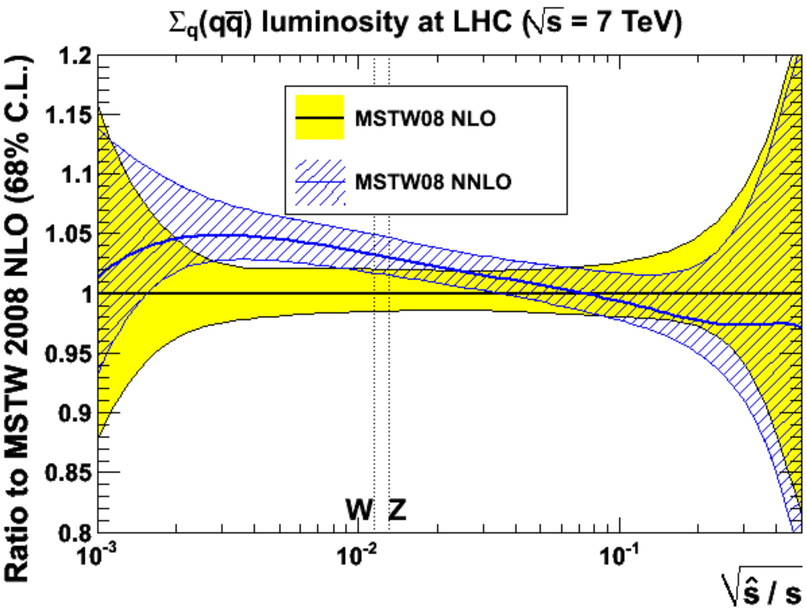
fractional uncertainty comparisons

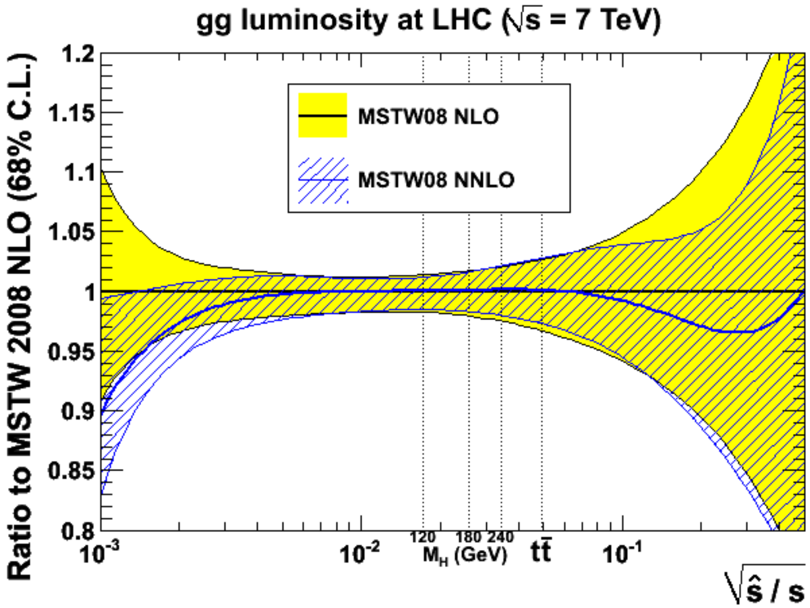




remarkably similar considering the different definitions of pdf uncertainties used by the 3 groups!

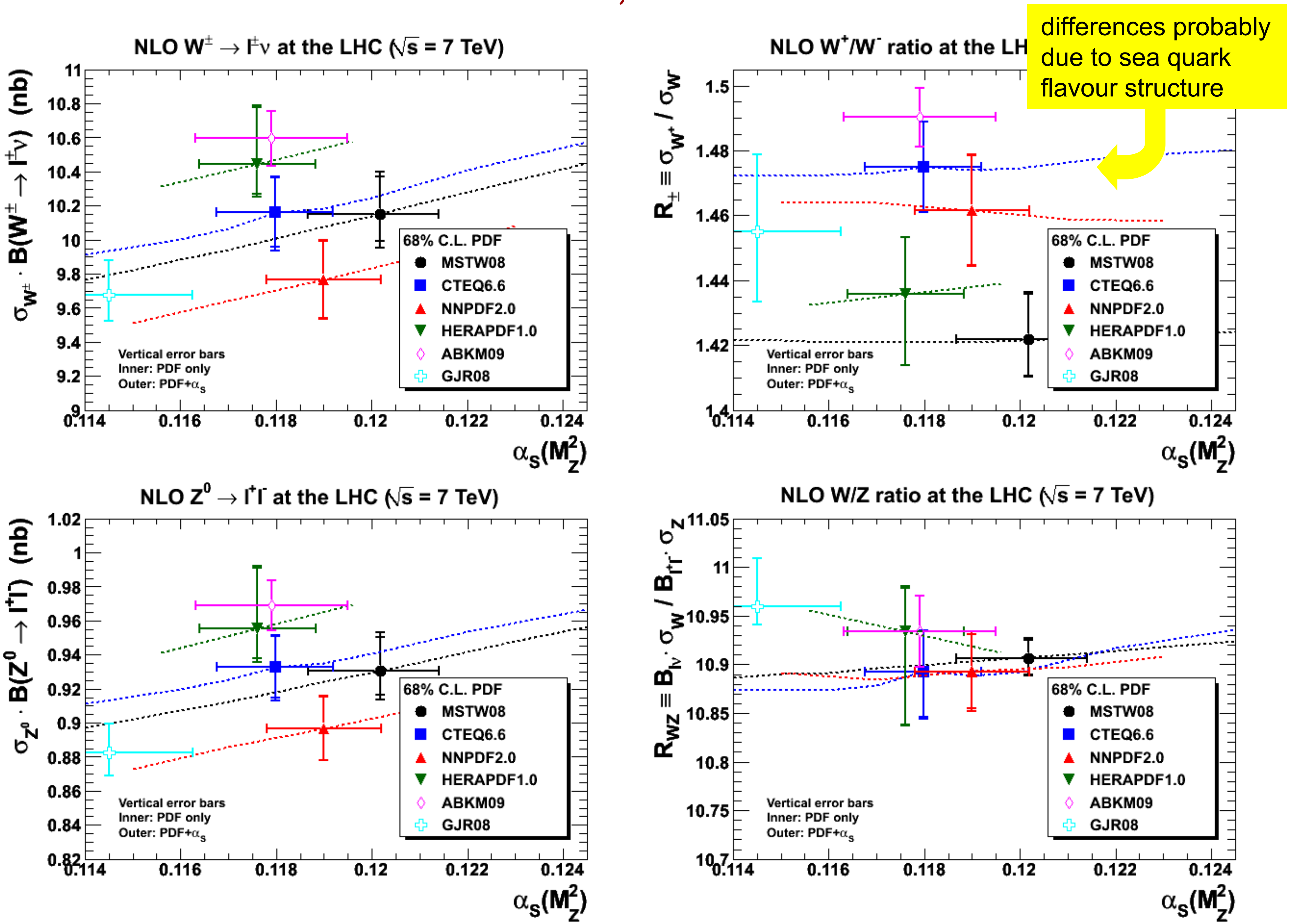
NLO and NNLO parton luminosity comparisons



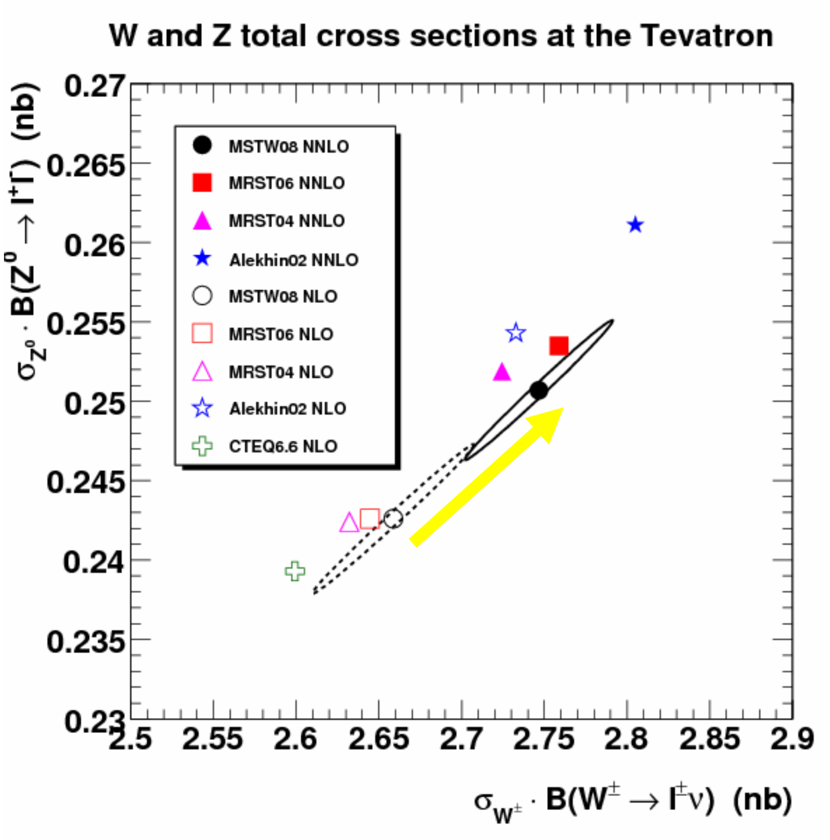


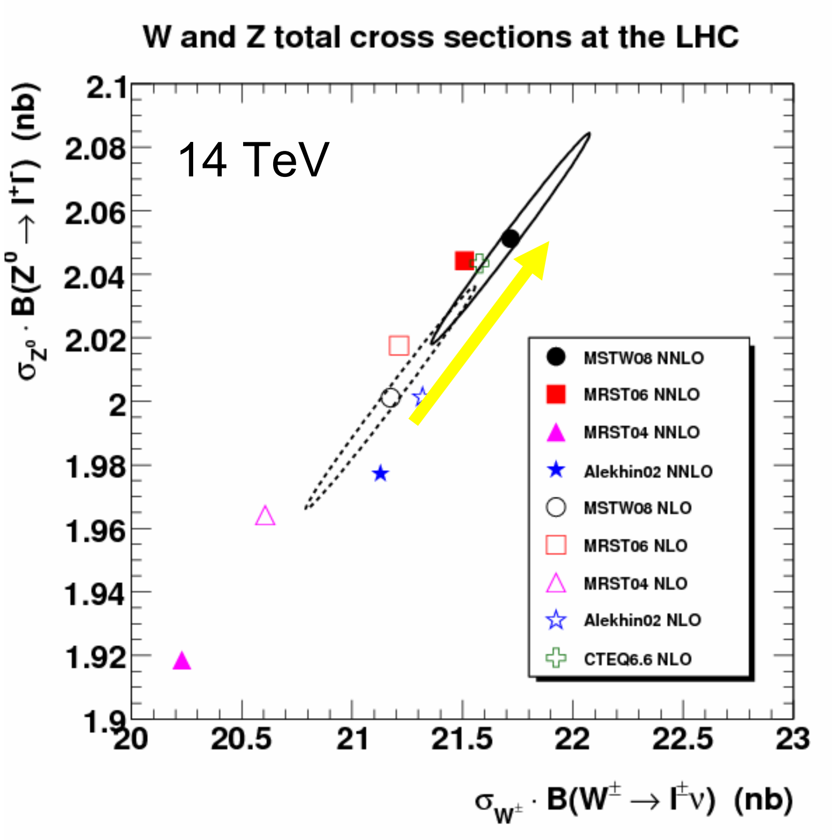
W, Z

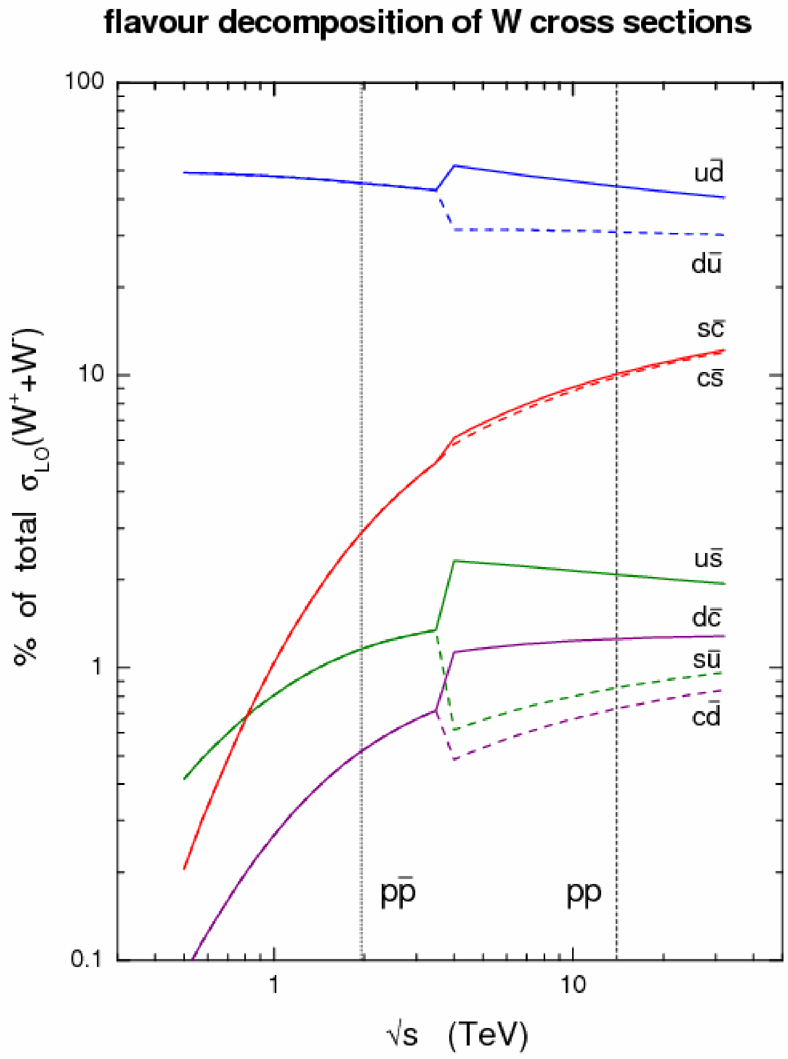
benchmark W,Z cross sections



predictions for σ(W,Z) @ Tevatron, LHC: NLO *vs.* NNLO







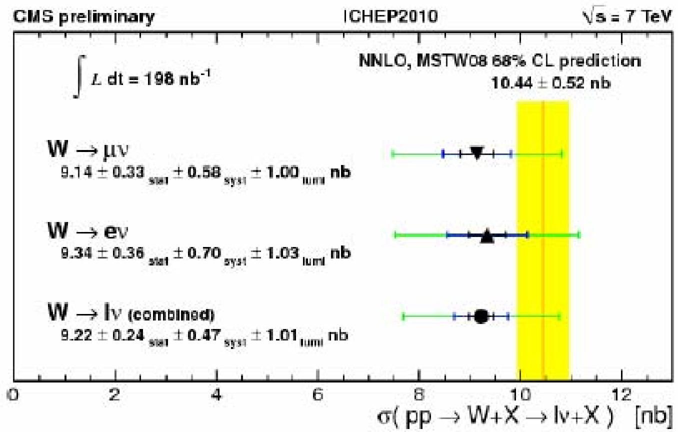
impact of sea quarks on the NLO W charge asymmetry ratio at 7 TeV:

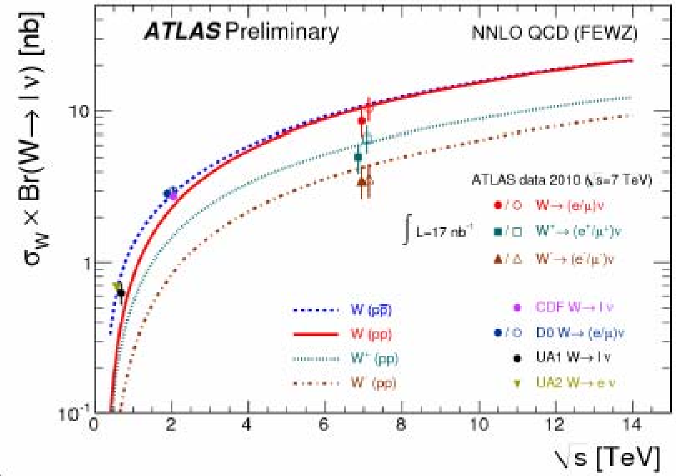
pdfs	$R(W^+/W^-)$
{udg} only	1.53
{udscbg} = MSTW08	1.42 ± 0.02
$\{udscbg\}_{sea}$ only	0.99
$\{udscbg\}_{sym.sea}$ only	1.00

at LHC, ~30% of W and Z total cross sections involves s,c,b quarks

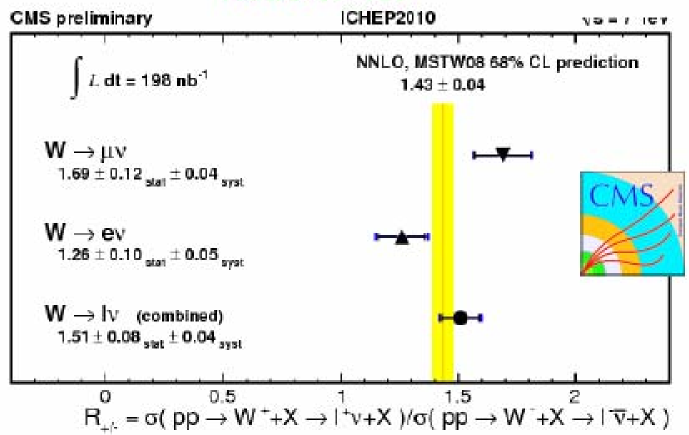
Inclusive W boson measurements: Summary

W→ℓν cross section





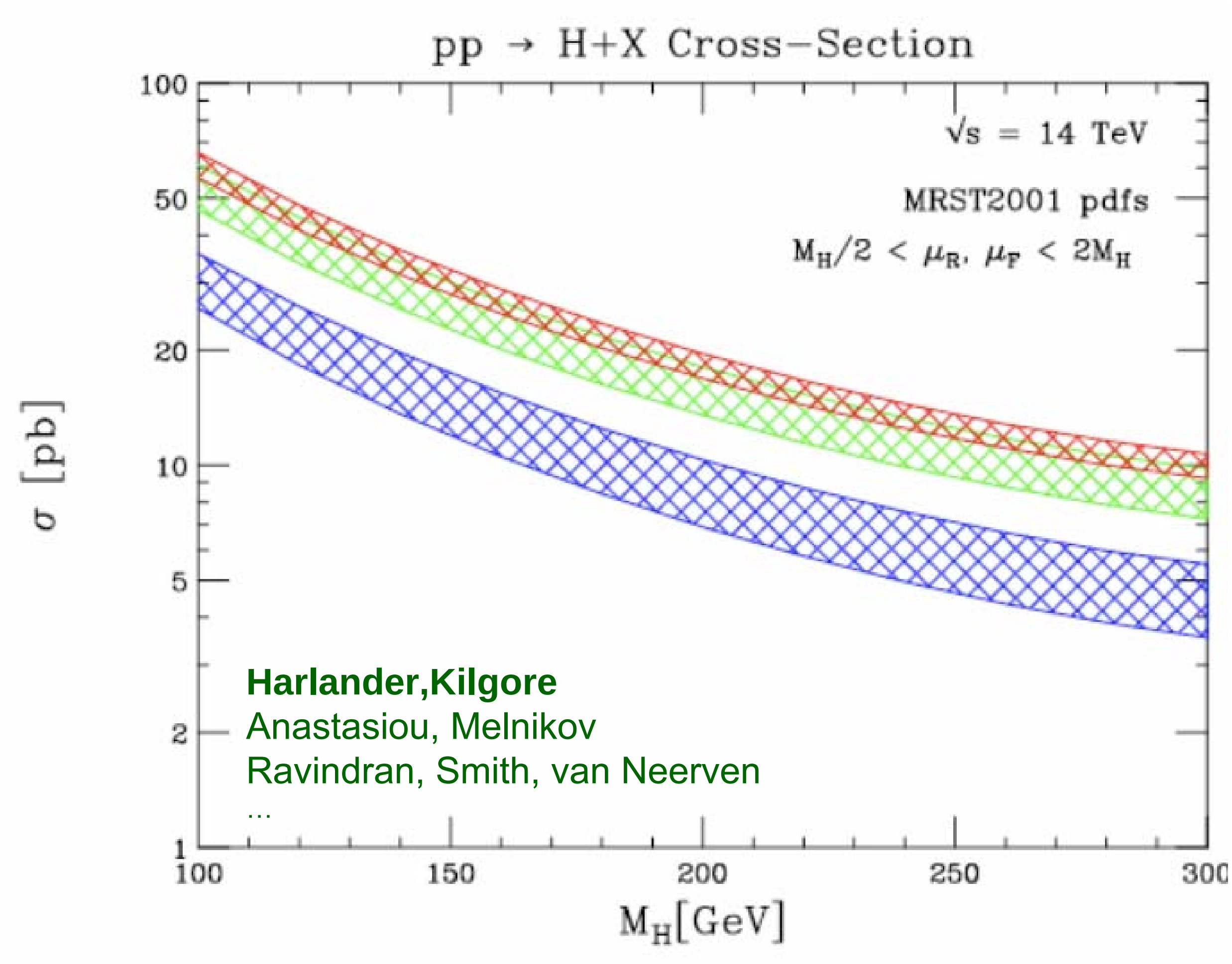
Ratio W+/W-



care needed with definition of 'total cross section' in these comparisons

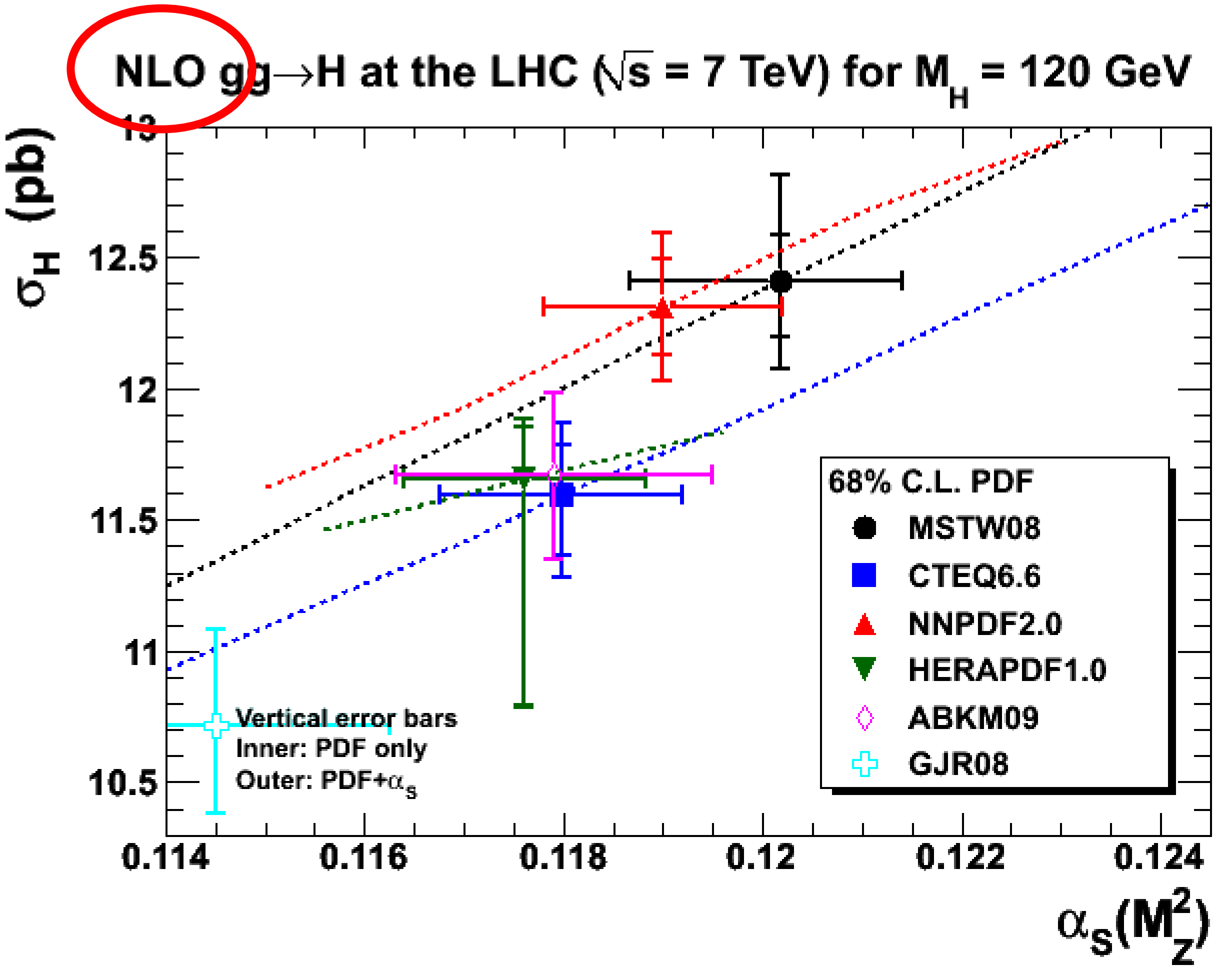
All the results are in agreement with the Standard Model expectations.

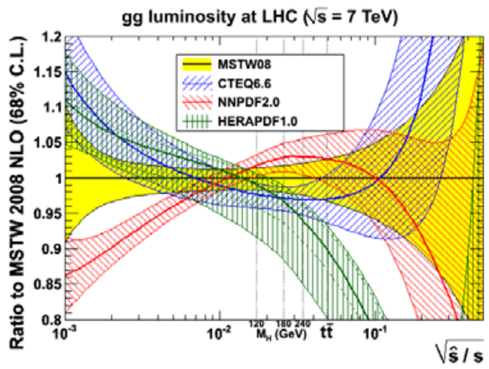
Higgs

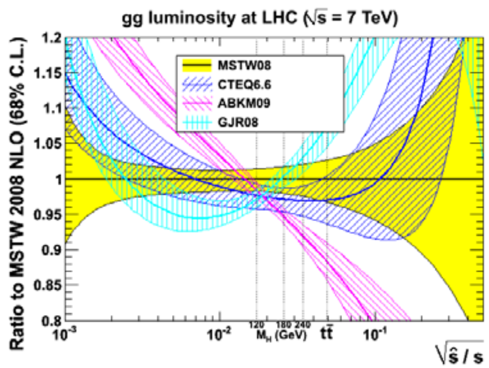


- only scale variation uncertainty shown
- central values calculated for a *fixed* set pdfs with a *fixed* value of $\alpha_S(M_Z)$

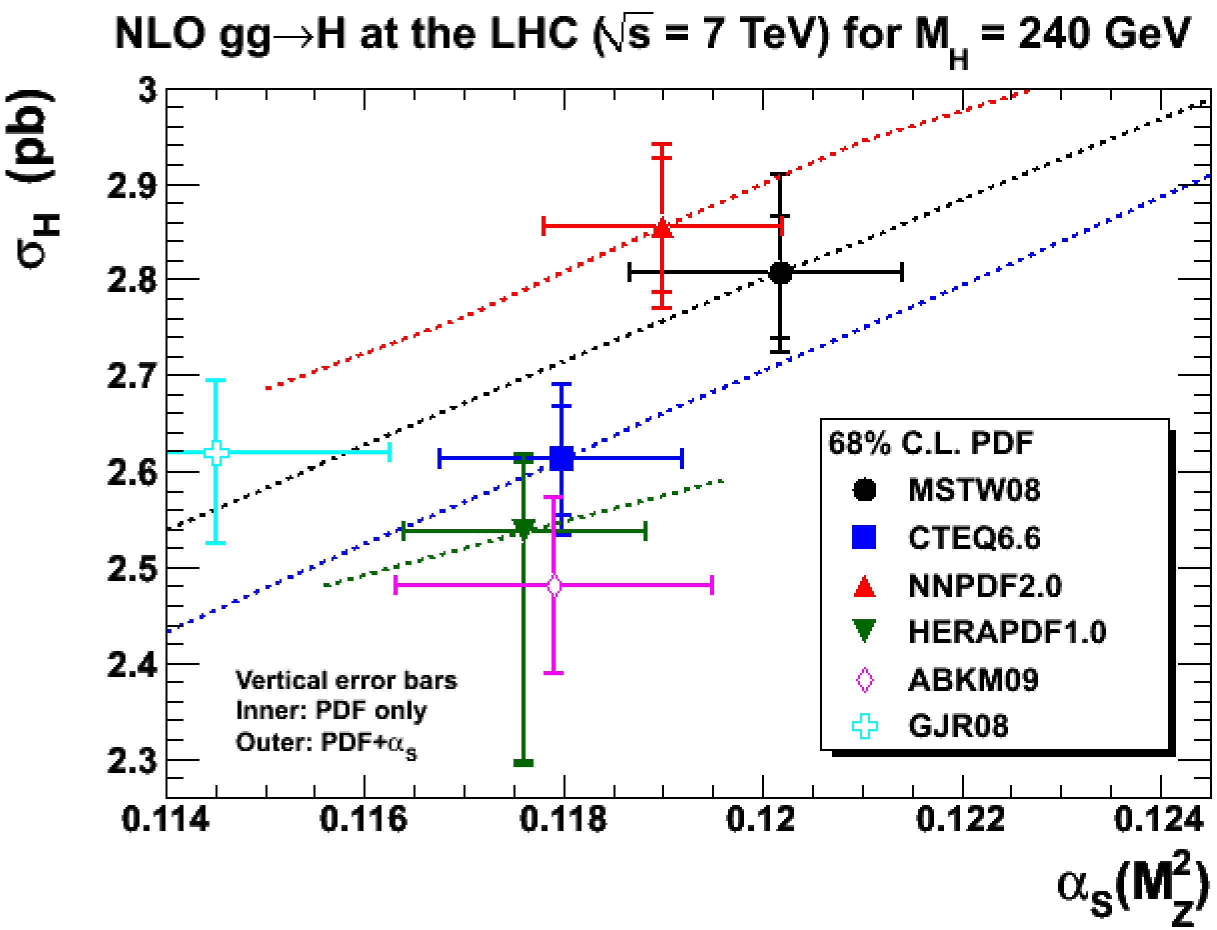
benchmark Higgs cross sections

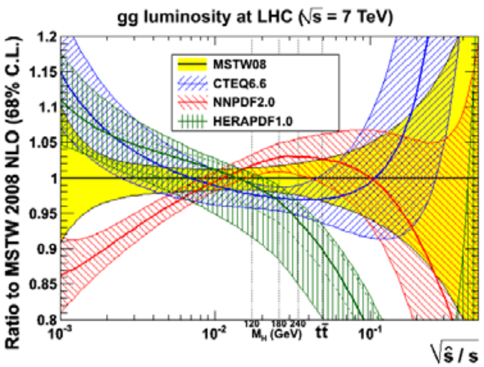


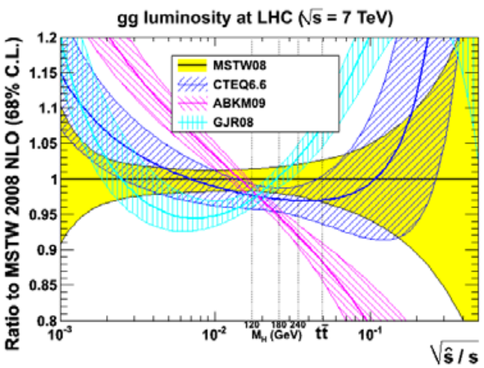


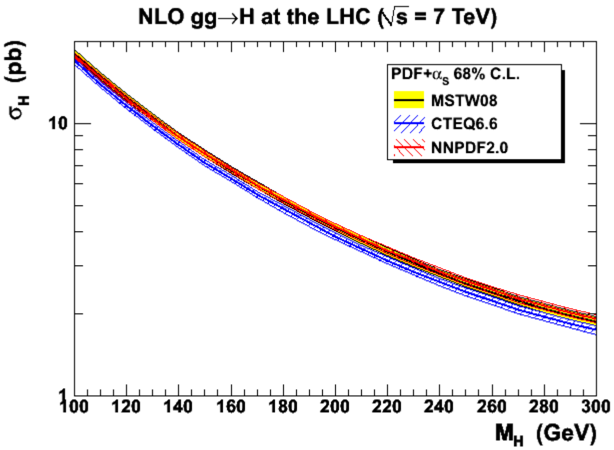


… differences from both pdfs AND α_S !


NLO gg→H at the LHC (√s = 7 TeV) for M_H = 240 GeV
σ_H (pb)
α_S(M_Z^2)
68% C.L. PDF
MSTW08
CTEQ6.6
NNPDF2.0
HERAPDF1.0
ABKM09
GJR08
Vertical error bars
Inner: PDF only
Outer: PDF+α_S


gg luminosity at LHC (√s = 7 TeV)
Ratio to MSTW 2008 NLO (68% C.L.)
MSTW08
CTEQ6.6
NNPDF2.0
HERAPDF1.0
√ŝ / s

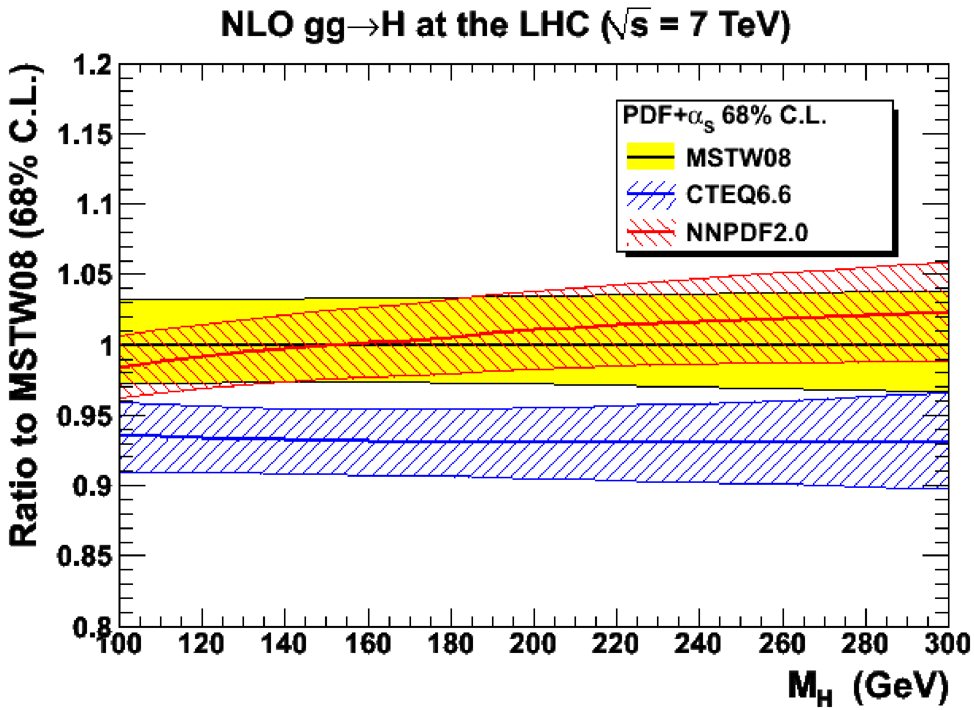

gg luminosity at LHC (√s = 7 TeV)
Ratio to MSTW 2008 NLO (68% C.L.)
MSTW08
CTEQ6.6
ABKM09
GJR08
√ŝ / s



How to define an overall 'best theory prediction'?! See LHC Higgs Cross Section Working Group meeting, 5-6 July, higgs2010.to.infn.it

small print

Central predictions use the values of $\alpha_S(M_Z)$ favoured by each PDF group, i.e. 0.1202 for MSTW08, 0.1180 for CTEQ6.6 and 0.1190 for NNPDF2.0. For MSTW08, $\alpha_S(M_Z)$ was determined simultaneously with the PDFs in the global fit. The *experimental* uncertainties on $\alpha_S(M_Z)$ are +0.0012/-0.0015 at 68% C.L The uncertainties on $\alpha_S(M_Z)$ for CTEQ6.6 and NNPDF2.0 are taken to be ±0.0012 at 68% C.L. The combined PDF+α_S uncertainty is calculated following the prescription recommended by each group, i.e. α_S uncertainties are simply added in quadrature for CTEQ6.6, while for NNPDF2.0 the exact prescription is used as explained in arXiv:1004.0962.



Note: (i) for MSTW08, uncertainty band similar at NNLO

(ii) everything here is at fixed scale $\mu=M_H$!

3

single pdfs – issues and outlook

(single) pdfs – issues and outlook

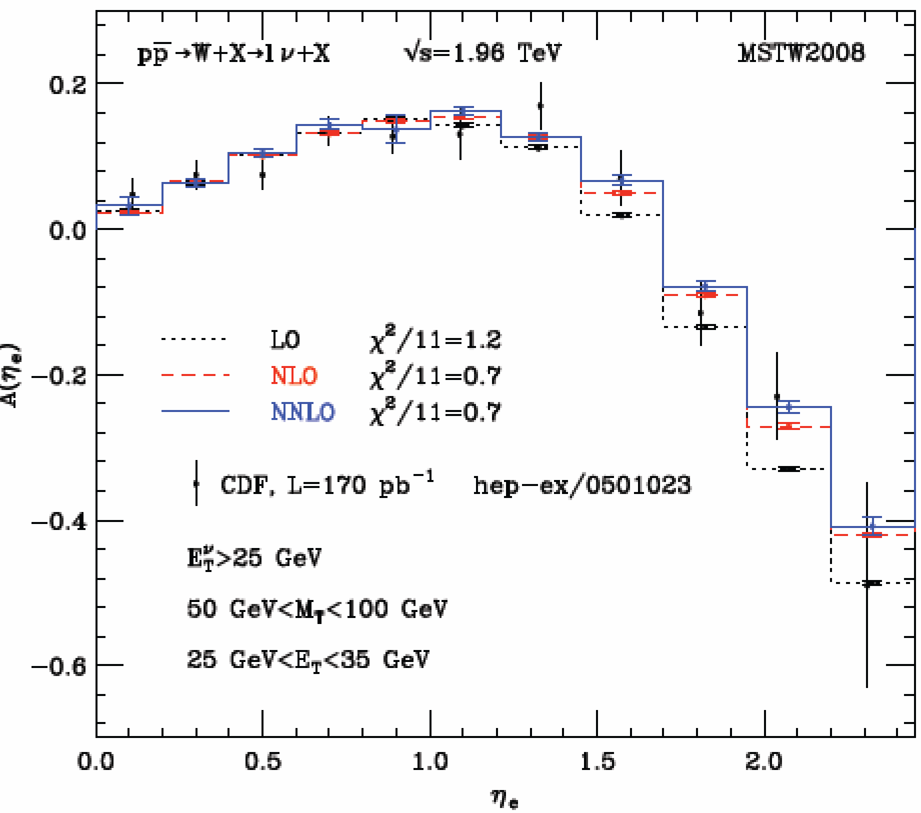
- continuing convergence between the various pdf sets

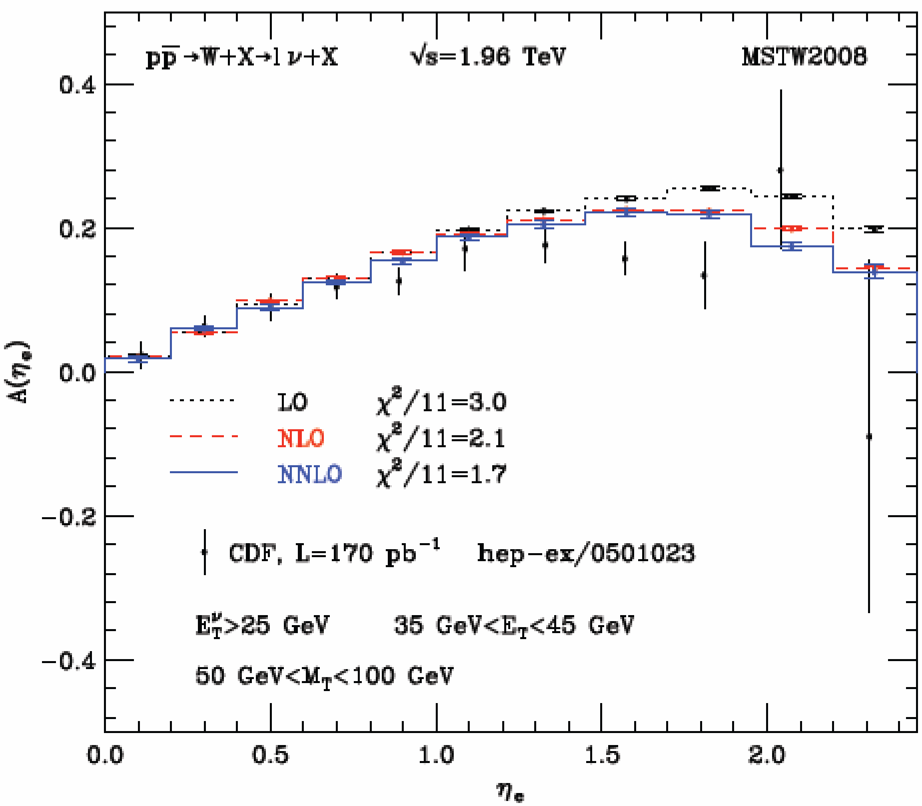
- outstanding issues include:
  - inclusion of combined HERA data (not yet in all fits)
  - difficulty of reconciling Run II Tevatron W asymmetry data
  - proper assessment of uncertainties due to treatment of heavy quark flavours (GM-VFNS optimal but not uniquely defined)
  - beyond NNLO? e.g. influence of $[\alpha_S \ln(1/x)]^n$ contributions
  - 'QED pdfs' (MSTW in preparation, *cf.* MRST 2004)

- much discussion (e.g. PDF4LHC workshops) among the pdf groups about how to define a 'overall best' theory prediction and uncertainty (be careful with `averaging' and `envelopes'!)

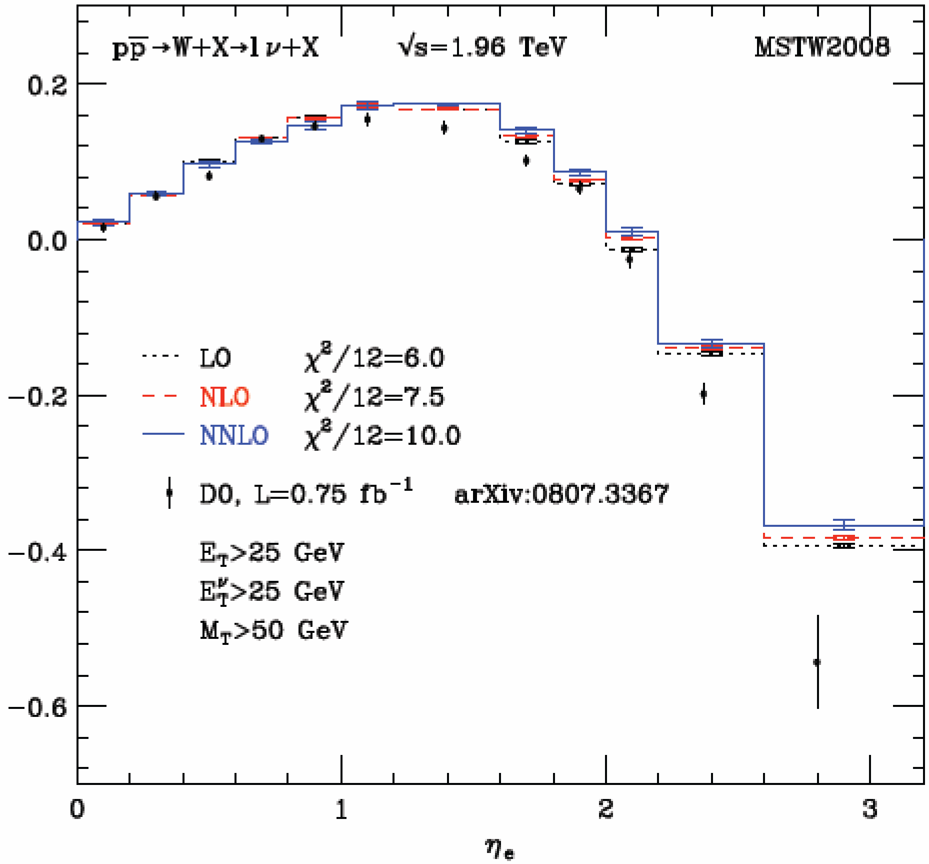
- eagerly awaiting *precision* cross sections at 7 TeV!

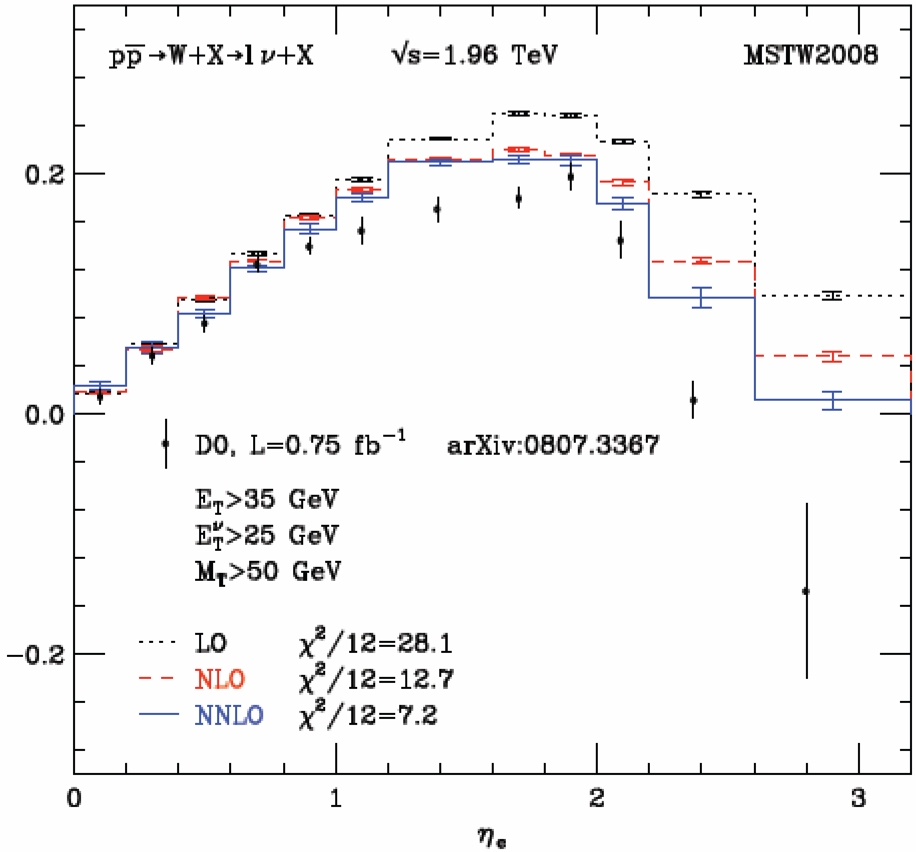
Lepton asymmetry and CDF data





Lepton asymmetry and new DØ data



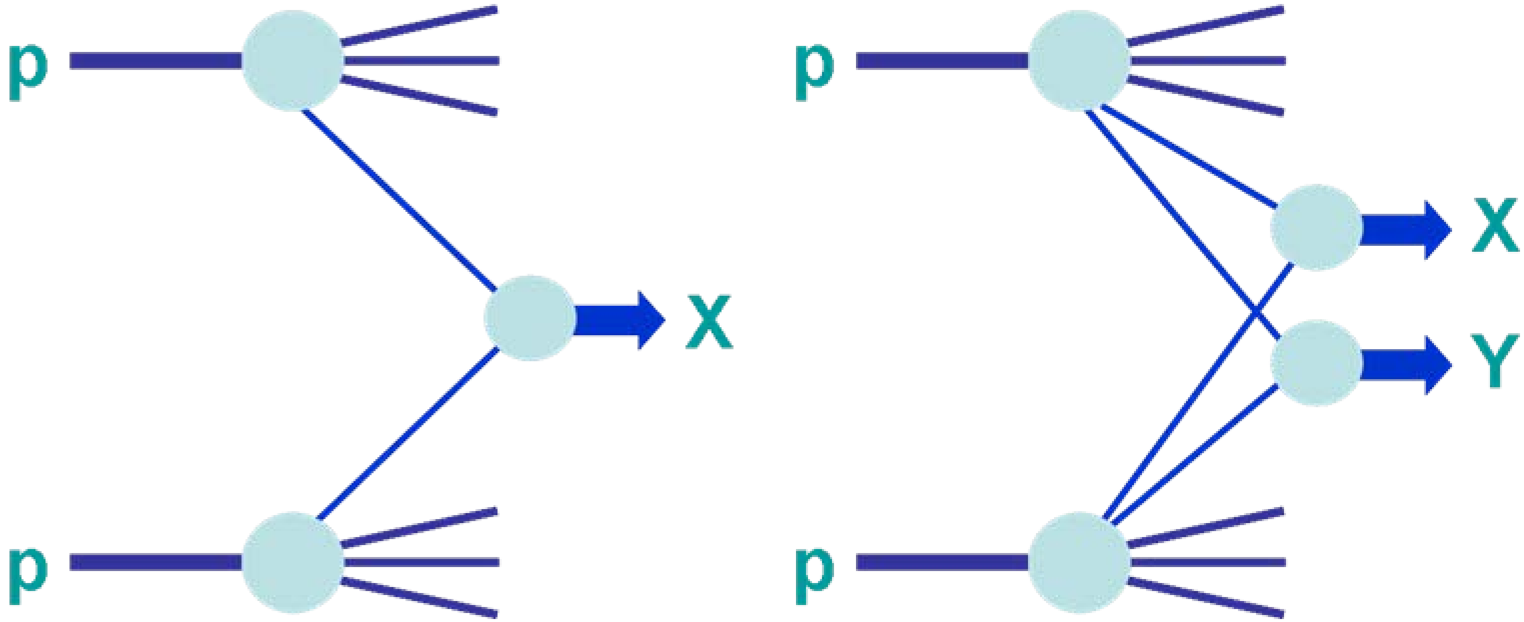


4

double parton distributions

For a `state of the art' overview of Multiple Parton Interactions, see the talks at the ongoing workshop:
indico.desy.de/conferenceDisplay.py?confId=3241

single and double hard parton scattering



$f_{a/A}(x; Q^2)$

$f_{ab/A}(x_1, x_2; Q_X^2, Q_Y^2)$

e.g. X = Y = W, $Q_X^2 \sim Q_Y^2 \sim M_W^2$

(Theoretical underpinning: see e.g. the talks by Treleani and Diehl at the Workshop)

double parton scattering: rates and topologies

- if we assume that the dPDFs factorise, i.e.

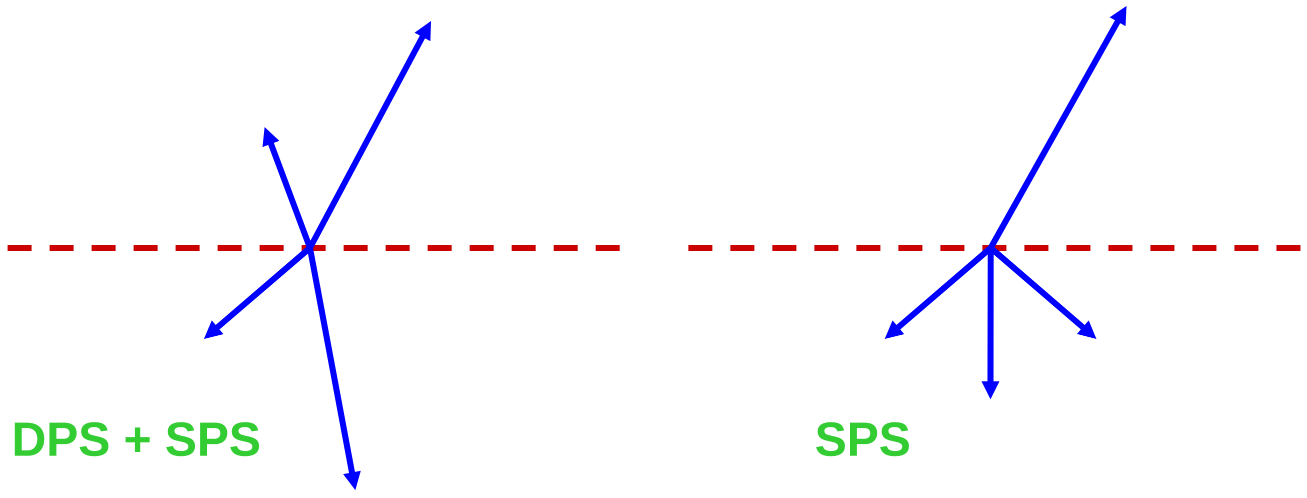
$$f_{ab/A}(x_1, x_2; Q_1^2, Q_2^2) = f_a(x_1, Q_1^2) f_b(x_2, Q_2^2)$$

- then we obtain

$$\sigma^{\mathrm{DPS}}(X,Y) = \frac{m}{2} \frac{\sigma_X \sigma_Y}{\sigma_{\mathrm{eff}}}$$

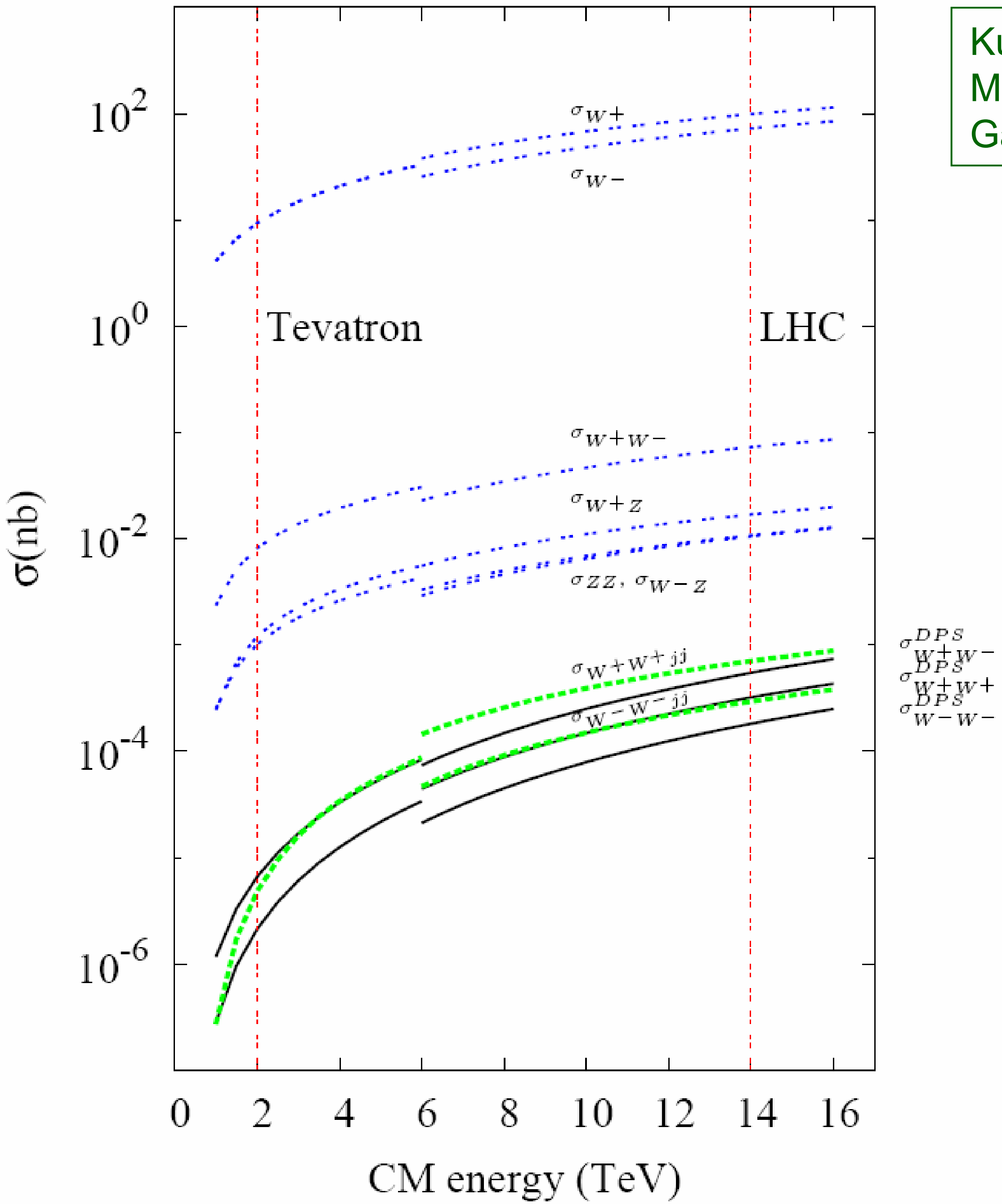
X,Y distinct: m=2
X,Y same: m=1

- studies of γ+3j production by CDF and D0 suggest $\sigma_{\mathrm{eff}} \approx 15$ mb
- but there is generally a SPS 'background', a+b → XY



- use `pairwise transverse momentum balance' (shape variable) as a signal for double parton scattering
- many final states have been studied*: X,Y = γj, 2j, W, bb, tt, H, …
- interesting example: same-sign W at LHC

*Del Fabbro, Treleani, Cattaruzza; Berger, Jackson, Shaughnessy; Maina; Hussein; Gaunt, Kom, Kulesza, S; …



Kulesza, S (1999)
Maina (2009)
Gaunt, Kom, Kulesza, S (2010)

Note:

$a + b \rightarrow W^+W^-$

but

$a + b \not\rightarrow W^+W^+$

instead

$q + q \rightarrow W^+W^+ + q'q'$

so same-sign W production could be a good place to look for DPS (with a lot of luminosity!)

dDGLAP

- the dPDFs satisfy a `double DGLAP' equation

$$\frac{dD_h^{j_1 j_2}(x_1,x_2;t)}{dt} = \frac{\alpha_s(t)}{2\pi}\Bigg[\sum_{j_1'}\int_{x_1}^{1-x_2}\frac{dx_1'}{x_1'}D_h^{j_1' j_2}(x_1',x_2;t)P_{j_1'\to j_1}\left(\frac{x_1}{x_1'}\right) + \sum_{j_2'}\int_{x_2}^{1-x_1}\frac{dx_2'}{x_2'}D_h^{j_1 j_2'}(x_1,x_2';t)P_{j_2'\to j_2}\left(\frac{x_2}{x_2'}\right) + \sum_{j'}D_h^{j'}(x_1+x_2;t)\frac{1}{x_1+x_2}P_{j'\to j_1 j_2}\left(\frac{x_1}{x_1+x_2}\right)\Bigg]$$

Kirschner 1979
Shelest, Snigirev, Zinovjev 1982
Snigirev 2003
Korotkikh, Snigirev 2004
Cattaruzza et al. 2005

- and note that $f_{ab}(x_1,x_2;Q_1^2,Q_2^2) = f_a(x_1,Q_1^2)f_b(x_2,Q_2^2)$
is **not** a solution, i.e. factorisation is broken (in fact this must be true since must have $x_1+x_2 < 1$ for momentum conservation)

Snigirev 2003
Korotkikh, Snigirev 2004
Cattaruzza et al. 2005

- the dPDFs and sPDFS are related by sum rules, e.g.

$$\sum_a \int_0^{1-x_2} dx_1 x_1 f_{ab}(x_1,x_2;Q^2) = (1-x_2)f_b(x_2,Q^2)$$

- a consistent LO package (GS09) is available

Gaunt, S 2009

MPI/dPDFs – issues and outlook

- soft multiple parton scattering: an essential part of MCs – needed to explain MB and UE
- hard double parton scattering has been observed, e.g. in γ+3j production need topological cuts to enhance DPS contribution: $\sigma_{eff} \sim 15$ mb is suggested
- lots of potential signals at LHC (4 jets, WW, …)
- double pdfs are of theoretical interest (dDGLAP, sum rules, correlations, factorisation breaking etc.) but testing these will be experimentally very challenging

extra slides

determination of best fit and uncertainties

- MSTW08 — 20 eigenvectors. Due to slight incompatibility of different sets (and perhaps to some extent parametrisation inflexibility) use 'dynamical tolerance' with inflated $\Delta\chi^2$ of 5 - 20 for eigenvectors

- CTEQ6.6 — 22 eigenvectors. Inflated $\Delta\chi^2=50$ for 1 sigma for eigenvectors (no normalization uncertainties in CTEQ6.6, *cf.* CT10)

- HERAPDF2.0 — 9 eigenvectors, use $\Delta\chi^2=20$. Additional model and parametrisation uncertainties

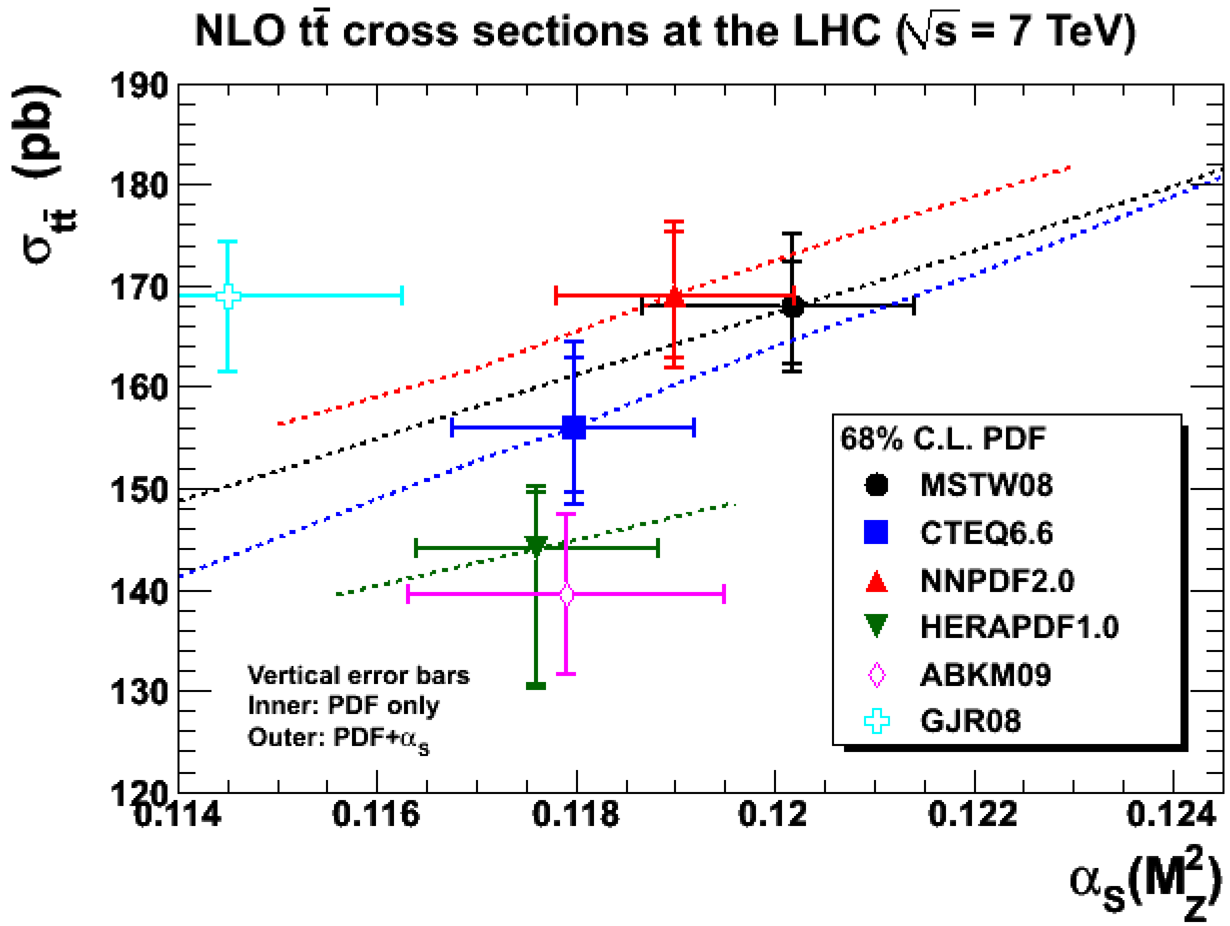
- ABKM09 — 21 parton parameters, use $\Delta\chi^2=1$

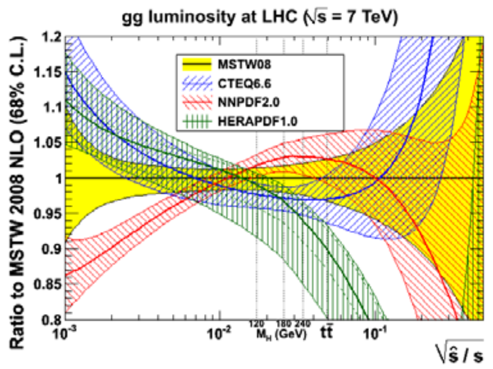
- GJR08 — 12 parton parameters. Use $\Delta\chi^2=20$. Impose strong theory ('dynamical parton') constraint on input form of pdfs.

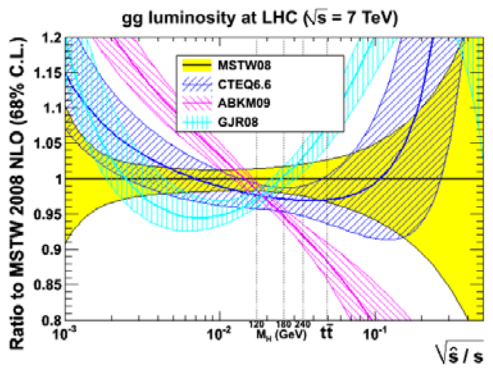
Note: NNPDF2.0 create many replicas of data and obtain PDF replicas in each case by fitting to training set and comparing to validation set → uncertainty determined by spread of replicas. Direct relationship to χ^2 in global fit not trivial.

top

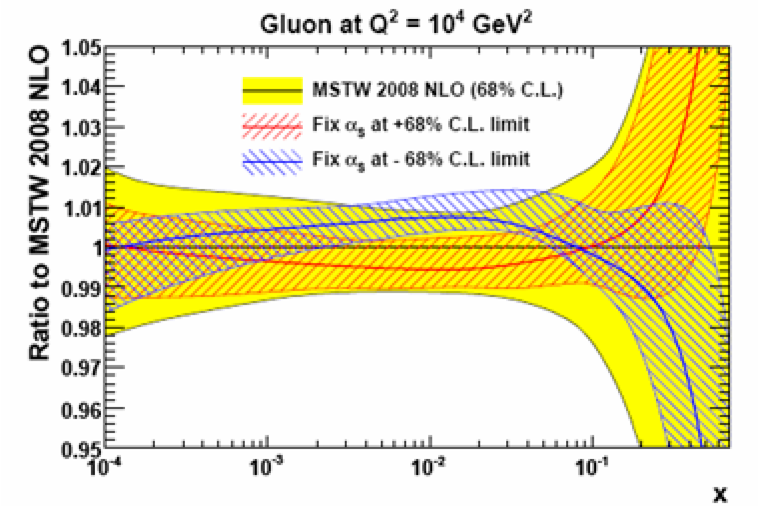
benchmark top cross sections

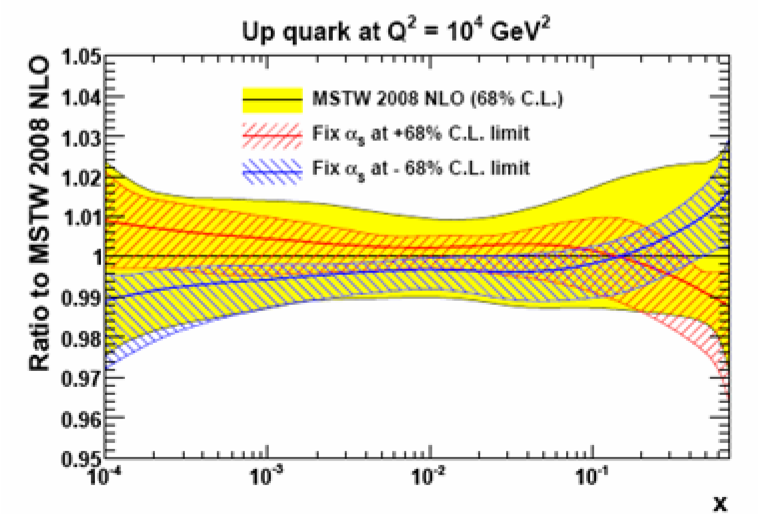






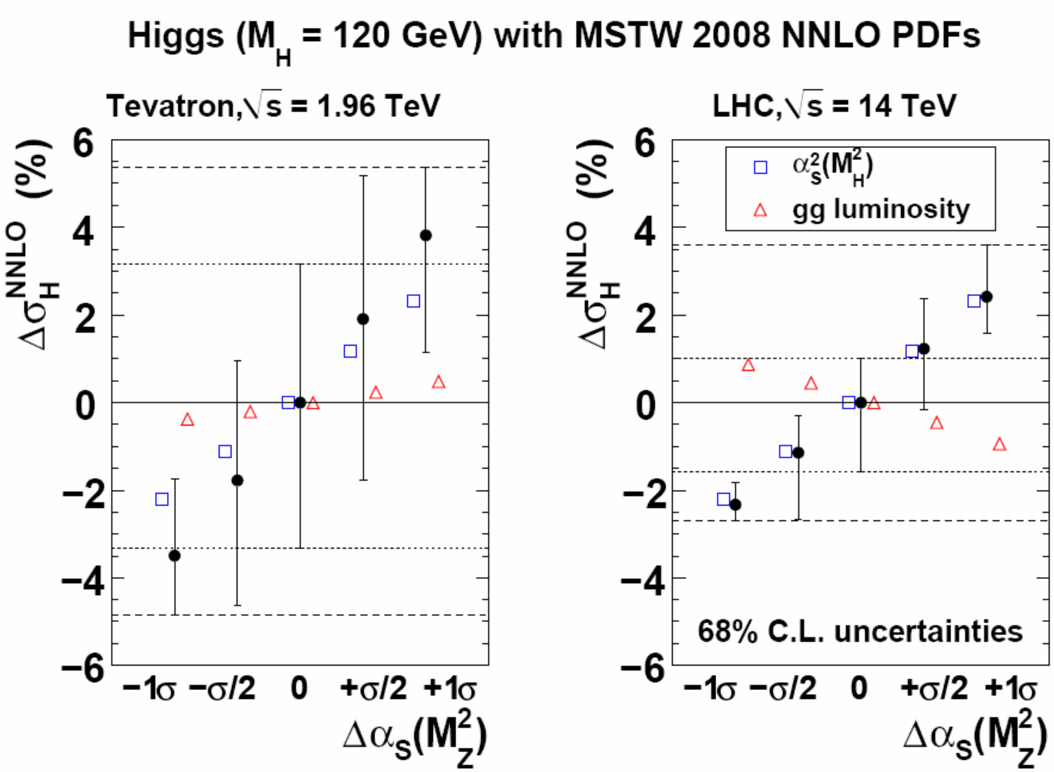
α_S - pdf correlations



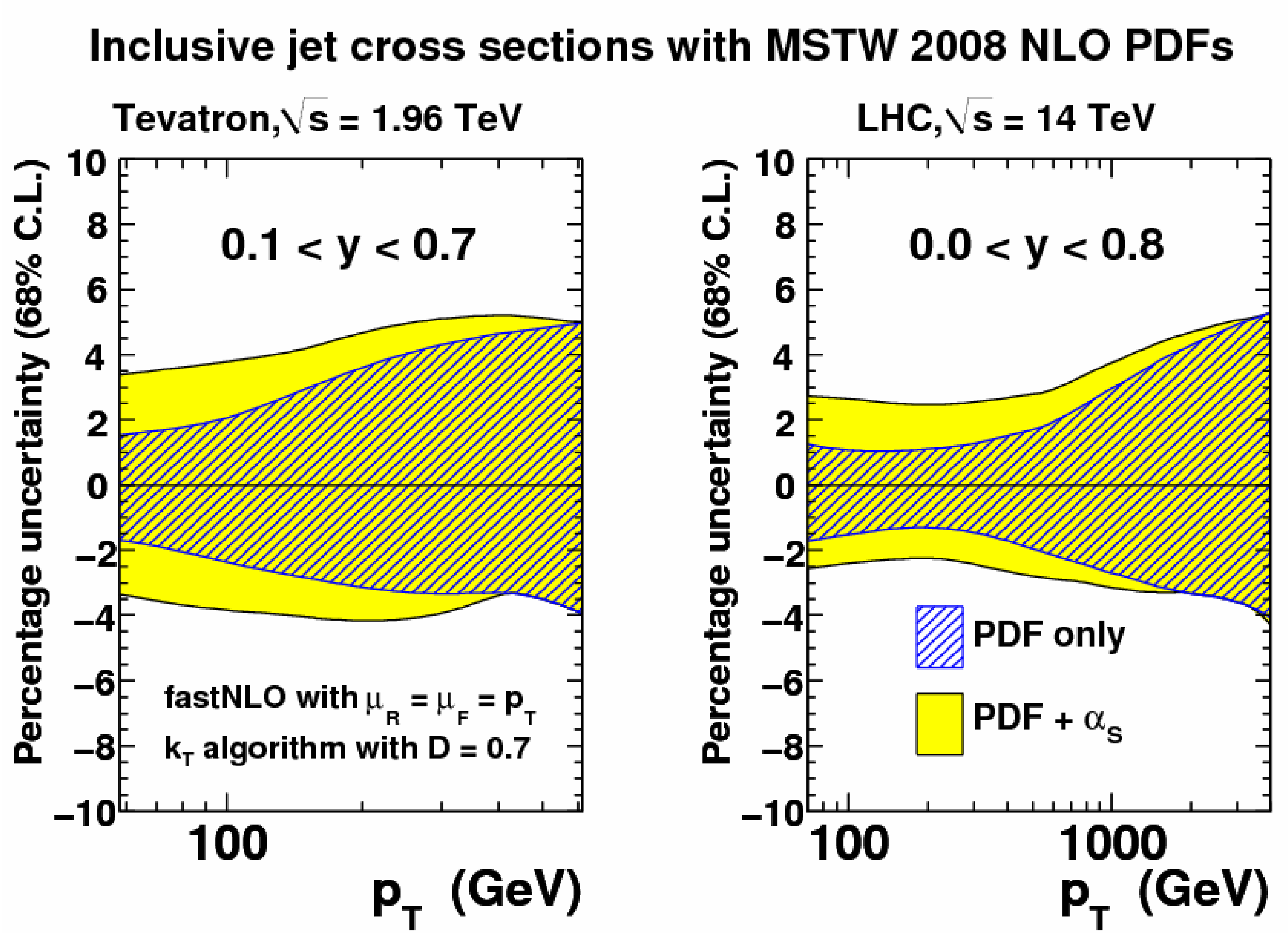


MSTW: arXiv:0905.3531

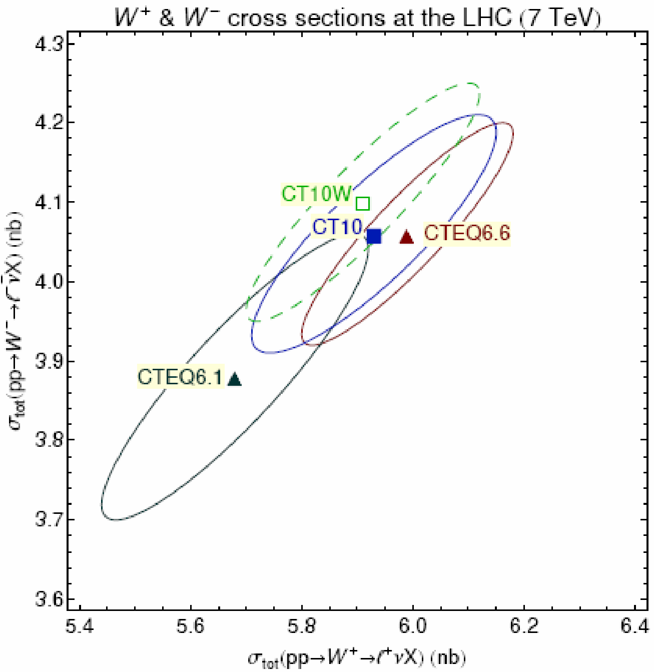
- care needed when assessing impact of varying α_S on cross sections ~ $(\alpha_S)^n$

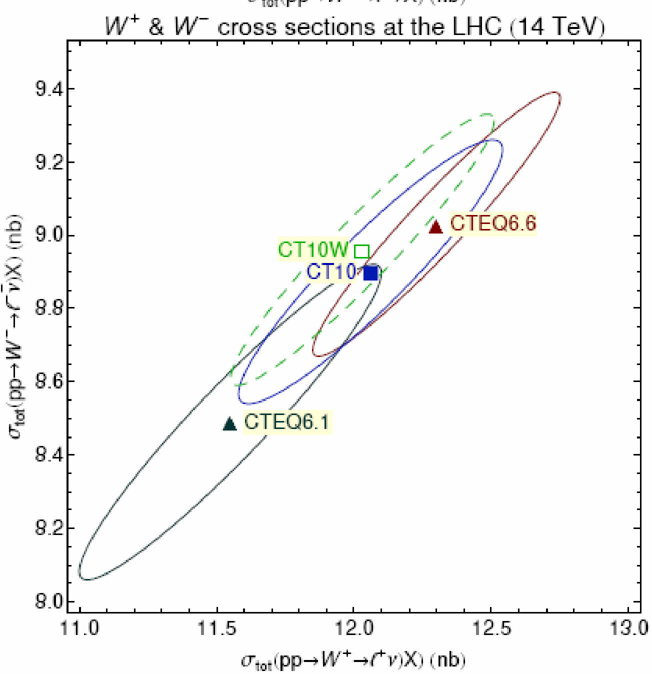


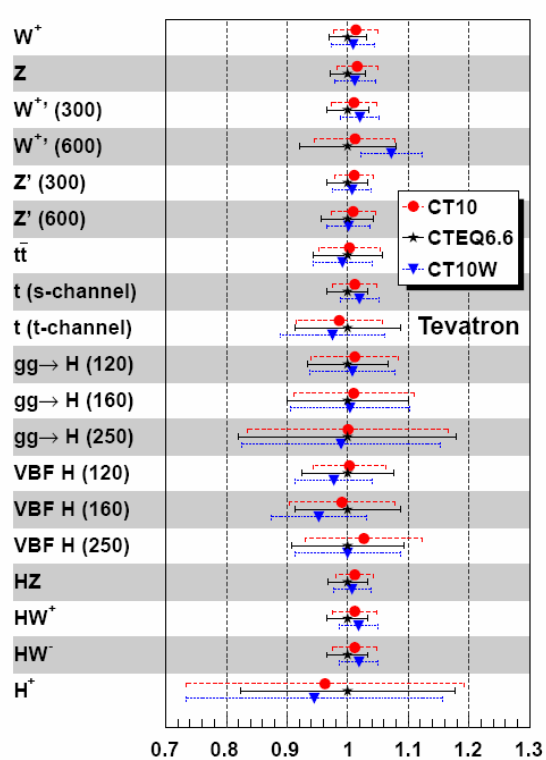
pdf + α_S uncertainties in jet cross sections

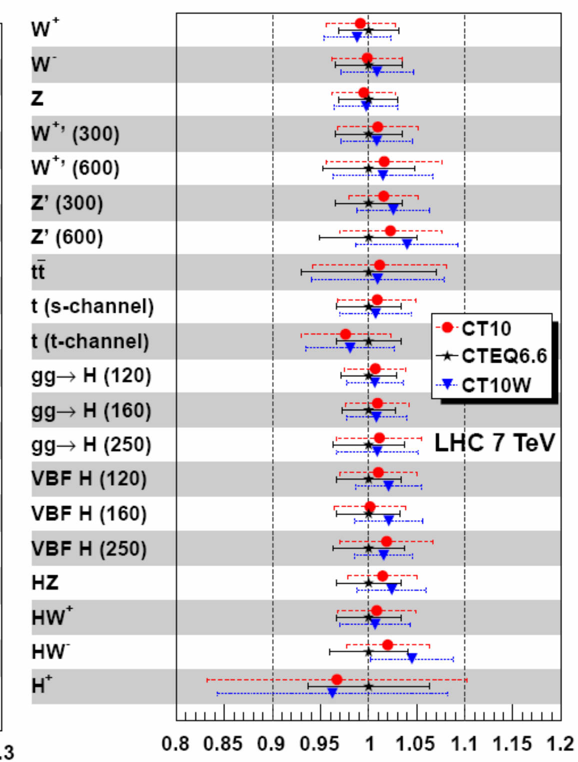


CTEQ6.6 vs. CT10, CT10W (NLO)



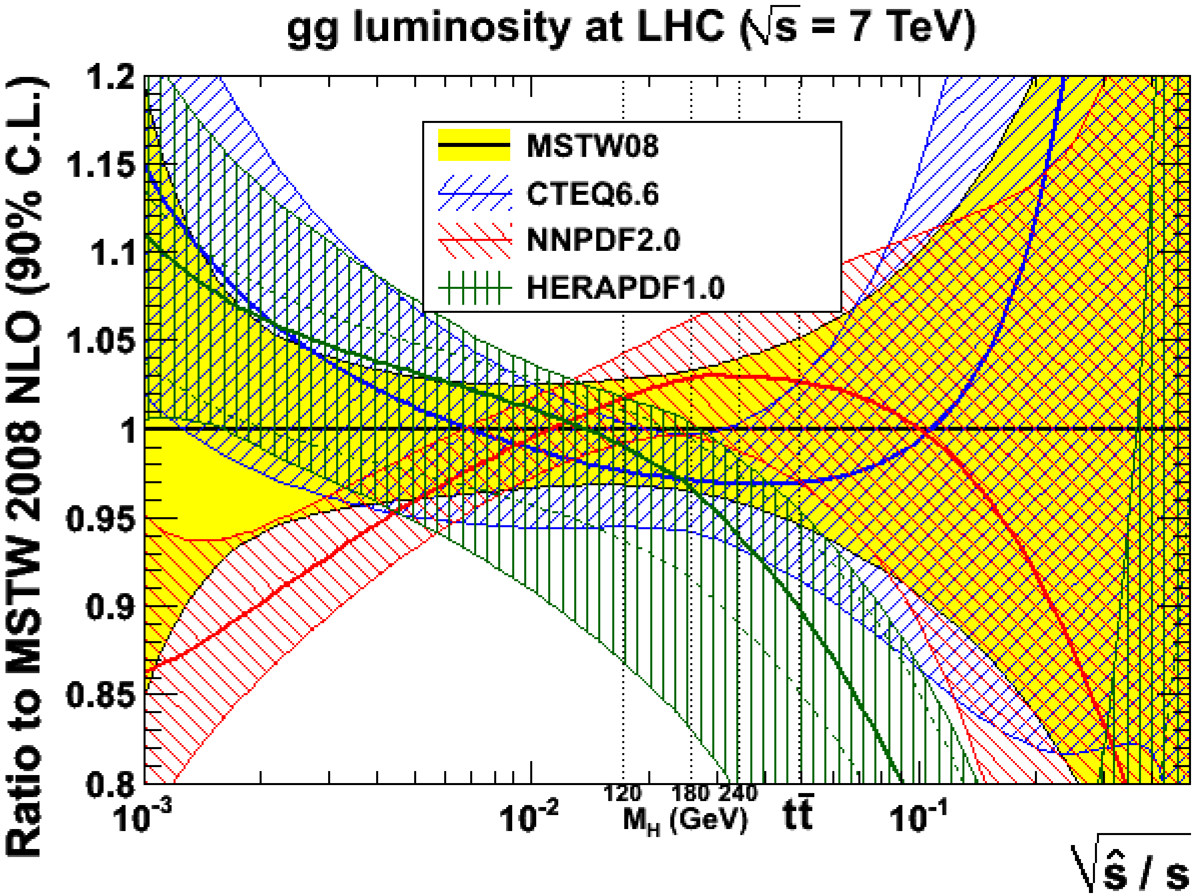


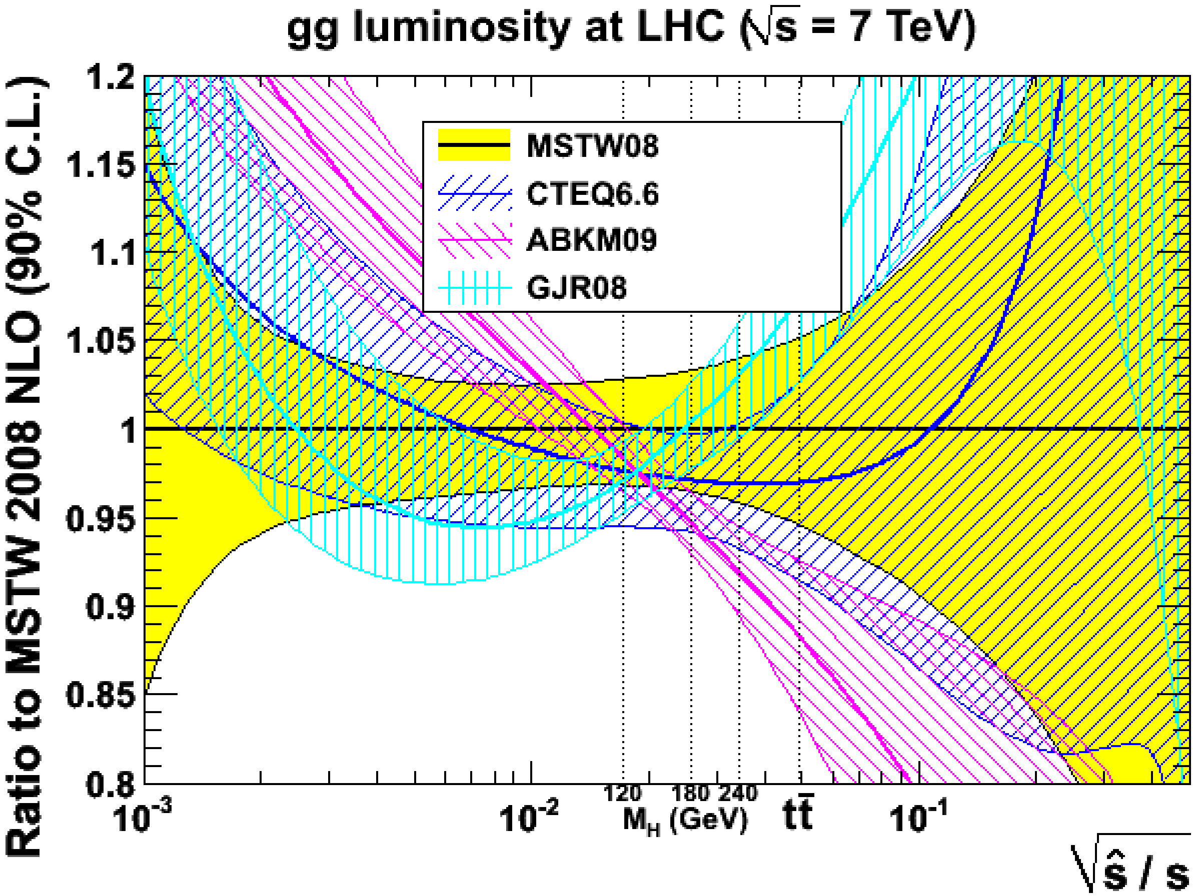




* CT10W: attempt to include recent D0 lepton asymmetry data in global fit
→ slightly different d/u

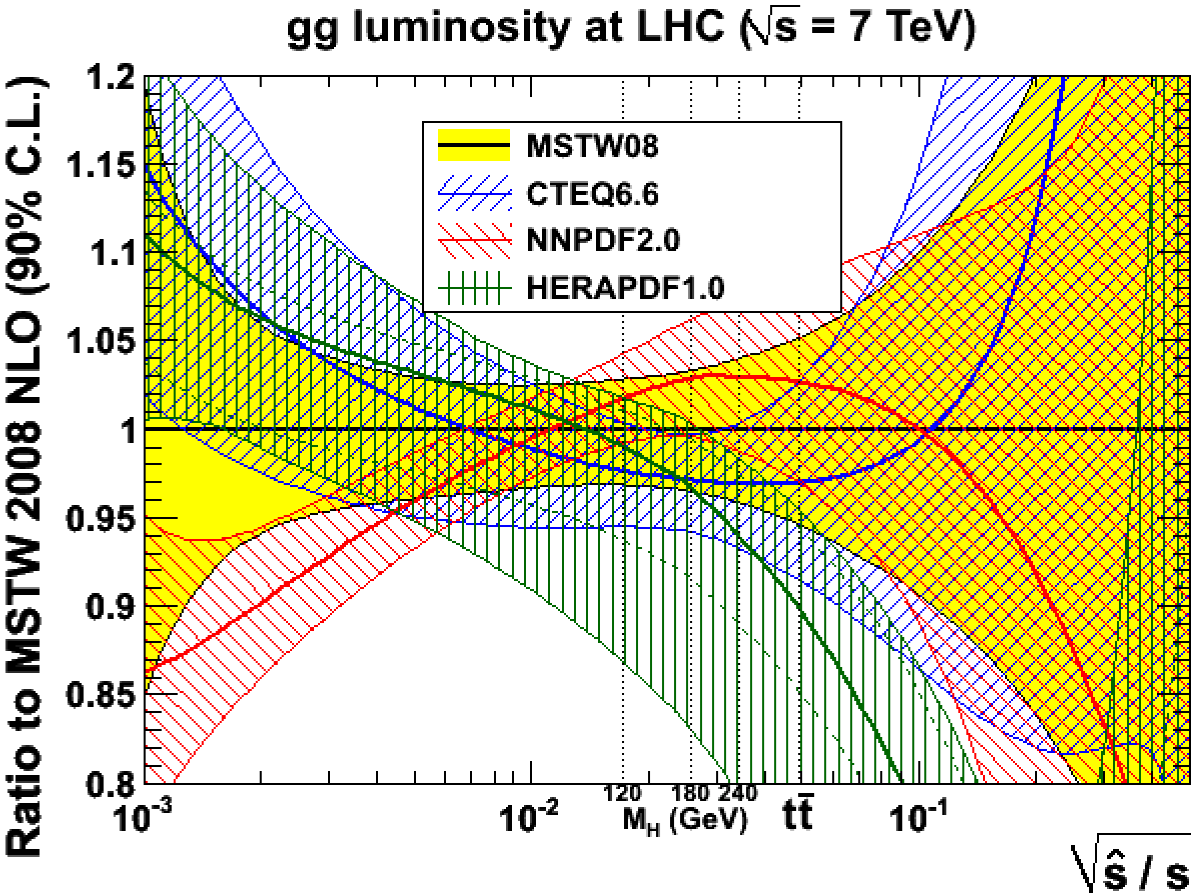
… the same at 90%cl

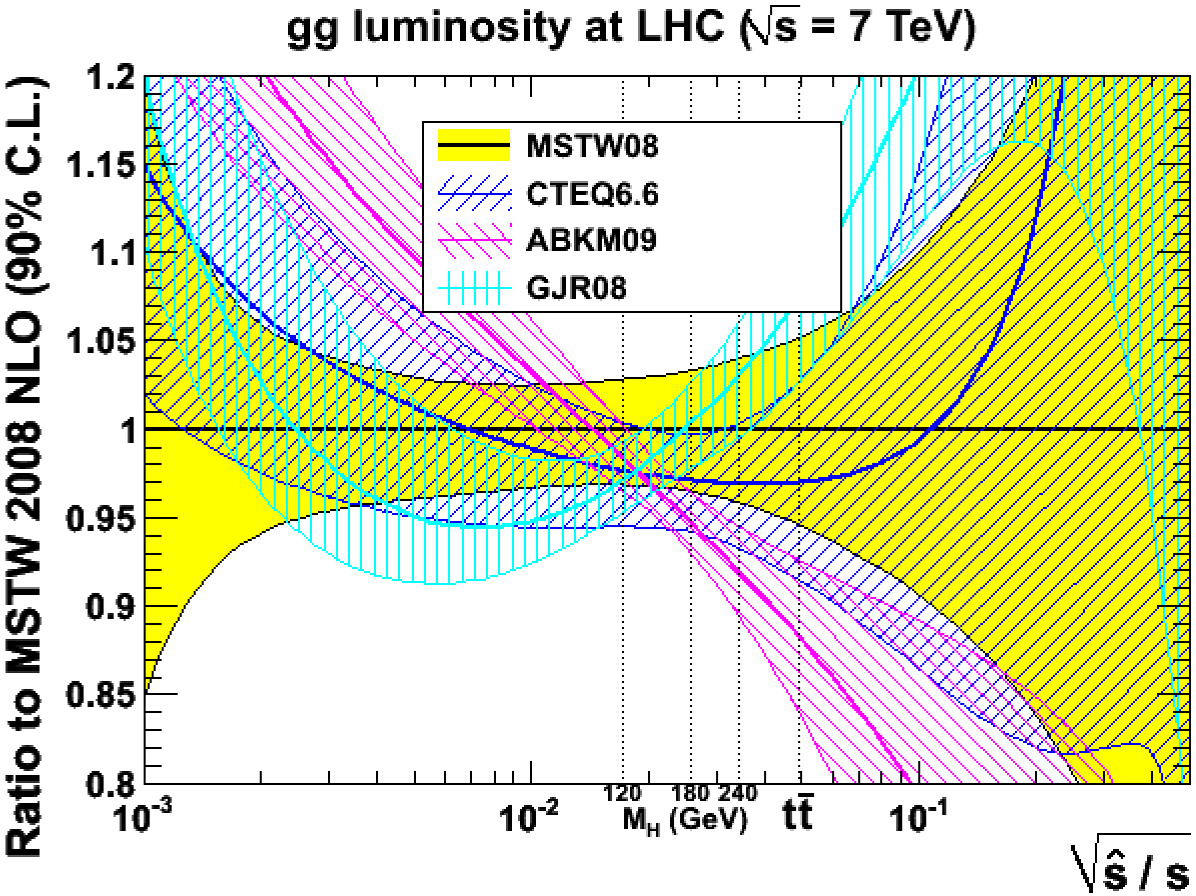




gg luminosity at LHC (√s = 7 TeV)
MSTW08
CTEQ6.6
ABKM09
GJR08
Ratio to MSTW 2008 NLO (90% C.L.)
1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
0.8
10^-3
10^-2
10^-1
120
180
240
M_H (GeV)
tt̄
√(ŝ / s)

… the same at 90%cl




gg luminosity at LHC ($\sqrt{s}$ = 7 TeV)
MSTW08
CTEQ6.6
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Ratio to MSTW 2008 NLO (90% C.L.)
1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
0.8
10^{-3}
10^{-2}
10^{-1}
120
180
240
M_H (GeV)
$t\bar{t}$
$\sqrt{\hat{s}/s}$

heavy quarks: charm, bottom, …

considered sufficiently massive to allow pQCD treatment: $g \to Q\overline{Q}$

distinguish two regimes:
(i) $Q^2 \sim m_H^2$ include full m_H dependence to get correct threshold behaviour
(ii) $Q^2 \gg m_H^2$ treat as ~massless partons to resum $\alpha_S^n \log^n(Q^2/m_H^2)$ via DGLAP

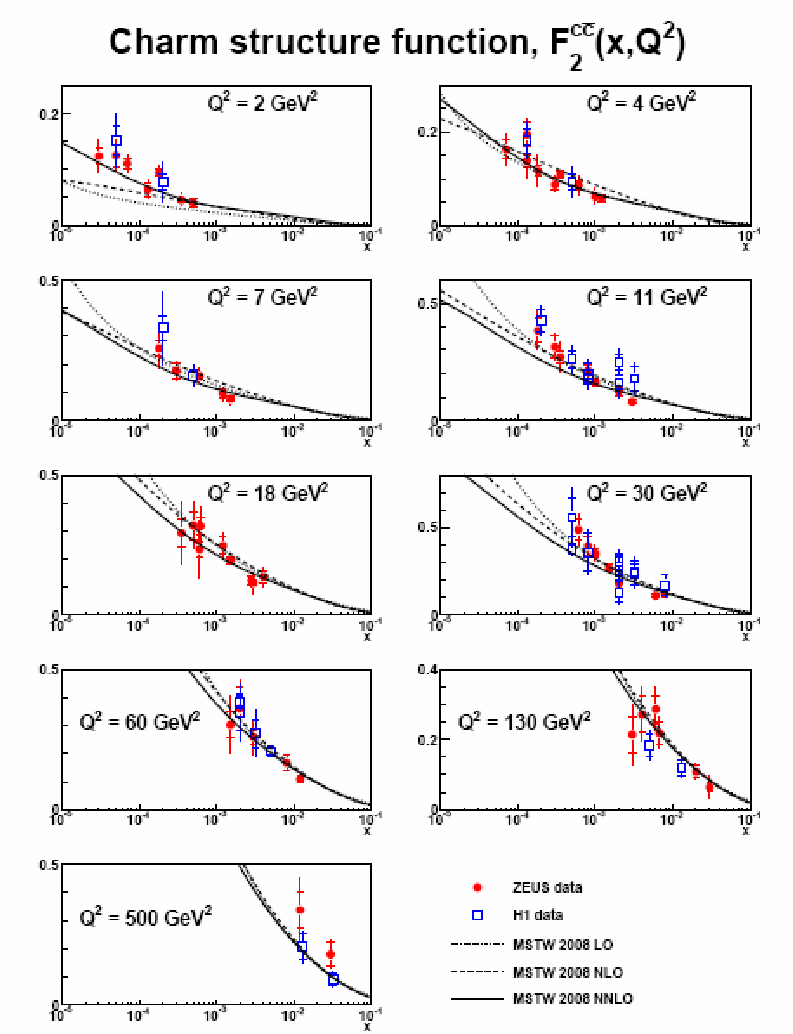
FFNS: OK for (i) only **ZM-VFNS:** OK for (ii) only

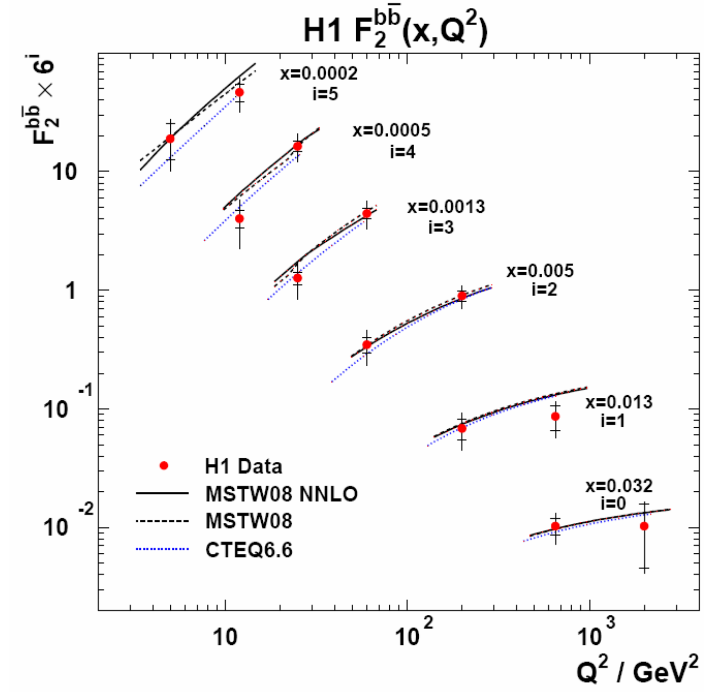
consistent **GM**(=general mass)-**VFNS** now available (e.g. ACOT(χ), RT, BMSN,…) which interpolates smoothly between the two regimes

Aivazis, Collins, Olness,Tung; Roberts, Thorne; Buza, Matiounine, Smith, Migneron, van Neerven, …

Note:
(i) the definition of these is tricky and non-unique (ambiguity in assignment of $O(m_H^2/Q^2)$ contributions), and the implementation of improved treatment (e.g. in going from MRST2004→MRST 2006 or CTEQ 6.1→6.5) can have a big effect on light partons
(ii) the *true* uncertainty on e.g. LHC predictions coming from ambiguities in the heavy quark treatment has yet to be quantified

charm and bottom structure functions





- MSTW 2008 uses *fixed* values of m_c = 1.4 GeV and m_b = 4.75 GeV in a GM-VFNS
- currently studying the sensitivity of the fit to these values, and impact on LHC cross sections

extrapolation uncertainties

theoretical insight for $x \to 0$:

$f \sim A\,x$

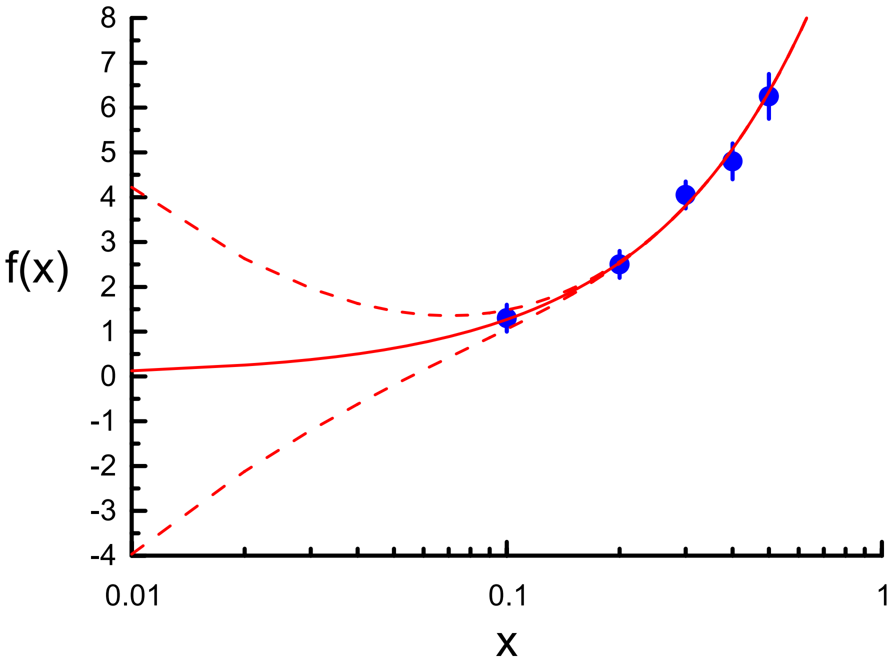
$f \sim A\,x^{\delta}$, $A > 0$

$f \sim A\,x^{\delta}$

no theoretical insight:

$f \sim ???$

…with only sum rules providing a constraint

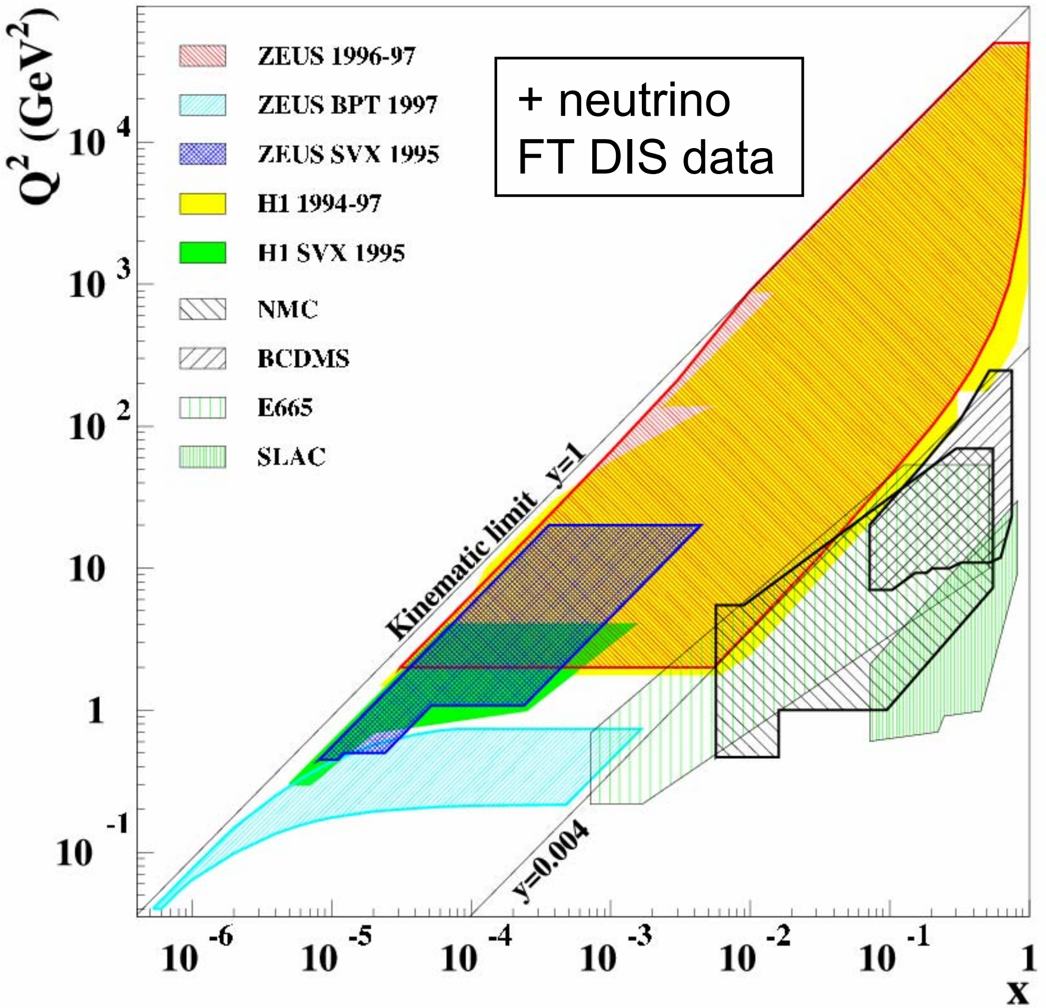


Examples:

(i) the MSTW negative small-x gluon at Q_0

(ii) the NNPDF 'parameter free' pdfs at small and large x

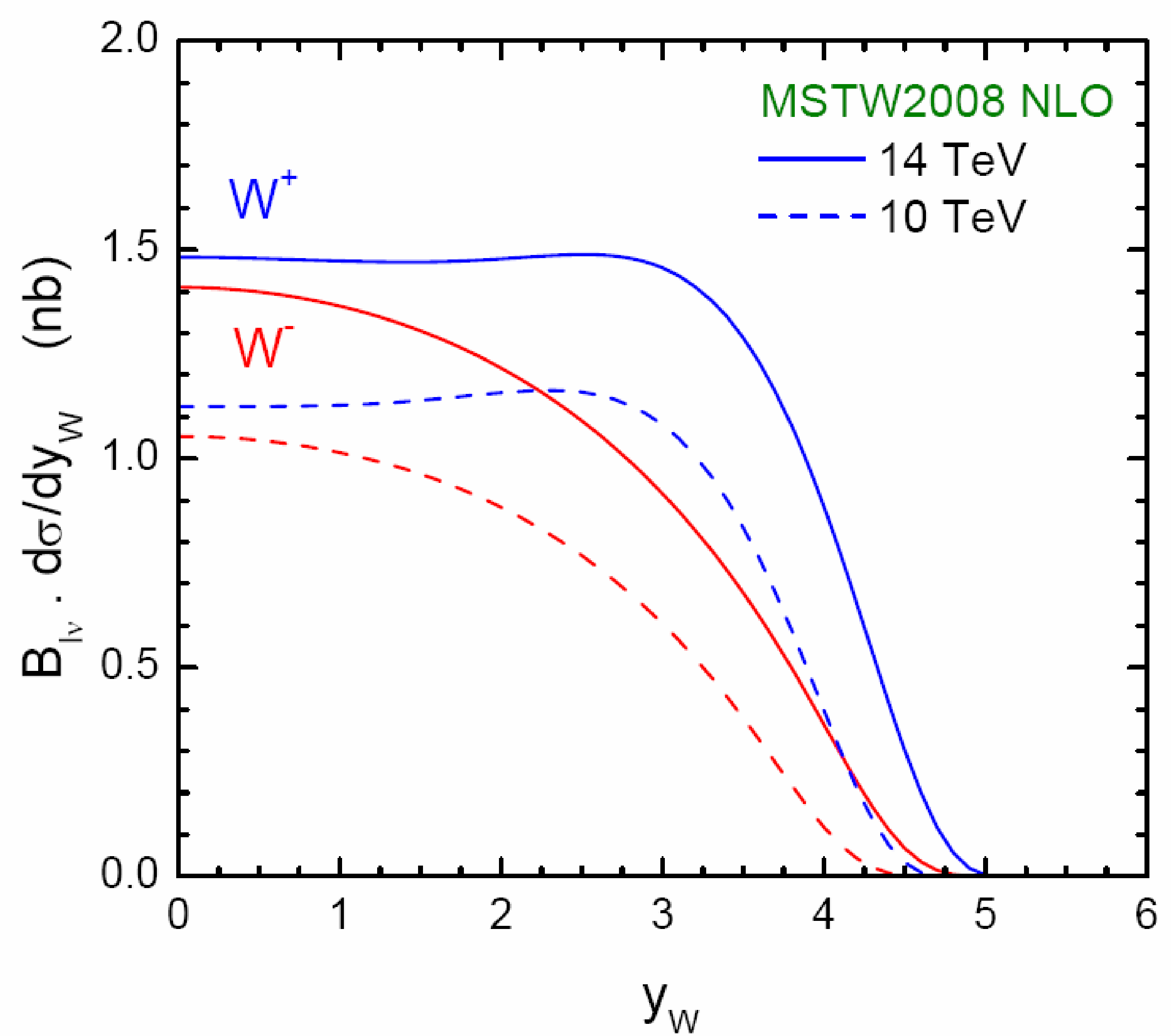
summary of DIS data



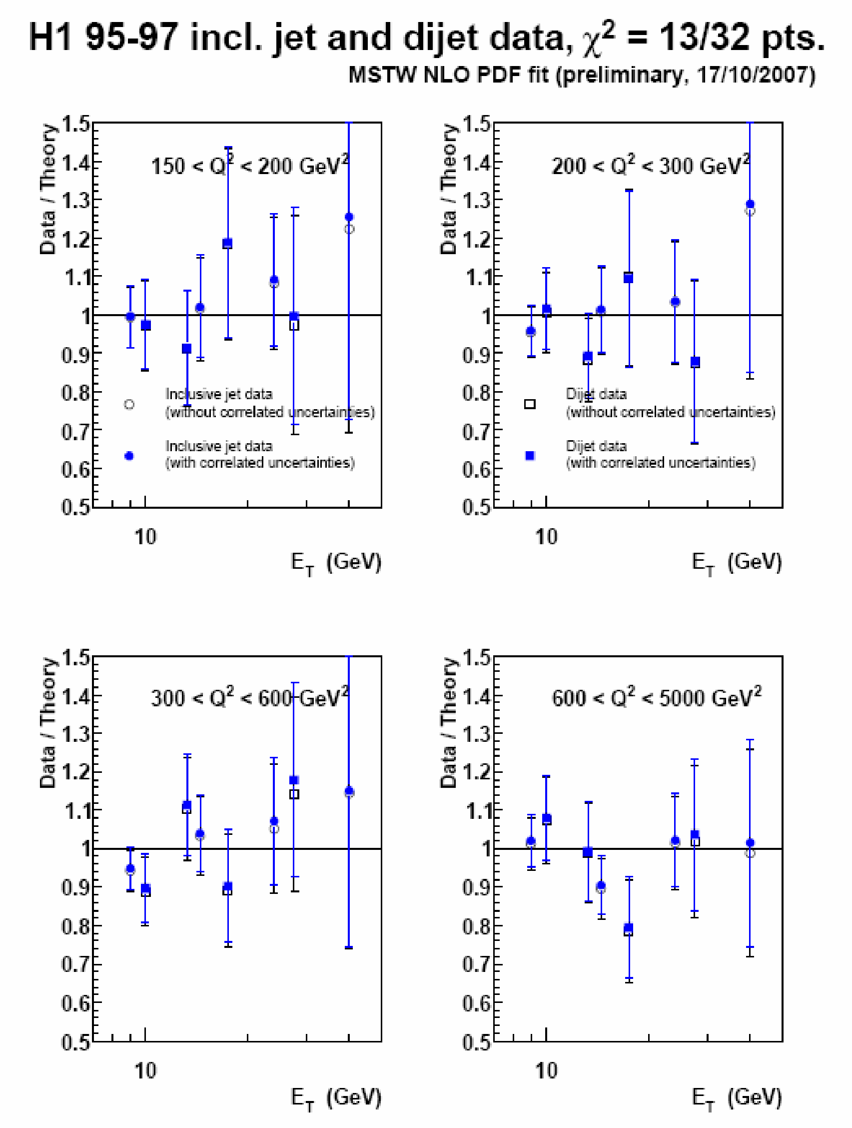
Note: must impose cuts on DIS data to ensure validity of leading-twist DGLAP formalism in analyses to determine pdfs, typically:

$Q^2 > 2 - 4\ GeV^2$

$W^2 = (1-x)/x\ Q^2 > 10 - 15\ GeV^2$



MSTW2008 NLO
14 TeV
10 TeV
W+
W-
B lν . dσ/dy W (nb)
y W
2.0
1.5
1.0
0.5
0.0
0
1
2
3
4
5
6

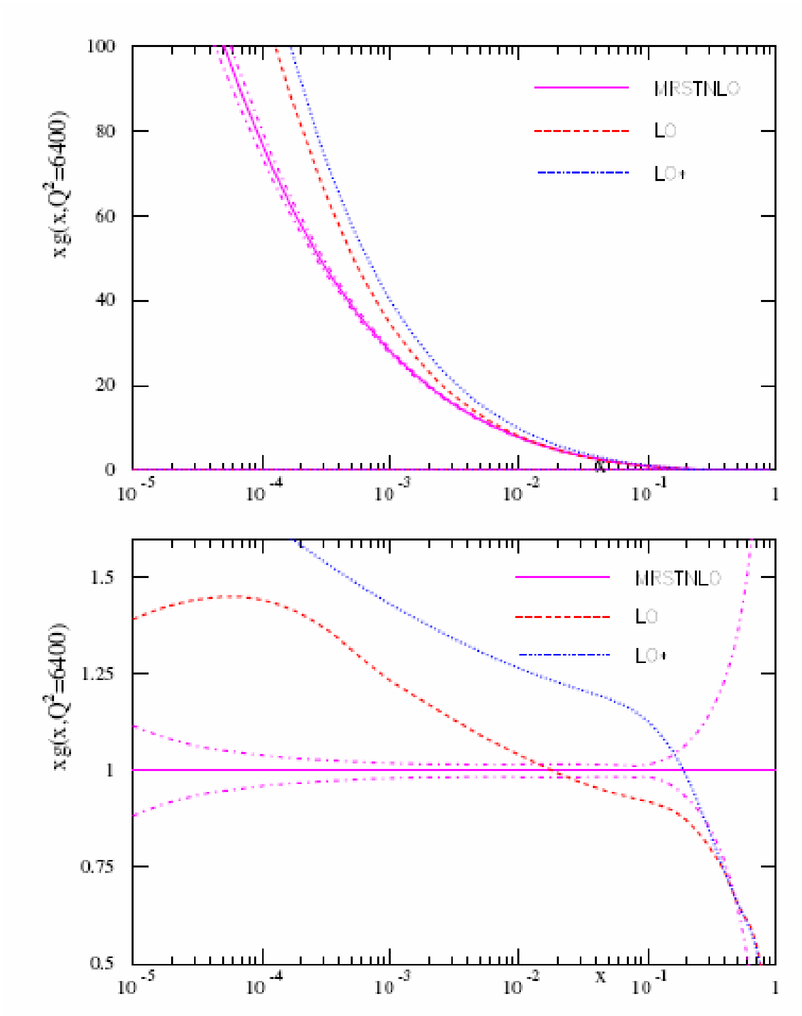


- only in NLO fit (no NNLO correction yet)

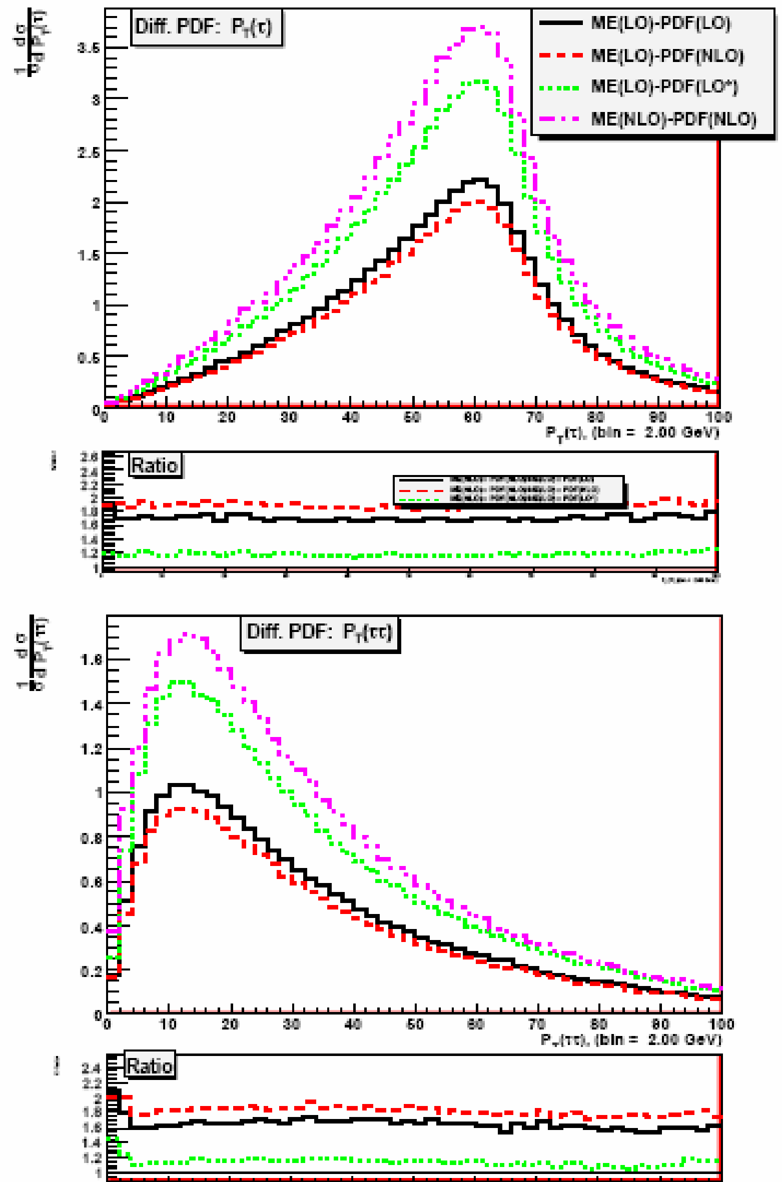
improved LO pdfs

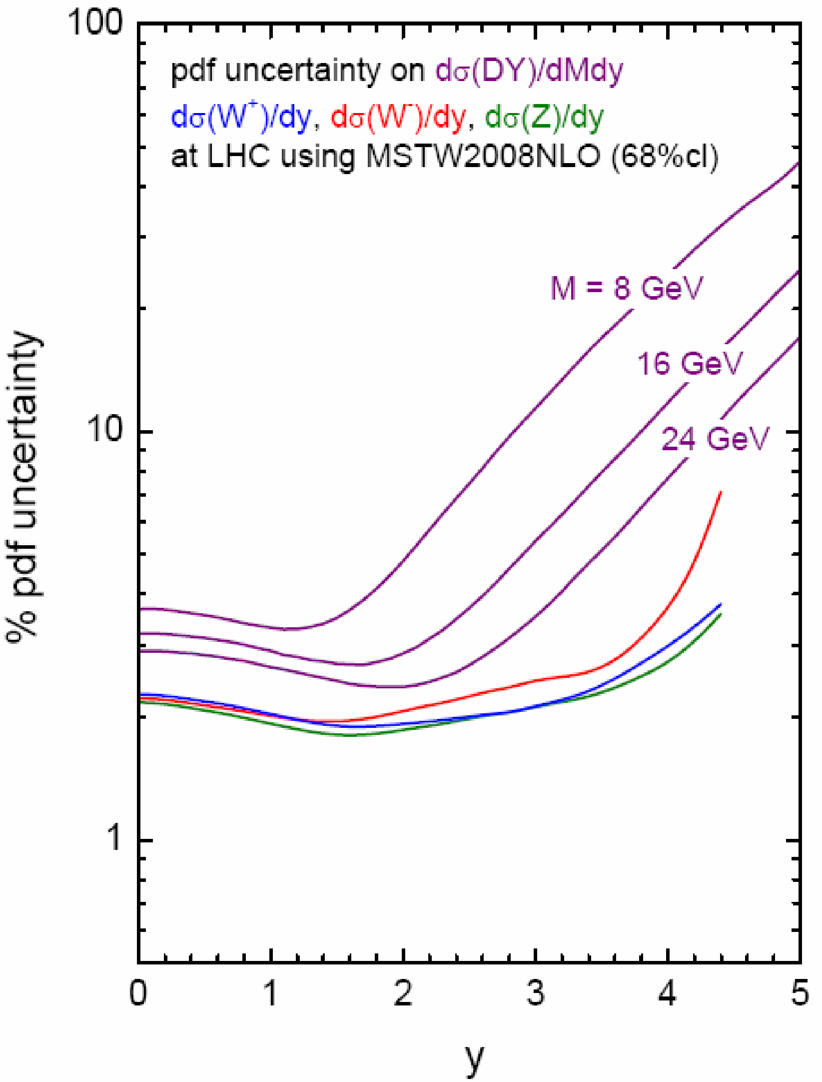
- conventional wisdom is to match pQCD order of pdfs with that of MEs
- but, in practice,
  - σ_{LO} = PDFs(LO) ⊗ ME(LO) can be different from σ_{NLO} = PDFs(NLO) ⊗ ME(NLO), in both shape and normalisation
  - LO pdfs have very poor χ^2 in (LO) global fit (no surprise: NLO corrections at large and small x are significant and preferred by the data)
- momentum conservation limits how much additional glue can be added to LO partons to compensate for missing NLO pQCD corrections (e.g. to get correct evolution rate of small-x quarks)
- therefore relax momentum conservation and redo LO fit; study the impact of this on χ^2, partons and cross sections
- e.g. Thorne & Shertsnev 2007: LO* partons
  - χ^2: 3066/2235 → 2691/2235, momentum conservation: 100% → 113%

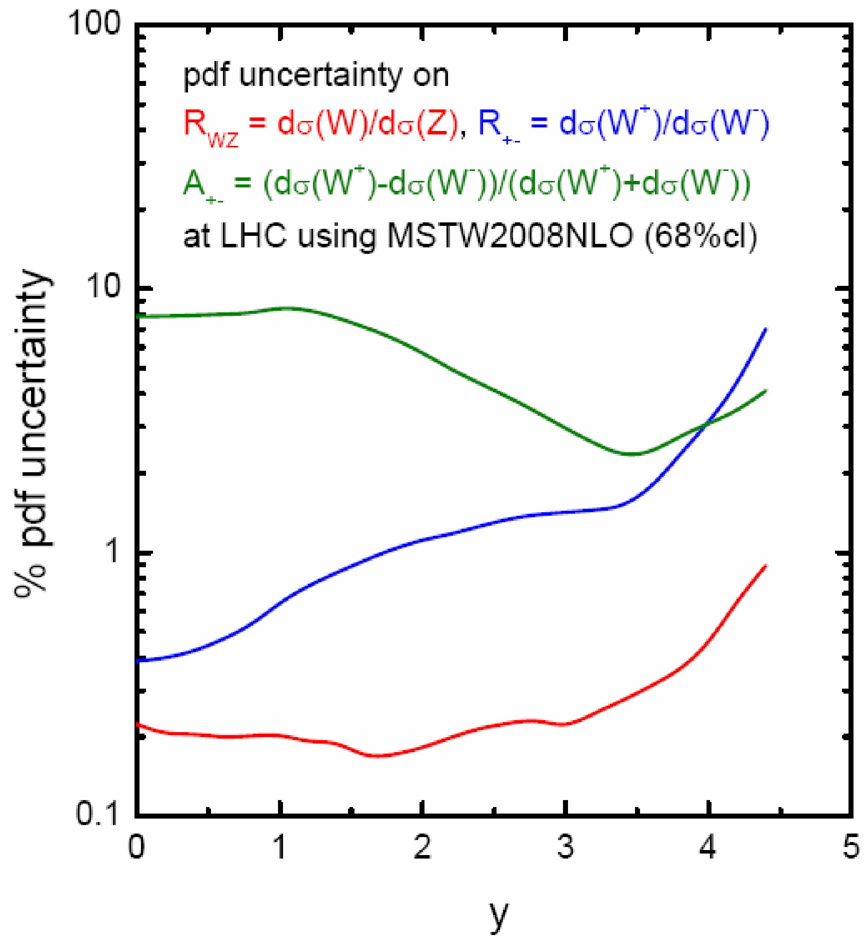
comparison of gluons at high Q^2



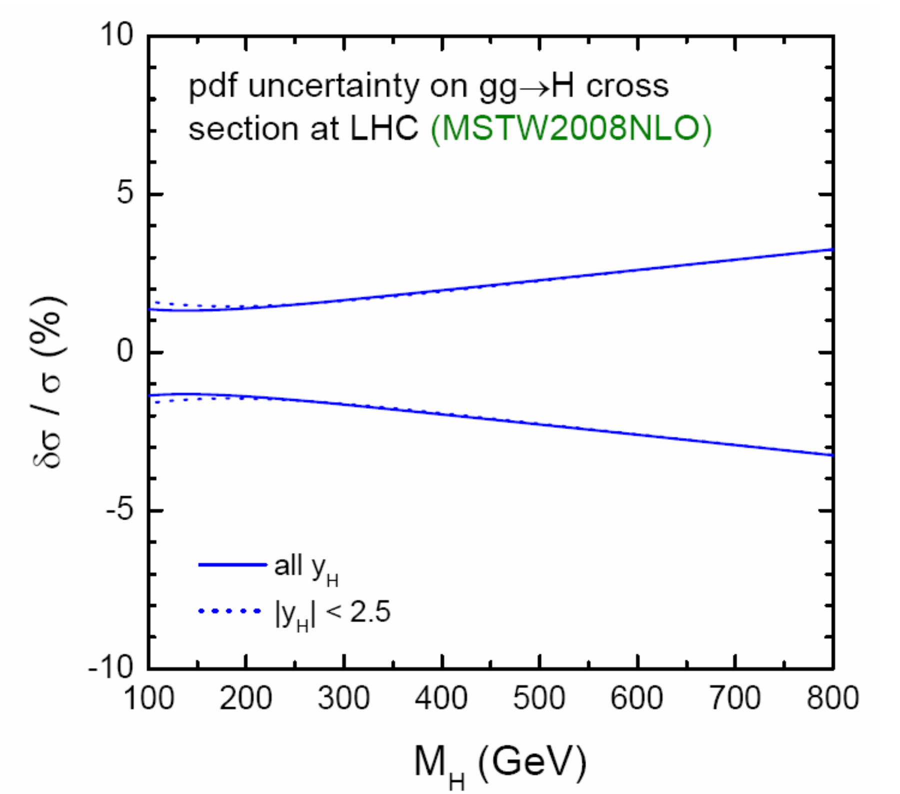
τ transverse momentum distribution in H → τ τ production at LHC

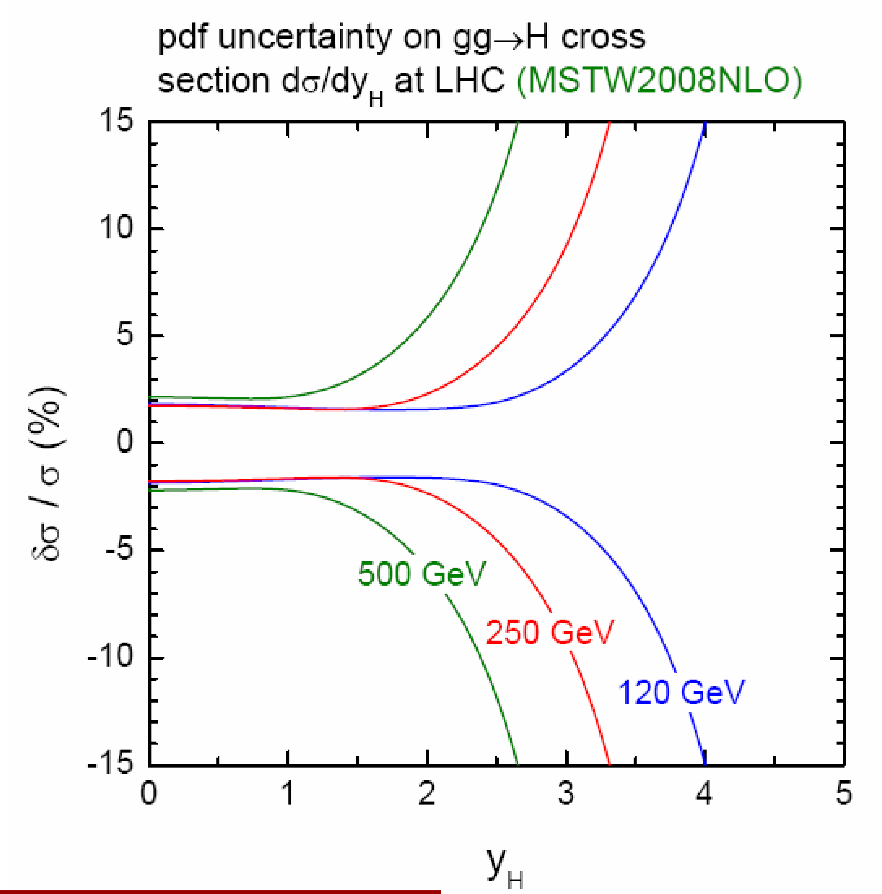



pdf uncertainty on dσ(DY)/dMdy
dσ(W+)/dy, dσ(W-)/dy, dσ(Z)/dy
at LHC using MSTW2008NLO (68%cl)
M = 8 GeV
16 GeV
24 GeV
% pdf uncertainty
y
100
10
1
0
1
2
3
4
5


pdf uncertainty on
R_WZ = dσ(W)/dσ(Z), R+- = dσ(W+)/dσ(W-)
A+- = (dσ(W+)-dσ(W-))/(dσ(W+)+dσ(W-))
at LHC using MSTW2008NLO (68%cl)
% pdf uncertainty
y
100
10
1
0.1
0
1
2
3
4
5

pdf uncertainty on σ(gg→H)

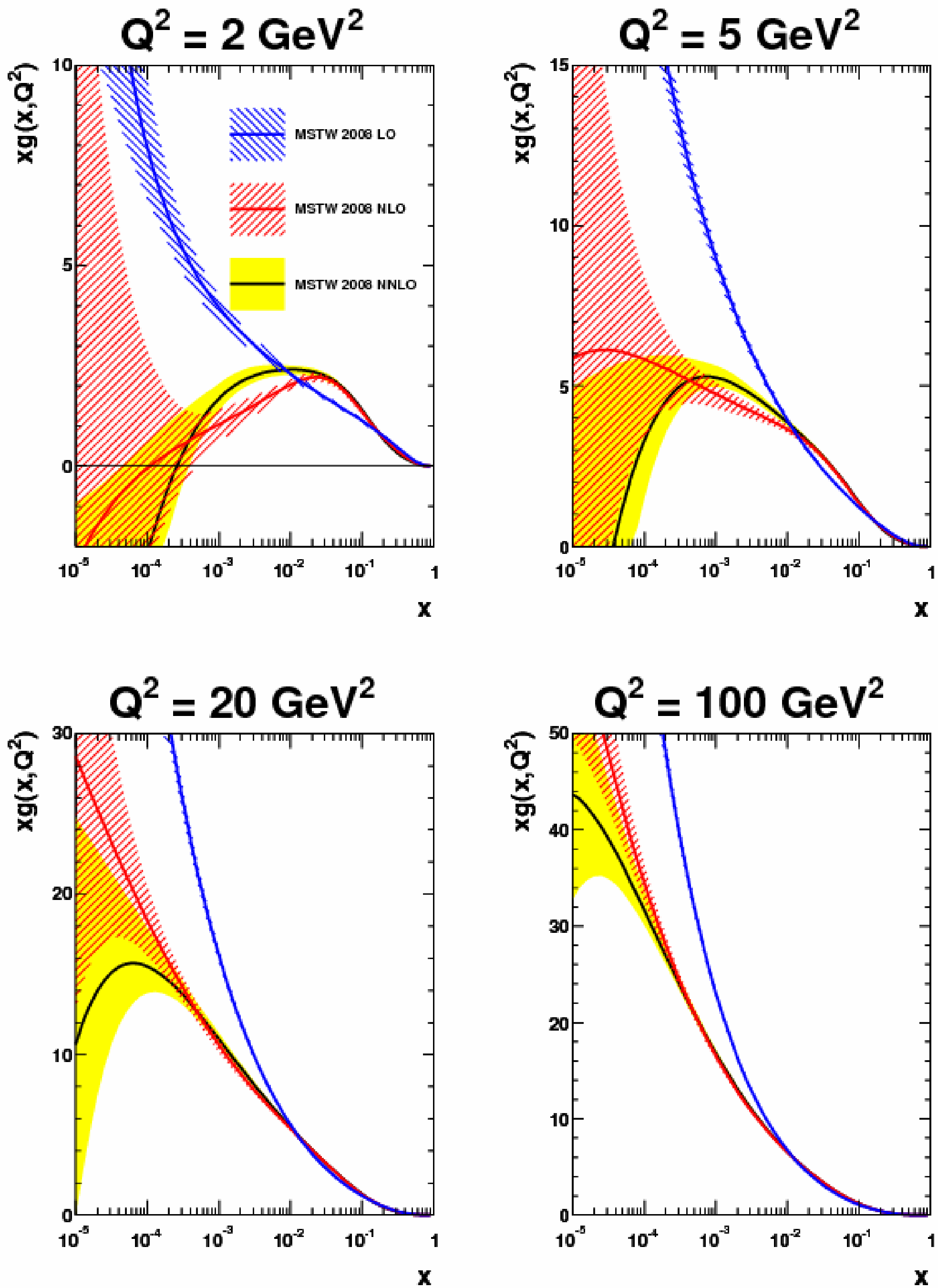




→ typically ± 2-3% pdf uncertainty, except near edges of phase space comparison of gluons extracted from LO, NLO, NNLO global fits

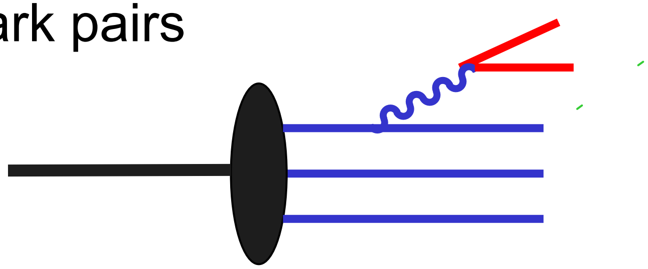
- large positive P_{qg} contributions at small x lead to smaller gluons at higher order

- clear instability at small x, Q^2 , and this is reflected in predictions for F_L (see later)



sea quarks

- the sea presumably arises when 'primordial' valence quarks emit gluons which in turn split into quark-antiquark pairs, with suppressed splitting into heavier quark pairs

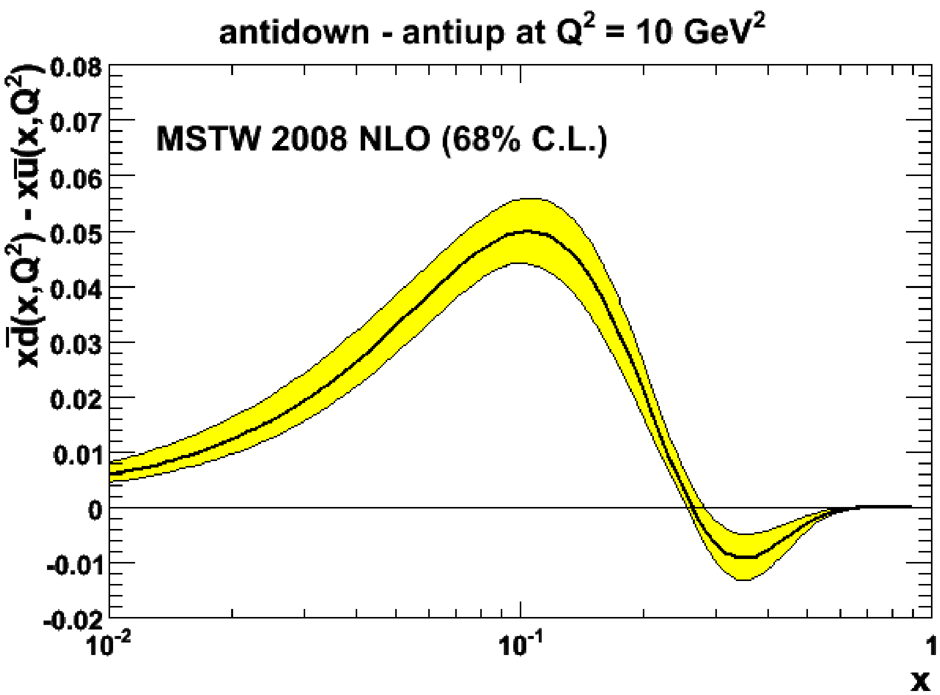


- so we naively expect

$$\bar{u} \approx \bar{d} > \bar{s} > \bar{c} > \ldots$$

- but why such a big d-u asymmetry? Meson cloud, Pauli exclusion, …?

The ratio of Drell-Yan cross sections for $pp, pn \rightarrow \mu^+\mu^- + X$ provides a measure of the difference between the *u* and *d* sea quark distributions



strange

earliest pdf fits had SU(3) symmetry: $s(x,Q_0^2) = \bar{s}(x,Q_0^2) = \bar{u}(x,Q_0^2) = \bar{d}(x,Q_0^2)$

later relaxed to include (constant) strange suppression (cf. fragmentation):

$$s(x,Q_0^2) = \bar{s}(x,Q_0^2) = \frac{\kappa}{2}\left[\bar{u}(x,Q_0^2) + \bar{d}(x,Q_0^2)\right]$$

with $\kappa = 0.4 - 0.5$

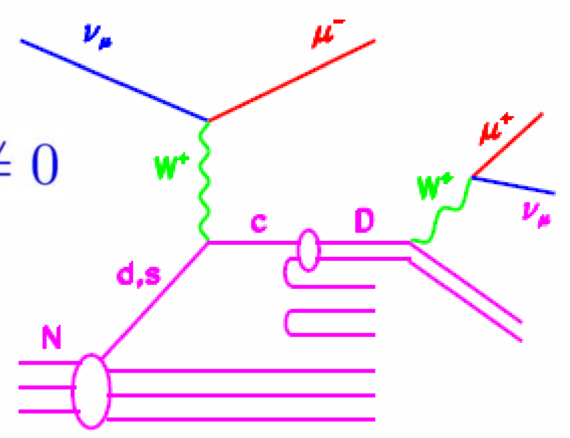
nowadays, dimuon production in υN DIS (CCFR, NuTeV) allows ‘direct’ determination:

$$\frac{d\sigma}{dxdy}\left(\nu_\mu(\bar{\nu}_\mu)N \to \mu^+\mu^- X\right) = B_c\,\mathcal{N}\mathcal{A}\,\frac{d\sigma}{dxdy}\left(\nu_\mu s(\bar{\nu}_\mu \bar{s}) \to c\mu^-(\bar{c}\mu^+)X\right)$$

in the range $0.01 < x < 0.4$

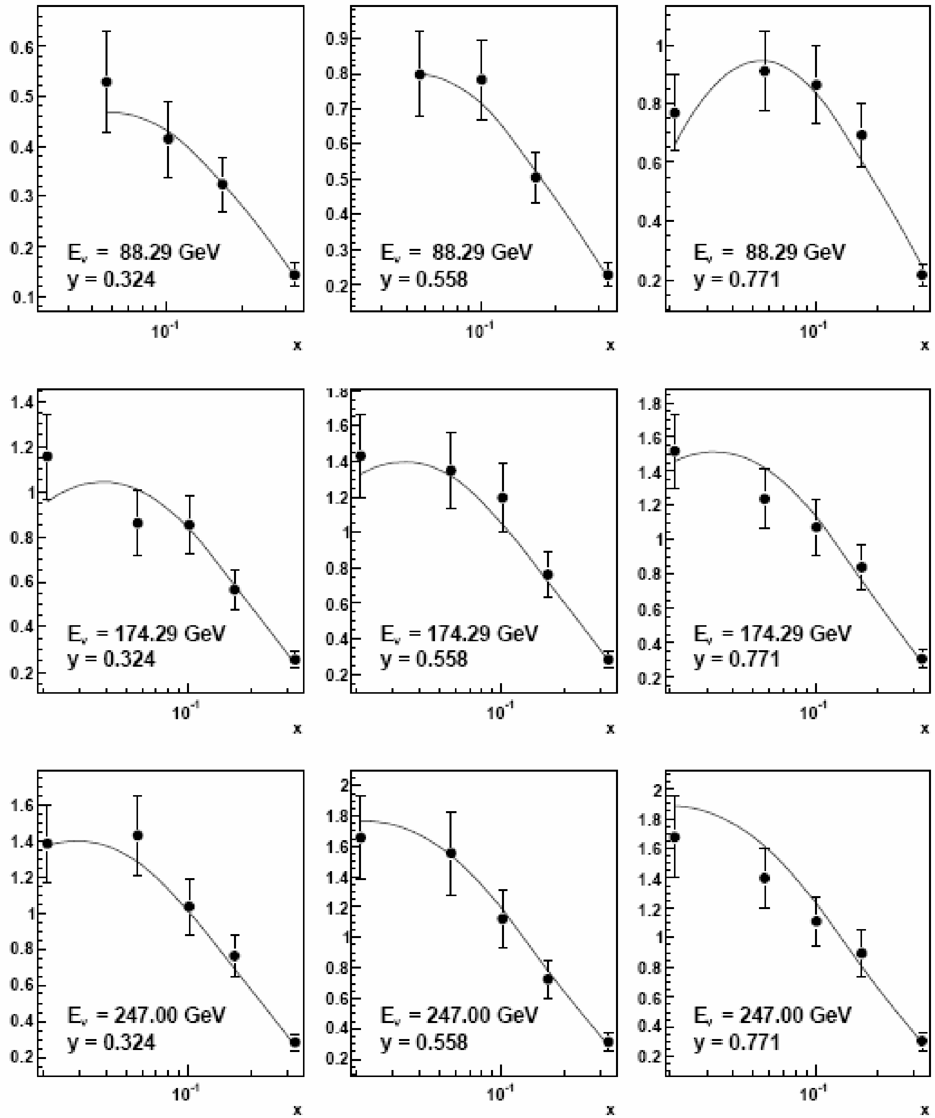
data seem to slightly prefer $s(x,Q_0^2) - \bar{s}(x,Q_0^2) \neq 0$

theoretical explanation?!

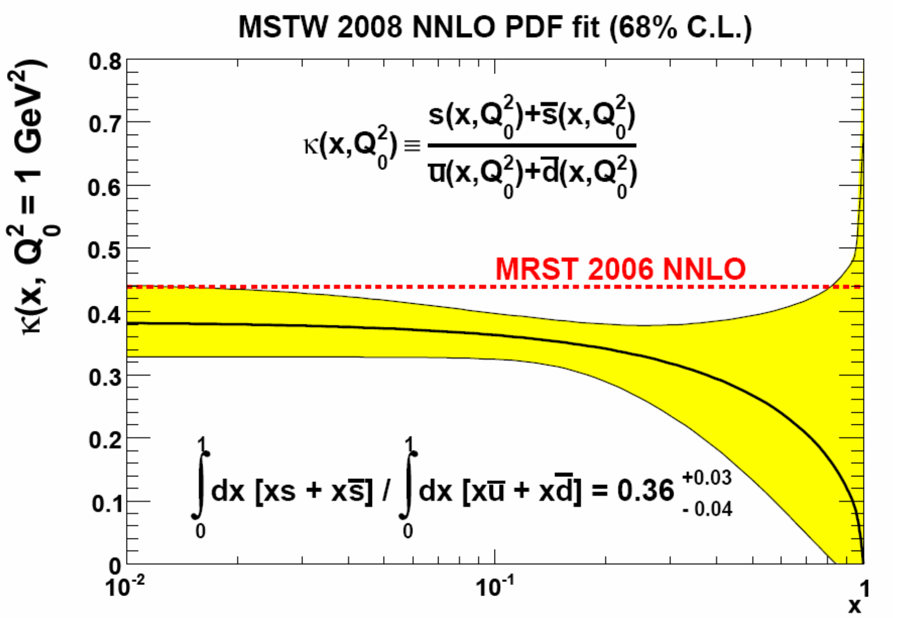


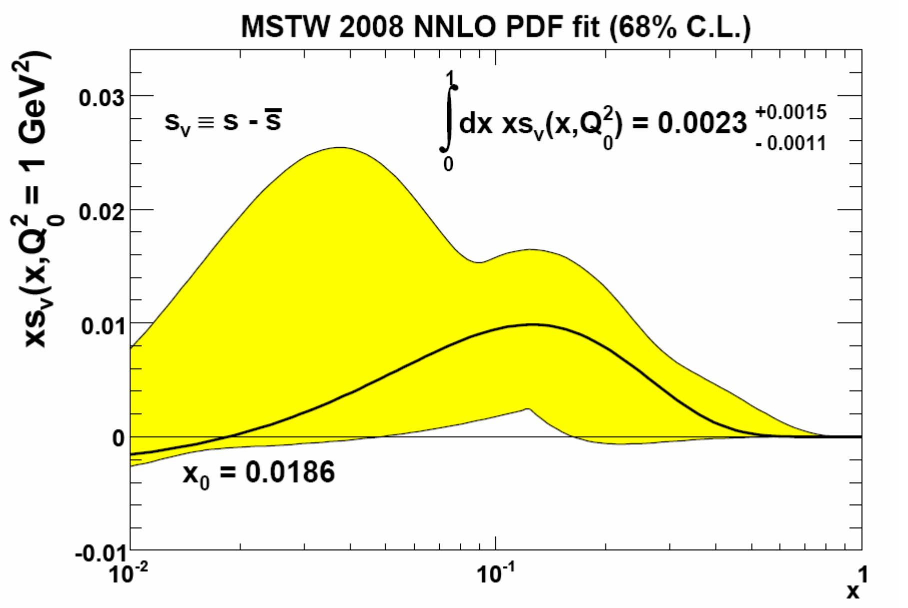
$$\text{NuTeV}\ \frac{100\pi}{G_F^2 M_N E_\nu} \frac{d\sigma}{dxdy}(\nu_\mu N \to \mu^+\mu^- X)\ \text{in GeV}^{-2}$$

MSTW 2008 NNLO PDF fit, χ^2 = 13 for 21 DOF



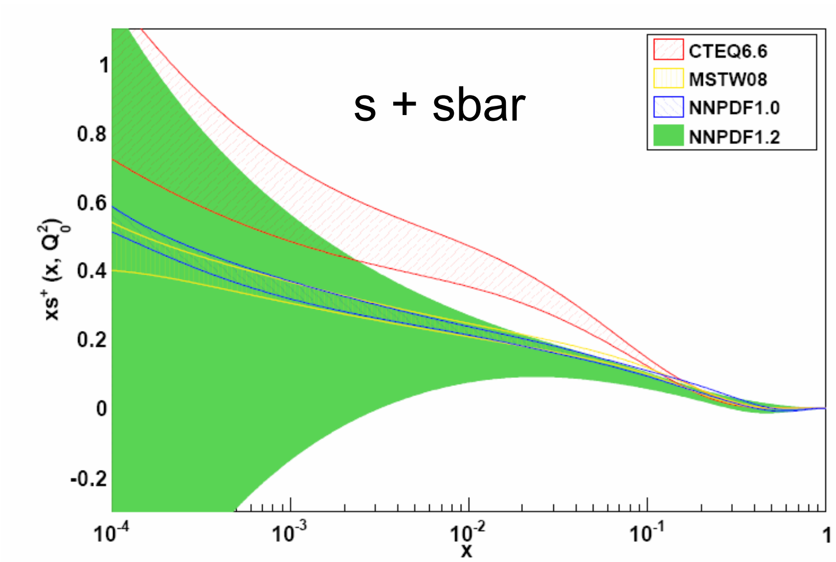
MSTW

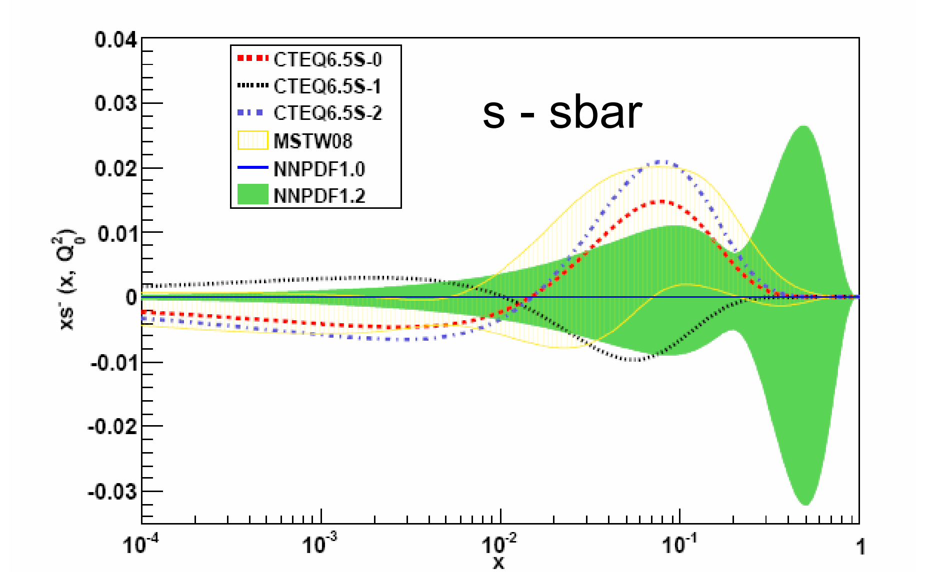

MSTW 2008 NNLO PDF fit (68% C.L.)
$\kappa(x, Q_0^2 = 1\ \mathrm{GeV}^2)$
$\kappa(x,Q_0^2) \equiv \frac{s(x,Q_0^2)+\bar{s}(x,Q_0^2)}{\bar{u}(x,Q_0^2)+\bar{d}(x,Q_0^2)}$
MRST 2006 NNLO
$\int_0^1 dx\,[xs + x\bar{s}] / \int_0^1 dx\,[x\bar{u} + x\bar{d}] = 0.36^{+0.03}_{-0.04}$
x


MSTW 2008 NNLO PDF fit (68% C.L.)
$xs_v(x,Q_0^2 = 1\ \mathrm{GeV}^2)$
$s_v \equiv s - \bar{s}$
$\int_0^1 dx\, xs_v(x,Q_0^2) = 0.0023^{+0.0015}_{-0.0011}$
$x_0 = 0.0186$
x

MSTW

strange quark in NNPDF





<u>Note:</u>

MSTW: assume u,d,s quarks have same x^{δ} behaviour as $x \to 0$

NuTeV $\sin^2\theta_W$ anomaly largely removed arXiv:0905.3531 [hep-ph]

NLO	$\alpha_S(M_Z^2)$ (expt. unc. only)
MSTW (this work)	$0.1202\ ^{+0.0012}_{-0.0015}$
CTEQ [2]	0.1170 ± 0.0047
H1 [23]	0.1150 ± 0.0017
ZEUS [48]	0.1183 ± 0.0028
Alekhin [57]	0.1171 ± 0.0015
BBG [58]	0.1148 ± 0.0019
GJR [59]	0.1145 ± 0.0018

NNLO	$\alpha_S(M_Z^2)$ (expt. unc. only)
MSTW (this work)	$0.1171\ ^{+0.0014}_{-0.0014}$
AMP [60]	0.1128 ± 0.0015
BBG [58]	$0.1134\ ^{+0.0019}_{-0.0021}$
ABKM [61]	0.1129 ± 0.0014
JR [62]	0.1158 ± 0.0035

- reasonable consistency between different analyses
- MSTW values slightly higher because of smaller low-x gluon needed for high-p_T Tevatron jet fit

α_S

MSTW (2009):
full global NLO and NNLO fit

CTEQ (2008):
full global NLO fit

H1 (2001):
H1 + BCDMS

ZEUS (2005):
ZEUS inc. DIS-JET + photoprodn.

BBG = Blumlein, Bottcher, Guffanti (2006):
non-singlet DIS analysis

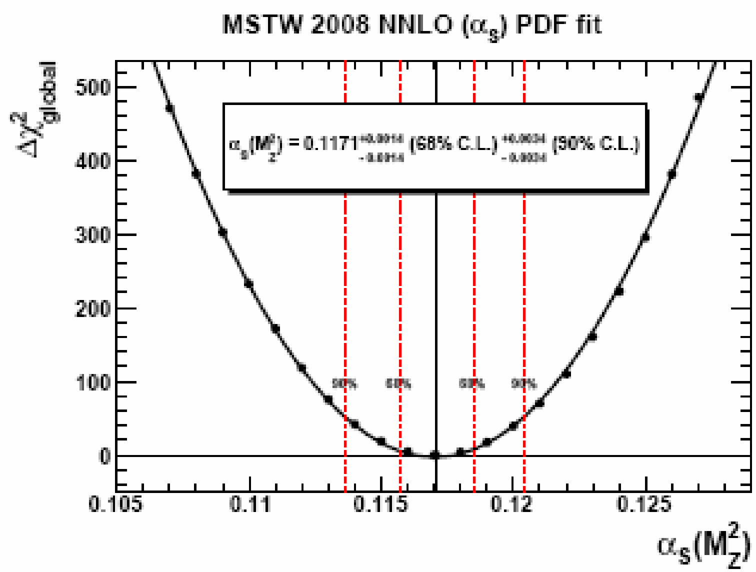
AMP = Alekhin, Melnikov, Petriello (2006):
DIS + DY

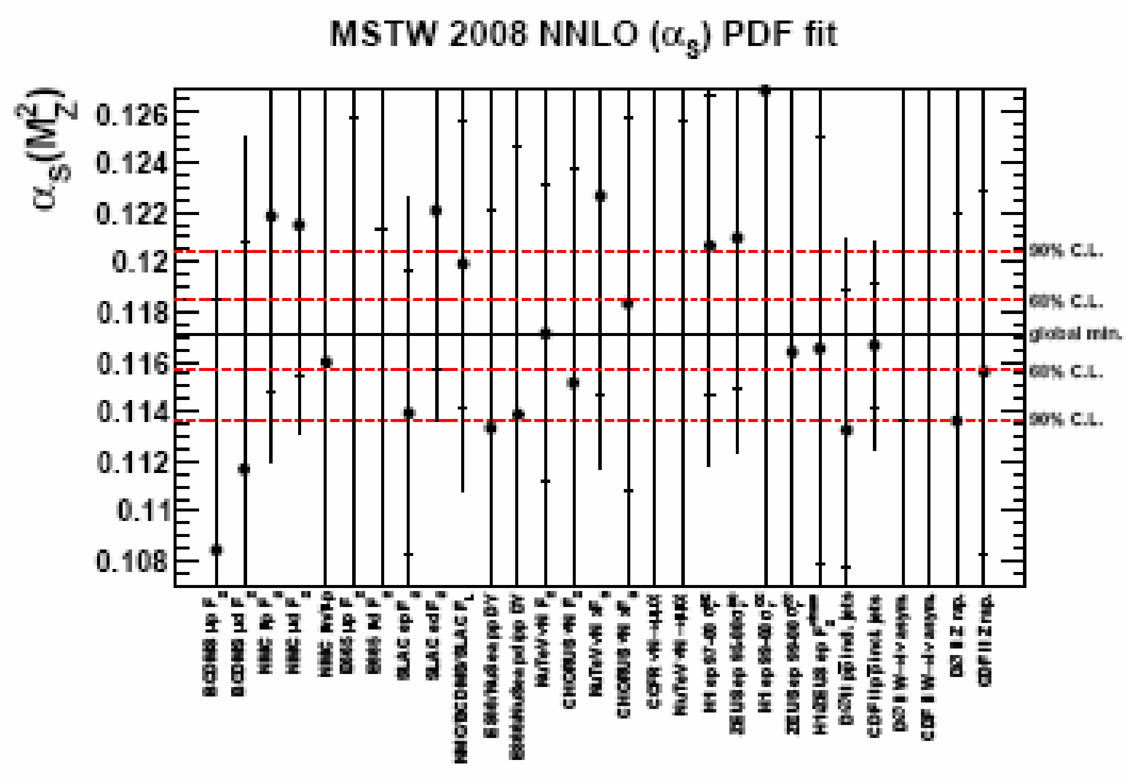
GJR = Gluck, Jimenez-Delgado, Reya (2008):
DIS + DY + Tevatron jet

JR = Jimenez-Delgado, Reya (2009):
DIS + DY

ABKM = Alekhin, Blumlein, Klein, Moch (2009):
DIS + DY

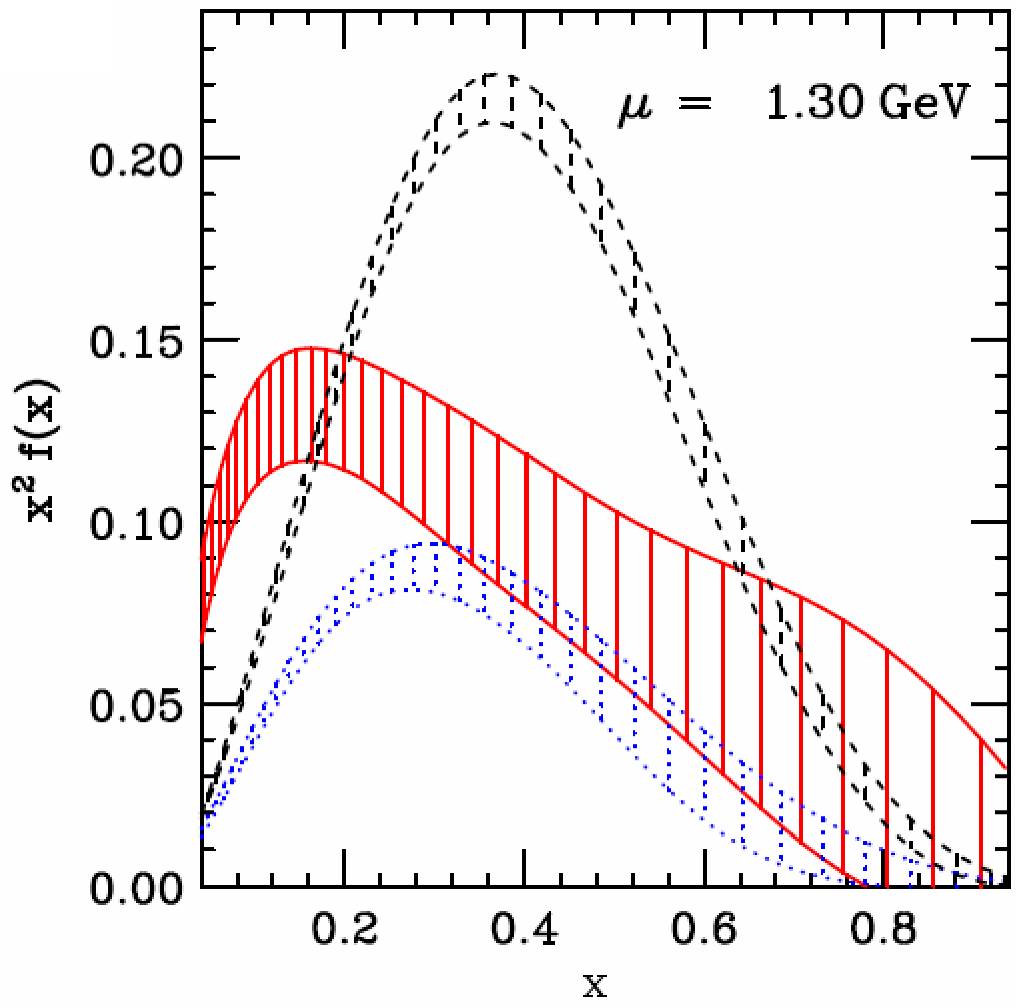
PDG(2008): $\alpha_S(M_Z^2) = 0.1176 \pm 0.002$


MSTW 2008 NNLO (α_S) PDF fit
$\Delta\chi^2_{global}$
500
400
300
200
100
0
0.105
0.11
0.115
0.12
0.125
$\alpha_S(M_Z^2)$
$\alpha_S(M_Z^2)$ = 0.1171 (68% C.L.) (90% C.L.)


MSTW 2008 NNLO (α_S) PDF fit
$\alpha_S(M_Z^2)$
0.126
0.124
0.122
0.12
0.118
0.116
0.114
0.112
0.11
0.108
90% C.L.
68% C.L.
global min.
68% C.L.
90% C.L.

CT09G fit to Run I and Run II jet data simultaneously, find much harder gluon (with more flexible parameterisation)

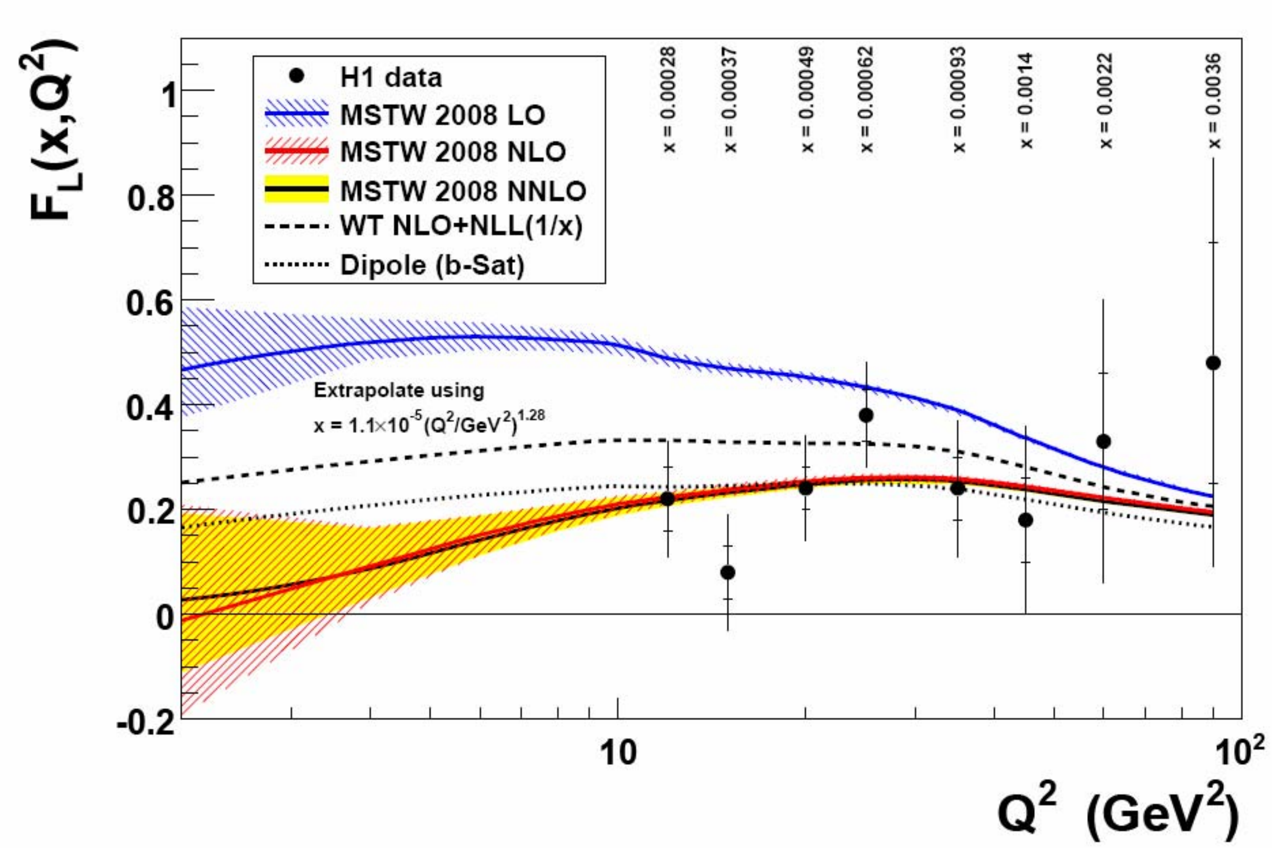
… harder than valence quarks?!

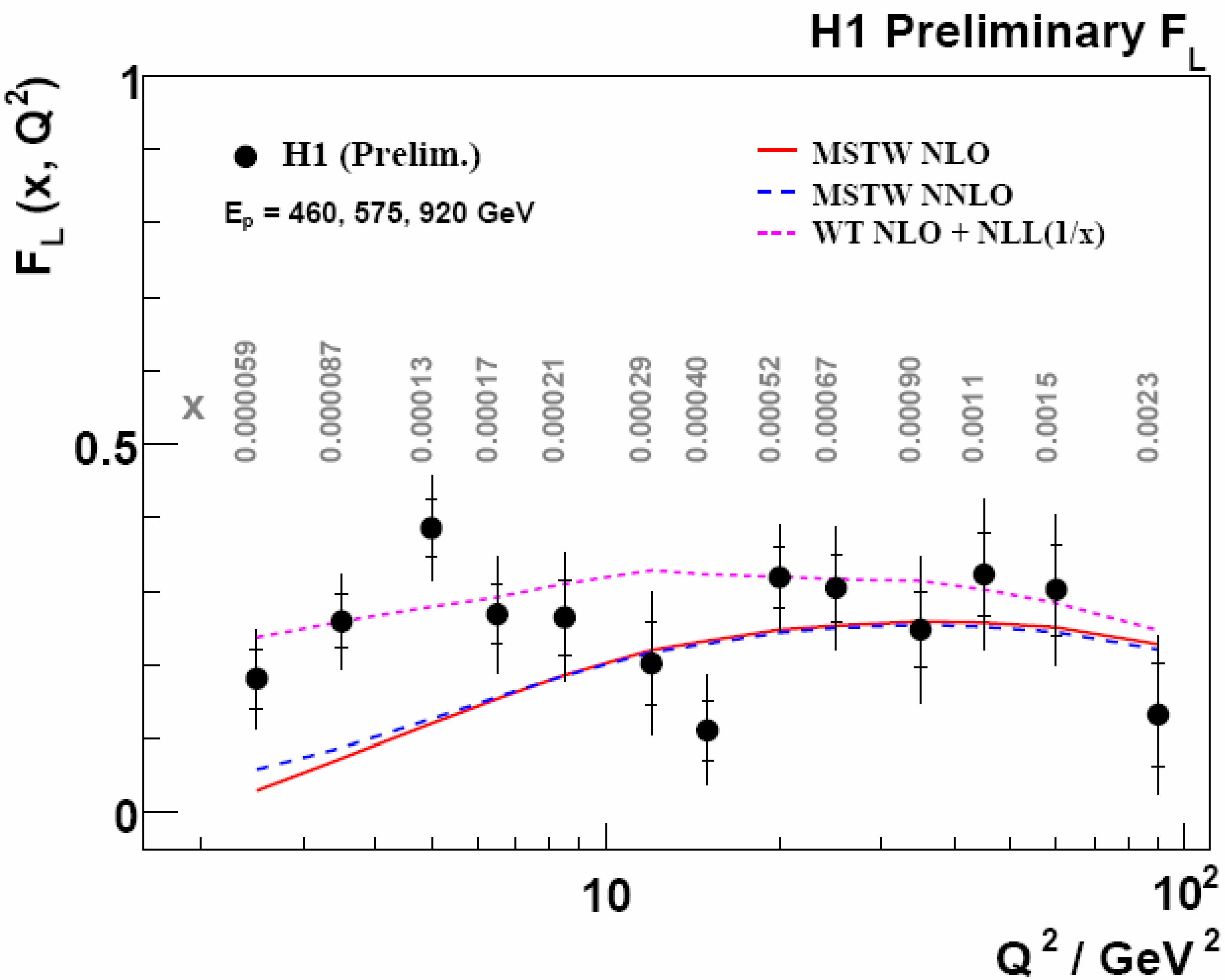


F_L

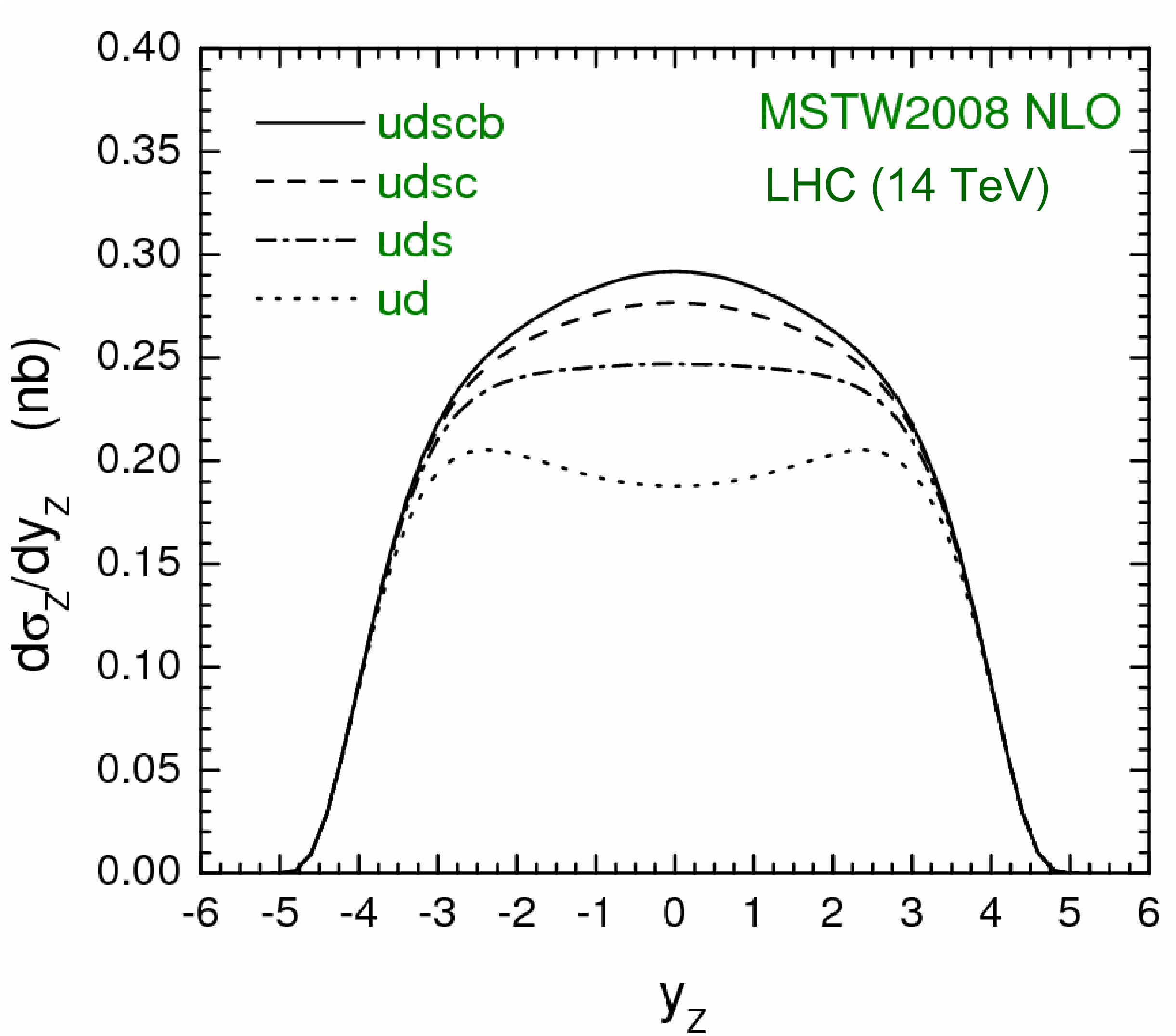
- an independent measurement of the small-x gluon
- a test of the assumptions in the DGLAP LT pQCD analysis of small-x F_2
- higher–order ln(1/x) and higher-twist contributions could be important

$$\frac{\partial F_2}{\partial \ln Q^2} \simeq \alpha_S P^{qg} \otimes g + \ldots$$
$$F_L \simeq \alpha_S C_{Lg} \otimes g + \ldots.$$





H1 Preliminary F_L
F_L (x, Q^2)
H1 (Prelim.)
E_p = 460, 575, 920 GeV
MSTW NLO
MSTW NNLO
WT NLO + NLL(1/x)
x
0.000059
0.000087
0.00013
0.00017
0.00021
0.00029
0.00040
0.00052
0.00067
0.00090
0.0011
0.0015
0.0023
1
0.5
0
10
10^2
Q^2 / GeV^2



udscb
udsc
uds
ud
MSTW2008 NLO
LHC (14 TeV)
dσ_Z/dy_Z (nb)
y_Z
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6

LHCb

Unique features

- pseudo-rapidity range 1.9 - 4.9
  - 1.9 - 2.5 complementary to ATLAS/CMS
  - > 2.5 unique to LHCb
- beam defocused at LHCb: 1 year of running = 2 fb^{-1}
- trigger on low momentum muons: $p > 8$ GeV, $p_T > 1$ GeV

→ access to unique range of (x, Q^2)

LHCb

→ detect forward, low p_T muons from $q\bar{q} \to \mu^+\mu^-$