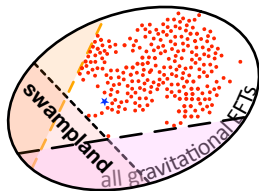
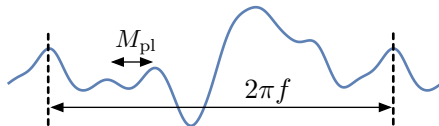
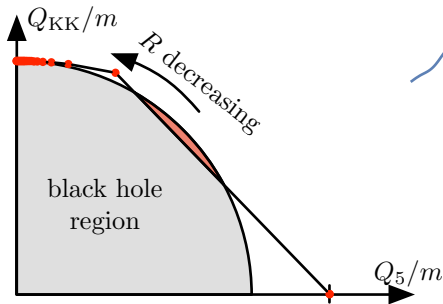


# The Weak Gravity Conjecture: from Quantum Gravity to the Real World



Ben Heidenreich

UMass **Amherst**

Physics

DESY Colloquium, May 31, 2022

# I. The Landscape and the Swampland

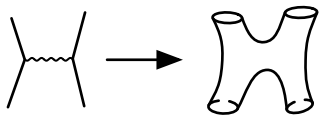
Quantum  
Mechanics + Gravity = ?

$$\Delta x \Delta p = \frac{\hbar}{2} \quad G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

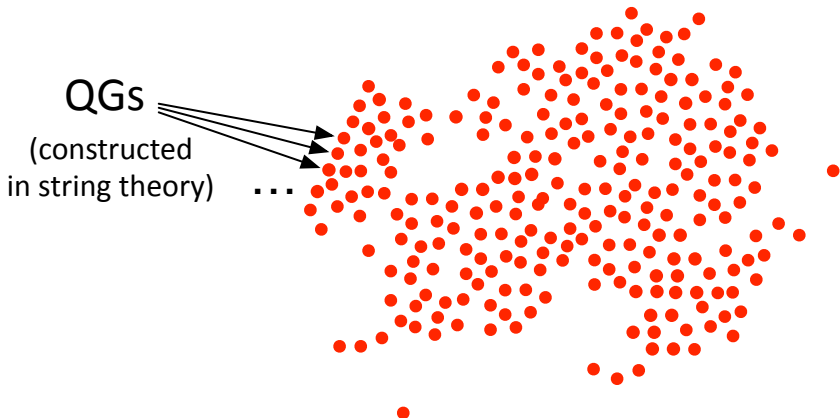
Yet to be understood from 1st principles

...but

String theory provides many **indirect**,  
exquisitely detailed insights

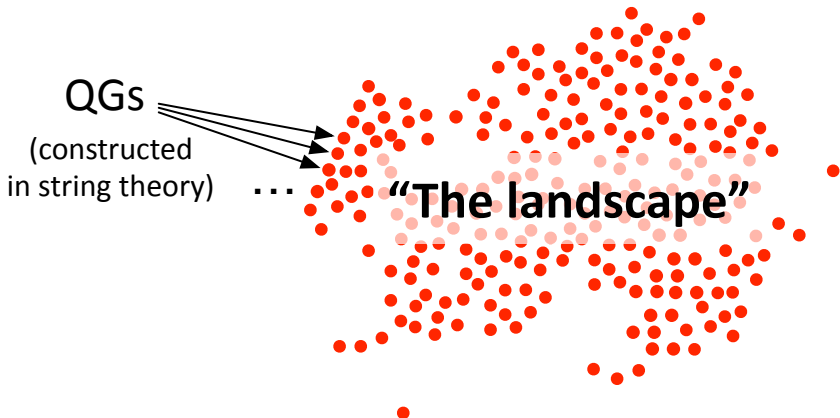


# Insight #1: There are **many** QG theories



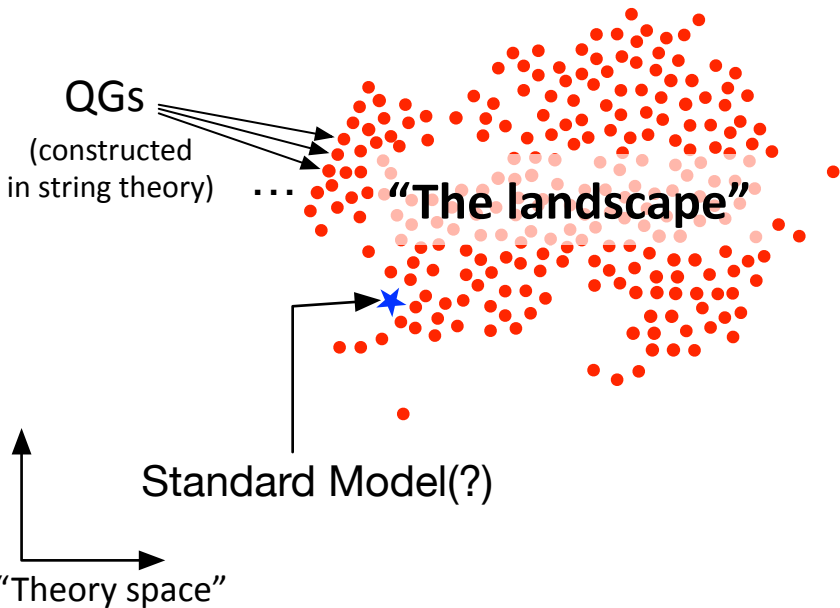
“Theory space”

# Insight #1: There are **many** QG theories

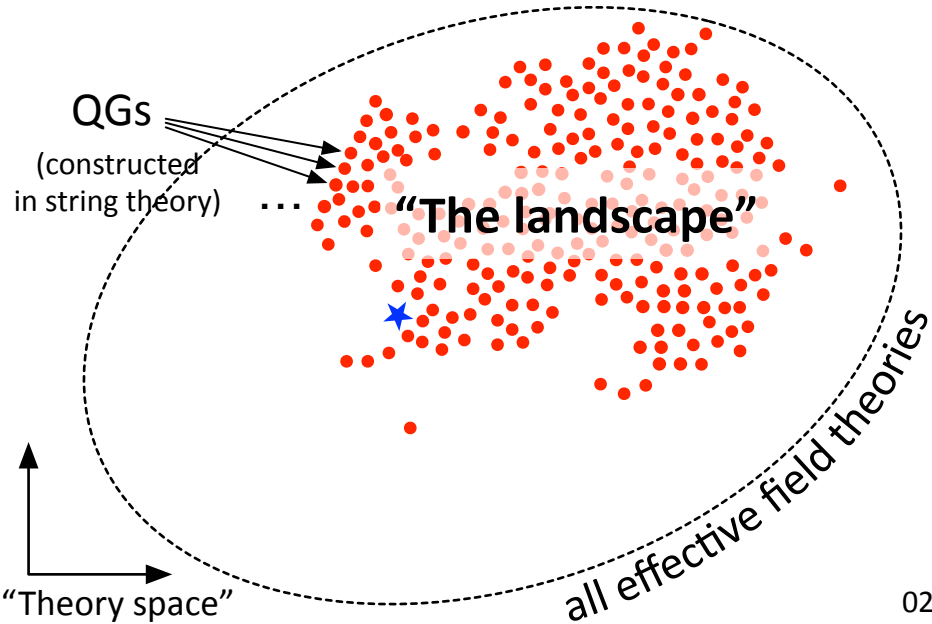


“Theory space”

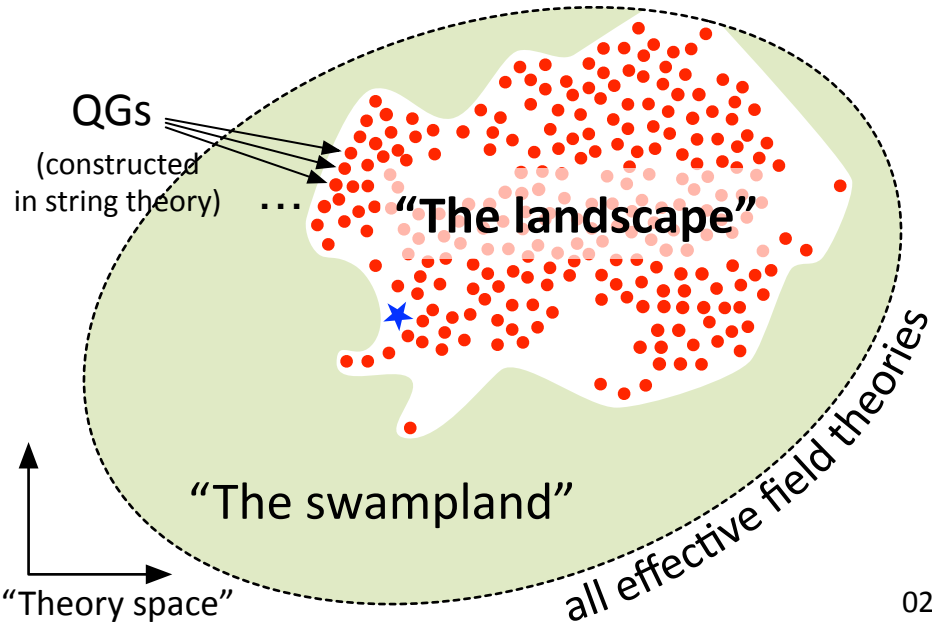
# Insight #1: There are **many** QG theories



# Insight #2: ...but not everything goes

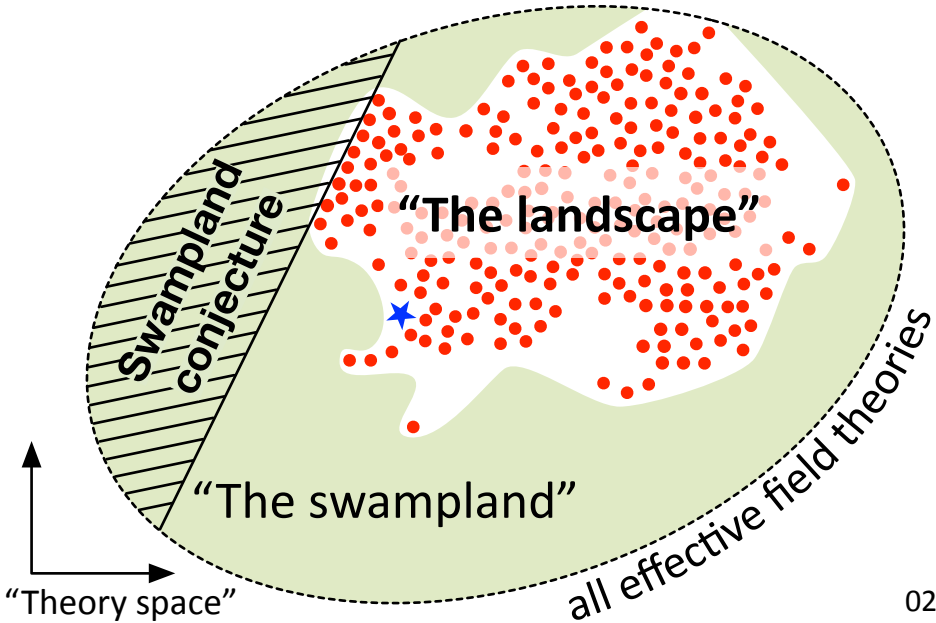


# Insight #2: ...but not everything goes





# Insight #2: ...but not everything goes

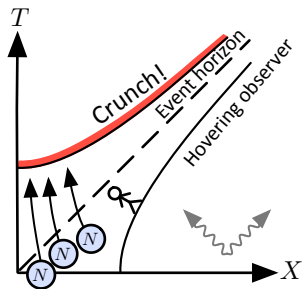
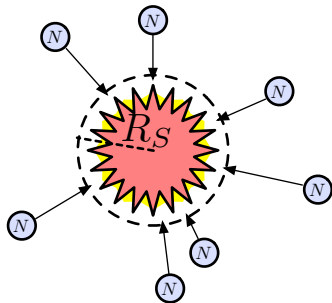


# Ex: “Theorem”

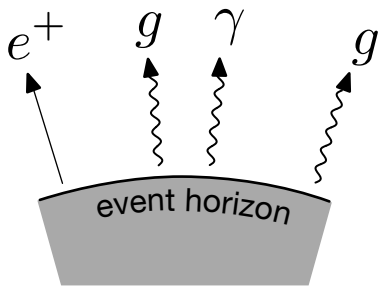
Quantum gravities cannot have global symmetries (e.g., baryon number)

## “Proof”

Create a black hole by colliding baryons



# Quantum black holes **glow**



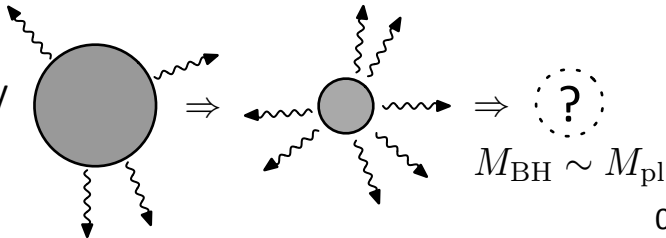
$$k_B T = \frac{\hbar c^3}{8\pi G M}$$

Hawking temperature

$$S = \frac{k_B c^3}{4\hbar G} A_{\text{Horizon}}$$

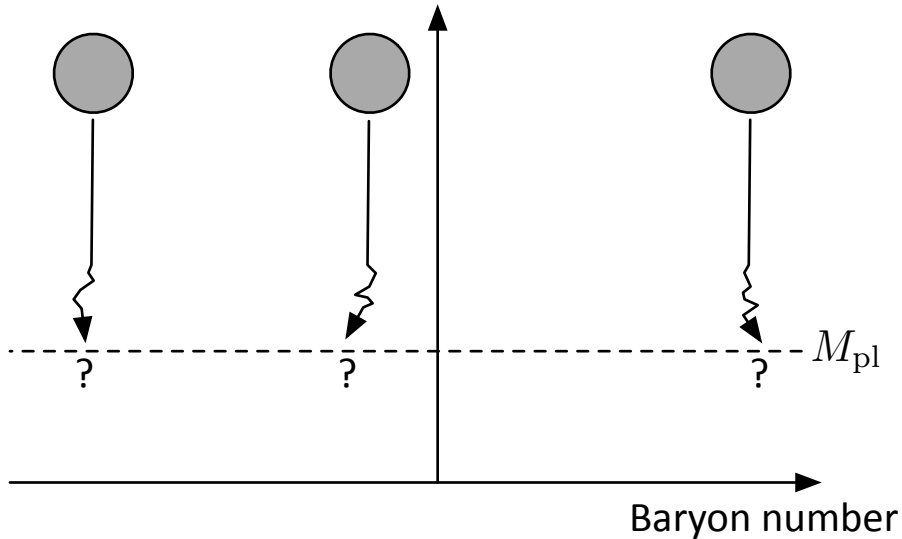
Bekenstein-Hawking entropy

Over time, they  
**evaporate:**



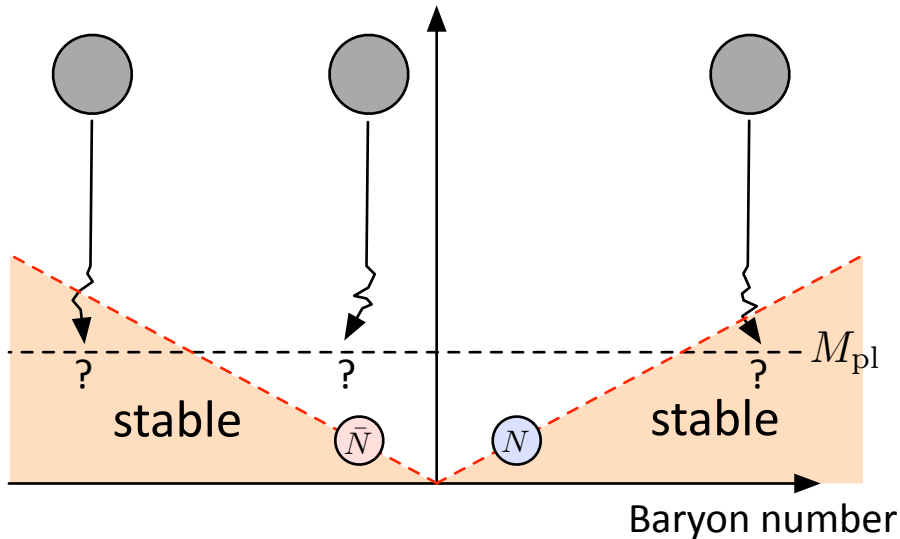
Remnants

Black Hole  
Mass



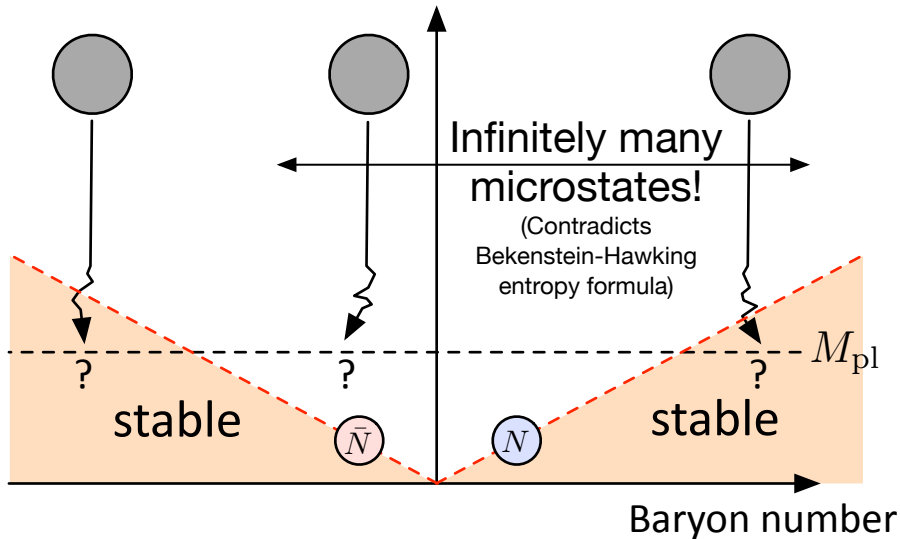
Remnants

Black Hole  
Mass



Remnants

Black Hole  
Mass

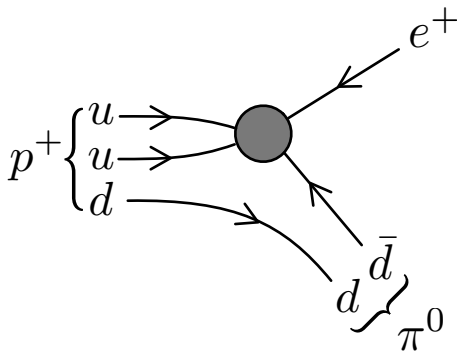


Baryon number is violated!

$$\text{e.g., } \mathcal{L}_{\text{eff}} \sim \frac{1}{M_{\text{pl}}^2} uude$$

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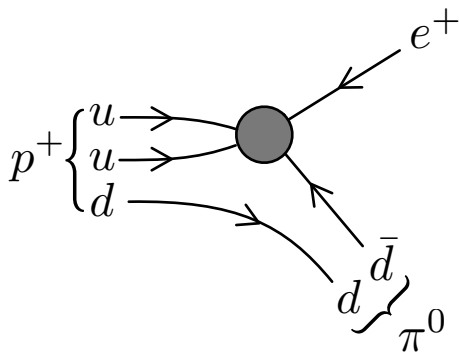


...proton decays!



# Baryon number is violated!

e.g.,  $\mathcal{L}_{\text{eff}} \sim \frac{1}{M_{\text{pl}}^2} u u d e$



$$\tau_{\text{pred}} \sim 10^{45} \text{ yrs}$$

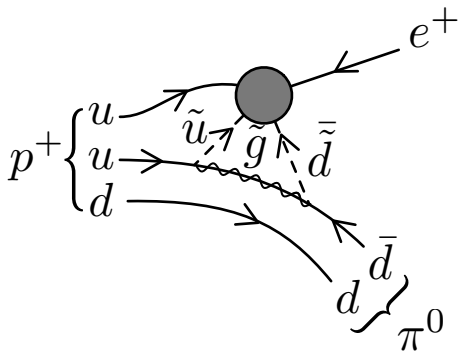
versus

$$\tau_{\text{obs}} \gtrsim 10^{34} \text{ yrs}$$

...proton decays!

# Baryon number is violated!

vs.  $\mathcal{L}_{\text{eff}} \sim \frac{1}{M_{\text{pl}}} u \tilde{u} \tilde{d} e$  (MSSM)



$\tau_{\text{pred}} \sim 10^{20}$  yrs

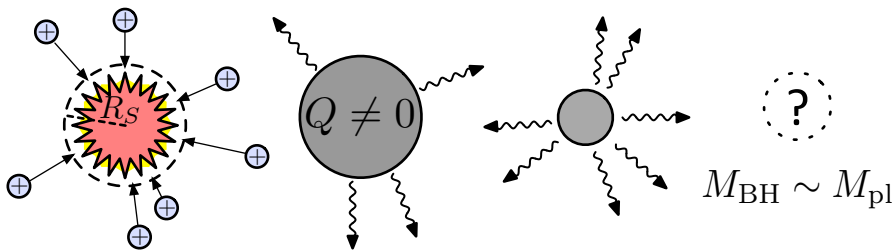
versus

$\tau_{\text{obs}} \gtrsim 10^{34}$  yrs!

...proton decays **too rapidly!**

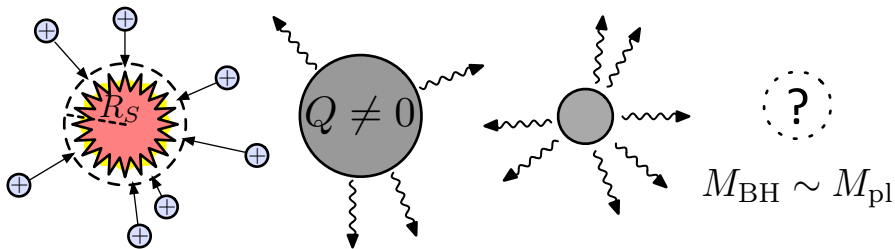
# Charged remnants?

What if the symmetry is gauged (coupled to a long range force) rather than global?

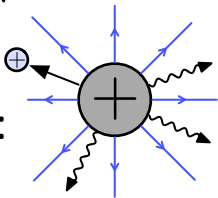


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What if the symmetry is gauged (coupled to a long range force) rather than global?

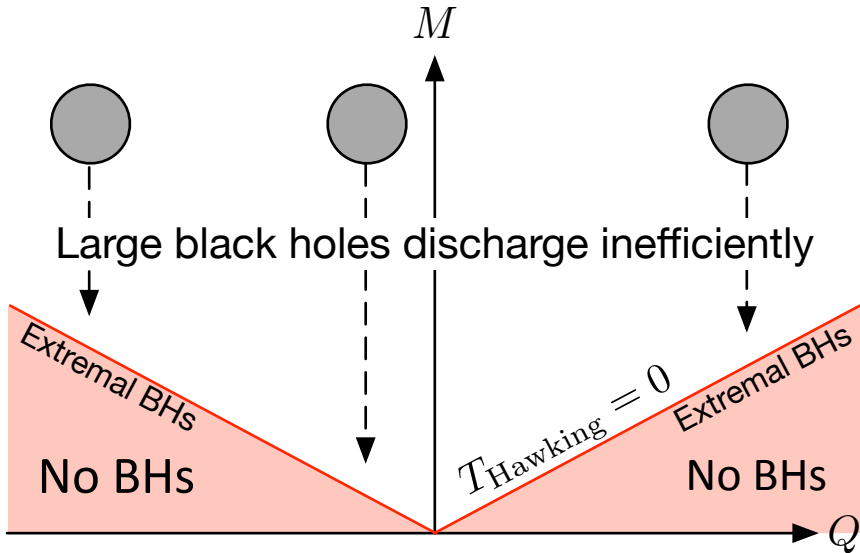


Now there is  
an electric field:

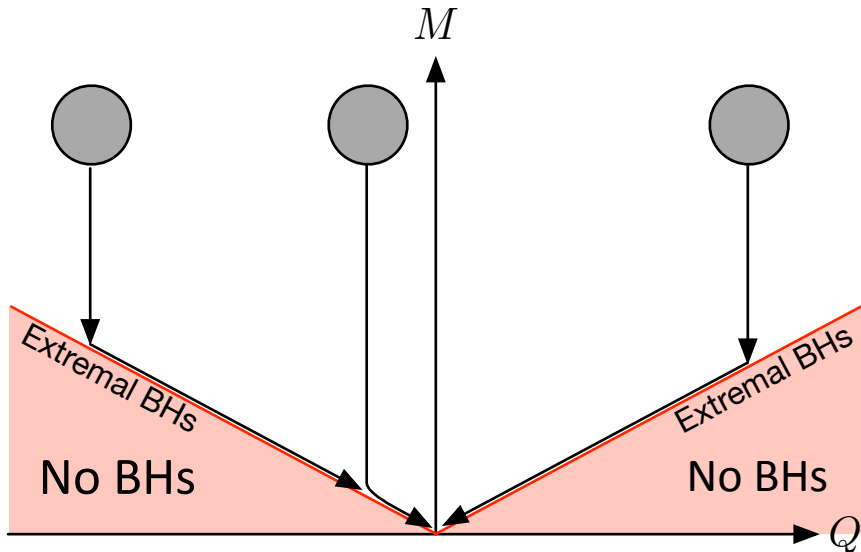


(Too much  
charge destroys  
the horizon)

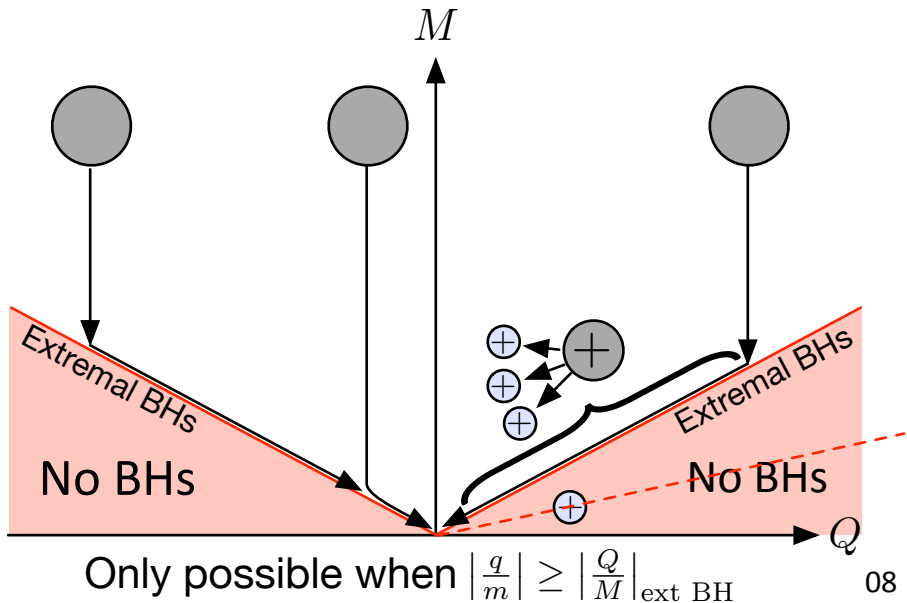
# Charged remnants?



# Charged remnants?



# Charged remnants?



# The Weak Gravity Conjecture (WGC)

(Arkani-Hamed, Motl, Nicolis, Vafa '06)

There is a charged particle with

$$\left| \frac{q}{m} \right| \geq \left| \frac{Q}{M} \right|_{\text{ext BH}}$$

(Otherwise would have charged remnants)



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$$\left| \frac{q}{m} \right| \geq \left| \frac{Q}{M} \right|_{\text{ext BH}}$$

(Otherwise would have charged remnants)

“Gravity is the weakest force!”

$|F_{\text{Coulomb}}| \geq |F_{\text{Newton}}|$  for identical pair

# The Weak Gravity Conjecture (WGC)

e.g., for electromagnetic forces

$$\frac{e^2}{4\pi} \sim \frac{\hbar c}{137} \gg Gm_e^2 \sim 10^{-45} \hbar c \quad \text{easily satisfied!}$$

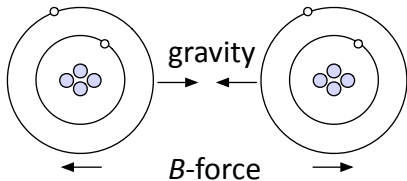
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e.g., for electromagnetic forces

$$\frac{e^2}{4\pi} \sim \frac{\hbar c}{137} \gg Gm_e^2 \sim 10^{-45} \hbar c \quad \text{easily satisfied!}$$

but for a “fifth force” such as  $B$  (really  $B-L$ )

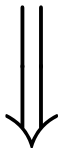
$$\frac{Q_N^2}{4\pi} \ll Gm_N^2 \sim 10^{-38} \hbar c \quad \text{not satisfied (by nucleons)!}$$



## II. The Weak Gravity Conjecture

# The Magnetic WGC (AMNV '06)

No (generalized) global symmetries

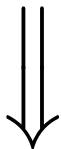


(e.g., 2104.07036  
& earlier “folk theorem”)

Magnetic monopoles exist!

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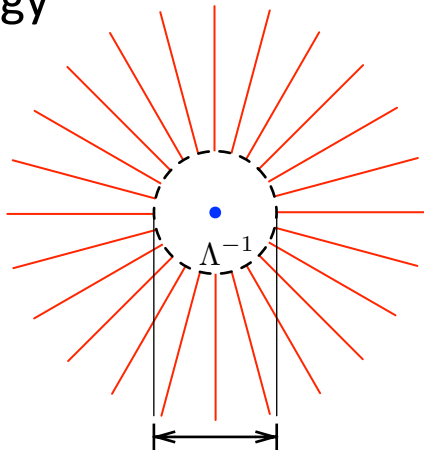
WGC for magnetic charge requires

$$\left| \frac{q_{\text{mag}}}{m} \right| \geq \left| \frac{Q_{\text{mag}}}{M} \right|_{\text{ext BH}}$$

# The Magnetic WGC (AMNV '06)

Monopole self-energy

$$m \gtrsim \frac{q_{\text{mag}}^2}{e^2} \Lambda$$



Scale of new physics 12

# The Magnetic WGC (AMNV '06)

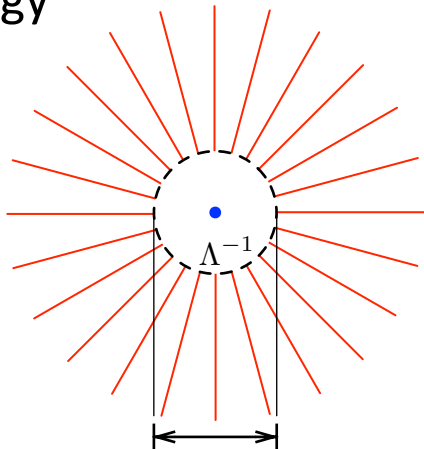
Monopole self-energy

$$m \gtrsim \frac{q_{\text{mag}}^2}{e^2} \Lambda$$

WGC

$$m < \sqrt{2} \frac{q_{\text{mag}}}{e} M_{\text{pl}}$$

$$\Rightarrow \boxed{\Lambda \lesssim e M_{\text{pl}}}$$



Scale of new physics 12



# The Magnetic WGC (AMNV '06)

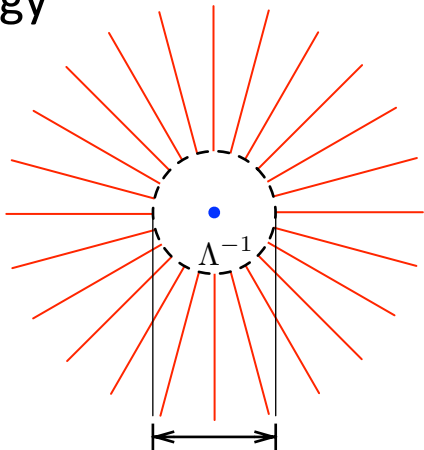
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WGC

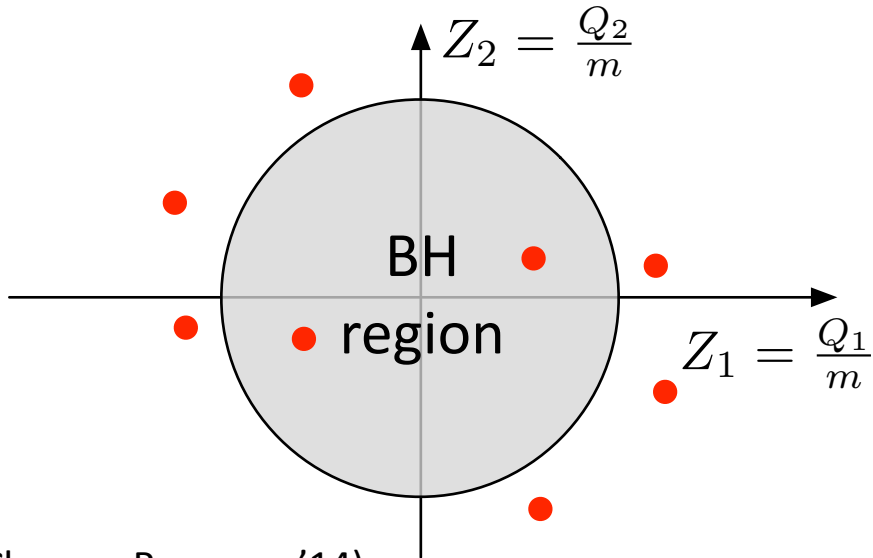
$$m < \sqrt{2} \frac{Q}{e} M_{\text{pl}}$$

$$\implies \boxed{\Lambda \lesssim e M_{\text{pl}}}$$



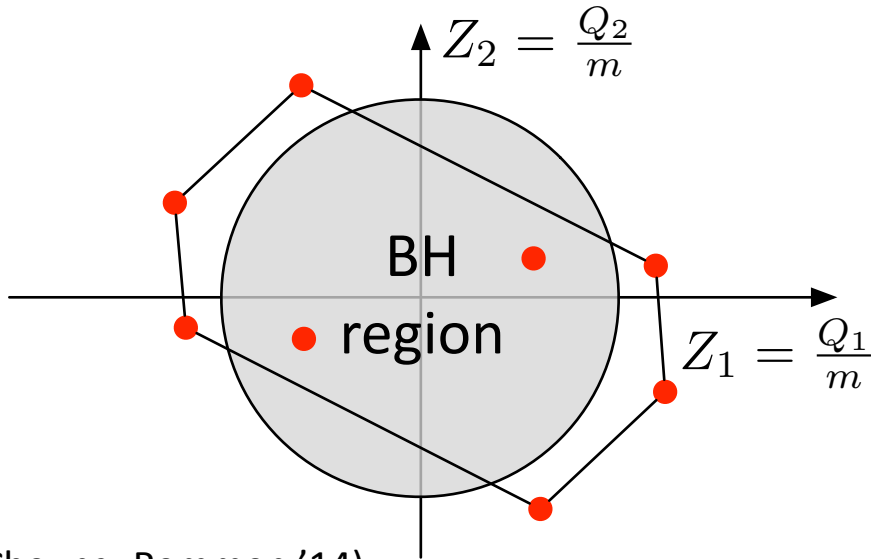
**What happens at new physics scale  $\Lambda$ ?**

# WGC w/ multiple photons



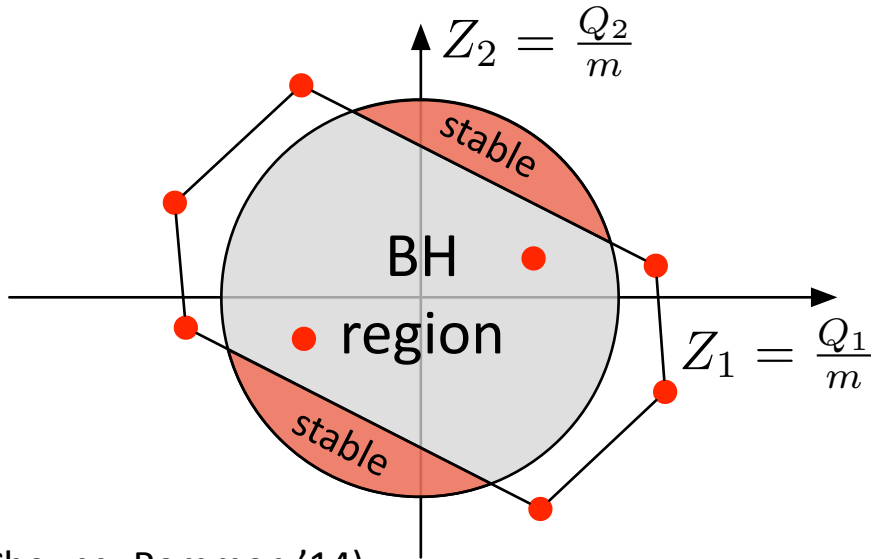
(Cheung, Remmen '14)

# WGC w/ multiple photons



(Cheung, Remmen '14)

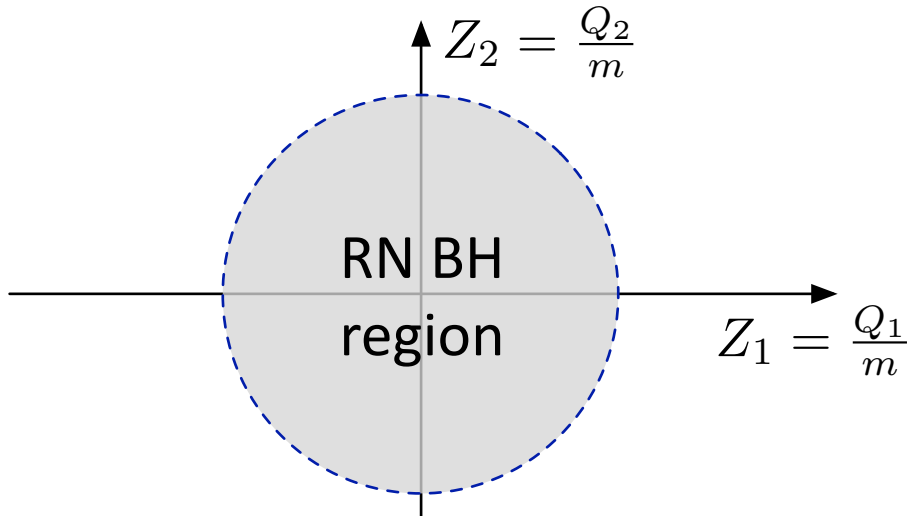
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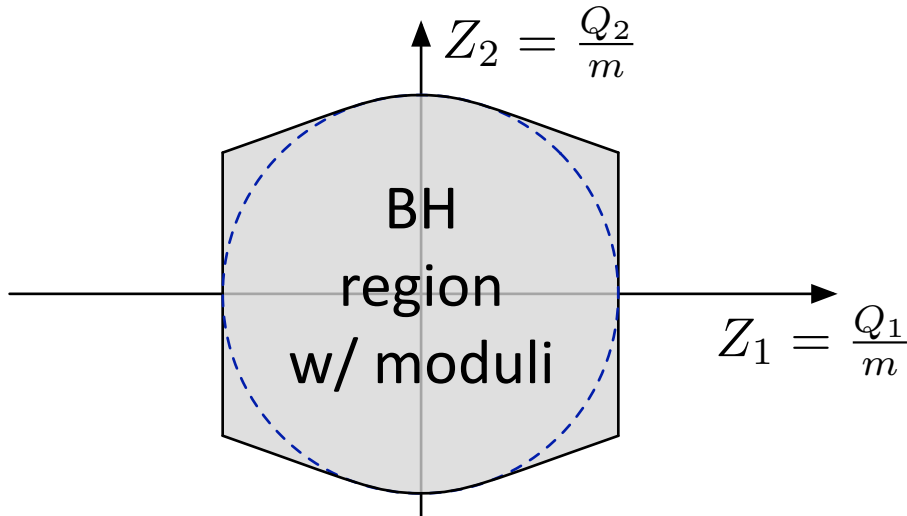
# The WGC with Moduli

BH, Reece  
Rudelius '15



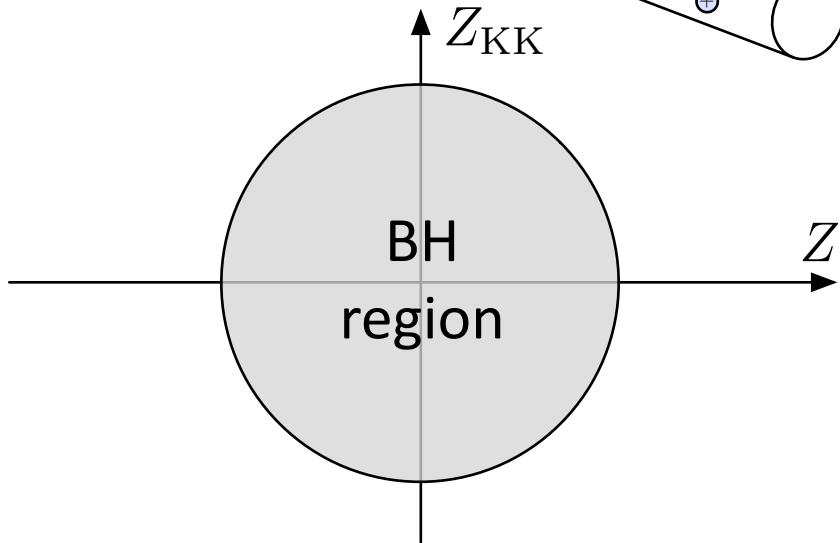
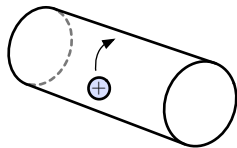
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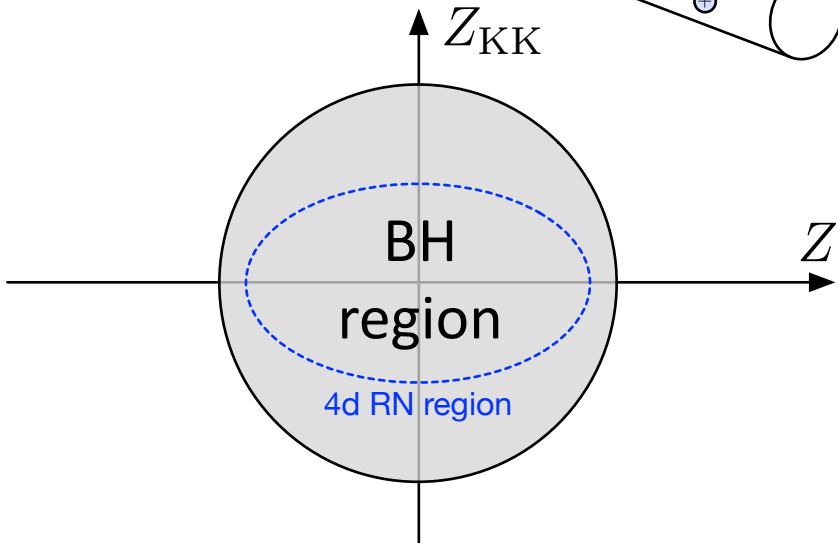
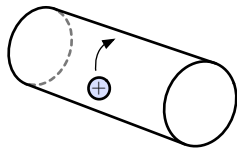


Ex. from Alim, BH, Rudelius, 2108.08309

# Circle compactification

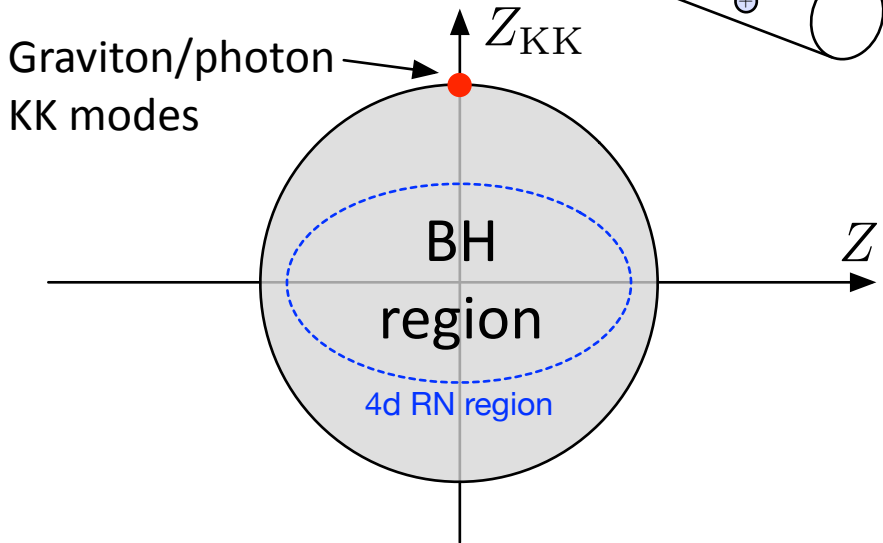


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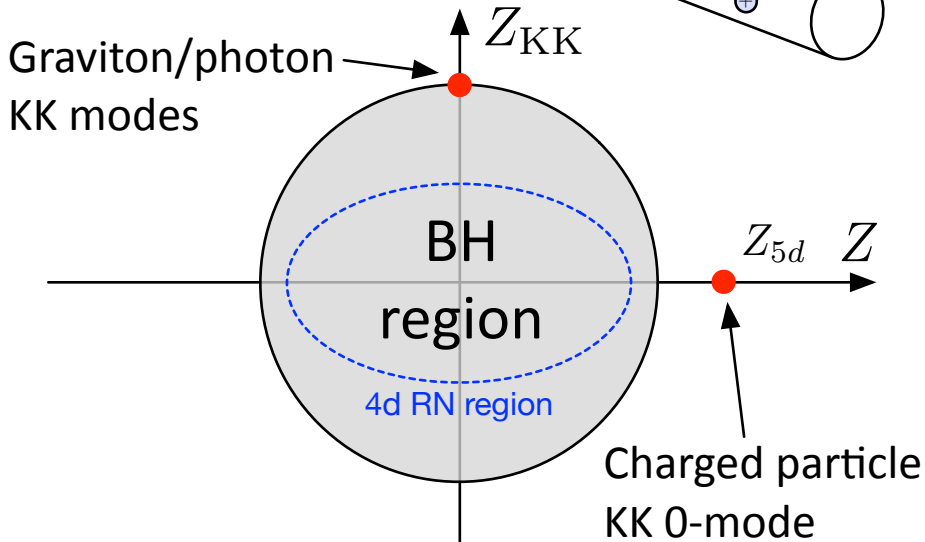




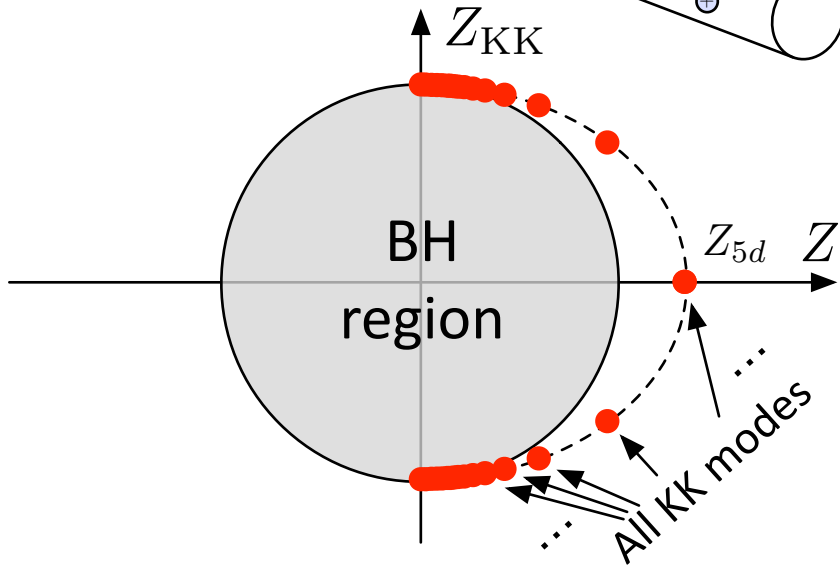
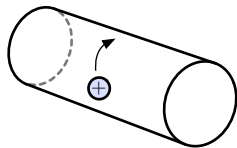
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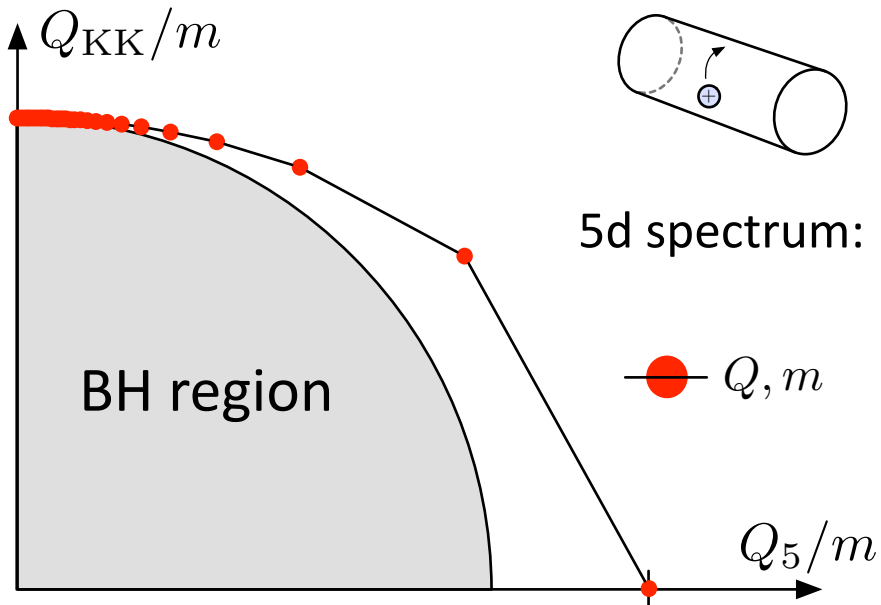


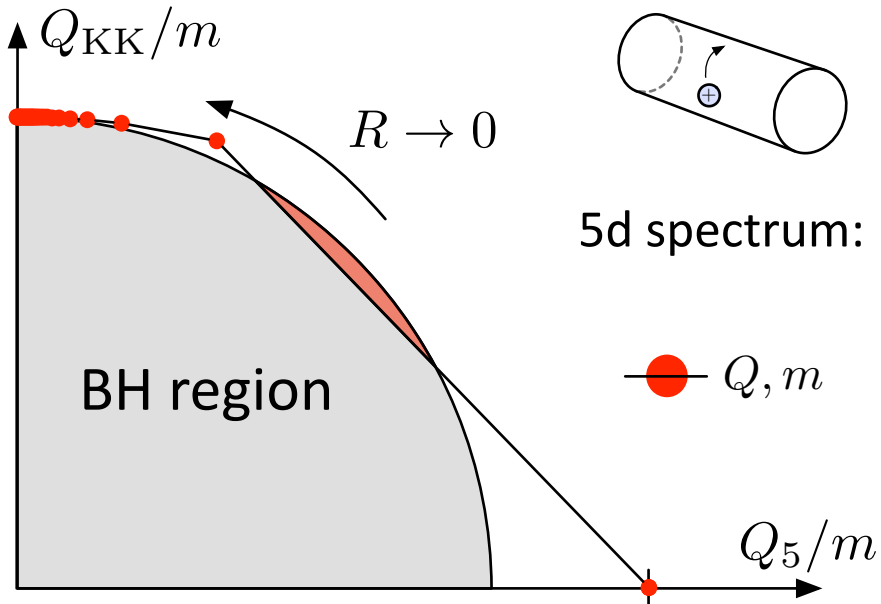
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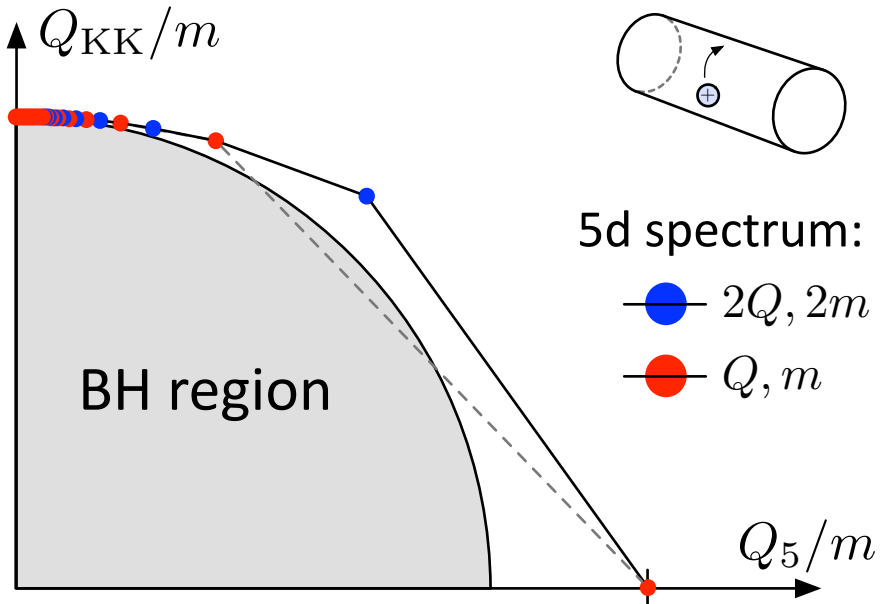


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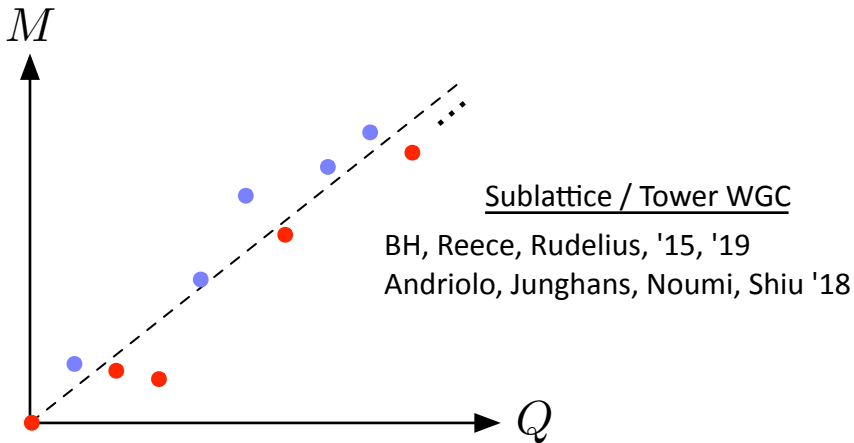




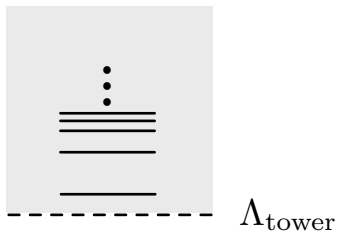




$WGC_{d-1}$  requires superextremal resonances of arbitrarily large charge!



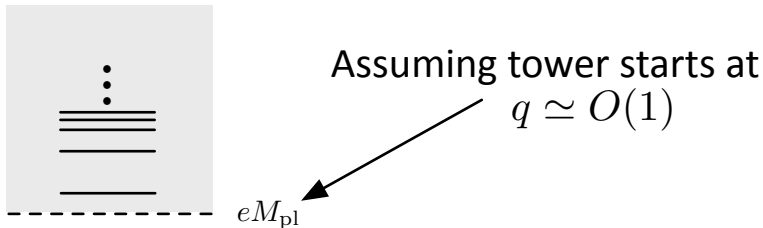
# sL/TWGC versus magnetic WGC



Light states

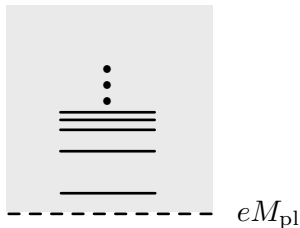


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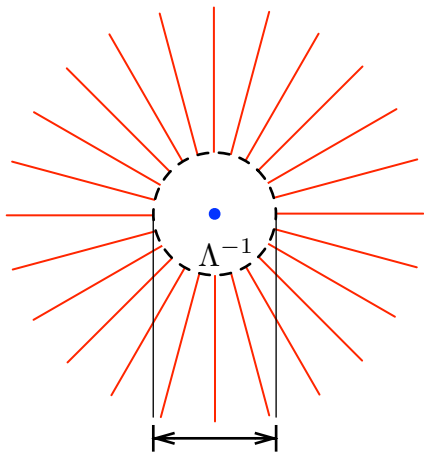


Light states

# sL/TWGC versus magnetic WGC

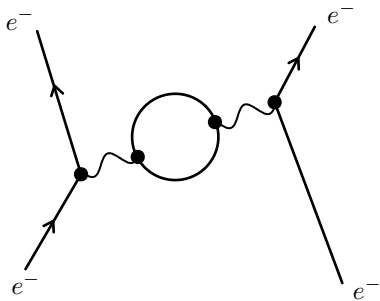
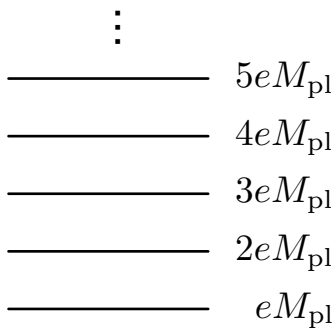


Light states



Tower provides required  
new physics!

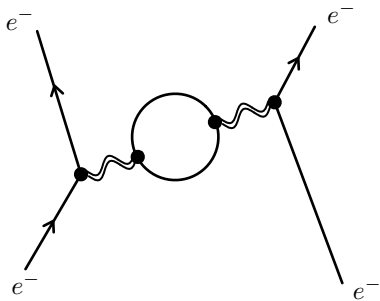
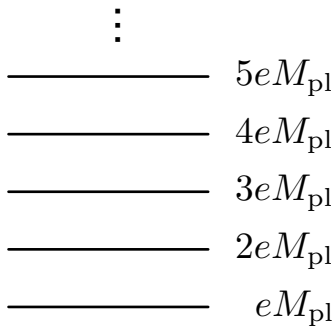
# Emergence



Loops of tower resonances  
renormalize elementary charge

$$e \rightarrow \infty \text{ as } \Lambda \rightarrow e_{\text{IR}}^{1/3} M_{\text{pl}}$$

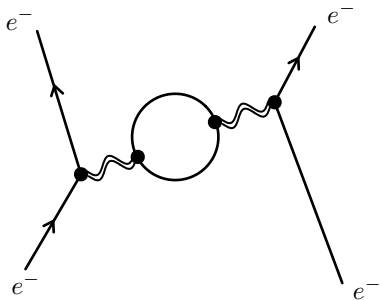
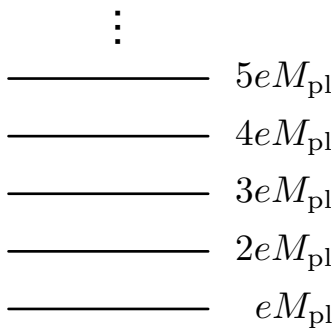
# Emergence



Gravitational interactions  
also get stronger

$$M_{\text{pl}} \rightarrow M_{\text{pl}}/\sqrt{N} \quad \text{for } N \text{ light species}$$

# Emergence



These scales match!

$$M_{\text{pl}}/\sqrt{N} \simeq e_{\text{IR}}^{1/3} M_{\text{pl}}$$

# Emergence

Gravity and electromagnetism both become highly non-linear at

$$\Lambda_{\text{QG}} \simeq e_{\text{IR}}^{1/3} M_{\text{pl}}$$

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Gravity and electromagnetism both become highly non-linear at

$$\Lambda_{\text{QG}} \simeq e_{\text{IR}}^{1/3} M_{\text{pl}}$$

Expectation: notion of spacetime breaks down at length scale  $l_{\text{QG}} = \hbar c / \Lambda_{\text{QG}}$

Spacetime concepts, like local (gauge) symmetries must disappear above this scale. They “emerge” for  $l \gg l_{\text{QG}}$ , due to screening effects of the tower of resonances.

# Emergence

Conversely, if gravity and electromagnetism both become highly non-linear at a common scale " $\Lambda_{QG}$ " then the WGC (approximately) follows!

Emergence implies the WGC!

Harlow '15

BH, Reece, Rudelius '17



# Emergence

Conversely, if gravity and electromagnetism both become highly non-linear at a common scale “ $\Lambda_{\text{QG}}$ ” then the WGC (approximately) follows!

Emergence implies the WGC!

Harlow ‘15

BH, Reece, Rudelius ‘17

e.g., Kaluza-Klein theory:

$$e_{\text{KK}}^{1/3} M_4 = \sqrt{2\pi}^{1/3} M_5$$

$$\implies \Lambda_{\text{QG}} \simeq M_5!$$

# The WGC in string theory

**Very often** true in string theory:

Lattice Weak Gravity Conjecture:

$\forall Q \in \Gamma, \exists$  a superextremal  
charged particle with charge  $Q$

BH, Reece, Rudelius, '15

# The WGC in string theory

**Very often** true in string theory:

Not always! BH, Reece  
Rudelius, '16

Lattice Weak Gravity ~~Conjecture~~:

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BH, Reece, Rudelius, '15

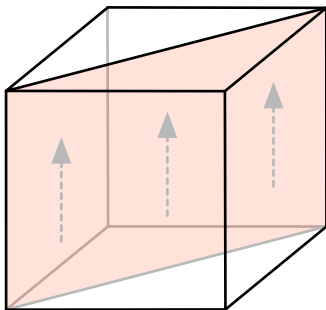
# Counterexample to LWGC

KK theory on  $T^2 / (\mathbb{Z}_2 \times \mathbb{Z}'_2)$

$$\mathbb{Z}_2 : \theta_{1,2} \rightarrow \theta_{1,2} + \pi$$

$$\mathbb{Z}'_2 : \theta_1 \rightarrow -\theta_1, \theta_3 \rightarrow \theta_3 + \pi$$

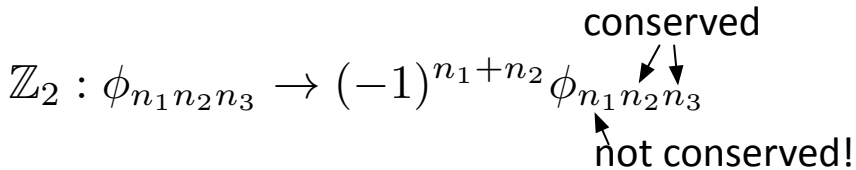
smooth  
(fixed pt free)  
orbifold!



$$\mathbb{Z}_2 : \phi_{n_1 n_2 n_3} \rightarrow (-1)^{n_1 + n_2} \phi_{n_1 n_2 n_3}$$

conserved

not conserved!

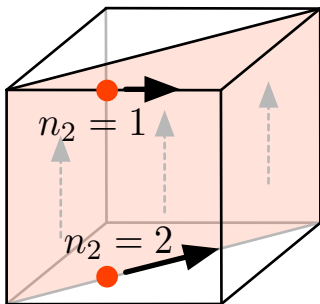


$$\mathbb{Z}_2 : \phi_{n_1 n_2 n_3} \rightarrow (-1)^{n_1 + n_2} \phi_{n_1 n_2 n_3}$$

conserved  
 ↙ ↘  
 not conserved!

$$m^2 = \frac{n_1^2}{R_1^2} + \frac{n_2^2}{R_2^2} + \frac{n_3^2}{R_3^2} > \frac{n_2^2}{R_2^2} + \frac{n_3^2}{R_3^2}$$

for  $n_2 \in 2\mathbb{Z} + 1$

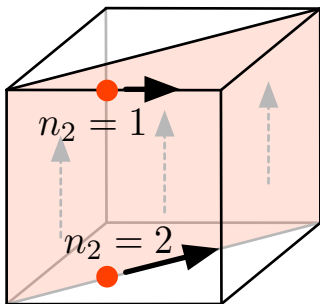


$$\mathbb{Z}_2 : \phi_{n_1 n_2 n_3} \rightarrow (-1)^{n_1 + n_2} \phi_{n_1 n_2 n_3}$$

conserved  
 ↙ ↘  
 not conserved!

$$m^2 = \frac{n_1^2}{R_1^2} + \frac{n_2^2}{R_2^2} + \frac{n_3^2}{R_3^2} > \frac{n_2^2}{R_2^2} + \frac{n_3^2}{R_3^2}$$

for  $n_2 \in 2\mathbb{Z} + 1$



Open question:

Is there some organizing principle to LWGC violation?

# The WGC in string theory

tree-level\*  
**Always** true in string theory:  
^

sub

Lattice Weak Gravity Conjecture:

^

$\exists$  a finite-index sublattice  $\Gamma_0 \subseteq \Gamma$   
s.t.  $\forall Q \in \Gamma_0, \exists$  a superextremal  
charged particle with charge  $Q$

BH, Reece, Rudelius, '16

\*In the electric NSNS sector



# Proof via modular invariance

(BH, Reece, Rudelius '16; Montero, Shiu, Soler '16; BH, Lotito 2207.xxxxx)

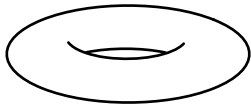
Tree-level spectrum:

$$\frac{\alpha'}{4} m^2 = \frac{1}{2} Q_L^2 + T_L = \frac{1}{2} Q_R^2 + T_R$$

Partition function (torus 0-pt amplitude):

$$Z(\mu, \tau) = \text{Tr}(q^{T + \frac{1}{2} Q^2} y^Q) \quad q = e^{2\pi i \tau}$$

$$Z(\mu + \rho) = Z(\mu), \quad \rho \in \Gamma^* \quad y = e^{2\pi i \mu}$$



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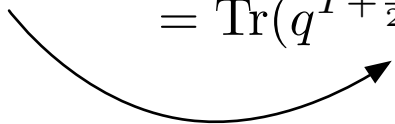
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Benjamin, Dyer, Fitzpatrick, Kachru '16:

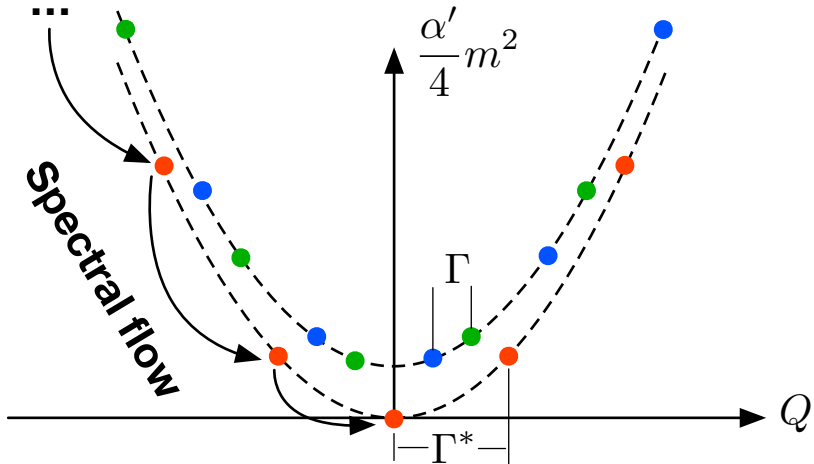
$$Z(\mu/\tau, -1/\tau) = e^{\pi i \frac{\mu^2}{\tau}} Z(\mu, \tau)$$

$$Z = \text{Tr}(q^{T+\frac{1}{2}Q^2} y^Q)$$
$$= \text{Tr}(q^{T+\frac{1}{2}(Q+k\rho)^2} y^{Q+k\rho})$$

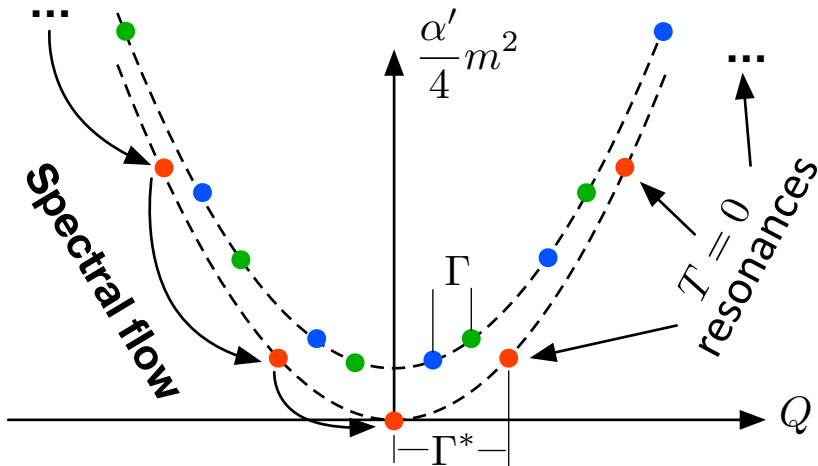


**Spectral flow**

$$\begin{aligned}
 Z &= \text{Tr}(q^{T+\frac{1}{2}Q^2} y^Q) \\
 &= \text{Tr}(q^{T+\frac{1}{2}(Q+k\rho)^2} y^{Q+k\rho})
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# The extremality bound & the RFC

BH, Reece, Rudelius '19; BH, Lotito 2207.xxxxx

Found a tower of resonances with:

$$\frac{\alpha'}{4} m^2 = \frac{1}{2} \max(Q_L^2, Q_R^2)$$

What is the relation to extremal BHs?

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BH, Lotito  
2207.xxxxx

**Proved in  
tree-level ST**



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BH, Lotito  
2207.xxxxx

...actually this proves (\*) is extremal

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What is the relation to extremal BHs?

A particle that is self-repulsive throughout moduli space is superextremal

(BH, Reece, Rudelius '19)

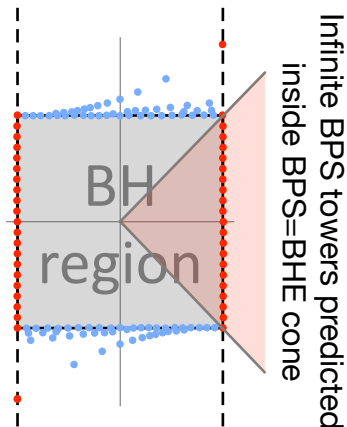
(Harlow, BH, Reece, Rudelius 2201.08380)

# Nonperturbative evidence via BPS states

Alim, BH, Rudelius 2108.08309

Gendler, BH, McAllister, Moritz, Rudelius 22xx.xxxxx

BPS prediction:

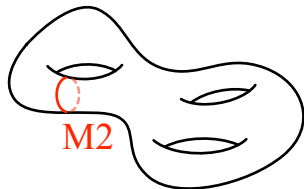
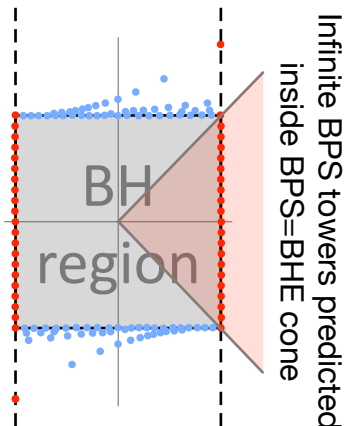


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BPS prediction:



Genus 0 GV invariants:

$q_Y \setminus q_X$	0	1	2	3
0	640	10032	288384	
1	64	6912	742784	75933184
2	0	14400	8271360	2445747712
3	0	6912	31344000	26556152064
4	0	640	48098560	130867460608
5	0	0	31344000	329212616704
6	0	0	8271360	445404149568
7	0	0	742784	329212616704
8	0	0	10032	130867460608
9	0	0	0	26556152064
10	0	0	0	2445747712
11	0	0	0	75933184
12	0	0	0	288384
13	0	0	0	0

$\tilde{X}$   
BPS=BHE cone

$\tilde{Y}$

# Nonperturbative evidence via BPS states

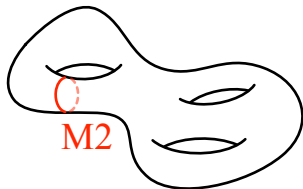
Alim, BH, Rudelius 2108.08309

Gendler, BH, McAllister, Moritz, Rudelius 22xx.xxxxx

WGC



Highly nontrivial  
predictions  
about CY geometry



Genus 0 GV invariants:

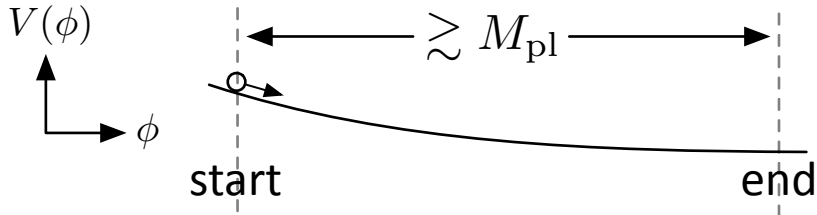
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$\tilde{Y}$  (vertical axis),  $\tilde{X}$  (horizontal axis), BPS=BHE cone (shaded region)

## III. Axion Potentials & the WGC

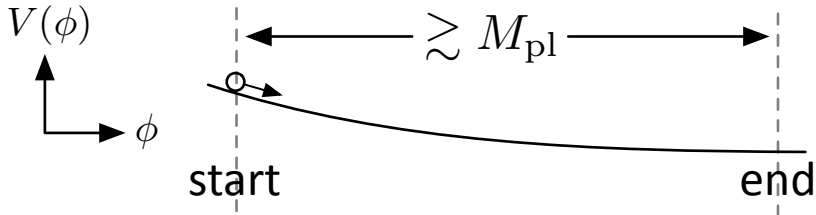
# Transplanckian scalars!?

Can a scalar field have a flat potential over a super-Planckian field range?



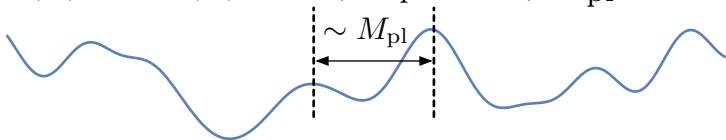
# Transplanckian scalars!?

Can a scalar field have a flat potential over a super-Planckian field range?



Generically **no**:

$$V_{\text{eff}}(\phi) \sim V_0(\phi) + \phi^5/M_{\text{pl}} + \phi^6/M_{\text{pl}}^2 + \dots$$





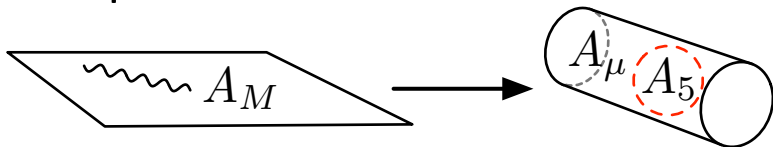
Axion loophole?  $\phi \cong \phi + 2\pi f$

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Example:



photon



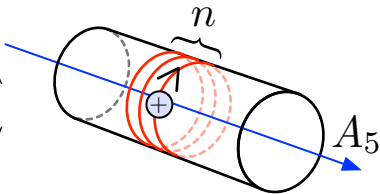
photon+axion

typical in string theory

# Axion Potential

Casimir  
effect

$$\sum_{\text{species}} \sum_n$$



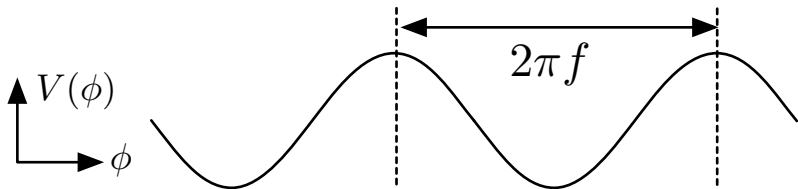
$$V = \sum_i c_i \underbrace{\cos(q_i \phi / f)}_{\text{Aharonov-Bohm phase}} e^{-2\pi R m_i} + \dots$$

Aharonov-Bohm phase

$$(Q_i \propto q_i \in \mathbb{Z}, A_5 \propto \phi)$$

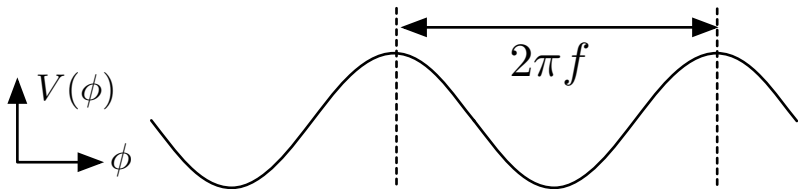
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“Natural inflation” (Freese, Frieman, Olinto '90)

Needs  $f \gtrsim 5-10 M_{\text{pl}}$  for successful inflation

$$V = \sum_i c_i \cos(q_i \phi / f) e^{-2\pi R m_i} + \dots$$

(s)LWGC implies particles with

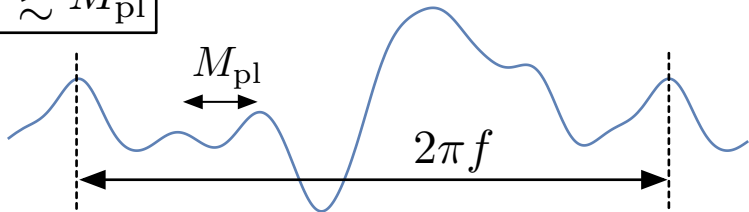
$$2\pi R m_i \lesssim \frac{M_{\text{pl}}}{f} q_i \quad \text{for every } q_i \in \mathbb{Z}$$

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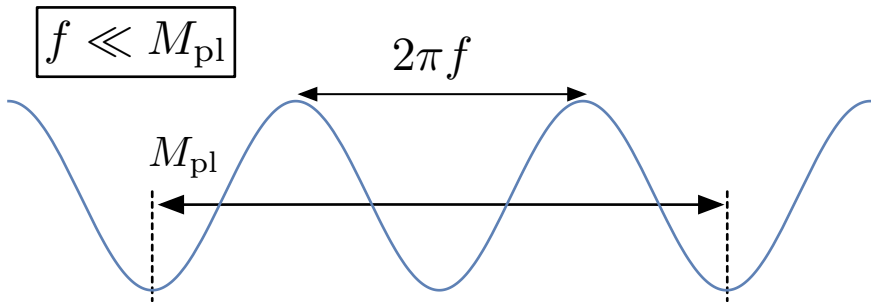
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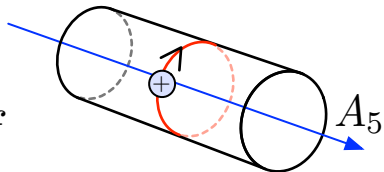


# The generalized WGC

Charged worldline wrapped around circle generates an “instanton”

“Action” is

$$S = 2\pi Rm \lesssim M_{\text{pl}}/f$$

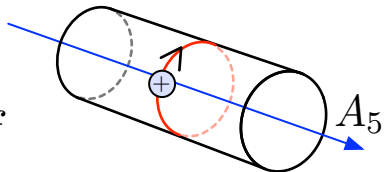


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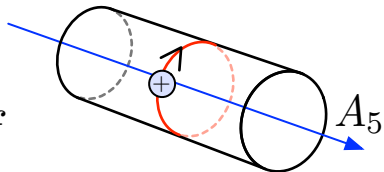
Generates harmonics in the potential, reducing effective field range!

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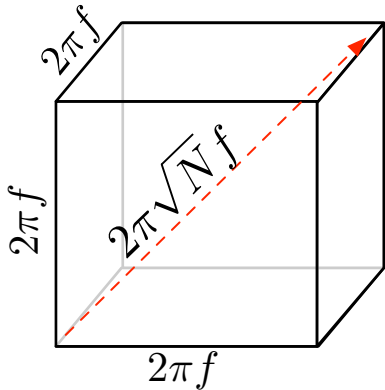
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ST:  $f \lesssim M_{\text{pl}}$  (Banks, Dine, Fox, Gorbatov '03)

1 axion:  $f \lesssim M_{\text{pl}}$

(Banks, Dine, Fox, Gorbатов '03)

$N \gg 1$  axions?



$$f_{\text{eff}} = \sqrt{N} f$$

(Dimopoulos, Kachru,  
McGreevy, Wacker '05)

# Random axion potentials

BH, Long, McAllister, Rudelius, Stout '19

$$\mathcal{L} = -\frac{1}{2}\delta_{ab}\partial_\mu\phi^a\partial^\mu\phi^b - \Lambda^4\sum_{\mathbf{Q}\in\Gamma}Z_{\mathbf{Q}}\exp(i\mathbf{Q}\cdot\phi)$$

$$\langle|Z_{\mathbf{Q}}|^2\rangle = e^{-2\mu|\mathbf{Q}|}$$

Focusing on potential along a ray:

$$\sigma_n^2(\bar{\mathbf{e}}) = \sum_{\mathbf{Q}\in\Gamma}^{\mathbf{Q}\cdot\bar{\mathbf{e}}=n} e^{-2\mu|\mathbf{Q}|}$$

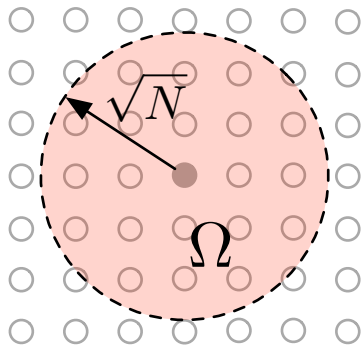
Using a continuum approximation

$$\text{vol } \phi \lesssim \text{vol } D_N(2\mu)$$

**Ball, not cube!**

# Random axion potentials

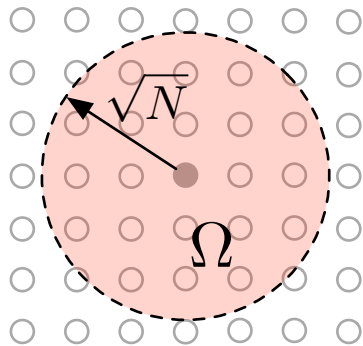
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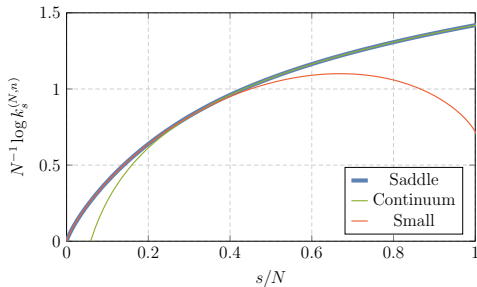
Many instantons contribute!

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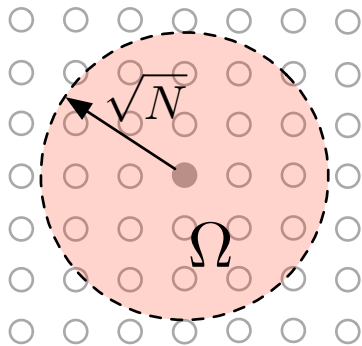
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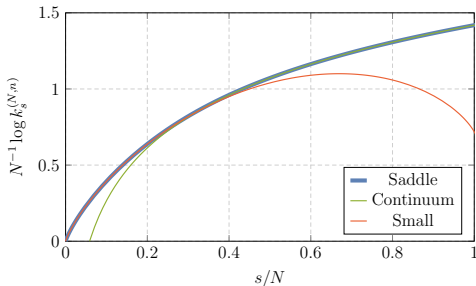
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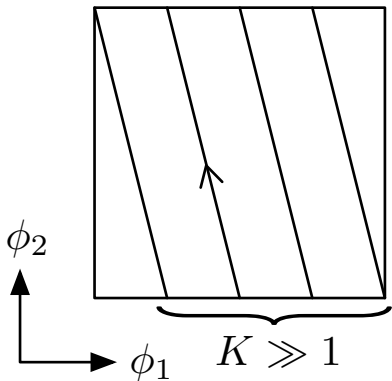
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$$\text{Still find: } f_{\text{eff}} \lesssim M_{\text{pl}}$$



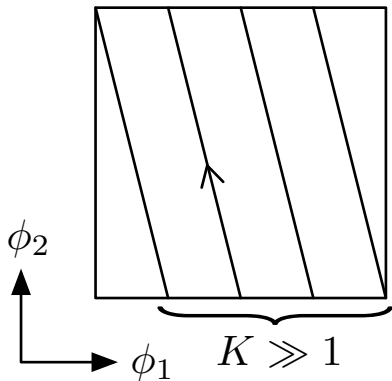
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(Kim, Nilles, Peloso '04)

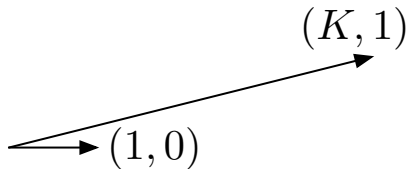


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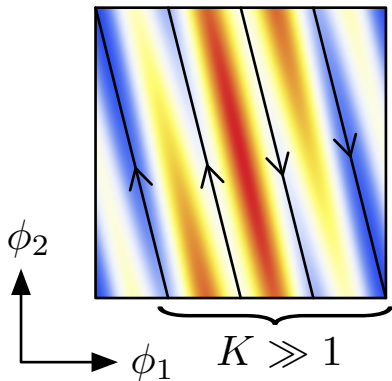


5d charge  
spectrum:

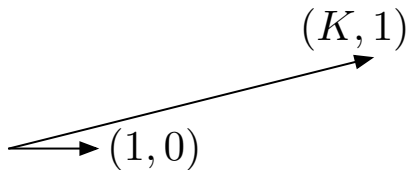


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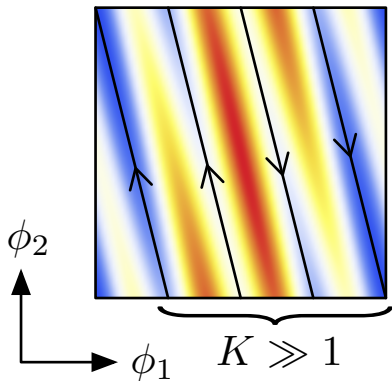
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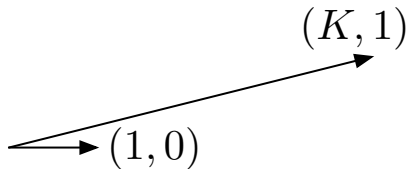
$$f_{\text{eff}} = \sqrt{K^2 + 1} f \simeq K f$$

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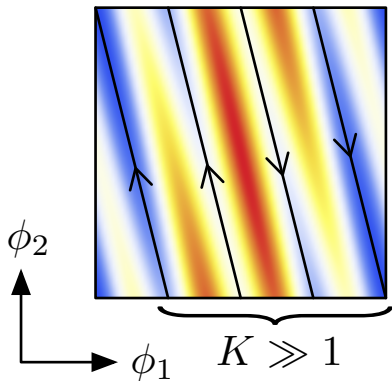
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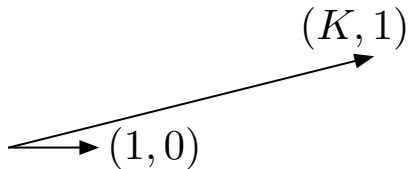
Requires tuning of charged spectrum

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(Kim, Nilles, Peloso '04)



5d charge  
spectrum:



Requires tuning of charged spectrum

Potential alternative: kinetic alignment

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...as well as providing a template for further exploration of the swampland and the landscape...

...even though there is still much more to be learned about the WGC itself!