

# PHYSICS OPPORTUNITIES FOR MUON COLLIDERS

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Pitt PACC, University of Pittsburgh

Colloquium at DESY, April 19, 2022









**With the Higgs discovery, completion of the SM:**

A relativistic, QM, renormalizable, self-consistent theory,  
valid up to an exponentially high scale! ...  $M_{Pl}$  ?

“... most of the grand underlying principles  
have been firmly established. (An eminent  
physicist remarked that) the future truths of  
physical science are to be looked for in the  
sixth place of decimals.”

**--- Albert Michelson (1894)**

Michelson–Morley experiments (1887):

“the moving-off point for the theoretical aspects  
of the second scientific revolution”

**Will History repeat itself (soon)?**



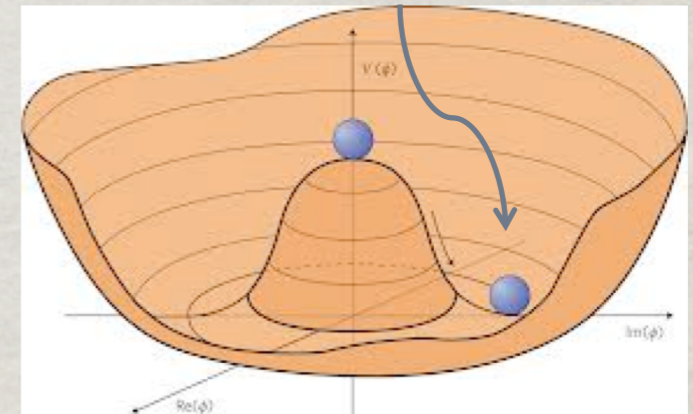
# MOTIVATION FOR ENERGY FRONTIER

## 1. Electroweak Symmetry Breaking, EW Superconductivity & Phase Transition

$$V(|\Phi|) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

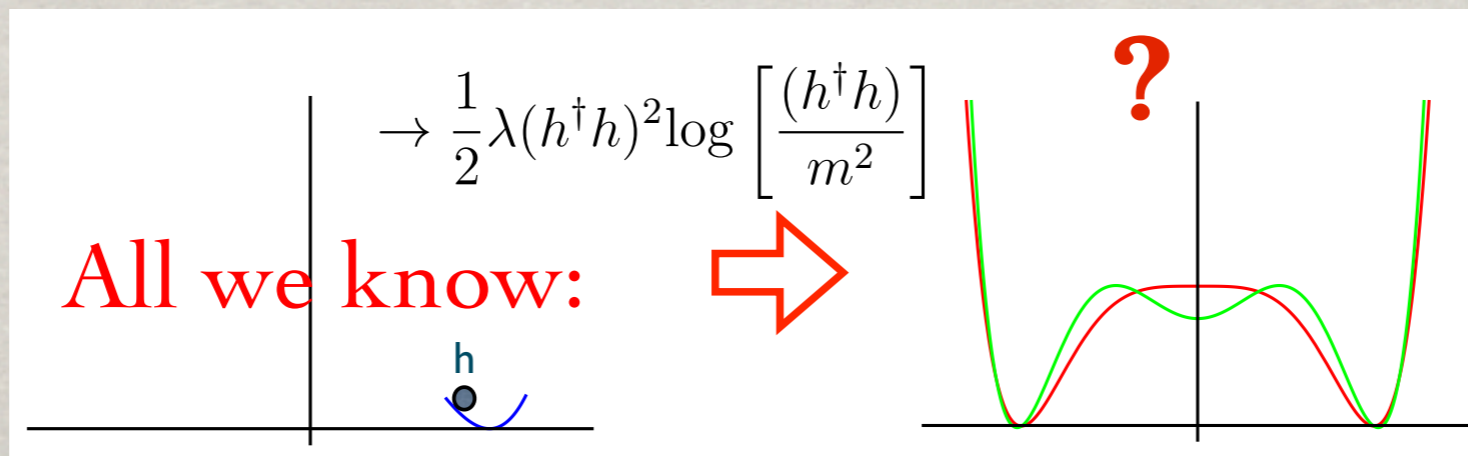
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It's like Landau-Ginzburg theory, but not!



- 😊 It is a relativistic QFT.
- 😞 No EW analogue for BCS as the underlying theory to understand the dynamics, to calculate  $\mu^2(\Lambda^2)$  &  $\lambda$ .

And the potential shape  $\rightarrow$  early universe cosmology!

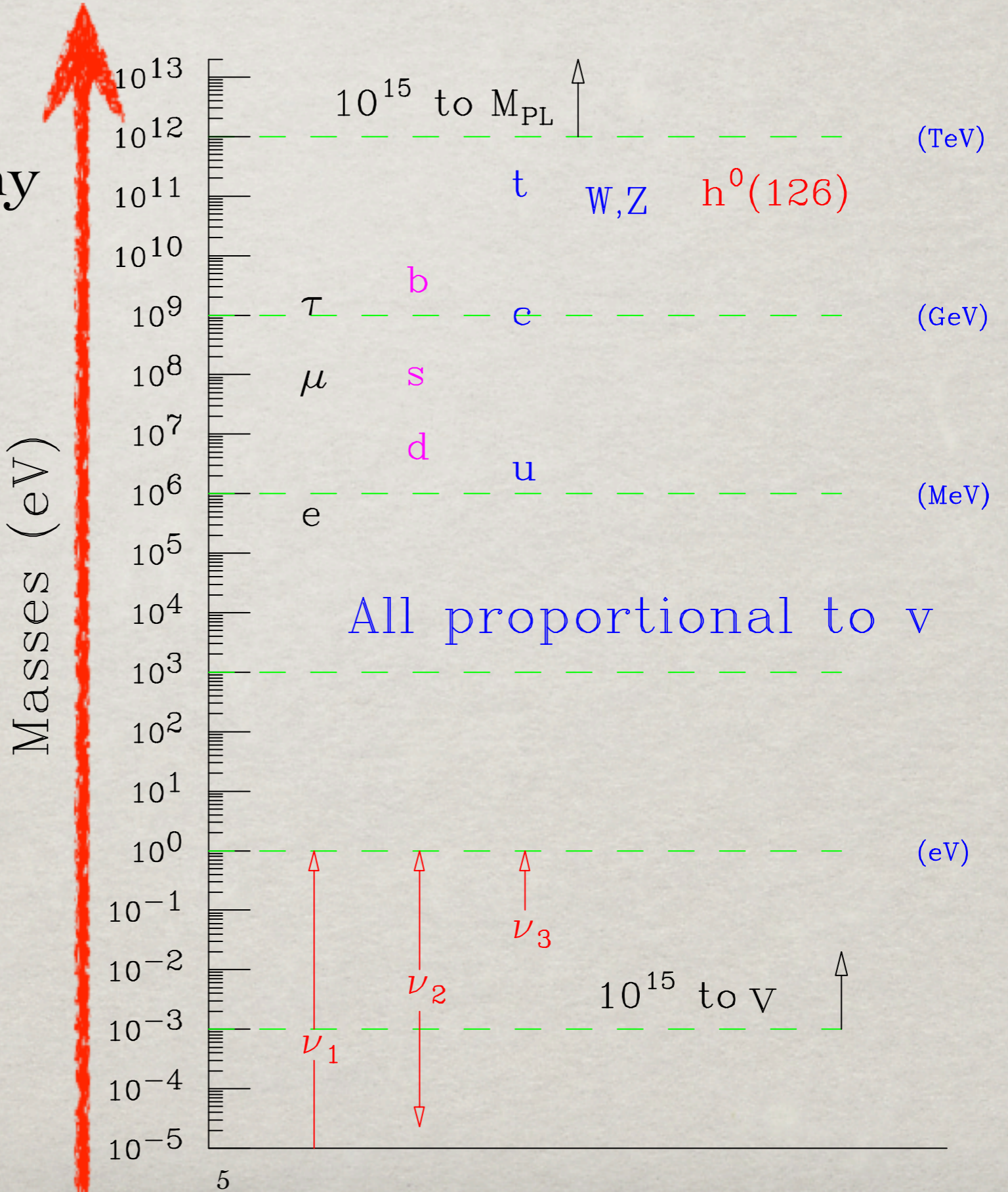




# 2. The “Flavor Puzzle”: fermion mass/mixing

- Particle mass hierarchy
- Patterns of quark, neutrino mixings
- Neutrino mass generation (seesaw)
- New CP-violation sources

Higgs is in a pivotal position.



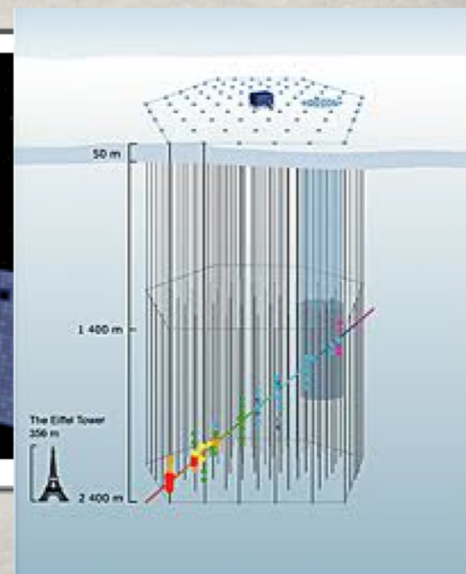
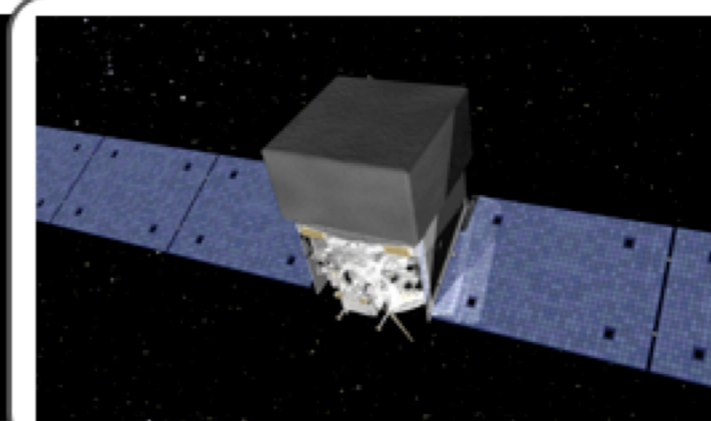
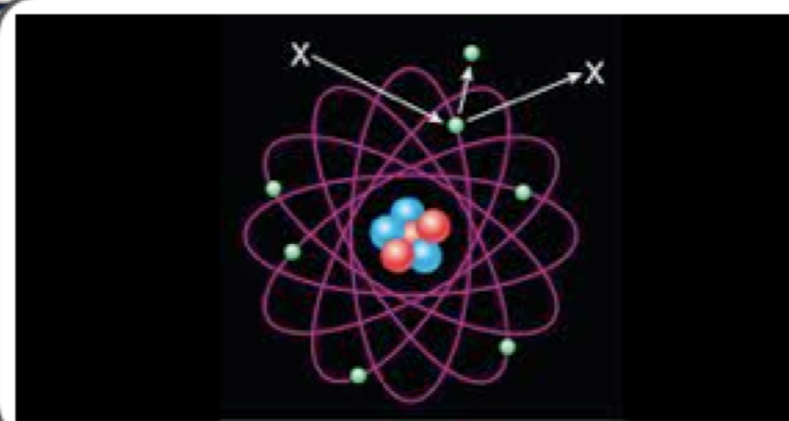
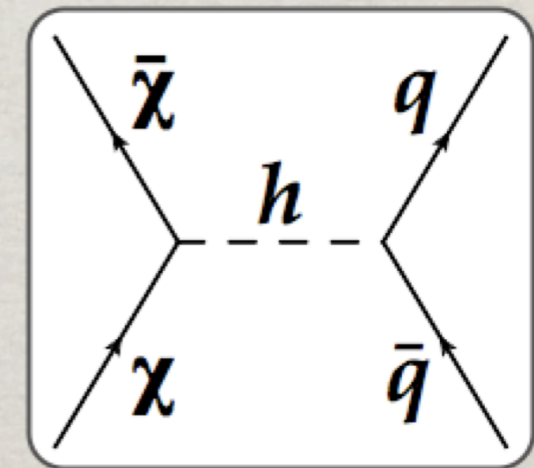
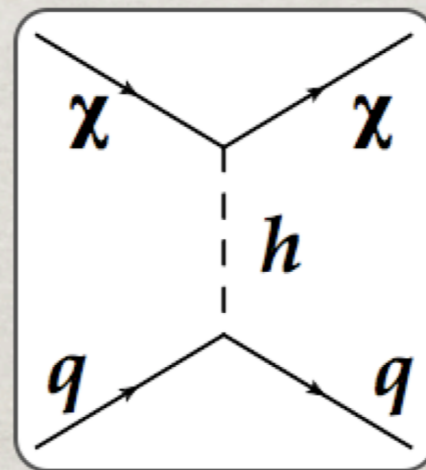
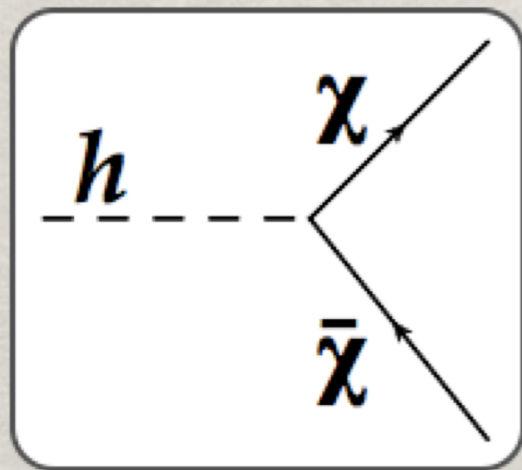


# 3. The Dark Sector: WIMP DM?

The nature of DM is among the most pressing issue.  
Weakly Interacting DM strongly motivated.

$H^\dagger H$  Higgs portal:  $k_s H^\dagger H S^* S, \frac{k_\chi}{\Lambda} H^\dagger H \bar{\chi} \chi.$

Dark matter at colliders    Direct detection    Indirect detection

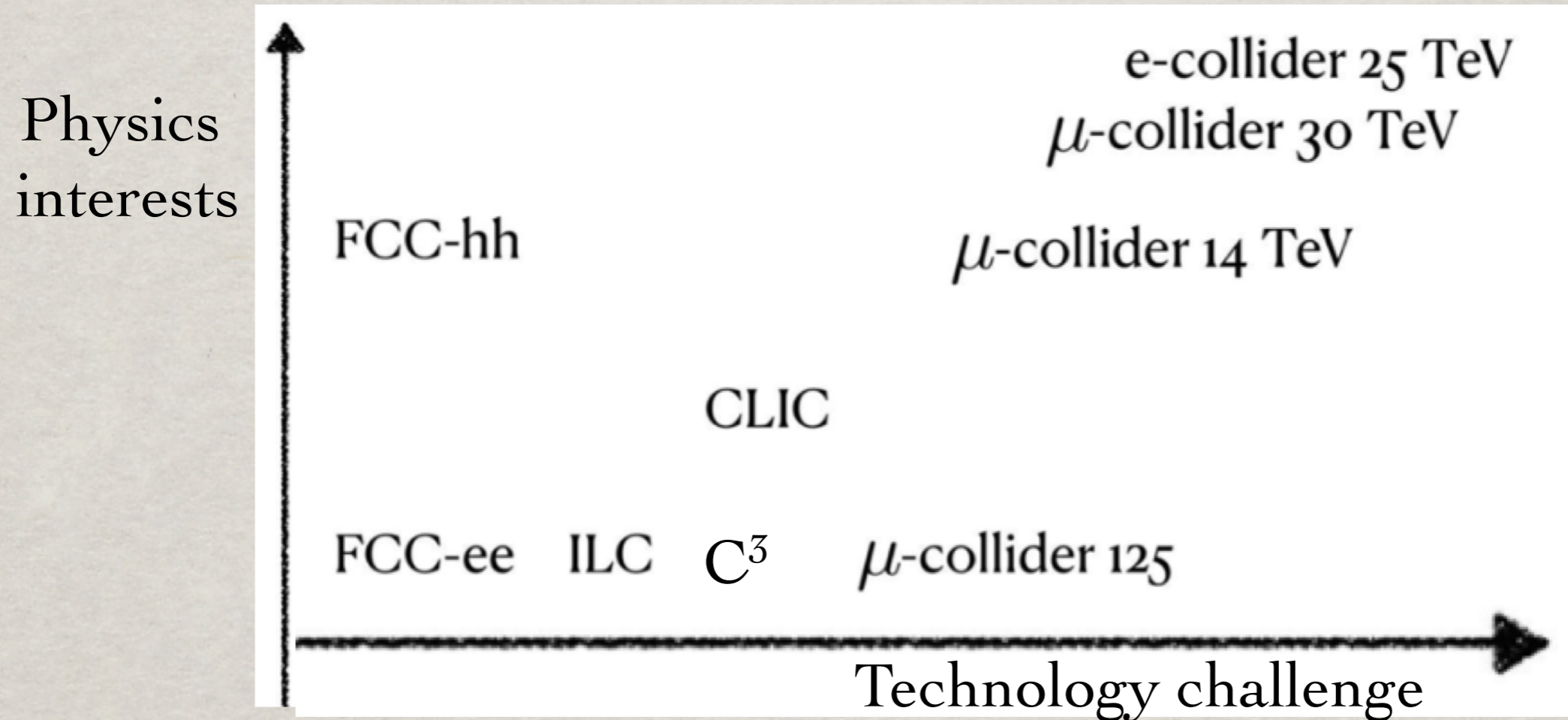




# Snowmass on Future Colliders:

Energy Frontier / Accelerator Frontier

<https://snowmass21.org>



Renewed interests:

- Muon colliders
- $C^3$ : Cool Copper Collider (Linear  $e^+e^-$ )



# A MUON COLLIDER

## Why muons?

Although sharing the same EW interactions,  
it isn't another electron:

$$m_{\mu} \approx 207 m_e$$

$$\tau(\mu \rightarrow e\bar{\nu}_e\nu_{\mu}) \approx 2.2 \mu s$$

$$c\tau \approx 660 m.$$

It is these features: heavy mass, short lifetime  
that dictate the physics.

Some early work in the 90's :

- *S-channel Higgs boson production at a muon collider*, Barger et al., PRL75 (1995).
- *$\mu^+ \mu^-$  Collider: Feasibility study*, Muon collider collaboration (July, 1996).
- *Higgs boson physics in the s-channel muon collider*, Barger et al., Phys Rep. 186 (1997).
- *Status of muon collider research*, Muon collider collaboration (Aug., 1999).
- *Recent progress on neutrino factory and muon collider research*, Muon collider collaboration (July, 2003).



- 😊 **Advantages of a muon collider**

- Much less synchrotron radiation energy loss than e's:

$$\Delta E \sim \frac{1}{R} \left( \frac{E}{m_\mu} \right)^4$$

which would allow a smaller and a circular machine:

- Unlike the proton as a composite particle,  
 $E_{CM}$  efficient in  $\mu^+\mu^-$  annihilation

- Much smaller beam-energy spread:

$$\Delta E/E \sim 0.01\% - 0.001\%$$



- 😞 **Disadvantages of a muon collider**

- Production: Protons on target  $\rightarrow$  pions  $\rightarrow$  muons:  
Require sophisticated scheme for  $\mu$  capture & transport

“Never play with an unstable thing!”

- Very short lifetime: in micro-second,

**Muons cooling in (x,p) 6-dimensions**

$\rightarrow$  Difficult to make quality beams and a high luminosity

[Note 👍:  $E_\mu \sim 1 \text{ TeV} \rightarrow \gamma \sim 10^4 \rightarrow \gamma\tau = 0.02 \text{ s} \rightarrow d=6,000 \text{ km}$ ]

- Beam Induced Backgrounds (BIB)

from the decays in the ring at the interacting point,

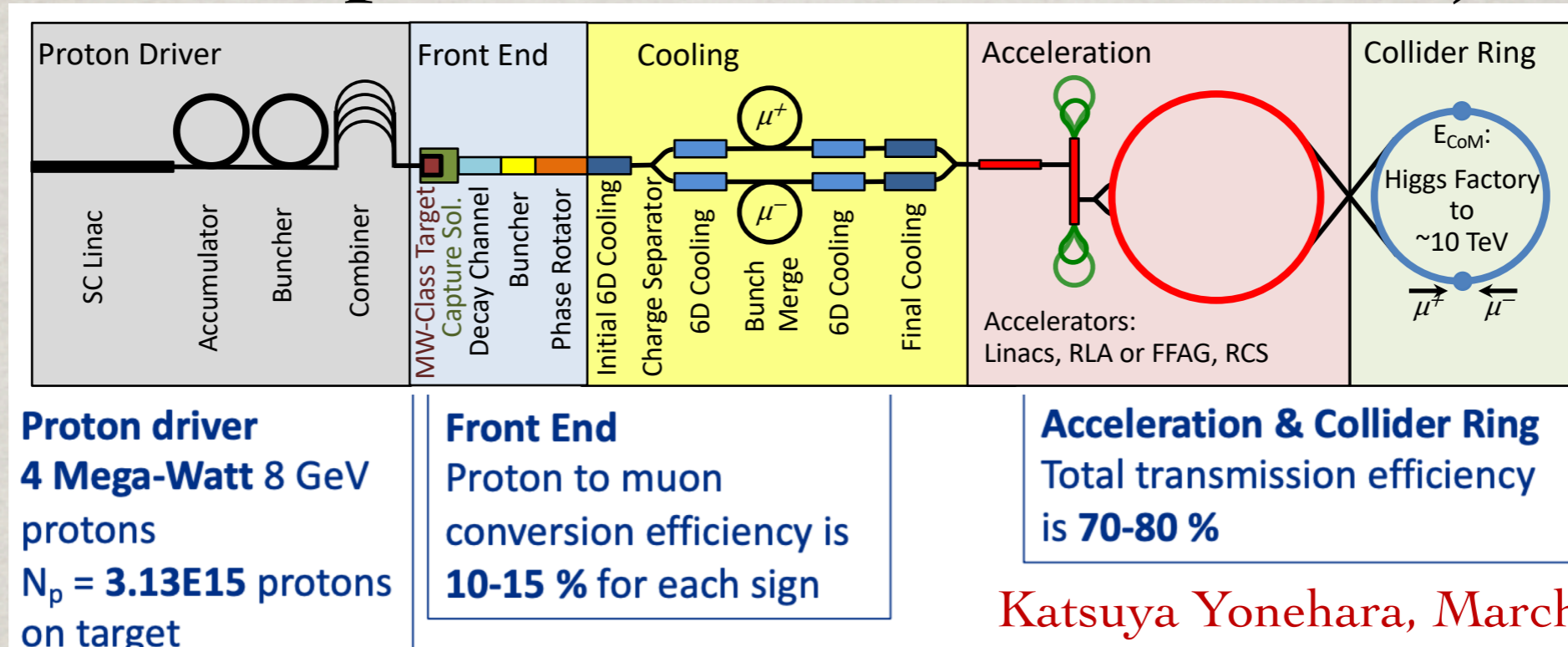
[Note 👍:  $\sigma_{pp}(\text{total}) \sim 100 \text{ mb}$ ;  $\sigma_{\mu\mu}(\text{total}) \sim 100 \text{ nb}$ ]

- Neutrino beam dump (environmental hazard)

$\sigma_\nu \sim G_F^2 E^2 \rightarrow \text{Shielding?}$



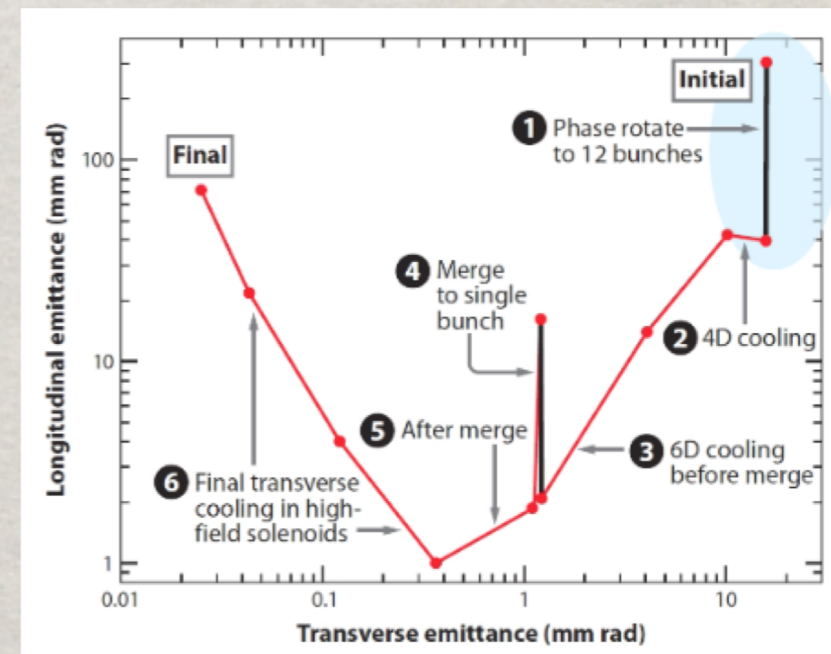
# Proton Driver Option: Muon Accelerator Project (MAP)



Katsuya Yonehara, March 2022

During 2011-2016, MAP collaboration formed:  
to address key feasibility issues for  $\mu C$

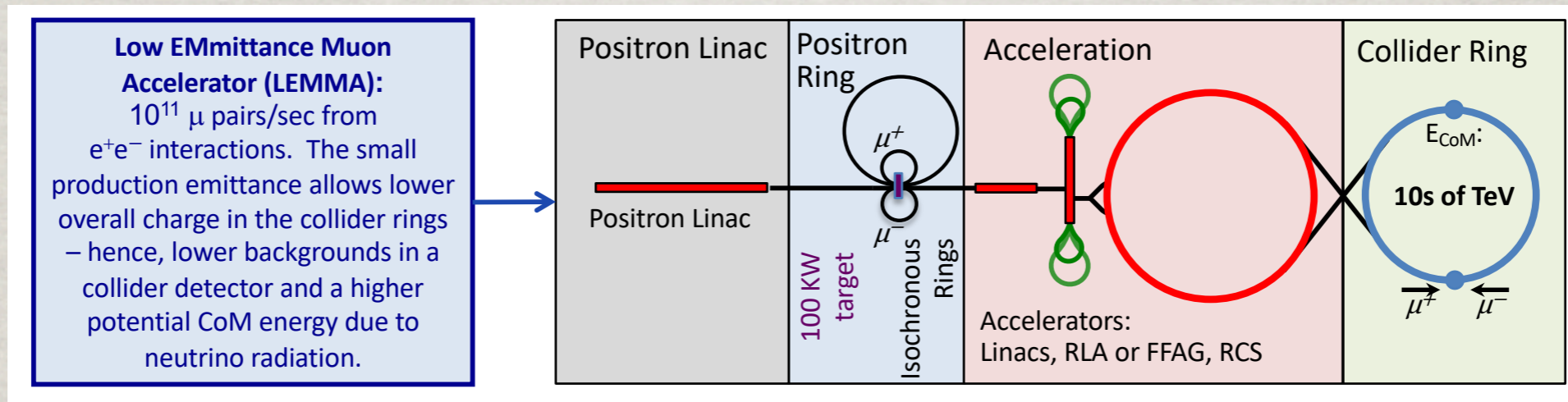
- Protons  $\rightarrow$  pions  $\rightarrow$  muons
- Transverse ionization cooling achieved by MICE
- Muon emittance exchange demonstrated at FNAL/RAL
- 6D cooling of 5-6 orders needed



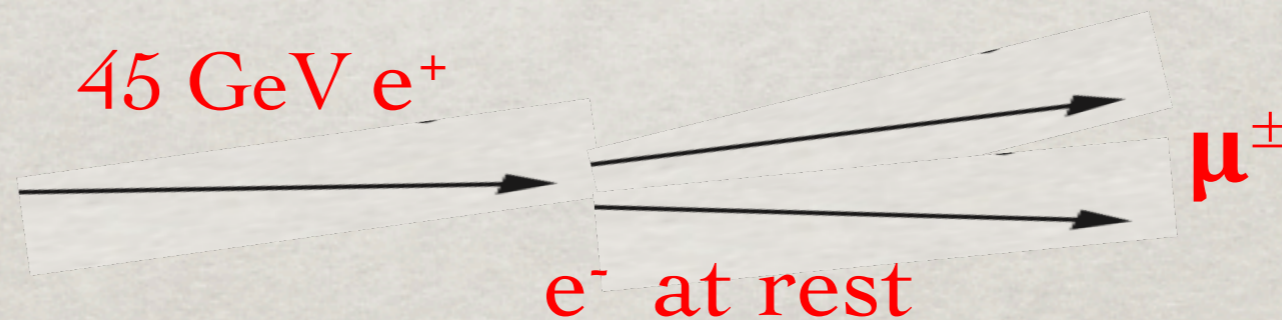
<https://arxiv.org/abs/1907.08562>, J.P. Delahaige et al., arXiv:1901.06150/



**LEMMA:**  $e^+e^-$  (at rest)  $\rightarrow \mu^+\mu^-$  (at threshold)



Low **EM**ittance **M**uon **A**ccelerator  
[web.infn.it/LEMMA](http://web.infn.it/LEMMA)



Cooling is not a problem;  
 but high luminosity is challenging!

J.P. Delahauge et al., arXiv:1901.06150



<https://muoncollider.web.cern.ch>

Fermilab on site:

## Site filler Accelerator

➤ **Largest**

**Radius is ~2.65 km**

- **~16.5 km Circumference**
- **~2/3 LHC**

**~RCS accelerator**

If  $B_{ave} = 3 T \rightarrow E_{\mu} = 2.4 TeV$   
 ( $B_{max} = 8T, B_{pulse} = \pm 2T$ )

**Doubled ?**

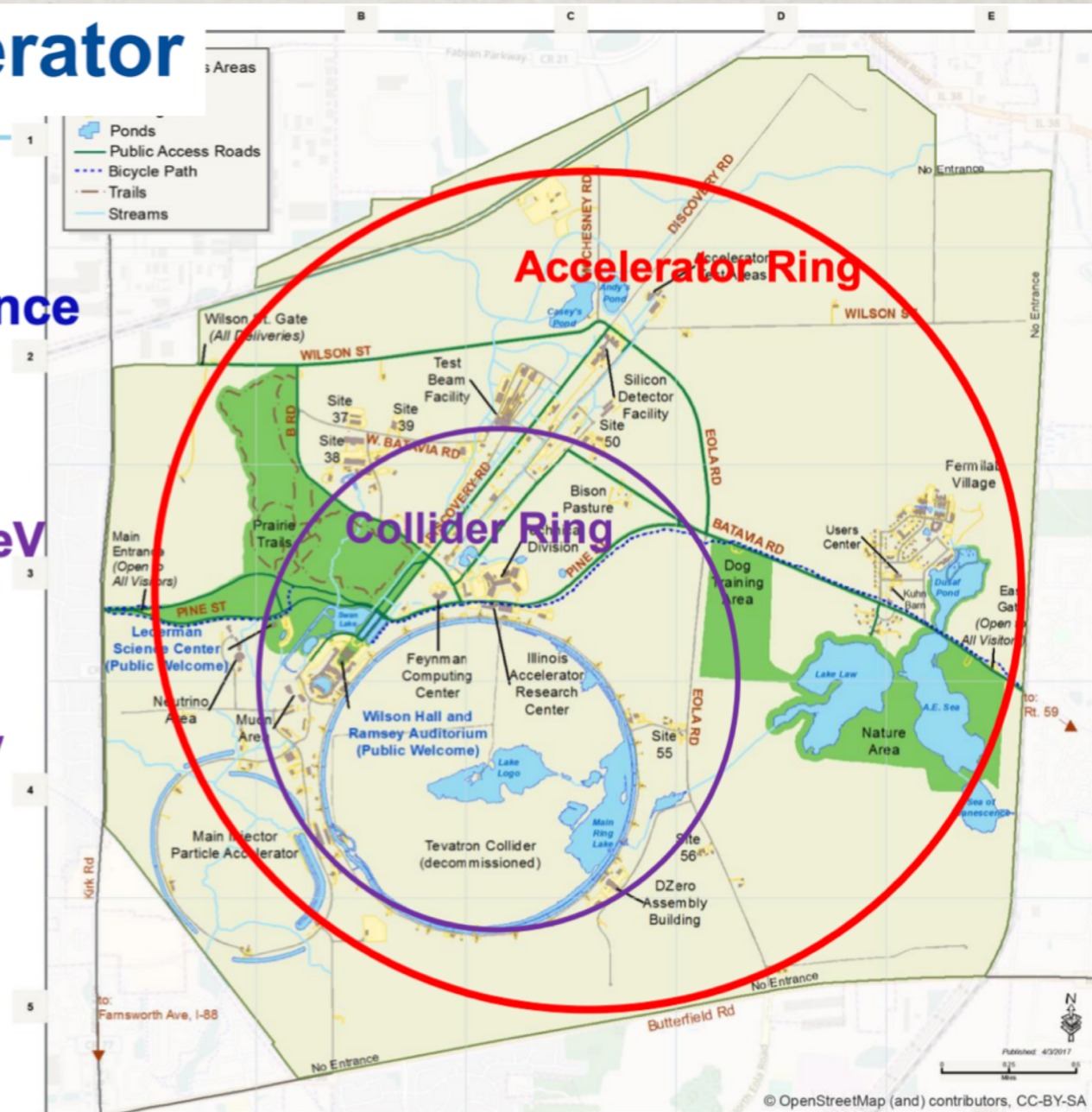
$B_{ave} = 6.3 T \rightarrow E_{\mu} = 5 TeV$   
 ( $B_{max} = 16T, B_{pulse} = \pm 4T$ )

**10 TeV collider**

**Collider Ring ~10 km**

$B_{ave} = 10 T$

$\tau_{\mu} = 0.104 s$



Daniel Schulte; Mark Palmer; Katsuya Yonehara talk, March 2022



# Collider benchmark points:

- The Higgs factory:

$$E_{\text{cm}} = m_H$$

$$L \sim 1 \text{ fb}^{-1}/\text{yr}$$

$$\Delta E_{\text{cm}} \sim 5 \text{ MeV}$$

Current Snowmass 2021 point:  $4 \text{ fb}^{-1}/\text{yr}$

Parameter	Units	Higgs
CoM Energy	TeV	0.126
Avg. Luminosity	$10^{34} \text{ cm}^{-2} \text{ s}^{-1}$	0.008
Beam Energy Spread	%	0.004
Higgs Production/ $10^7$ sec		13'500
Circumference	km	0.3

- Multi-TeV colliders:

Lumi-scaling scheme:  $\sigma L \sim \text{const.}$

$$L \gtrsim \frac{5 \text{ years}}{\text{time}} \left( \frac{\sqrt{s}_\mu}{10 \text{ TeV}} \right)^2 2 \cdot 10^{35} \text{ cm}^{-2} \text{ s}^{-1} \quad 1 \text{ ab}^{-1} / \text{yr}$$

The aggressive choices:

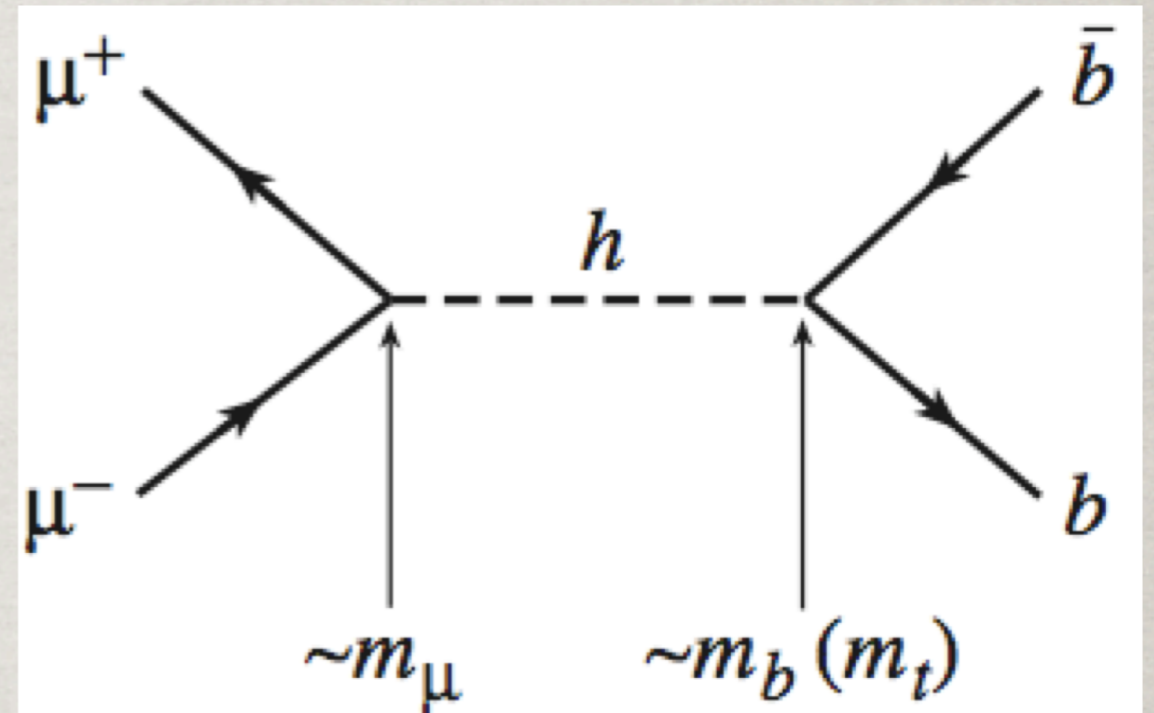
$$\sqrt{s} = 3, 6, 10, 14, 30 \text{ and } 100 \text{ TeV}, \quad \mathcal{L} = 1, 4, 10, 20, 90, \text{ and } 1000 \text{ ab}^{-1}$$

European Strategy, arXiv:1910.11775; arXiv:1901.06150; arXiv:2007.15684.



# A HIGGS FACTORY

Resonant Production:



$$\sigma(\mu^+ \mu^- \rightarrow h \rightarrow X) = \frac{4\pi \Gamma_h^2 \text{Br}(h \rightarrow \mu^+ \mu^-) \text{Br}(h \rightarrow X)}{(\hat{s} - m_h^2)^2 + \Gamma_h^2 m_h^2}.$$

$$\begin{aligned} \sigma_{peak}(\mu^+ \mu^- \rightarrow h) &= \frac{4\pi}{m_h^2} \text{BR}(h \rightarrow \mu^+ \mu^-) \\ &\approx 71 \text{ pb at } m_h = 125 \text{ GeV.} \end{aligned}$$

About **O(70k)** events produced per **fb<sup>-1</sup>**



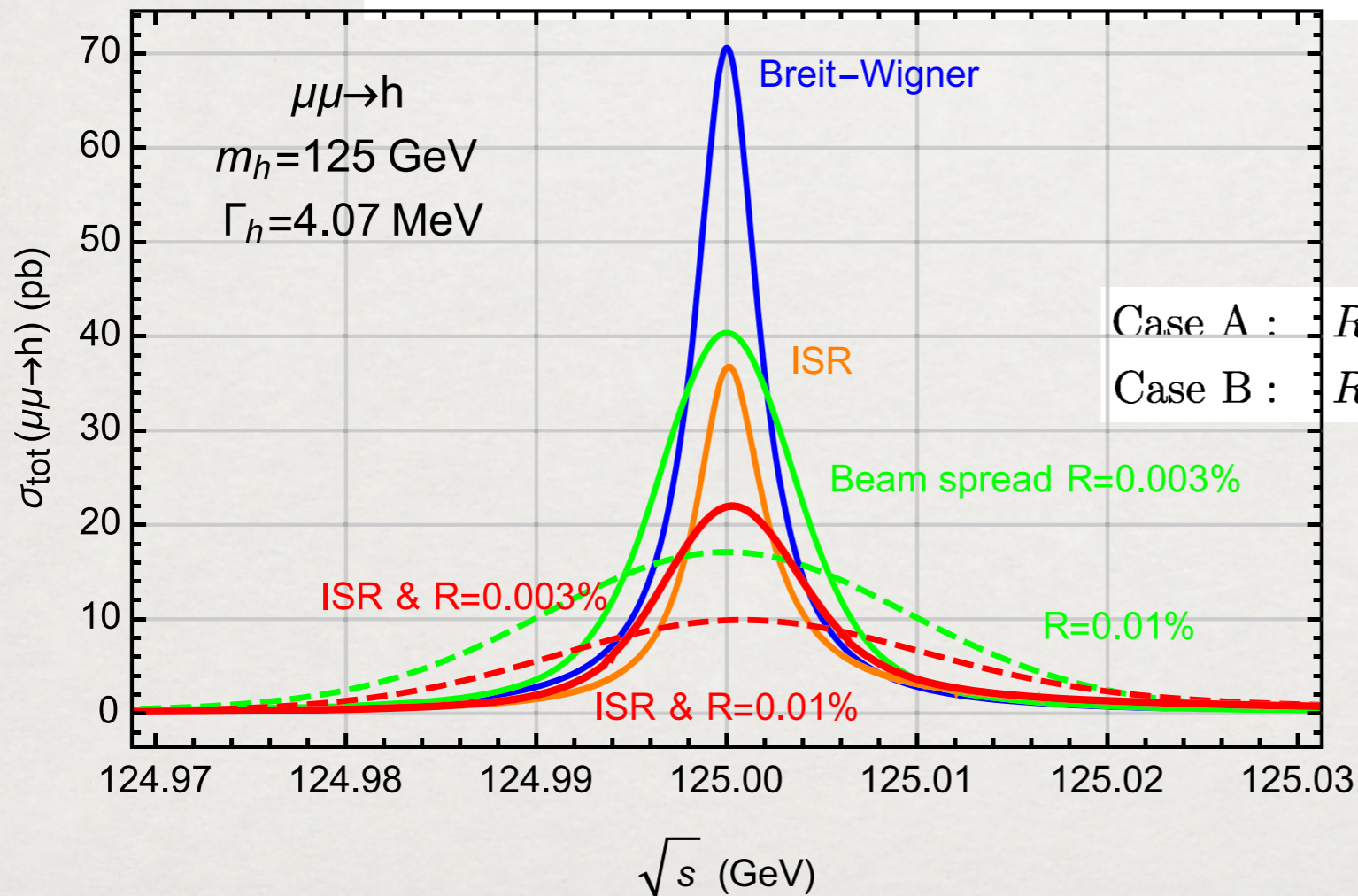
# At $m_h=125$ GeV, $\Gamma_h=4.2$ MeV

$$\frac{\exp[-(\sqrt{\hat{s}} - \sqrt{s})^2/(2\sigma_{\sqrt{s}}^2)]}{\sqrt{2\pi}\sigma_{\sqrt{s}}}$$

$$\frac{4\pi\Gamma(h \rightarrow \mu\mu)\Gamma(h \rightarrow X)}{(\hat{s} - m_h^2)^2 + m_h^2[\Gamma_h^{\text{tot}}]^2}$$

$$\sigma_{\text{eff}}(s) = \int d\sqrt{\hat{s}} \frac{dL(\sqrt{s})}{d\sqrt{\hat{s}}} \sigma(\mu^+ \mu^- \rightarrow h \rightarrow X)$$

$$\propto \begin{cases} \Gamma_h^2 B / [(s - m_h^2)^2 + \Gamma_h^2 m_h^2] & (\Delta \ll \Gamma_h), \\ B \exp\left[-\frac{(m_h - \sqrt{s})^2}{2\Delta^2}\right] \left(\frac{\Gamma_h}{\Delta}\right) / m_h^2 & (\Delta \gg \Gamma_h). \end{cases}$$



**“Muon Collider Quartet”:**  
 Barger-Berger-Gunion-Han  
 PRL & Phys. Report (1995)

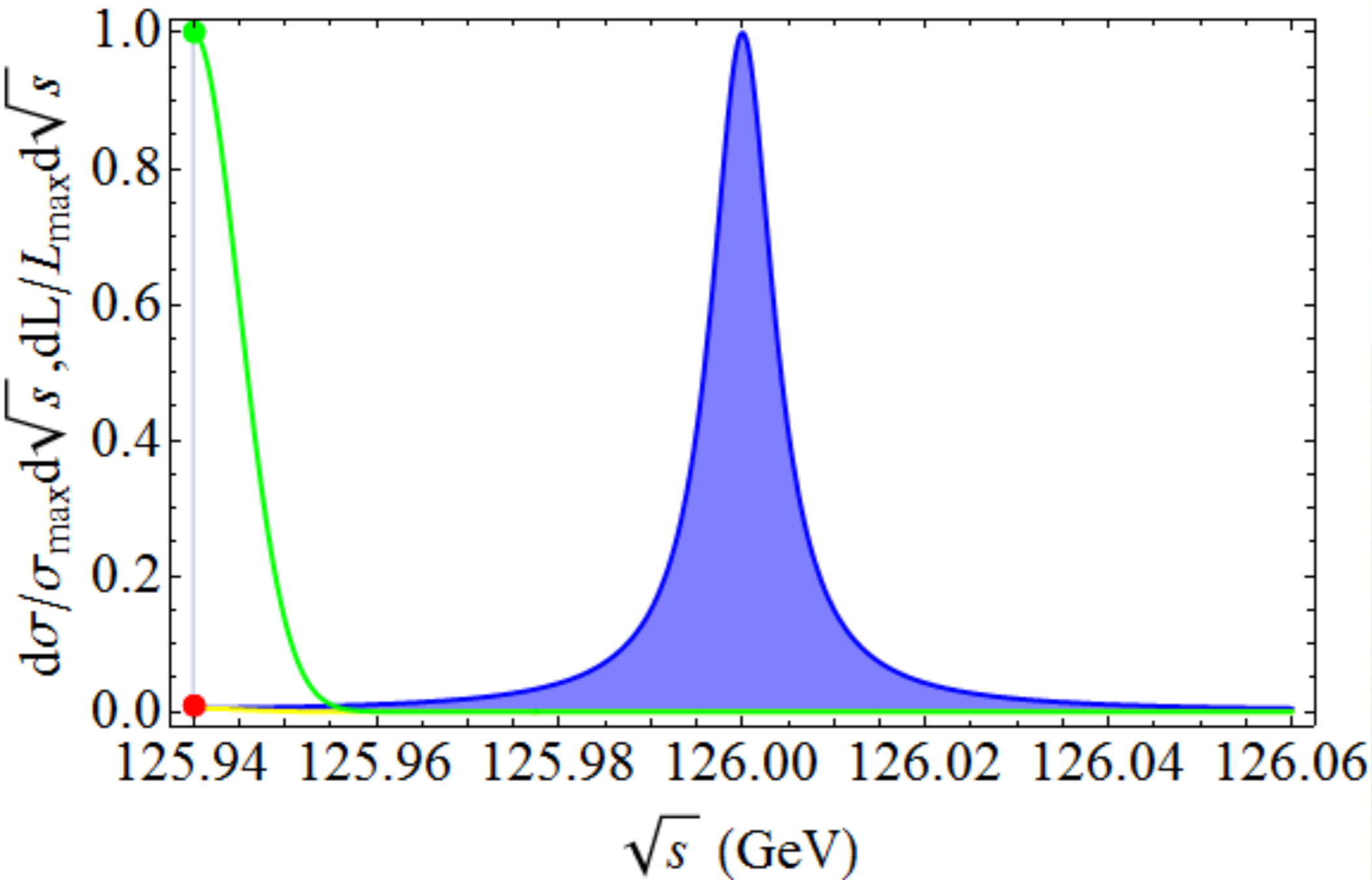
Case A :  $R = 0.01\%$  ( $\Delta = 8.9$  MeV),  $L = 0.5$  fb $^{-1}$ ,  
 Case B :  $R = 0.003\%$  ( $\Delta = 2.7$  MeV),  $L = 1$  fb $^{-1}$ .

TH, Liu: 1210.7803;  
 Greco, TH, Liu: 1607.03210



# Ideal, conceivable case:

$$(\Delta = 5 \text{ MeV}, \quad \Gamma_h \approx 4.2 \text{ MeV})$$



An optimal fitting would reveal  $\Gamma_h$



# Achievable accuracy at the Higgs factory:

TABLE I. Effective cross sections (in pb) at the resonance  $\sqrt{s} = m_h$  for two choices of beam energy resolutions  $R$  and two leading decay channels, with the SM branching fractions  $\text{Br}_{b\bar{b}} = 56\%$  and  $\text{Br}_{WW^*} = 23\%$  [9]. **a cone angle cut:  $10^\circ < \theta < 170^\circ$**

R (%)	$\mu^+ \mu^- \rightarrow h$	$h \rightarrow b\bar{b}$		$h \rightarrow WW^*$	
	$\sigma_{\text{eff}}$ (pb)	$\sigma_{\text{Sig}}$	$\sigma_{\text{Bkg}}$	$\sigma_{\text{Sig}}$	$\sigma_{\text{Bkg}}$
0.01	16	7.6		3.7	
0.003	38	18	15	5.5	0.051

**Good  $S/B > 1$ ,  $S/\sqrt{B} \rightarrow$  % accuracies**

**Table 3**

Fitting accuracies for one standard deviation of  $\Gamma_h$ ,  $B$  and  $m_h$  of the SM Higgs with the scanning scheme for two representative luminosities per step and two benchmark beam energy spread parameters.

$\Gamma_h = 4.07$ MeV	$L_{\text{step}}$ ( $\text{fb}^{-1}$ )	$\delta\Gamma_h$ (MeV)	$\delta B$	$\delta m_h$ (MeV)
$R = 0.01\%$	0.05	0.79	3.0%	0.36
	0.2	0.39	1.1%	0.18
$R = 0.003\%$	0.05	0.30	2.5%	0.14
	0.2	0.14	0.8%	0.07

**$\sim 3.5\%$**

TH, Liu: 1210.7803;

Greco, TH, Liu: 1607.03210



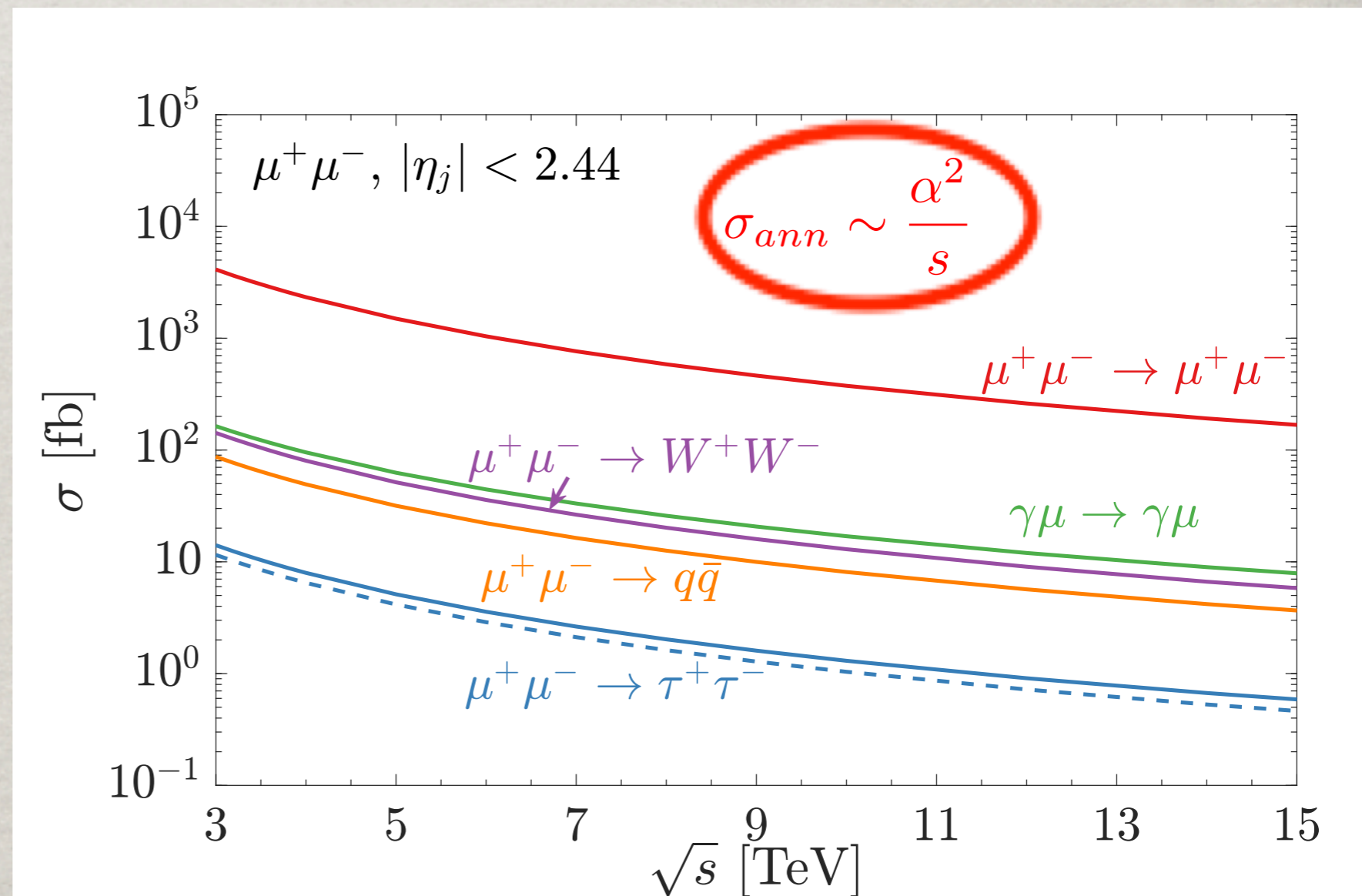
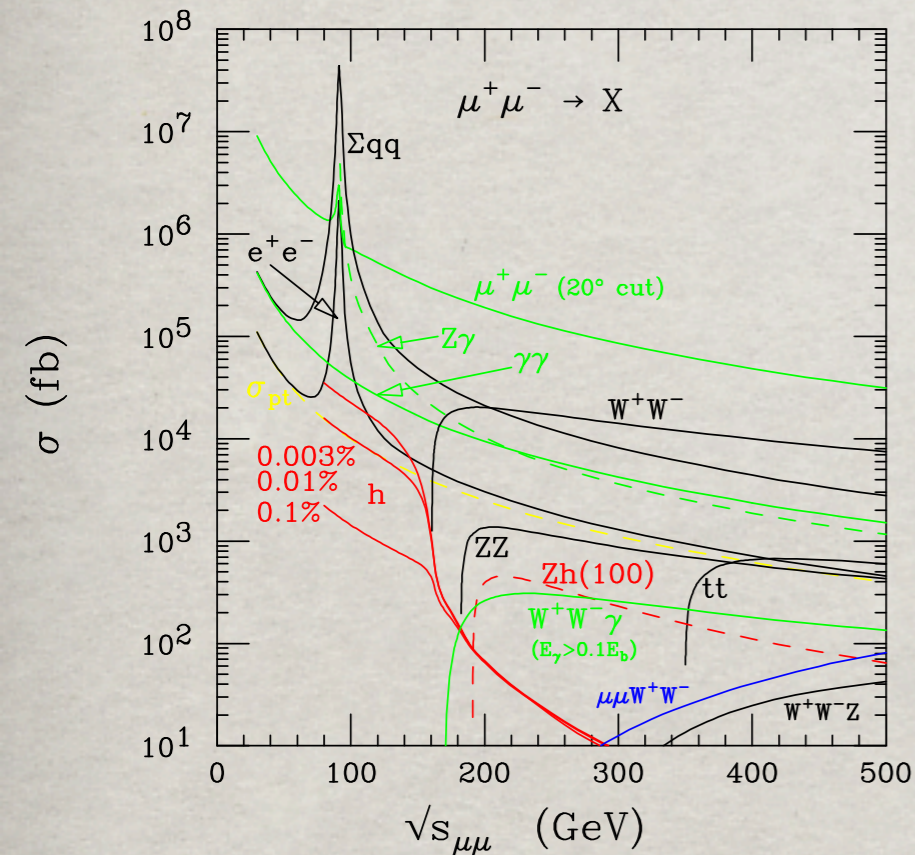
# A MULTI-TeV MUON COLLIDER

Exciting new energy-frontier!

$$\frac{v}{E} : \frac{v (250 \text{ GeV})}{10 \text{ TeV}} \approx \frac{\Lambda_{QCD} (300 \text{ MeV})}{10 \text{ GeV}}$$

$$v/E, m_t/E, M_W/E \rightarrow 0!$$

Leading-order  $\mu^+\mu^-$  annihilation:

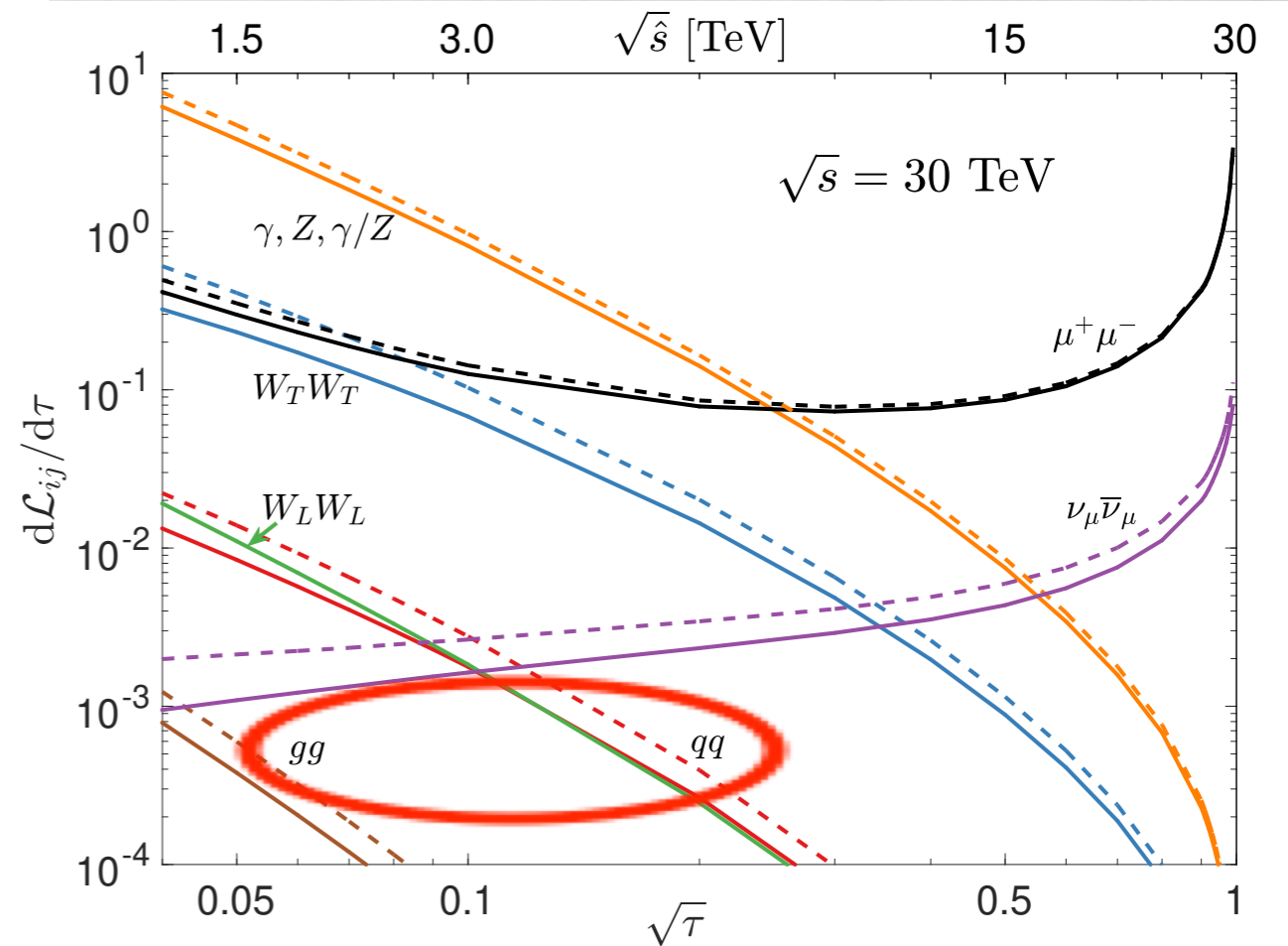
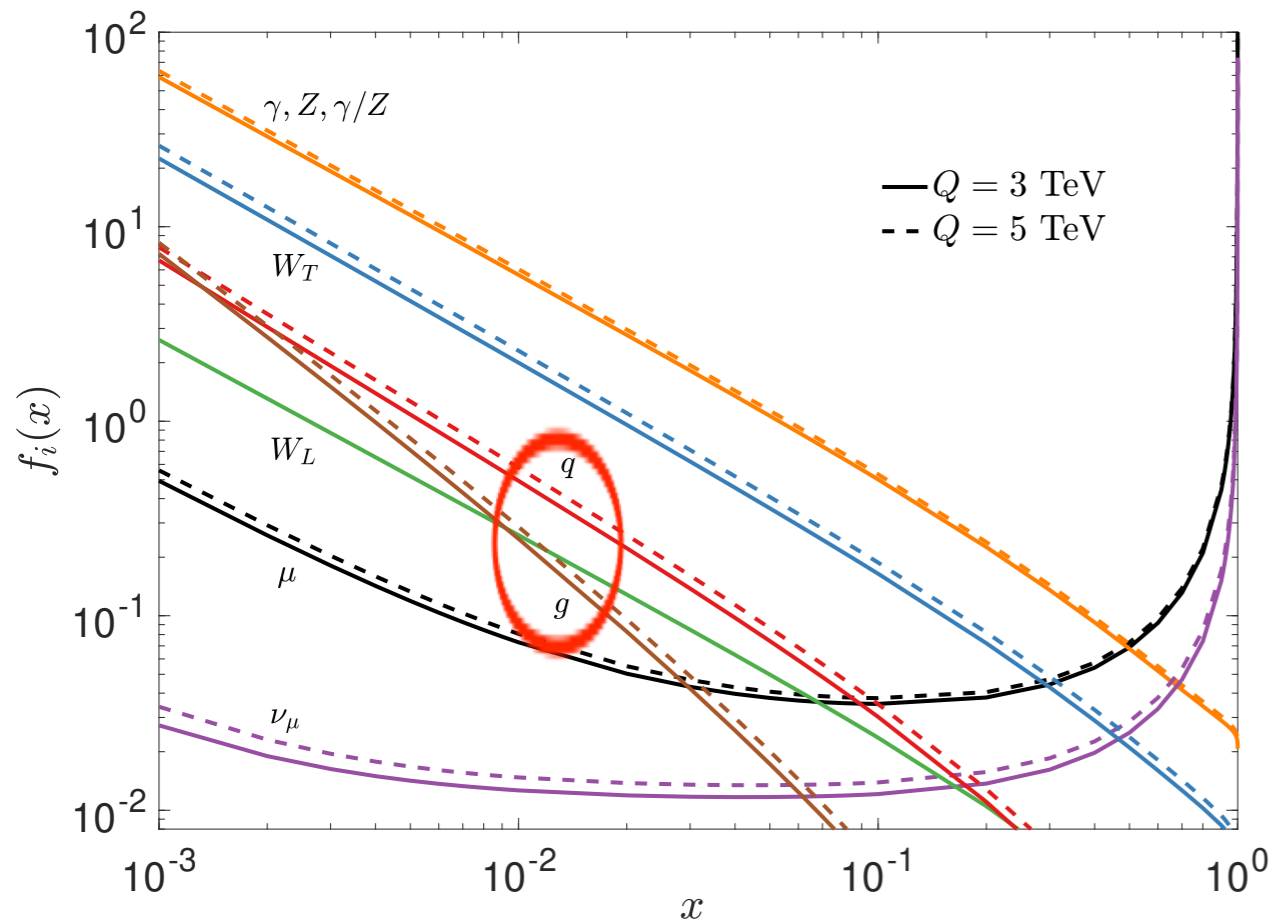




# • EW PDFs at a muon collider:

Collinear splitting phenomena dominate,

“partons” dynamically generated:  $\frac{df_i}{d \ln Q^2} = \sum_I \frac{\alpha_I}{2\pi} \sum_j P_{i,j}^I \otimes f_j$



$\mu^\pm$ : the valance.  $\ell_R, \ell_L, \nu_L$  and  $B, W^{\pm,3}$ : LO sea.

Quarks: NLO; gluons: NNLO.

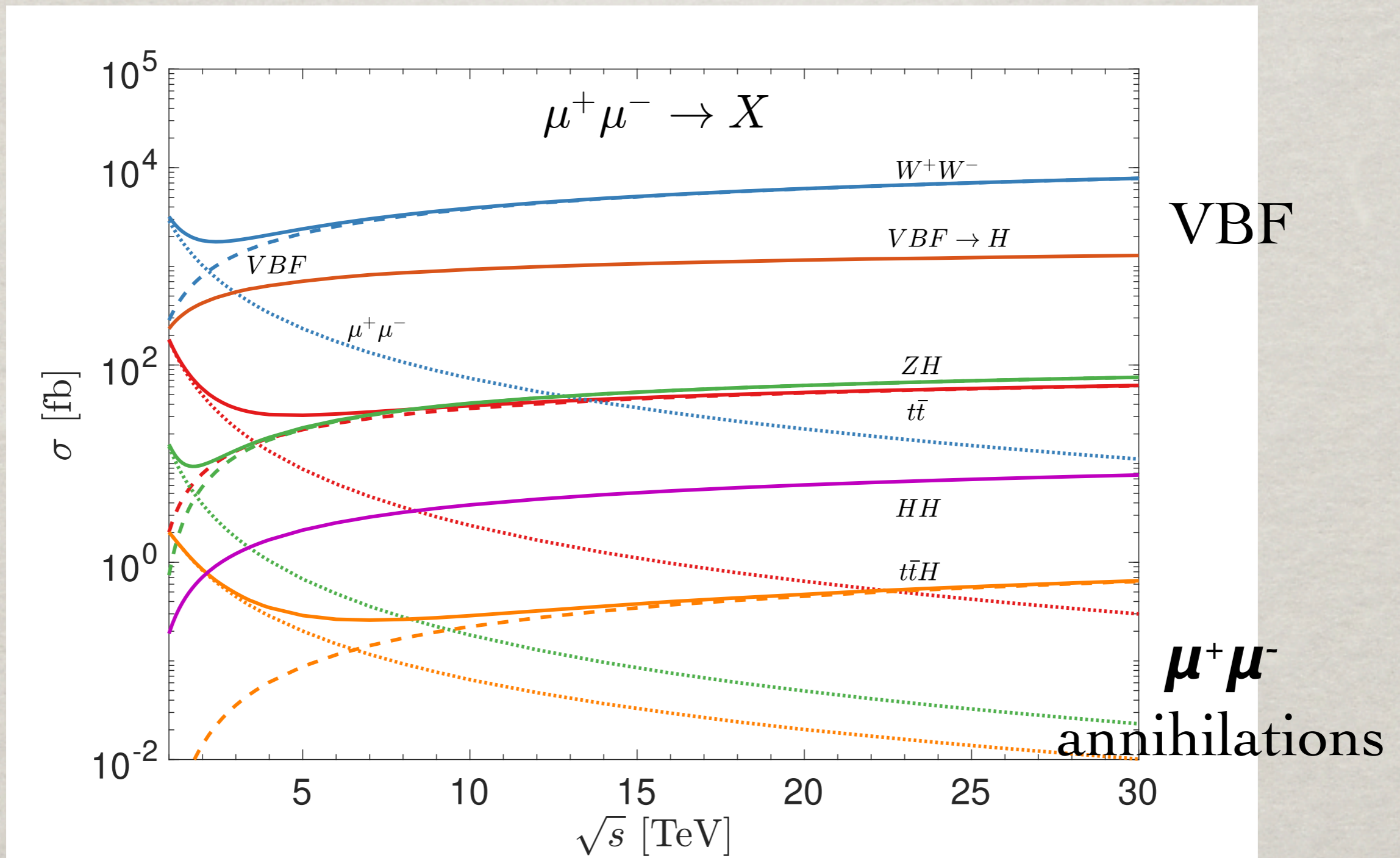
TH, Yang Ma, Keping Xie, arXiv:2007.14300



- “Semi-inclusive” processes

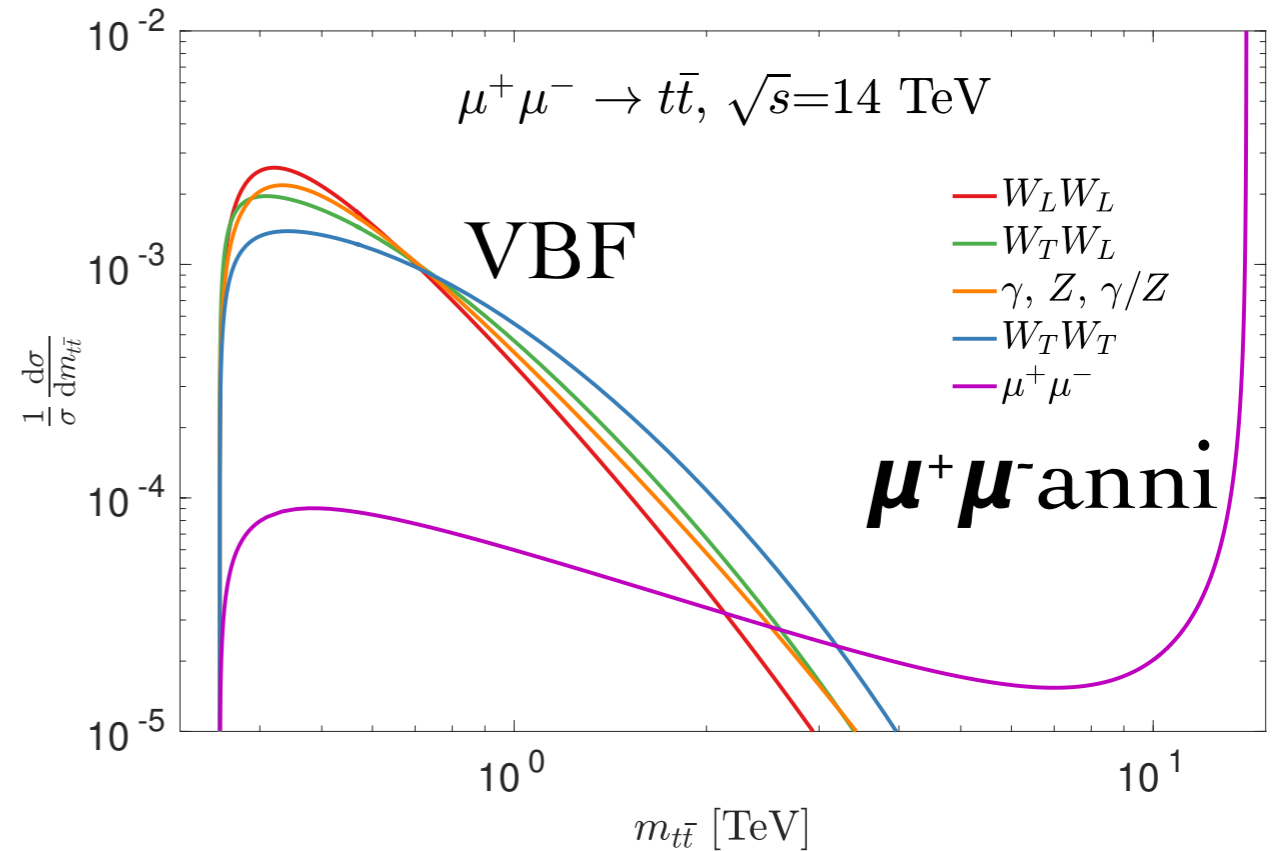
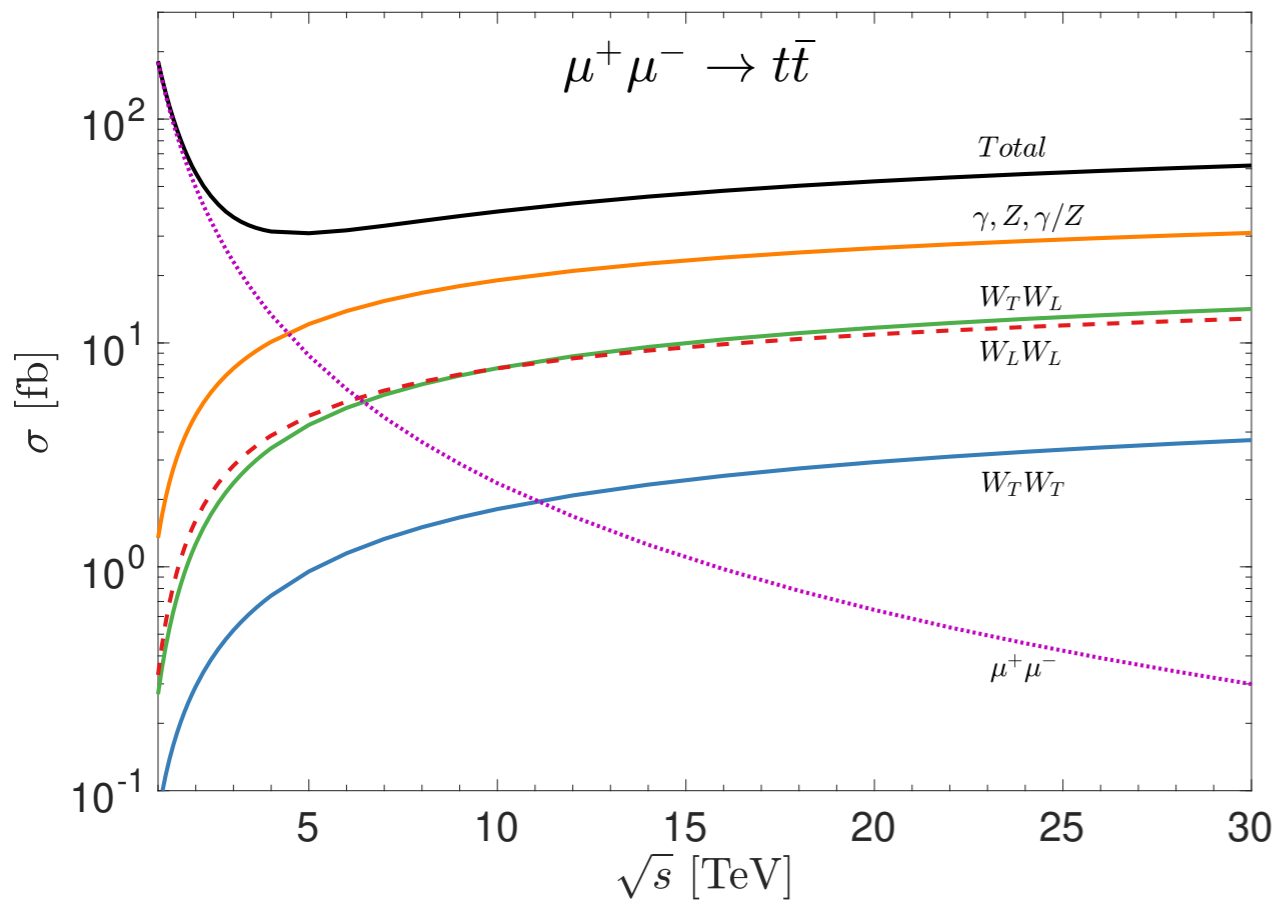
Just like in hadronic collisions:

$\mu^+\mu^- \rightarrow$  exclusive particles + remnants





- Underlying sub-processes:

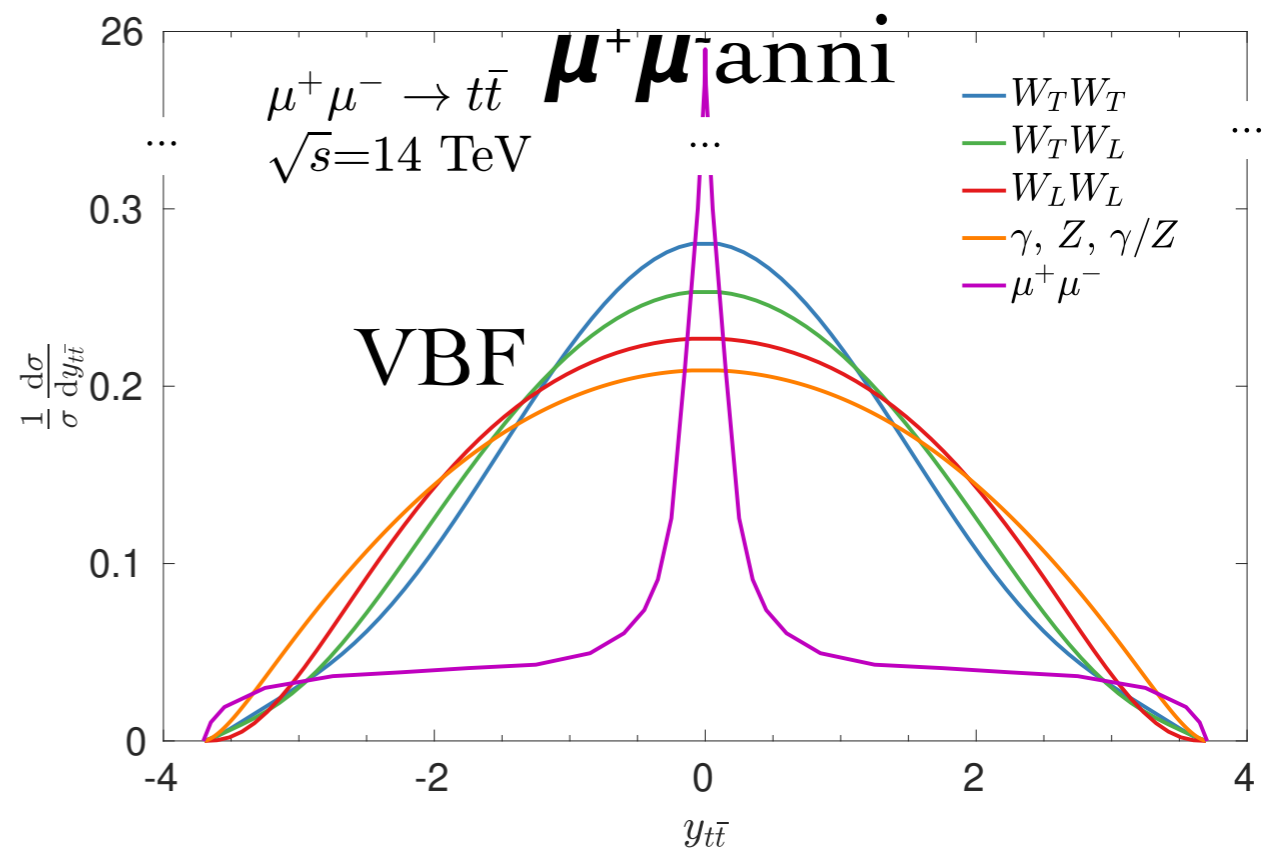


Decomposed partonic contributions

$\mu^+ \mu^-$  Collider:

“Buy one, get one free”

Annihilation + VBF



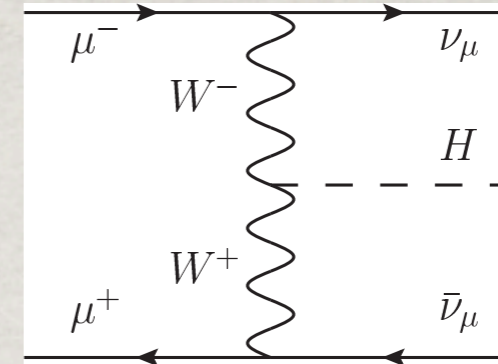


# • Precision Higgs Physics

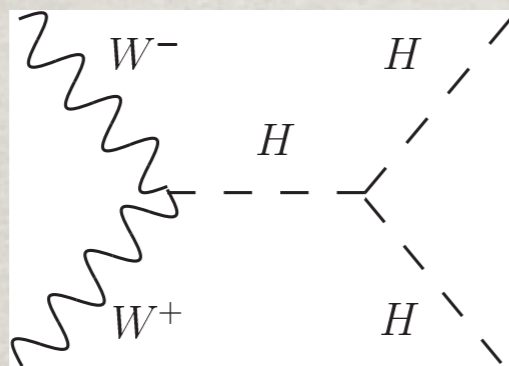
$$\mu^+ \mu^- \rightarrow \nu_\mu \bar{\nu}_\mu H \quad (WW \text{ fusion}),$$

$$\mu^+ \mu^- \rightarrow \mu^+ \mu^- H \quad (ZZ \text{ fusion}).$$

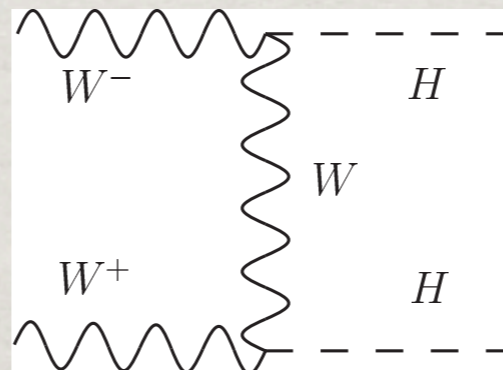
## WWH / ZZH couplings



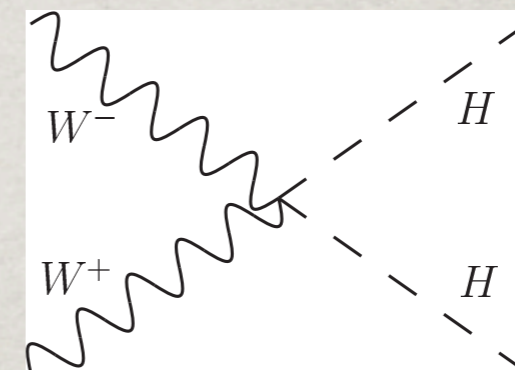
## HHH / WWHH couplings:



(a)



(b)



(c)

$\sqrt{s}$ (TeV)	3	6	10	14	30
benchmark lumi ( $\text{ab}^{-1}$ )	1	4	10	20	90
$\sigma$ (fb): $WW \rightarrow H$	490	700	830	950	1200
$ZZ \rightarrow H$	51	72	89	96	120
$WW \rightarrow HH$	0.80	1.8	3.2	4.3	6.7
$ZZ \rightarrow HH$	0.11	0.24	0.43	0.57	0.91
$WW \rightarrow ZH$	9.5	22	33	42	67
$WW \rightarrow t\bar{t}H$	0.012	0.046	0.090	0.14	0.28
$WW \rightarrow Z$	2200	3100	3600	4200	5200
$WW \rightarrow ZZ$	57	130	200	260	420

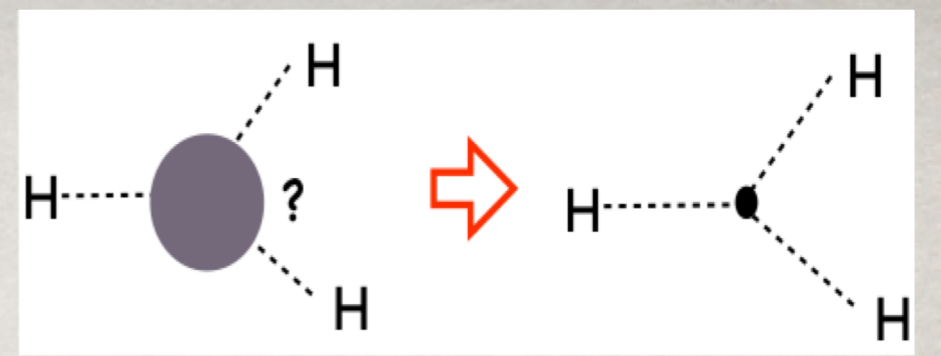
10M H

500k HH

TH, D. Liu, I. Low,  
X. Wang, arXiv:2008.12204



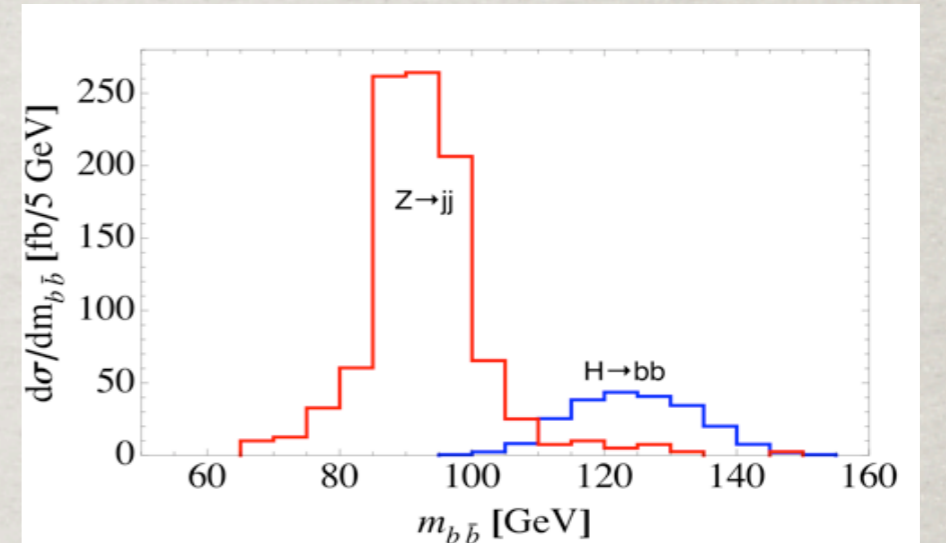
# Achievable accuracies



Leading channel  $H \rightarrow bb$ :

$$\Delta E/E = 10\%.$$

$$10^\circ < \theta_{\mu^\pm} < 170^\circ.$$



$$\mathcal{L} \supset \left( M_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} M_Z^2 Z_\mu Z^\mu \right) \left( \kappa_V \frac{2H}{v} + \kappa_{V_2} \frac{H^2}{v^2} \right) - \frac{m_H^2}{2v} \left( \kappa_3 H^3 + \frac{1}{4v} \kappa_4 H^4 \right)$$

$\sqrt{s}$ (lumi.)	3 TeV (1 ab <sup>-1</sup> )	6 (4)	10 (10)	14 (20)	20 (90)	Comparison
$WWH$ ( $\Delta\kappa_W$ )	0.26%	0.12%	0.073%	0.050%	0.023%	0.1% [41]
$\Lambda/\sqrt{c_i}$ (TeV)	4.7	7.0	9.0	11	16	(68% C.L.)
$ZZH$ ( $\Delta\kappa_Z$ )	1.4%	0.89%	0.61%	0.46%	0.21%	0.13% [17]
$\Lambda/\sqrt{c_i}$ (TeV)	2.1	2.6	3.2	3.6	5.3	(95% C.L.)
$WWHH$ ( $\Delta\kappa_{W_2}$ )	5.3%	1.3%	0.62%	0.41%	0.20%	5% [36]
$\Lambda/\sqrt{c_i}$ (TeV)	1.1	2.1	3.1	3.8	5.5	(68% C.L.)
$HHH$ ( $\Delta\kappa_3$ )	25%	10%	5.6%	3.9%	2.0%	5% [22, 23]
$\Lambda/\sqrt{c_i}$ (TeV)	0.49	0.77	1.0	1.2	1.7	(68% C.L.)

**Table 7:** Summary table of the expected accuracies at 95% C.L. for the Higgs couplings at a variety of muon collider energies and luminosities.



# • WIMP Dark Matter

## (a conservative SUSY scenario)

Consider the “minimal EW dark matter”: **an EW multi-plet**

- The lightest neutral component as DM
- Interactions well defined  $\rightarrow$  pure gauge
- Mass upper limit predicted  $\rightarrow$  thermal relic abundance

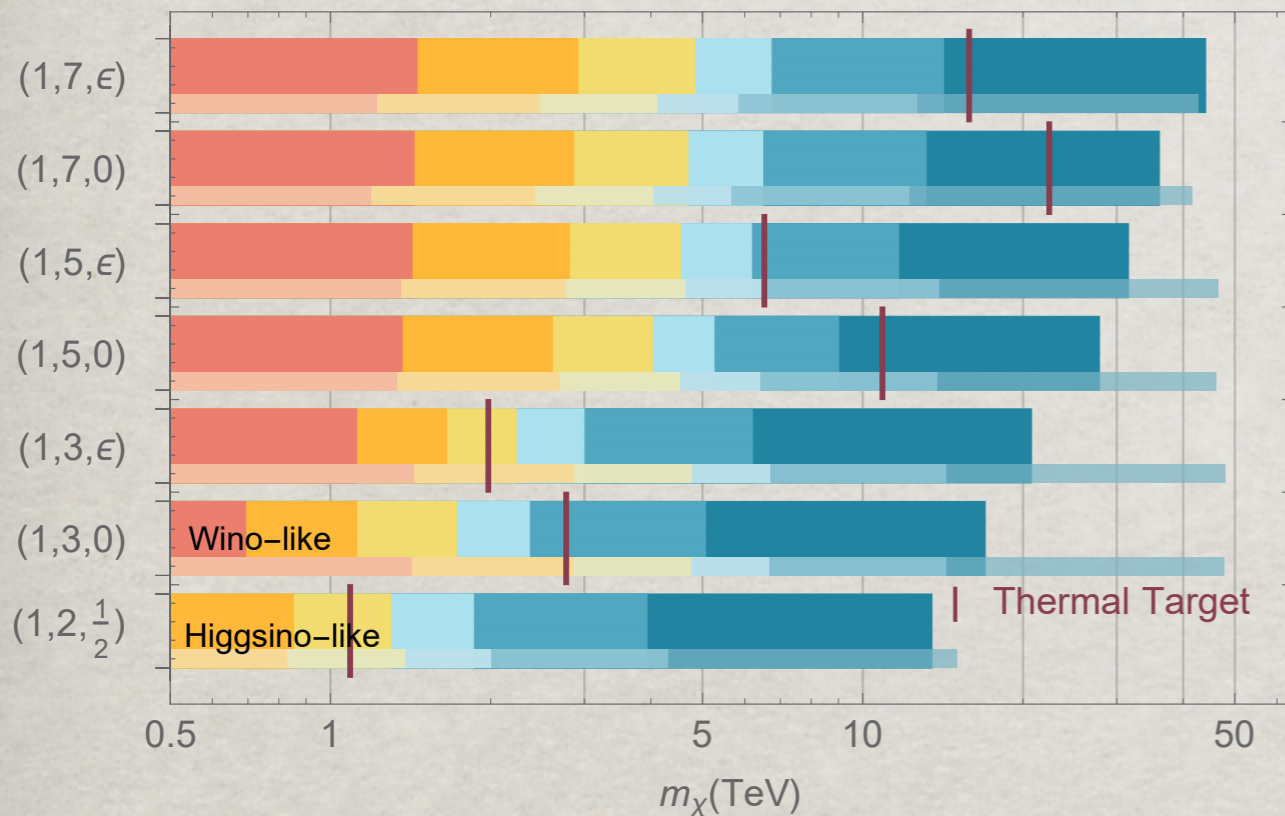
Model (color, $n$ , $Y$ )		Therm. target
(1,2,1/2)	Dirac	1.1 TeV
(1,3,0)	Majorana	2.8 TeV
(1,3, $\epsilon$ )	Dirac	2.0 TeV
(1,5,0)	Majorana	14 TeV
(1,5, $\epsilon$ )	Dirac	6.6 TeV
(1,7,0)	Majorana	23 TeV
(1,7, $\epsilon$ )	Dirac	16 TeV

Cirelli, Fornengo and Strumia:  
[hep-ph/0512090](https://arxiv.org/abs/hep-ph/0512090), 0903.3381;  
 TH, Z. Liu, L.T. Wang, X. Wang:  
[arXiv:2009.11287](https://arxiv.org/abs/2009.11287)

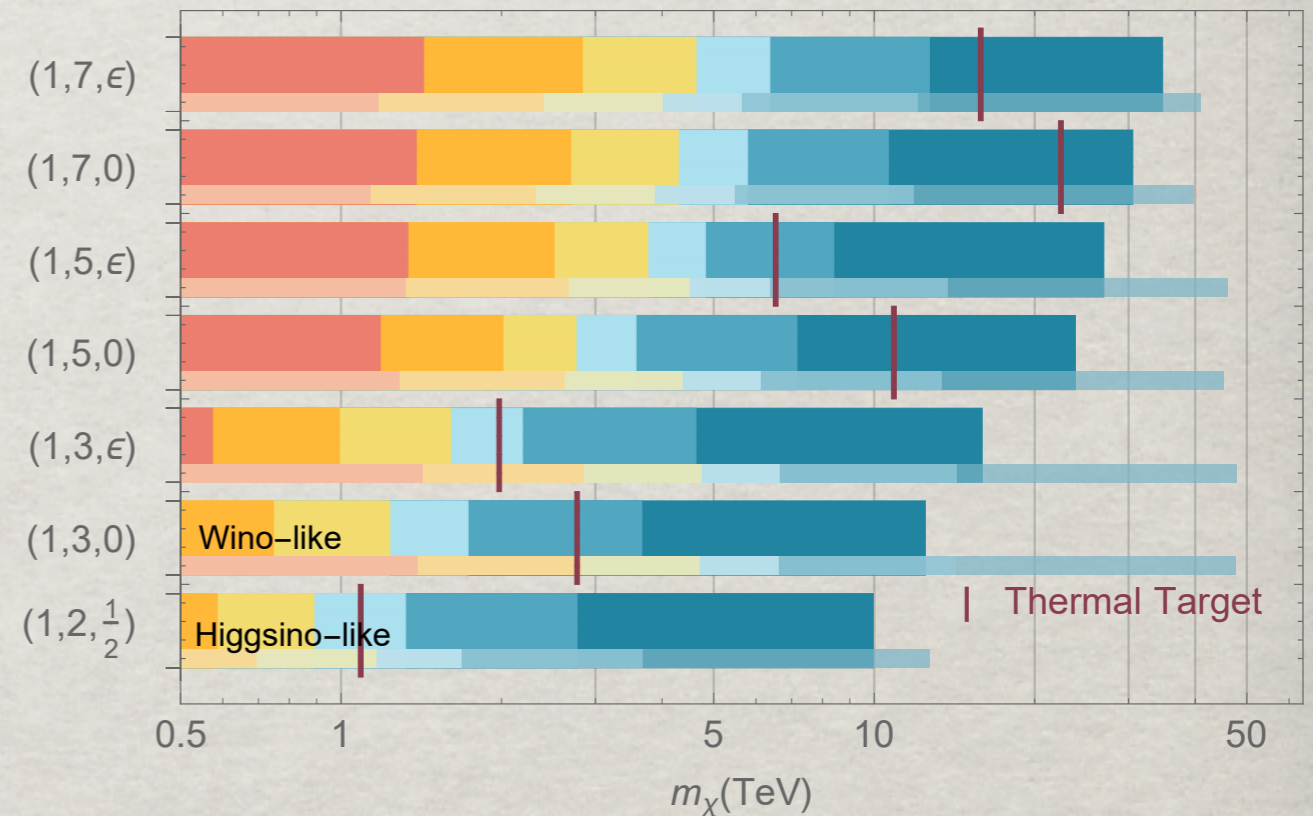


# The mass reach for minimal WIMP DM:

Muon Collider  $2\sigma$  Reach ( $\sqrt{s} = 3, 6, 10, 14, 30, 100$  TeV)



Muon Collider  $5\sigma$  Reach ( $\sqrt{s} = 3, 6, 10, 14, 30, 100$  TeV)



$E_{CM} \approx 14$  TeV enough to cover  $n \leq 3$  multiplets.  
Higher energy needed to cover higher multiplets.

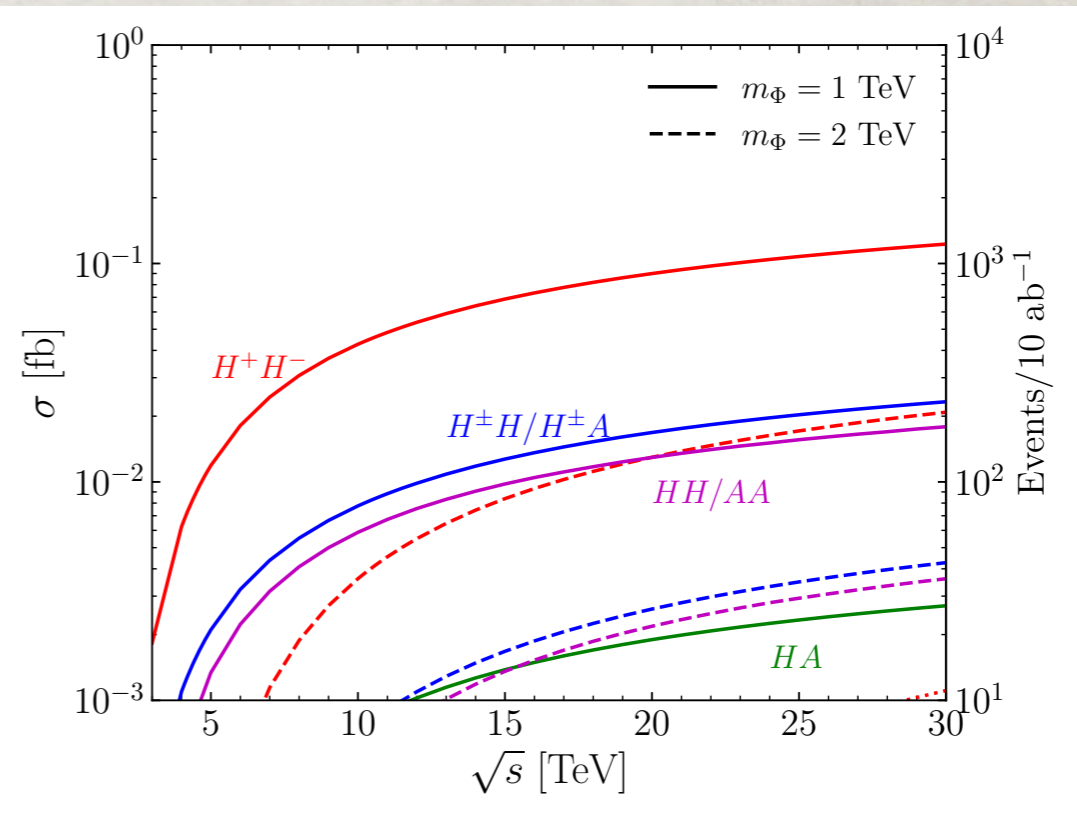
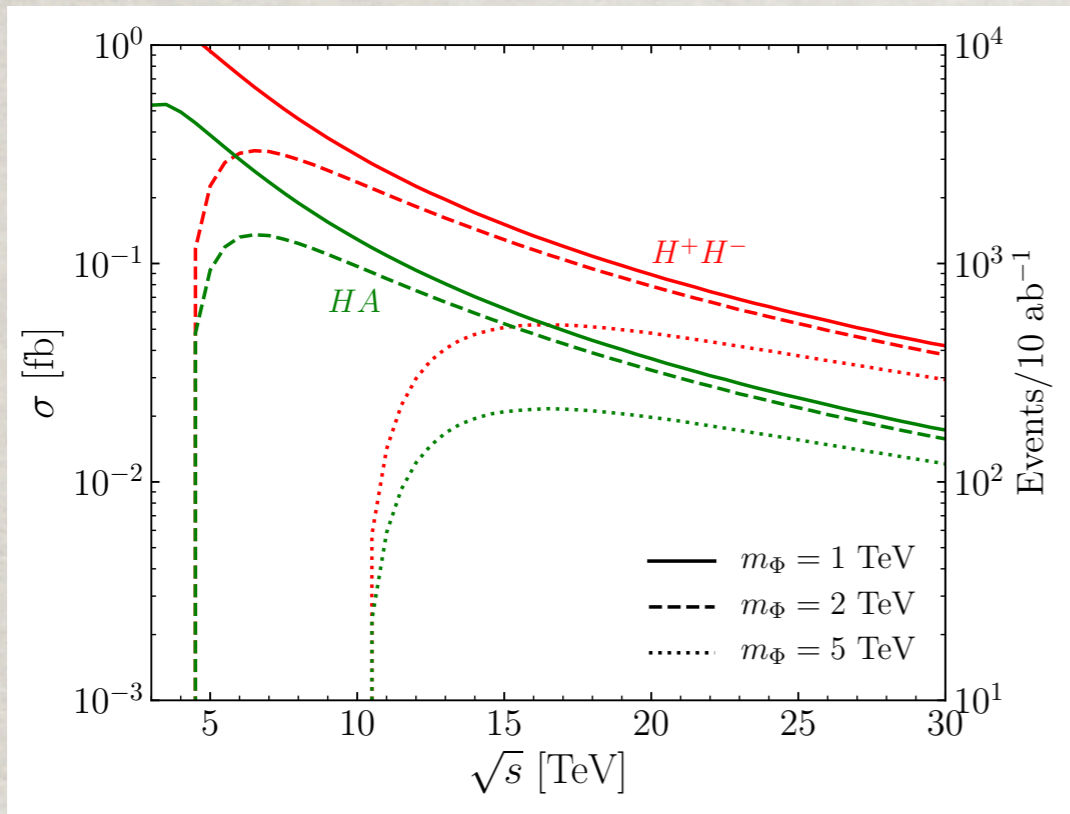
TH, Z. Liu, L.T. Wang, X. Wang: arXiv:2009.11287



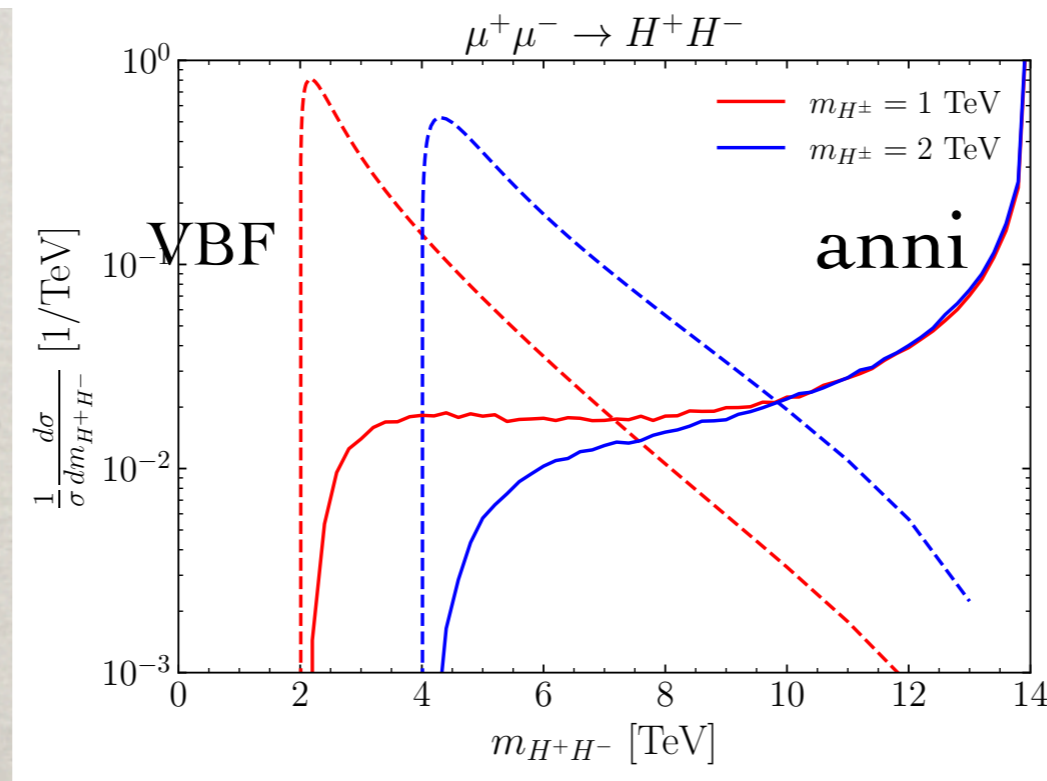
# • Heavy Higgs Bosons Production

annihilation

VBF



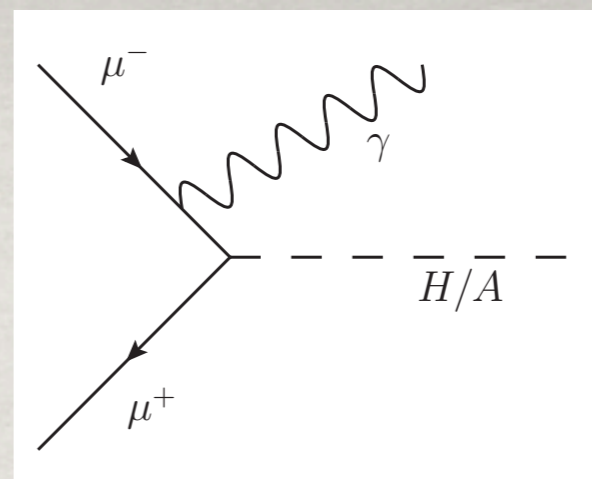
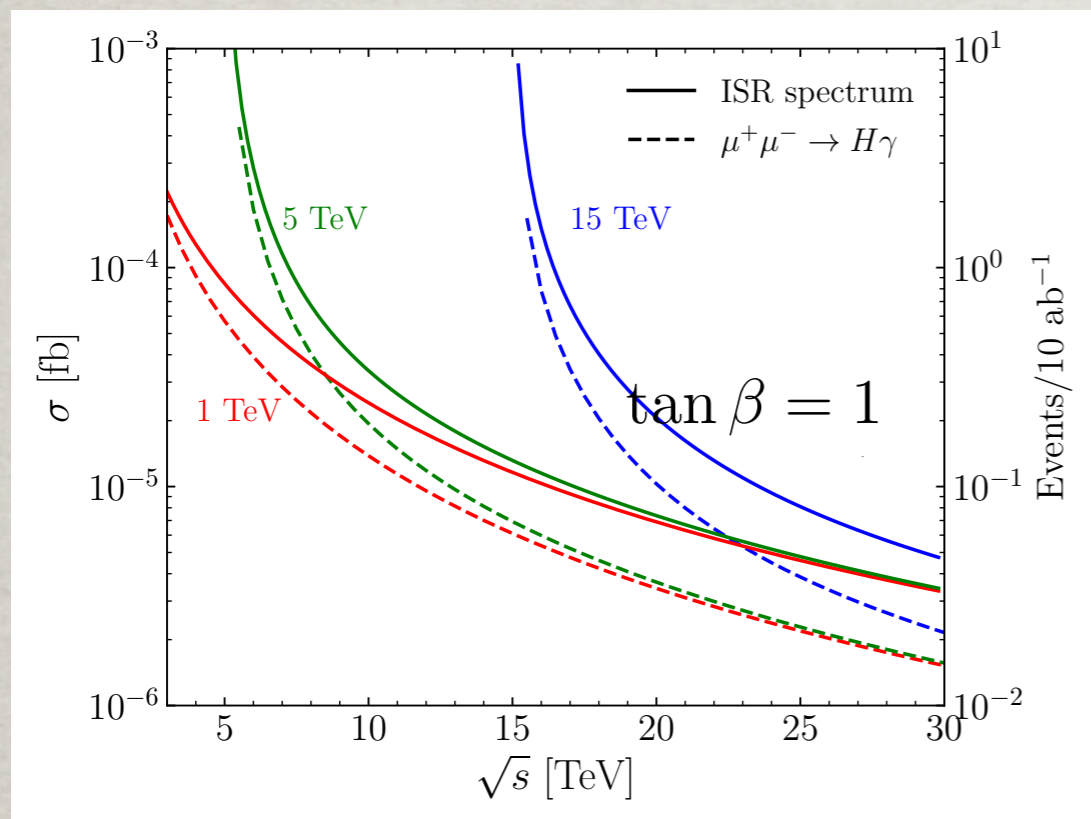
VBF



TH, S. Li, S. Su,  
W. Su, Y. Wu:  
arXiv:2102.08386.



# Radiative returns:



$$\hat{\sigma}(\mu^+ \mu^- \rightarrow H) = \frac{\pi Y_\mu^2}{4} \delta(\hat{s} - m_H^2) = \frac{\pi Y_\mu^2}{4s} \delta(\tau - \frac{m_H^2}{s})$$

$$f_{e/\ell}(x) = \frac{\alpha}{2\pi} \frac{1+x^2}{1-x} \log \frac{s}{m_\mu^2}$$

$$\sigma = 2 \int dx_1 f_{e/\ell}(x_1) \hat{\sigma}(\tau = x_1) = \frac{\alpha Y_\mu^2}{4s} \frac{s + m_H^4/s}{s - m_H^2} \log \frac{s}{m_\mu^2}$$

Depending on the coupling,  
one may reach the kinematic limit:

$$M_H \sim E_{\text{cm}}$$

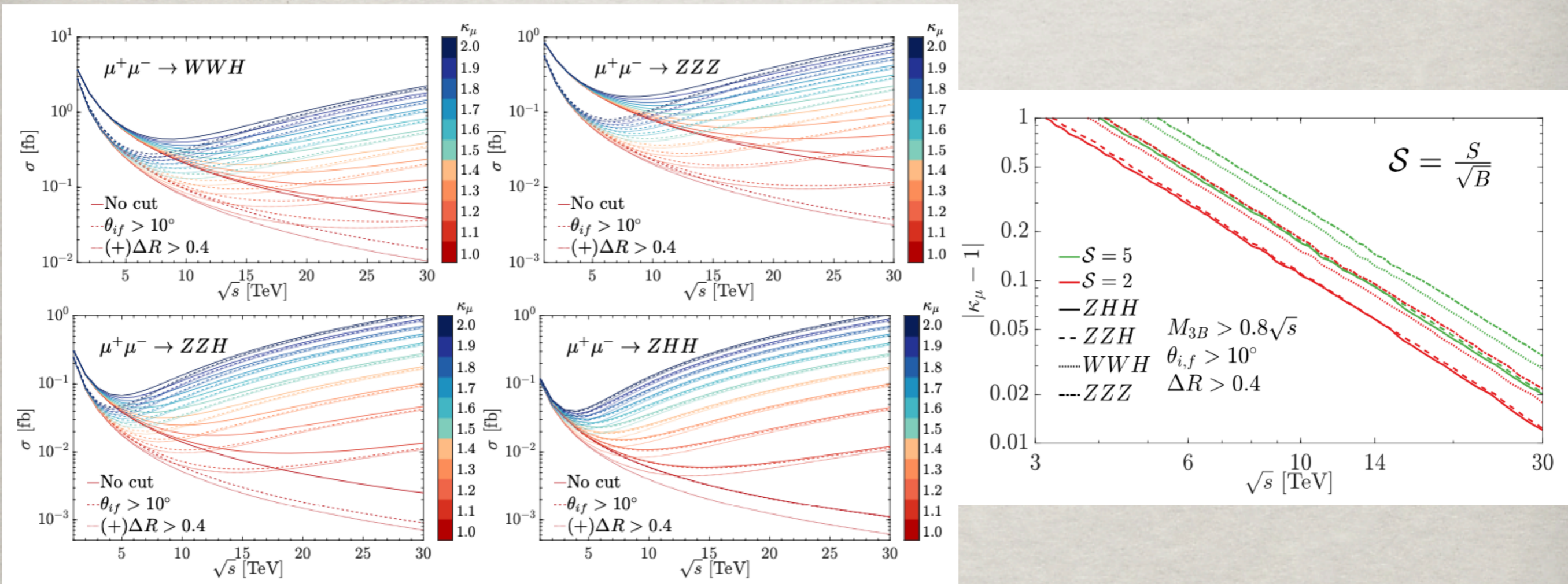
TH, S. Li, S. Su, W. Su, Y. Wu, arXiv:2102.08386;  
TH, Z. Liu et al., arXiv:1408.5912.



- **Model-independent sensitivity:**

$$Y_{\mu} = \kappa_{\mu} Y_{\mu}^{\text{SM}}$$

New physics modification of SM leads to notable deviation, eventually violate unitarity.



TH, Wolfgang Kilian, Nils Kreher, Yang Ma, Juergen Reuter, Tobias Striegl and Keping Xie: arXiv:2108.05362.



# Lots of recent works!

-- my apologies not to cover properly

D. Buttazzo, D. Redogolo, F. Sala, arXiv:1807.04743 (VBF to Higgs)

A. Costantini, F. Maltoni, et al., arXiv:2005.10289 (VBF to NP)

M. Chiesa, F. Maltoni, L. Mantani, B. Mele, F. Piccinini, and X. Zhao,  
arXiv:2005.10289 (SM Higgs)

R. Capdevilla, D. Curtin, Y. Kahn, G. Krnjaic,  
arXiv:2006.16277; arXiv:2101.10334 (g-2, flavor)

P. Bandyopadhyay, A. Costantini et al., arXiv:2010.02597 (Higgs)

D. Buttazzo, P. Paradisi, arXiv:2012.02769 (g-2)

W. Yin, M. Yamaguchi, arXiv:2012.03928 (g-2)

R. Capdevilla, F. Meloni, R. Simoniello, and J. Zurita, arXiv:2012.11292 (MD)

D. Buttazzo, F. Franceschini, A. Wulzer, arXiv:2012.11555 (general)

G.-Y. Huang, F. Queiroz, W. Rodejohann,  
arXiv:2101.04956; arXiv:2103.01617 (flavor)

W. Liu, K.-P. Xie, arXiv:2101.10469 (EWPT)

H. Ali, N. Arkani-Hamed, et al, arXiv:2103.14043 (Muon Smasher's Guide)

Richard Ruiz et al., arXiv:2111.02442 (MadGraph5)

... ..



# Summary

- **High energy muon-collider is a new endeavor:**  
Challenging technology; interdisciplinary to other fields; great physics potential!
- **s-channel Higgs factory:**
  - Direct measurements on  $Y_\mu$  &  $\Gamma_H$
  - Other BRs comparable to  $e^+e^-$  Higgs factories
- **Multi-TeV colliders:**
  - Unprecedented accuracies for  $WWH$ ,  $WWHH$ ,  $H^3$ ,  $H^4$
  - Bread & butter SM EW physics in the new territory
  - New particle ( $Q, H\dots$ ) mass coverage  $M_H \sim (0.5 - 1)E_{\text{cm}}$
  - Decisive coverage for minimal WIMP DM  $M \sim 0.5 E_{\text{cm}}$
  - Complementary to Astro/Cosmo/GW & to FCC-hh:

**Exciting journey ahead!**