The entropy of Hawking radiation

Juan Maldacena

Institute for Advanced Study

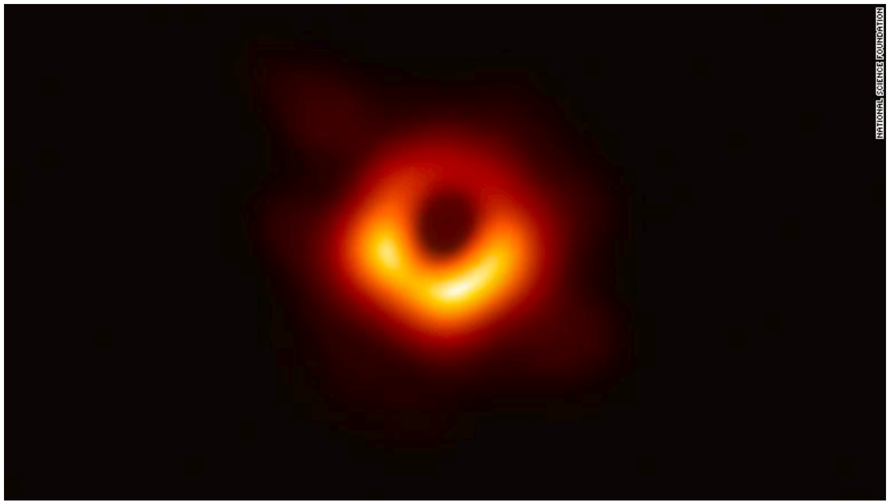
We will discuss recent progress on the black hole information problem

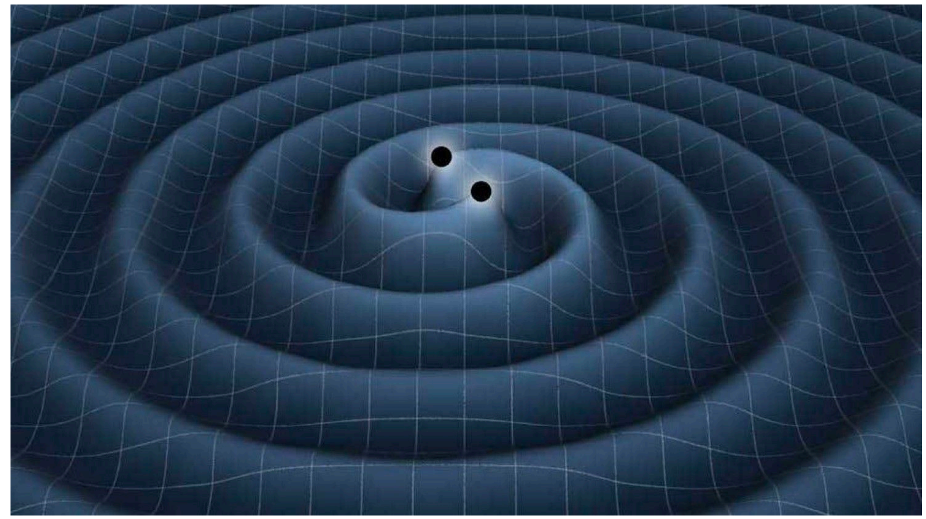
Outline

- Black hole entropy = area of horizon
- The fine grained gravitational entropy formula. Entropy = Minimal area
- Compute the entropy of radiation coming out of black holes.
- Get a result consistent with information conservation (as opposed to information loss).

This will not be historical, will hopefully be pedagogical…

Black holes have been in the news





We will mainly talk about Quantum aspects of black holes

Black hole

Schwarzschild 1917

$$ds^2 = -(1 - \frac{r_s}{r})dt^2 + \frac{dr^2}{(1 - \frac{r_s}{r})} + r^2 d\Omega_2^2$$

$$r_s = 2G_N M/c^2$$

Time component of the metric goes to zero at some radius = Schwarzschild radius. (r_s sets the ``size'' of the black hole). An observer at constant r, feels an infinite gravitational force!

Einstein: ``this does not make any physical sense''

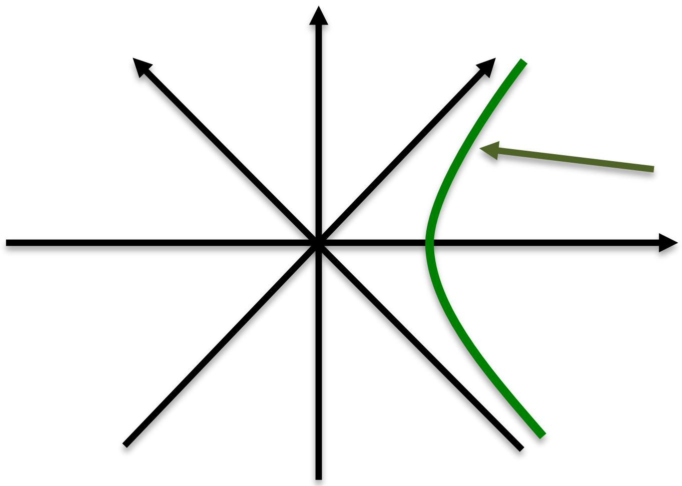
In other coordinates: (expanding around the Schwarzschild radius)

$$ds^2 = d\rho^2 - \rho^2 d\tau^2$$

metric near $r = r_s$ in the t and r directions

The metric near r=r_s is flat

$$ds^2 = d\rho^2 - \rho^2 d\tau^2 = -(dx^0)^2 + (dx^1)^2 \ , \qquad x^0 = \rho \sinh \tau \ , \qquad x^1 = \rho \cosh \tau$$



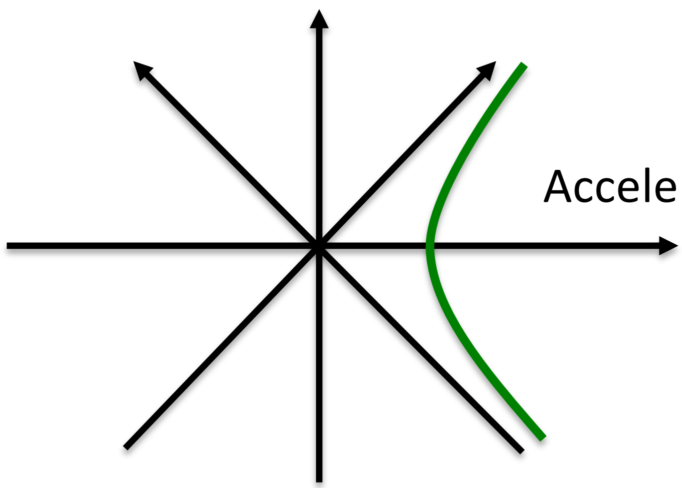
Constant r trajectory → accelerated observer

Looks locally like flat space
Observer at constant radial position near ρ =0 → very large acceleration.
If she does not accelerate → no force!. Just a coordinate singularity!

Time translation → acts like a boost at the horizon.

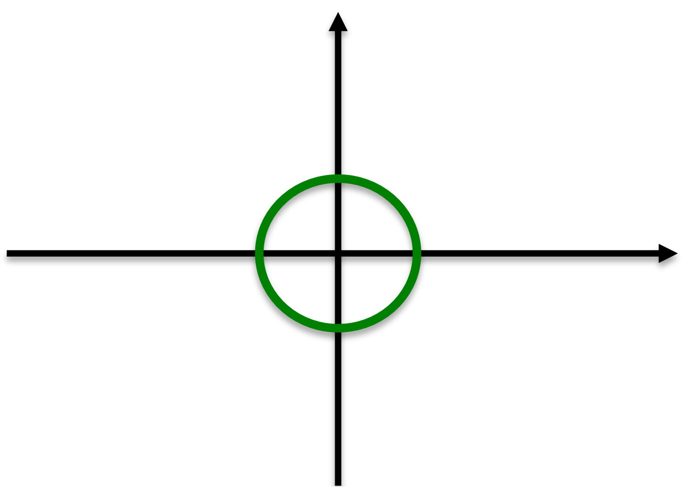
Euclidean analogy

$$ds^2 = d\rho^2 - \rho^2 d\tau^2 = -(dx^0)^2 + (dx^1)^2 \ , \qquad x^0 = \rho \sinh \tau \ , \qquad x^1 = \rho \cosh \tau$$



Accelerated observer, at constant ρ.

$$ds^2 = d\rho^2 + \rho^2 d\theta^2 = (dx_E^0)^2 + (dx^1)^2 \ , \qquad x_E^0 = \rho \sin \theta \ , \qquad x^1 = \rho \cos \theta$$



Circle at constant ρ.

Finite temperature and circles in Euclidean time

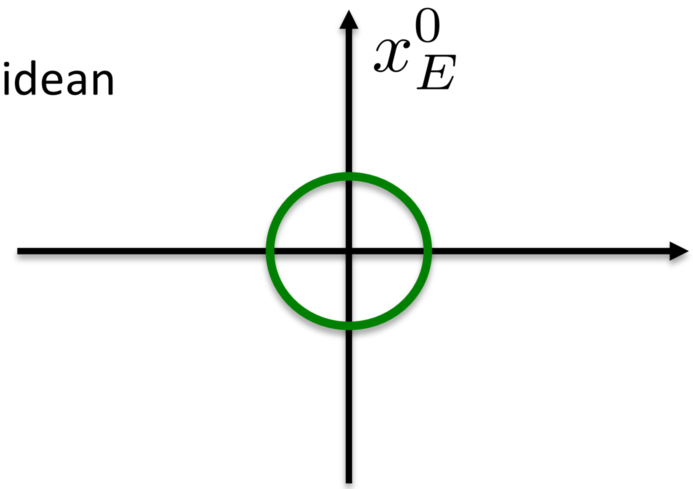
Thermal partition function:

$Z = Tr[e^{-\beta H}]$ = evolution in Euclidean time on a circle of length β

A theory on a Euclidean circle is related to a system is thermal equilibrium.

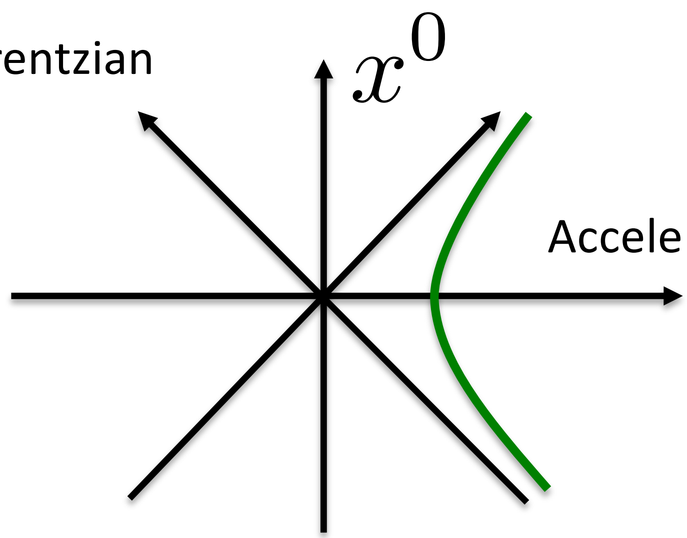
$$T = \frac{1}{\beta} = \frac{1}{\text{Length of Euclidean circle}}$$

Euclidean



Circle in Euclidean time → finite temperature

Lorentzian



Accelerated observer measures a temperature!

$$T = \frac{1}{\beta} = \frac{1}{2\pi r}$$

Unruh
Bisognano Wichmann
Hartle Hawking

Observer who only has access to the right wedge → sees a mixed state, due to entanglement in the vacuum.

So far we discussed the near horizon geometry.

Now let’s go back to the full black hole geometry

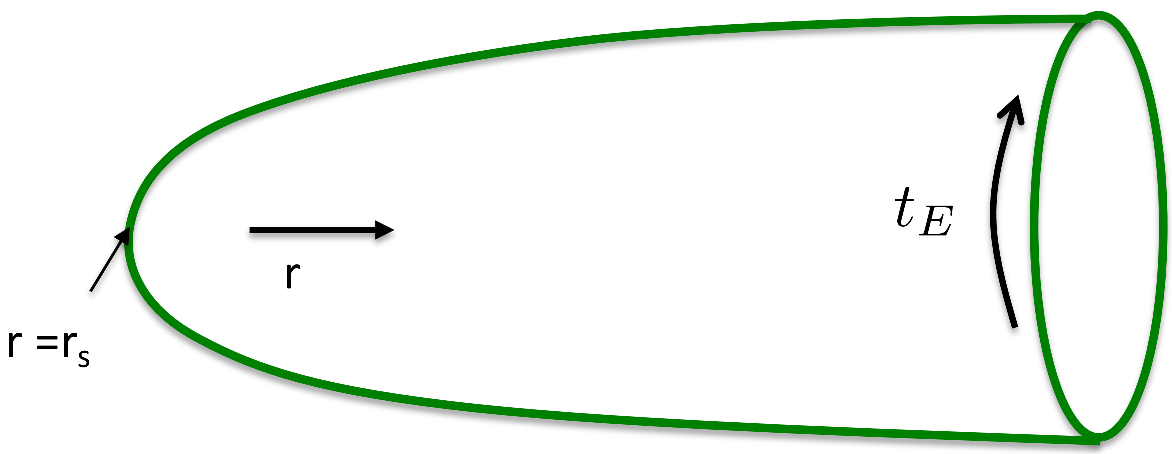
Euclidean black hole

$$ds^2 = -(1 - \frac{r_s}{r})dt^2 + \frac{dr^2}{(1 - \frac{r_s}{r})} + r^2 d\Omega_2^2$$

$$ds^2 = (1 - \frac{r_s}{r})dt_E{}^2 + \frac{dr^2}{(1 - \frac{r_s}{r})} + r^2 d\Omega_2^2$$

Hawking

$$t_E = t_E + \beta \ , \qquad \beta = 4\pi r_s ,$$ = inverse temperature far away



``cigar''

Gibbons
Hawking

Quiz

- How big is a ``white” black hole?

$$T = \frac{1}{4\pi r_s}$$

$$kT = \frac{\hbar c}{4\pi r_s}$$

Entropy

- Use first law:

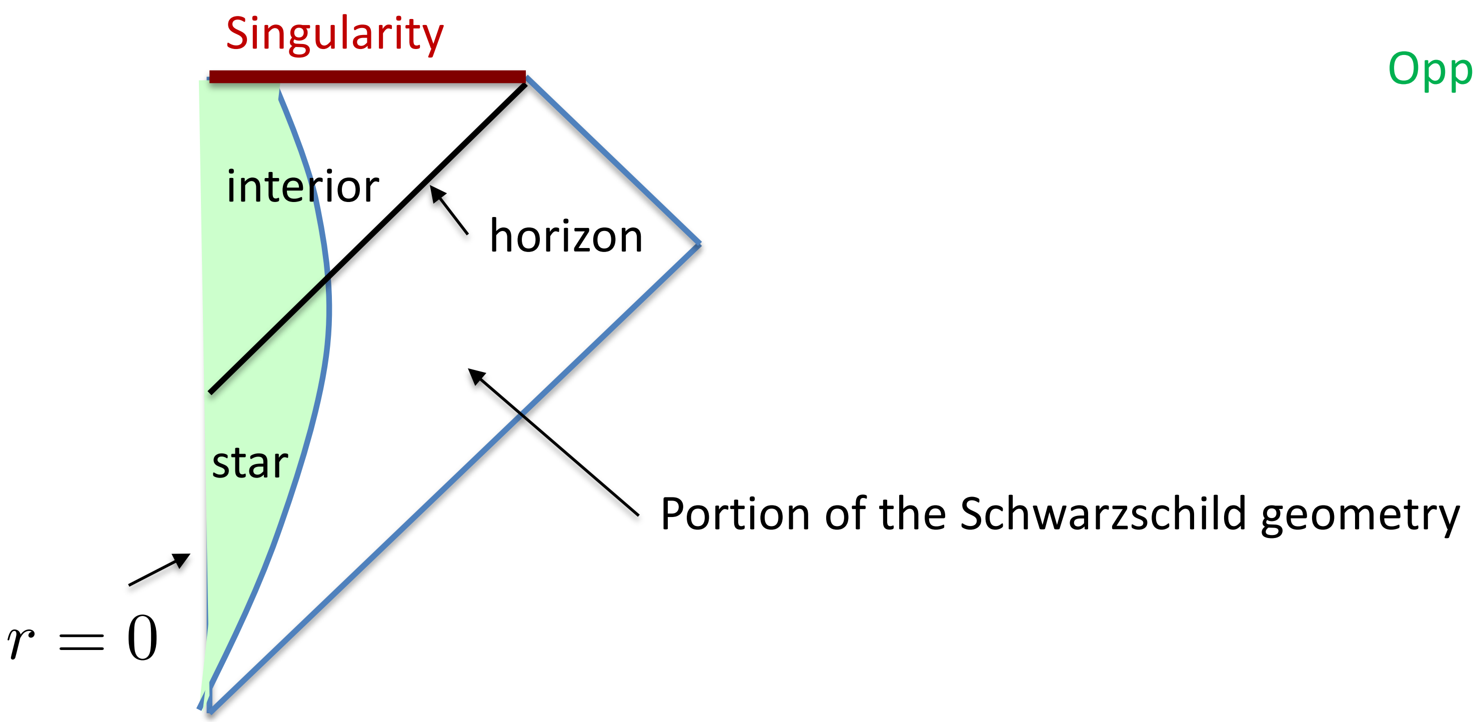
$$dS = \frac{dE}{T} = \frac{dM}{T}\ , \qquad r_s = G_N M/c^2$$

$$S = \frac{\text{Area}}{4G_N} = \frac{\text{Area}}{4l_p^2} = \frac{4\pi r_s^2}{4l_p^2}$$

A black hole is a thermodynamic object!

Let us discuss in more detail the geometry of a black hole

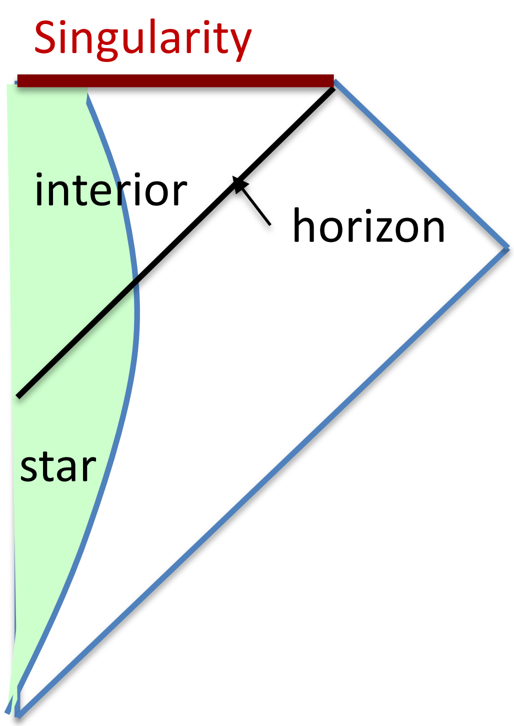
Geometry of a Black Hole made from collapse



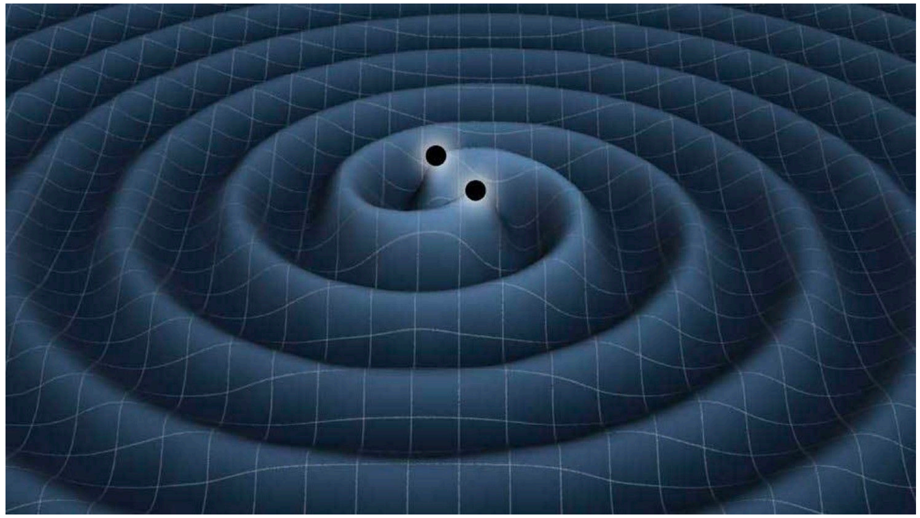
Oppenheimer Snyder 1939

Horizon Area law

Area law: The area of a black hole horizon always increases. Hawking



Starts with small area and it grows to larger area

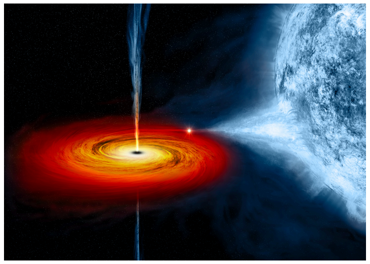


The area law puts an upper bound on the energy that can be emitted in gravitational waves when two black holes collide

Area law ensures the 2nd Law of thermodynamics

$$S = \frac{\text{Area}_H}{4G_N}$$

Generalized entropy



$$S = \frac{\text{Area}_H}{4G_N} + S_{\text{matter}}$$

Bekenstein 70's

Question

- When a black hole emits Hawking radiation, it loses energy, so its area becomes smaller.
- What happens to the entropy ?

Question

- When a black hole emits Hawking radiation, it loses energy, so its area becomes smaller.
- What happens to the entropy ?

$$S = \frac{\text{Area}_H}{4G_N} + S_{\text{matter}} = \frac{\text{Area}_H}{4G_N} + S_{\text{QFT}}$$

↑

Includes the entropy of quantum fields

Bombelli, Koul, Lee,Sorkin 1986

Obeys the 2^{nd} Law

Wall 2010

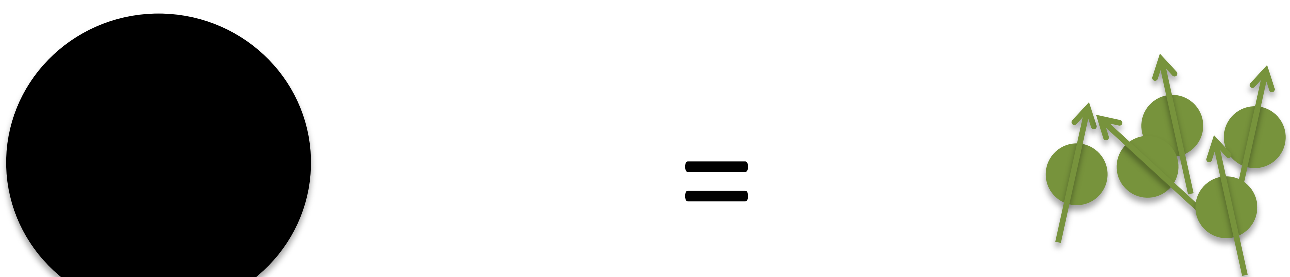
These results have inspired a

Central hypothesis

Black holes as quantum systems

Central hypothesis

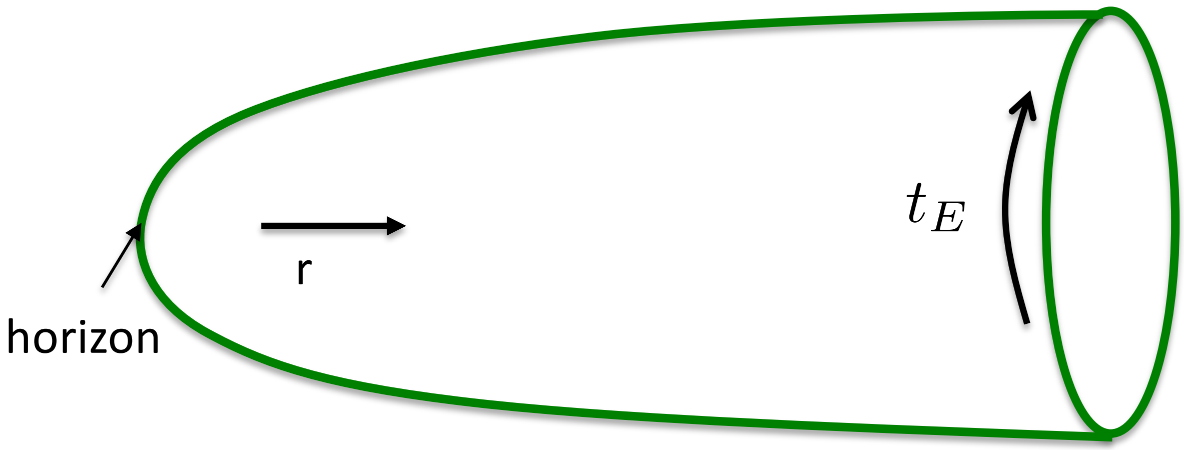
- A black hole seen from the outside can be described as a quantum system with S degrees of freedom (qubits). S = Area/4 (l_p =1)
- It evolves according to unitary evolution, seen from outside.



…in other words

- If one includes $A/4G_N$ "mysterious" qubits, then the black hole can be described as an ordinary quantum system.

Thermodynamics



``cigar''

Gibbons
Hawking

$$Z = Tr[e^{-\beta H}] \sim e^{-I_{grav}}$$

$$I_{grav} \propto -\frac{1}{G_N}\int \sqrt{g}R + \cdots$$

Tells us the answer but does not tell us what microstates we are counting

Evidence

1) Entropy counting

Special black holes, in special theories (supersymmetric) can be counted precisely using strings/D-branes → reproduce the Area formula. (+ also corrections to this formula)

Strominger Vafa

using results by

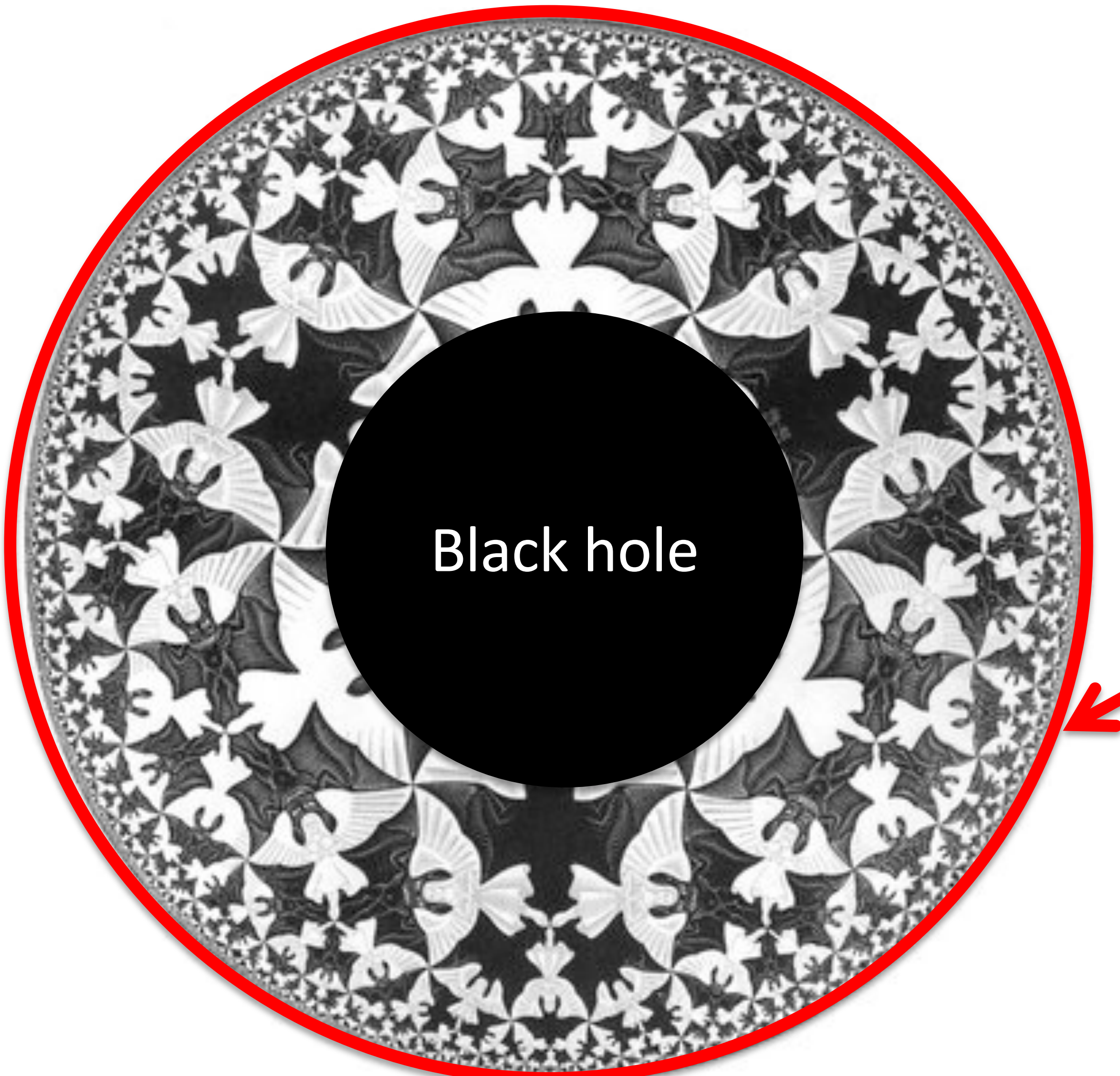
J. Polchinski

…

Sen

…

2) AdS/CFT...



Black hole in a box.
Evolving unitarity.

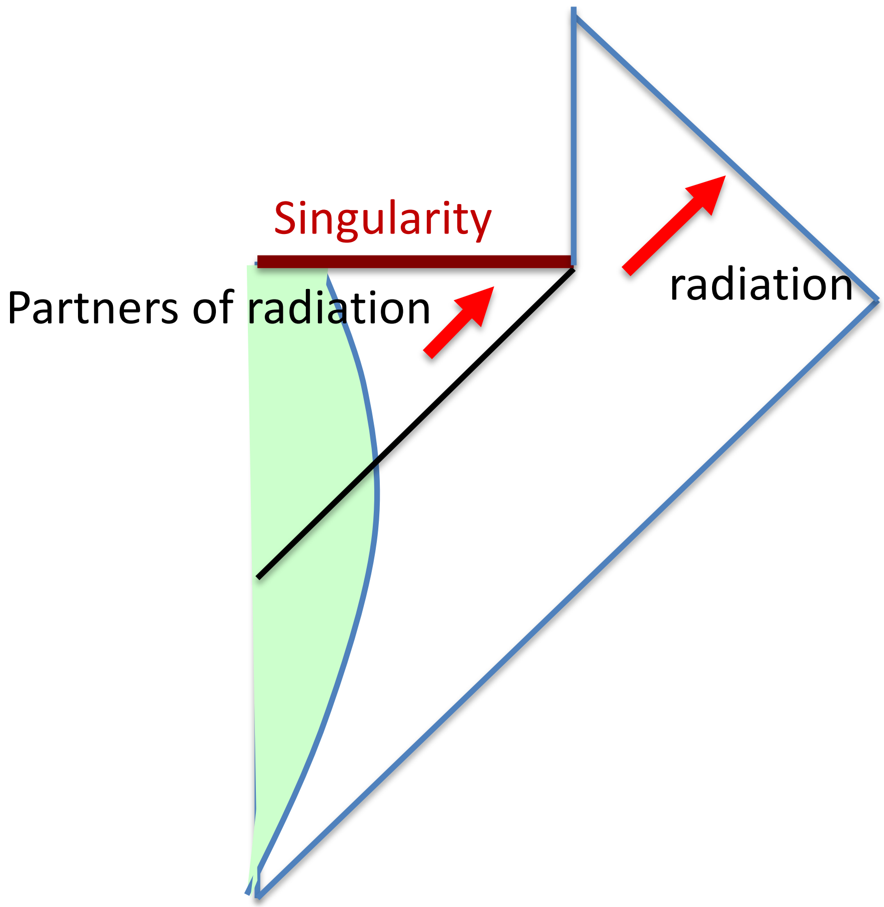
Hot fluid made out of
very strongly
interacting particles.

but

Hawking 1976 :

This can’t possibly be true!

Geometry of an evaporating black hole made from collapse



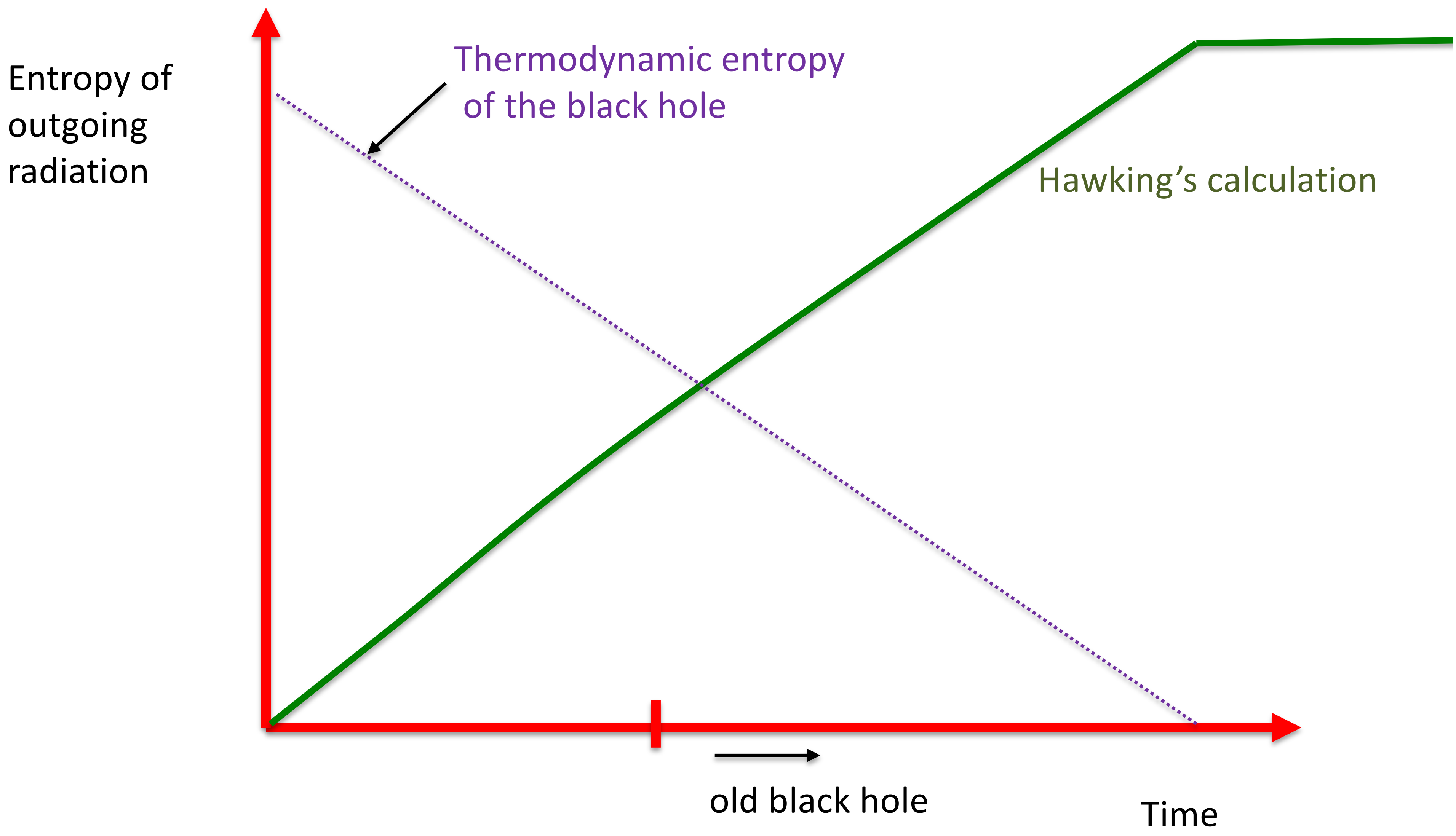
The radiation is entangled with partners of radiation.

Since we do not measure the interior we get a large entropy for the radiation.

A pure state seems to go a mixed state.

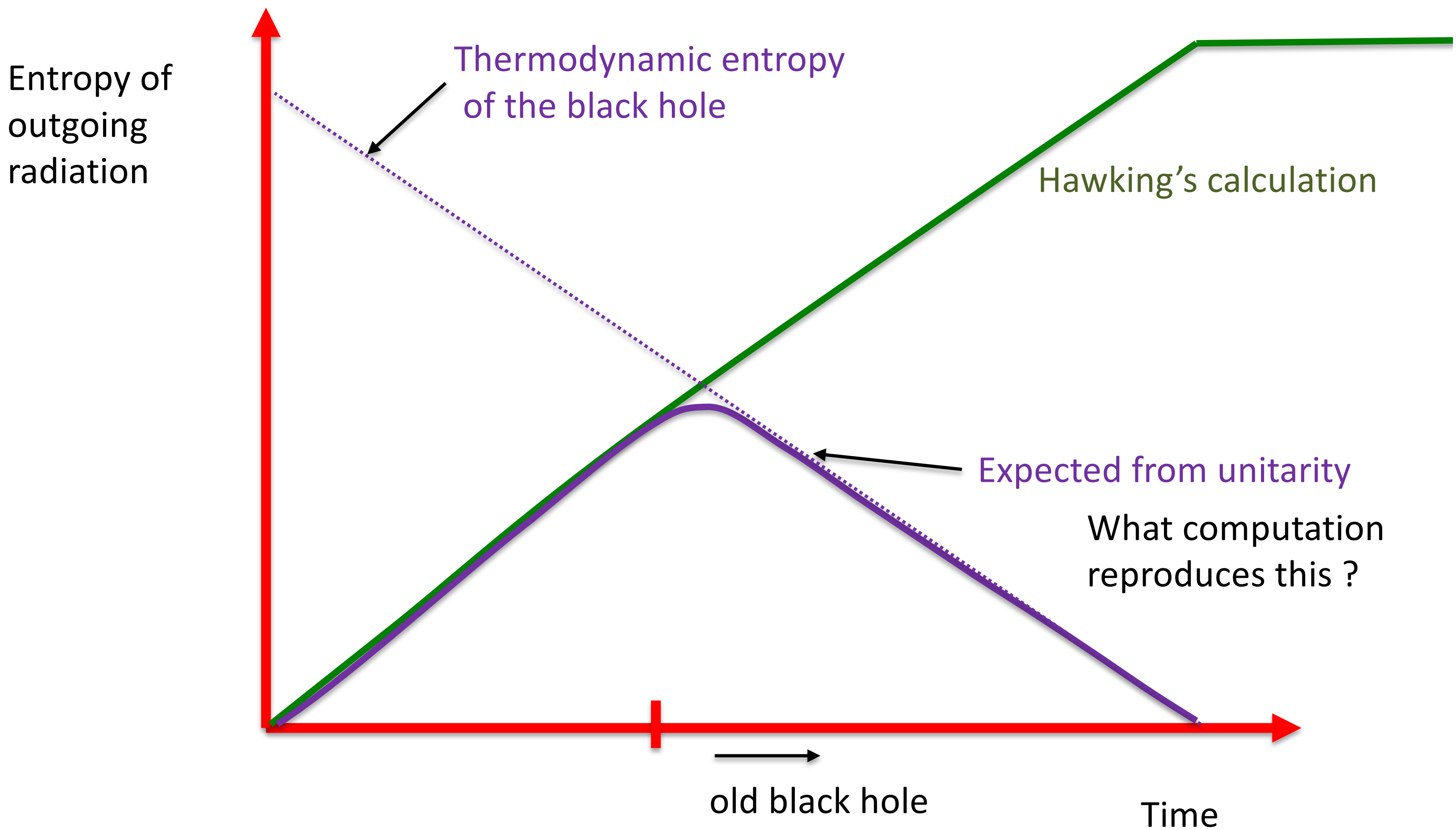
The Hawking curve

Compute the fine grained entropy of the radiation as it comes out of the black hole (formed by a pure state)



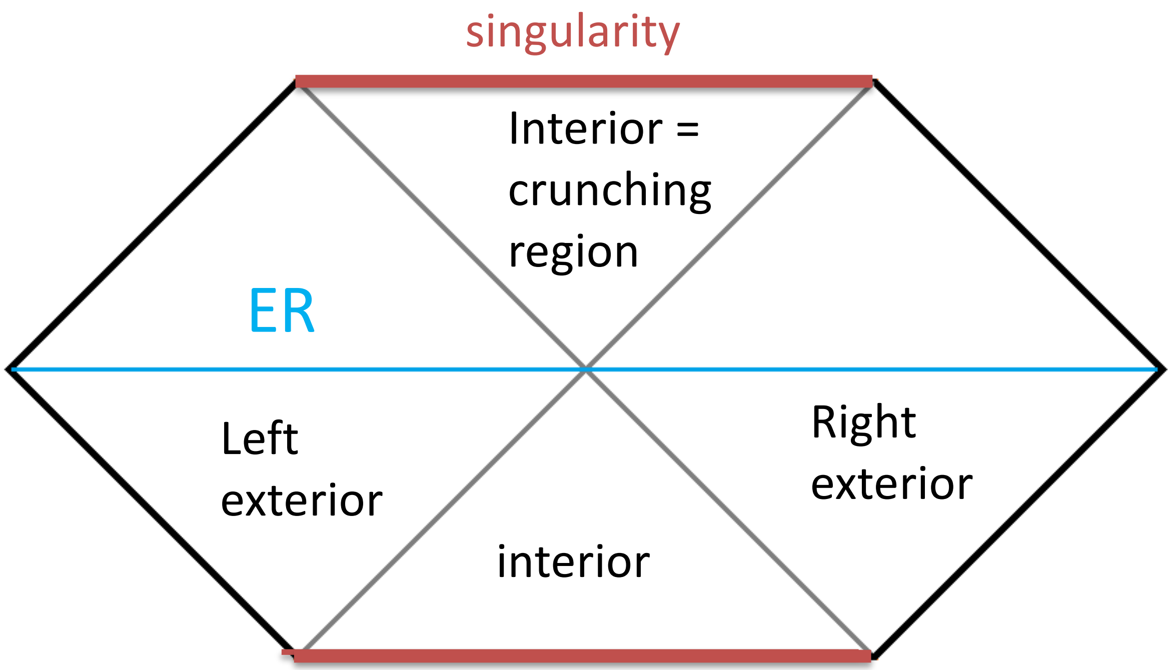
The Hawking curve vs. the Page curve

Compute the fine grained entropy of the radiation as it comes out of the black hole (formed by a pure state)



There were other apparent paradoxes with the black hole entropy formula.

Full Schwarzschild solution



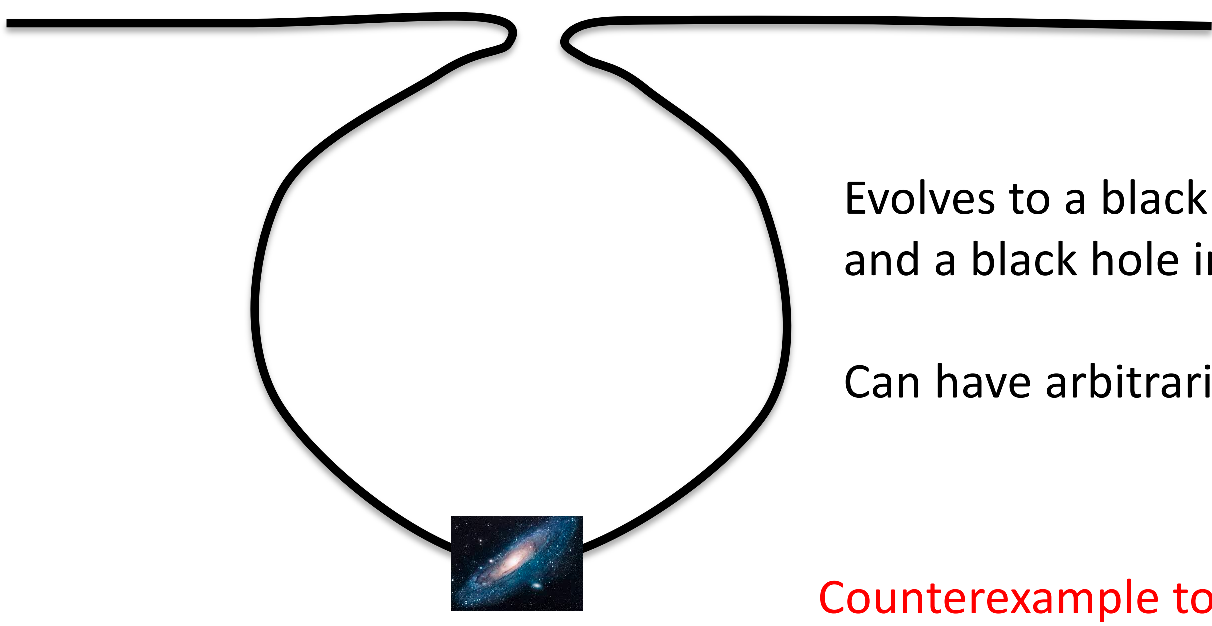
Eddington, Lemaitre, Einstein,
Rosen, Finkelstein,
Kruskal

Vacuum solution. No matter.
Two exteriors, sharing the interior.

``Bags of Gold”

Wheeler

Initial slice:



Evolves to a black hole as seen from the outside
and a black hole in a closed universe.

Can have arbitrarily large amount of entropy ``inside’’

Counterexample to the statement that
Area entropy counts the entropy “inside”

Wikipedia article:

The holographic principle resolves the black hole information paradox within the framework of string theory.[4] However, there exist classical solutions... "Wheeler's bags of gold". …. are not yet fully understood.[5]

We will see how to resolve these confusions

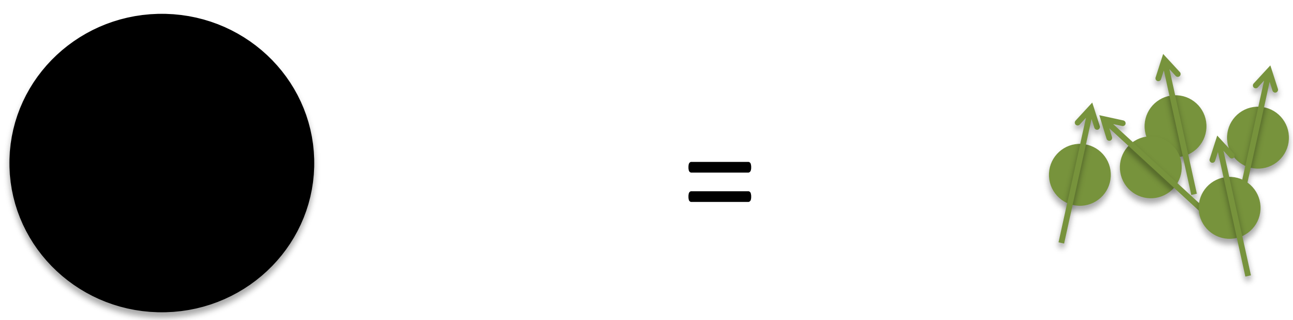
- These confusions involve the black hole interior.
- They involve computations of ``fine grained entropy'', not thermodynamic entropy.
- They confusions involve entanglement.

Back to basics

Black holes as quantum systems

Central hypothesis

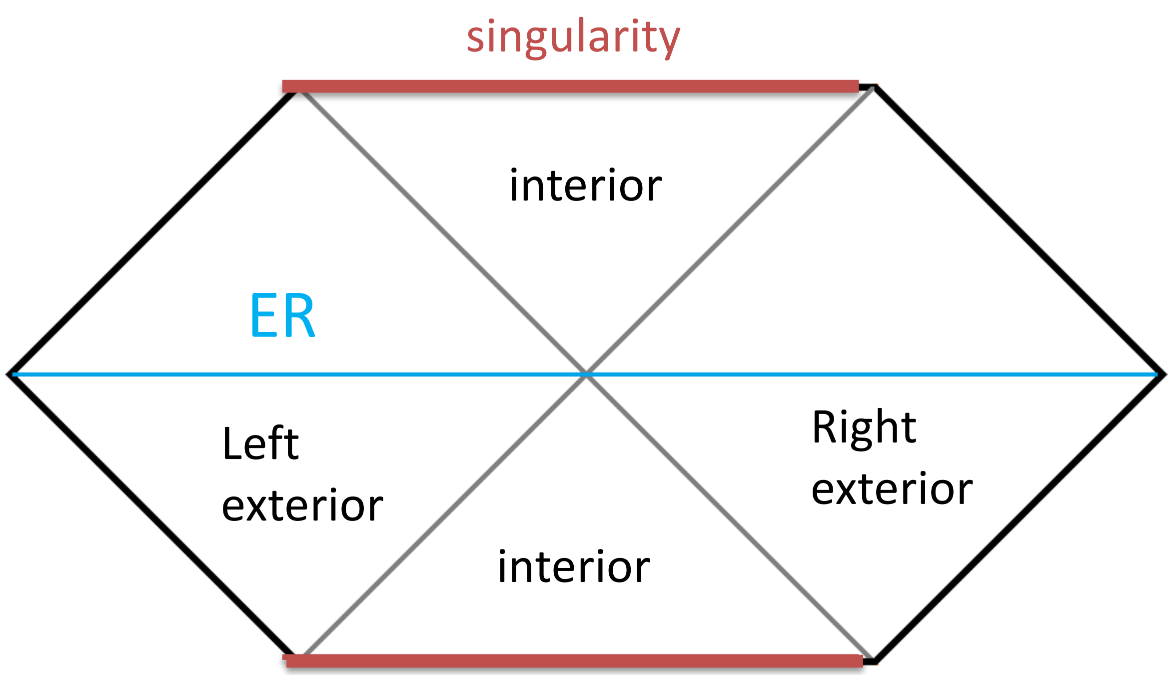
- A black hole seen from the outside can be described as a quantum system with S degrees of freedom (qubits). S = Area/4 (l_p =1)
- It evolves according to unitary evolution, seen from outside.



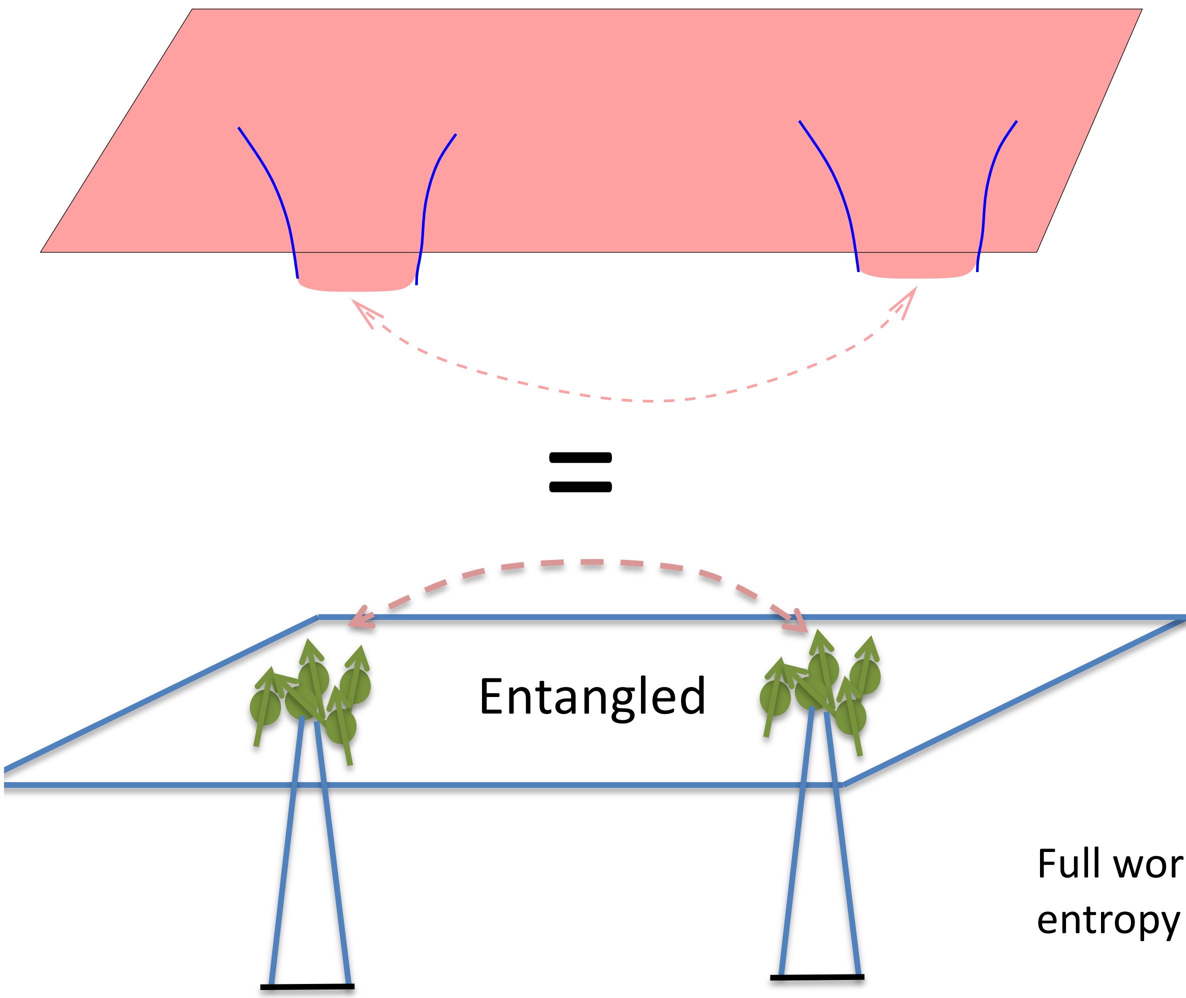
It is only a statement about the black hole as seen from the outside !

No statement has been made about the inside (yet).

Full Schwarzschild solution as a wormhole



Wormholes and entangled states



Connected through the interior

ER = EPR

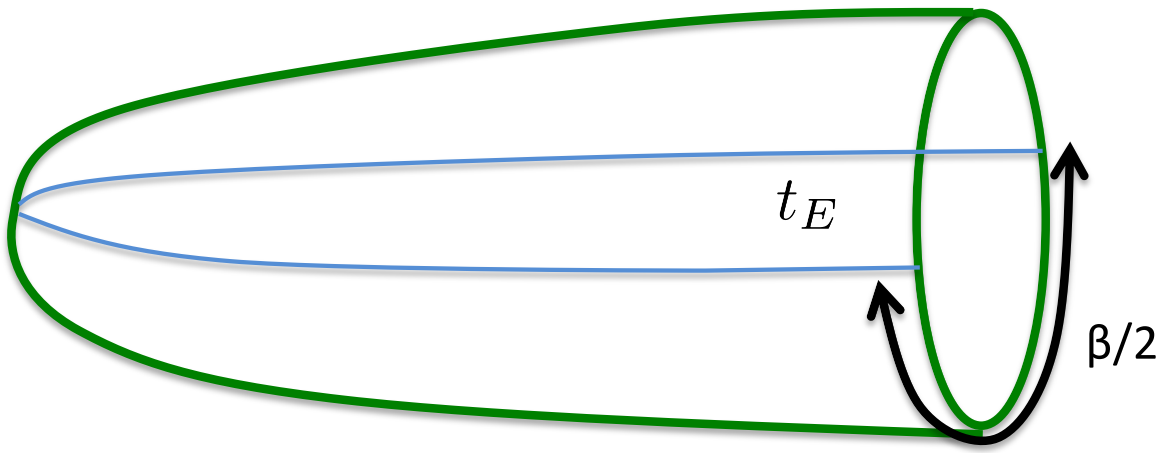
W. Israel
J.M.
JM Suskind

Full wormhole is in a pure state,
entropy of each black hole arises from entanglement

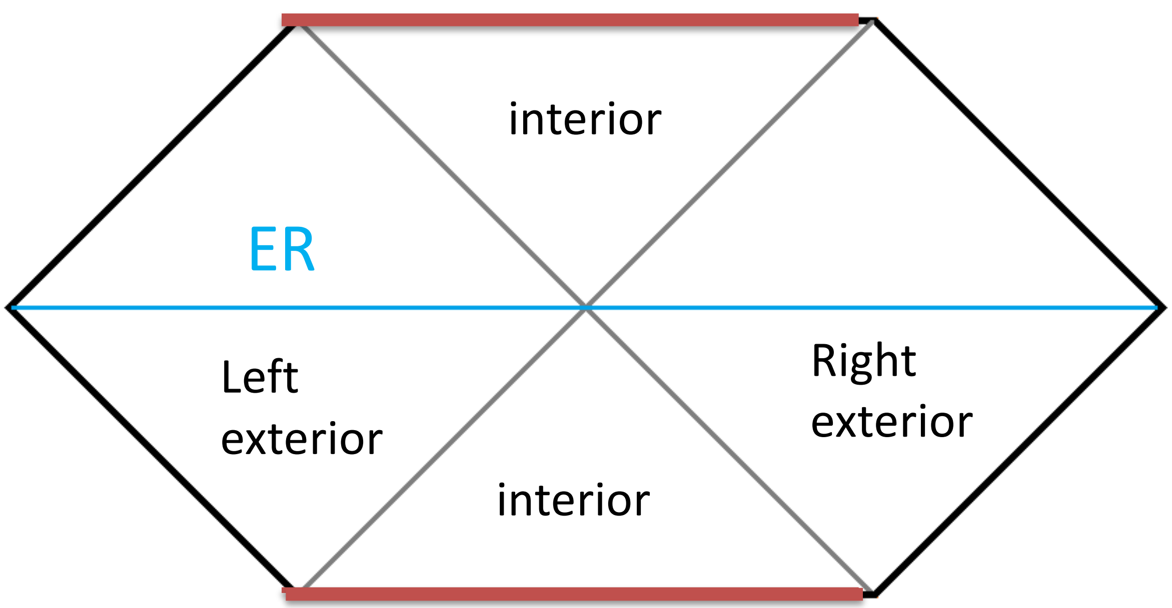
In a particular entangled state

$$|TFD\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L |E_n\rangle_R$$

Euclidean black hole



Hartle
Hawking



The rest of the paradoxes involve understanding fine grained entropy

Two notions of entropy

- Coarse grained entropy = thermodynamic entropy. Obeys 2nd law. Arises from ``sloppiness”

$$S = \max_{\tilde{\rho}} \left(-Tr[\tilde{\rho} \log \tilde{\rho}] \right) \quad , \qquad\qquad Tr[A\tilde{\rho}] = Tr[A\rho]$$

Subset of observables, ``simple observables”

- Fine grained entropy. Remains constant under unitary time evolution. (sometimes called ``entanglement’’ entropy)

$$S = -Tr[\rho \log \rho]$$

For the moment we will be talking about the entropy of the black hole as seen from the outside.

This is the entropy of the quantum system that appeared in our “central hypothesis”

The horizon area computes thermodynamic entropy

How can we compute the fine grained one ?

Fine grained gravitational entropy

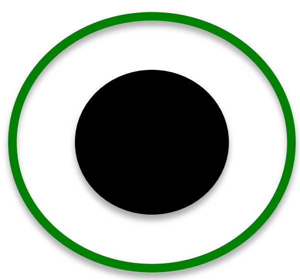
Ryu-Takayanagi 2006
Hubeny, Rangamani, Takayanagi 2007
Engelhardt, Wall 2014

$$S = \min \left\{ \text{ext} \left[\frac{\text{Area}}{4 l_p^2} + S_{\text{matter}} \right] \right\}$$

Follows from AdS/CFT rules:
Lewkowycz, JM , Faulkner,Dong,...

We need to find an extremal area. The ``smallest'' extremal area.

More precise: minimal area along a spatial slice (Cauchy slice) and maximal among all possible Cauchy slices



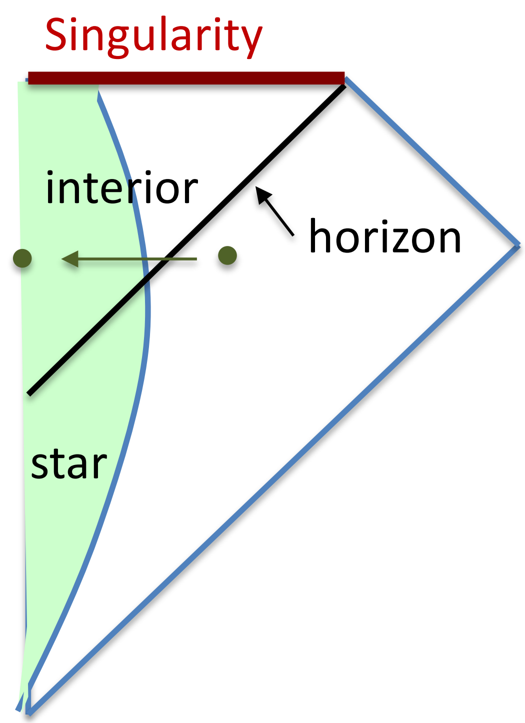
Start with a surface going around the horizon and shrink it as much as possible.

We are allowed to take the surface to the inside.

It is the fine grained entropy of the quantum system that describes the black hole as seen from the outside.

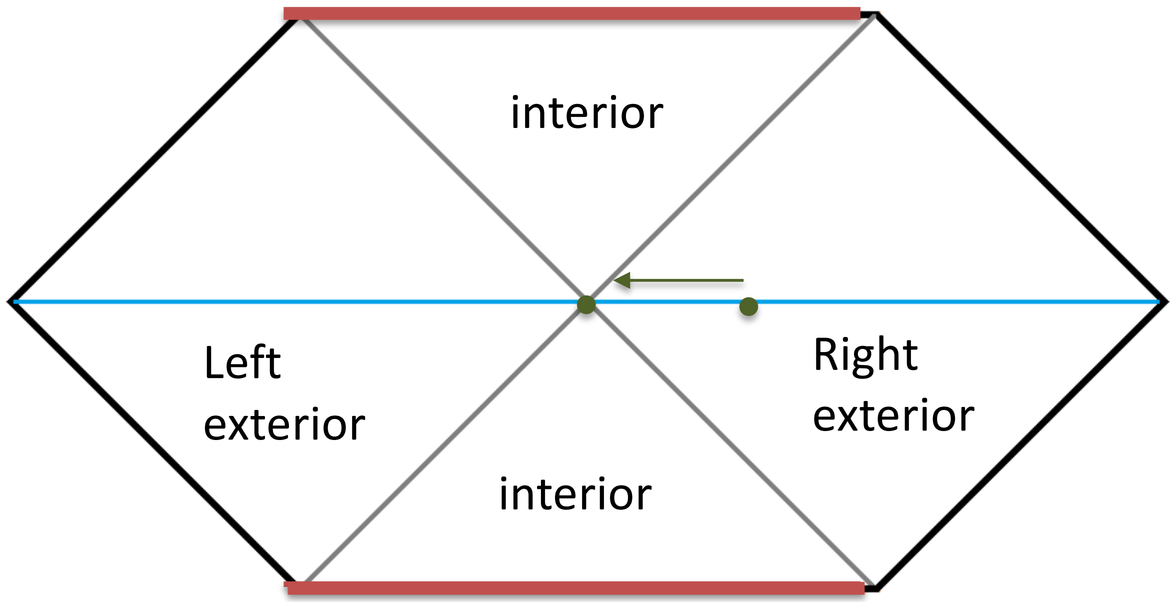
Examples

Extremal surface for a black hole formed from collapse



Zero area.
Entropy = entropy of matter that makes up the star

Extremal surface for the Schwarzschild wormhole



The fine grained entropy for one side of the Schwarzschild wormhole is equal to the Area.

There are intermediate cases…

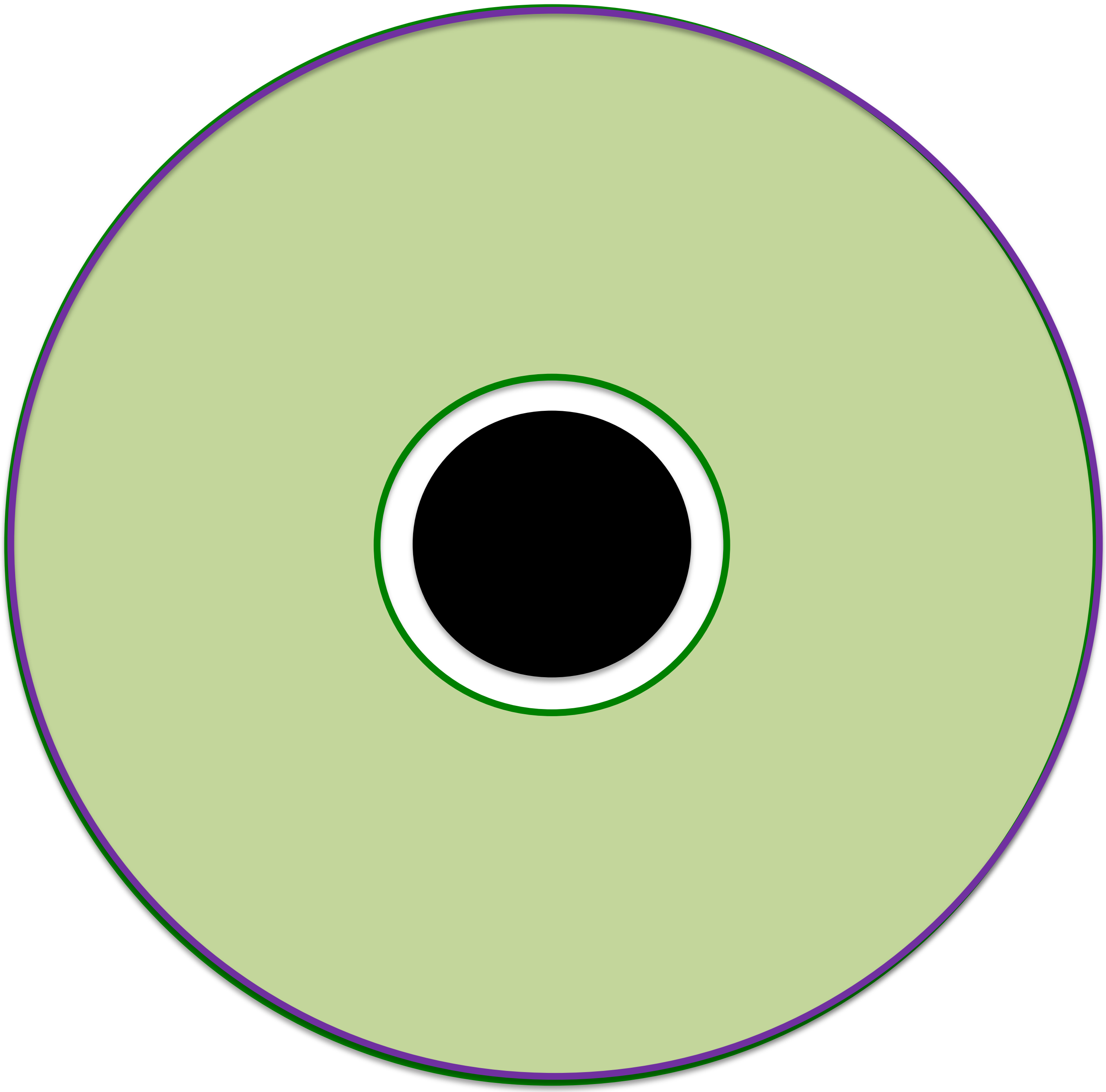
You should be surprised by the claim that there is a formula for the <u>fine grained</u> entropy

Entanglement wedge

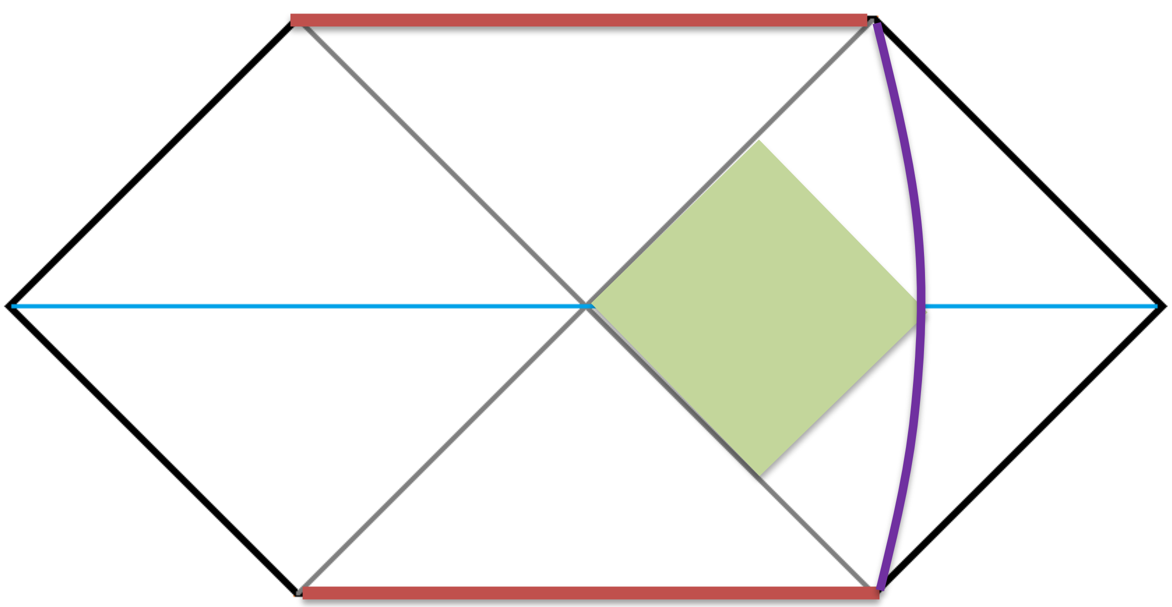
- We keep track of the region swept by the surface as it tries to extremize the entropy.

- That region outside the quantum extremal surface is called the ``entanglement wedge''.

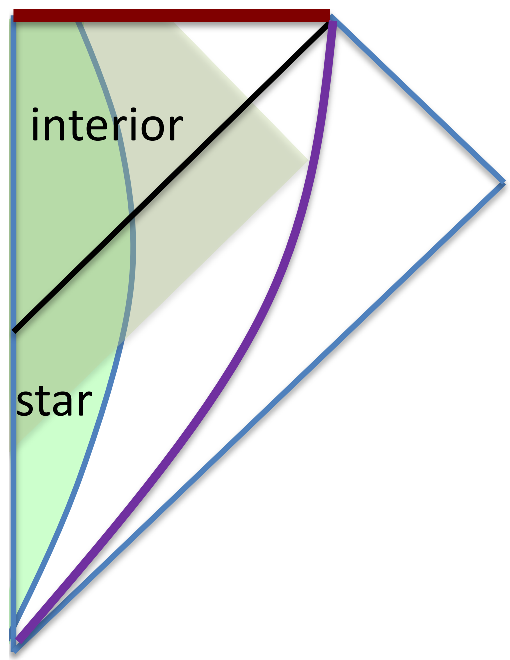
Czech, Karczmarek, Nogueira, Van Raamsdonk,
Wall, Headrick, Hubeny, Lawrence, Rangamani



As we move the surface inwards,
we keep track of the entropy in the fields outside



Here the extremal surface
Is the bifurcation surface



Entanglement wedge covers the interior.

We saw that there was a quantum system that describes the black hole from the outside.

How much of the spacetime does this quantum system describe ?

- Only the outside ?
- A portion of the inside ?
- Which portion ?

We will introduce a new hypothesis

Entanglement wedge reconstruction hypothesis

- The quantum system describes everything that is included in the entanglement wedge.

- We can recover the state of a (probe) qubit inside the entanglement wedge.

- Recovery is state dependent and similar to quantum error correction.

- Many consistency checks

  Almheiri, Dong, Harlow, Jafferis, Lewkowycz, J.M., Suh, Wall, Faulkner….

- <u>Usual:</u> If you do simple observations → see everything outside the horizon.

- <u>New:</u> If you do arbitrarily complex observations → see everything outside the "minimal surface".

We will now argue that this removes some apparent paradoxes

``Bags of Gold"

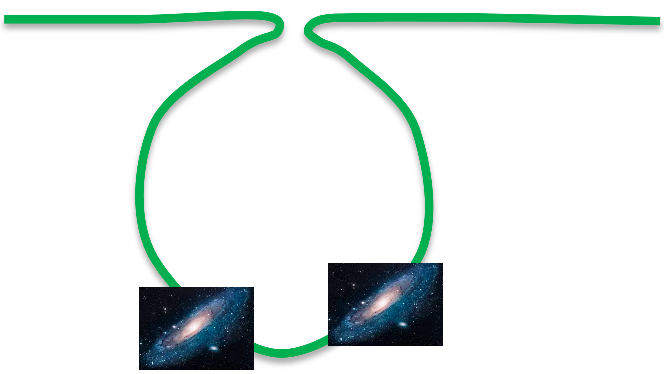
Marolf, Wall

Two possibilities:

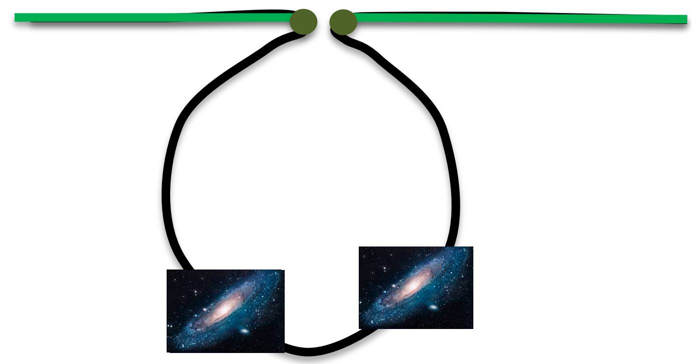
A) Bag has little entropy due to matter (compared with the area of the black hole horizon)

B) Bag has lots of entropy due to matter

Little matter entropy inside



Lots of matter entropy inside



Entanglement wedge covers all.

Could still be very big, but since the number of states is small, and the map can be state dependent, this is not a problem

Minimal surface at the neck.

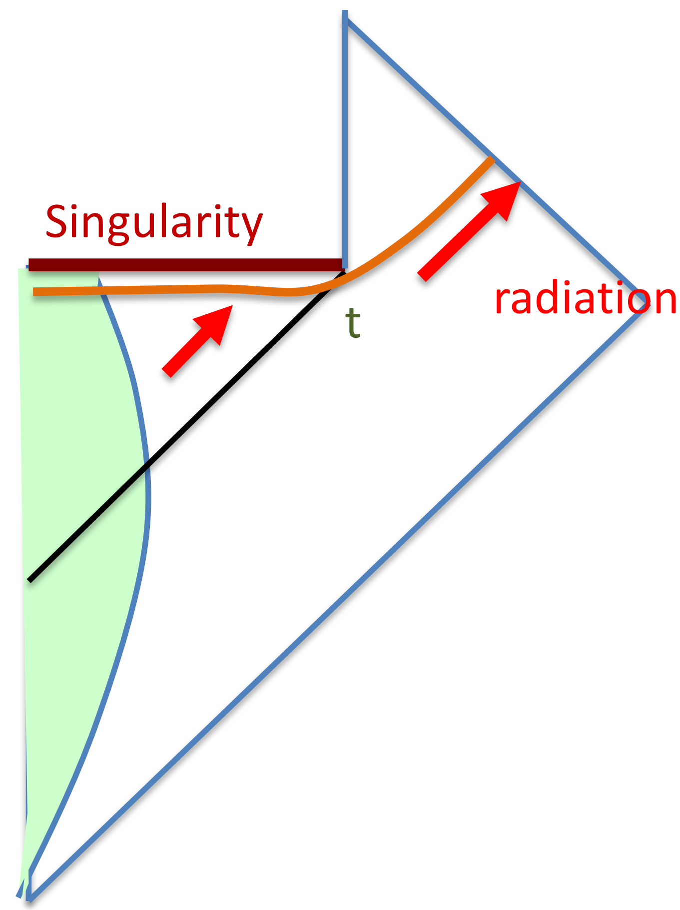
The size of entanglement wedge depends on the (fine grained) entropy, not on the energy, of the matter inside.

Old evaporating black hole

Geometry of an evaporating black hole made from collapse

Entropy on the orange slice (nice slice), inside the black hole, could be much bigger than the area at t, for a black hole that has evaporated for a long time.

The geometry and entropy on the orange slice is somewhat similar to the bag of gold.

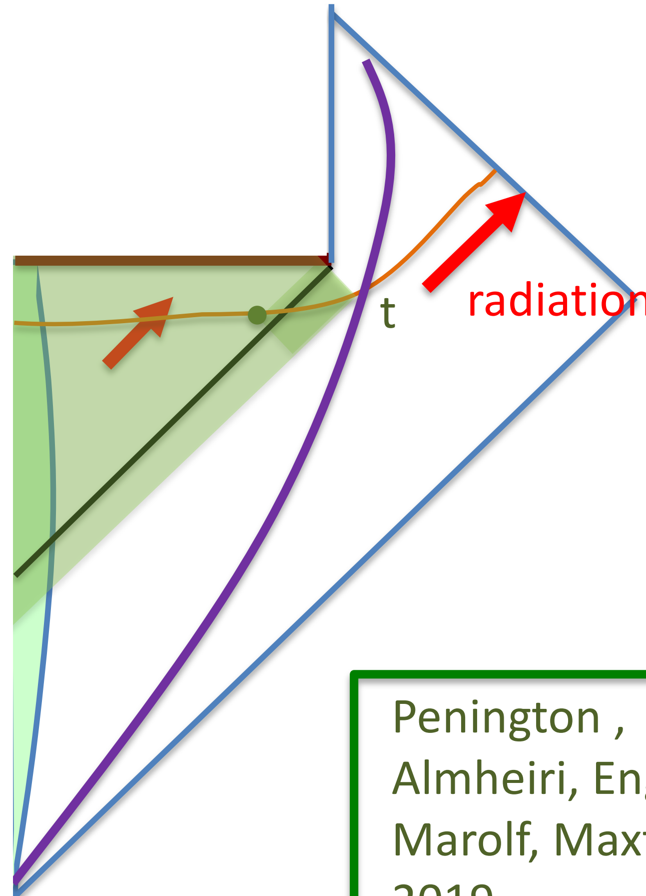


Old evaporating black hole

There is a second extremal surface.

The entanglement wedge
Includes only a small part of the interior,
just behind the horizon.

It is crucial to include the
quantum contribution.



Penington ,
Almheiri, Engelhardt,
Marolf, Maxfield ;
2019

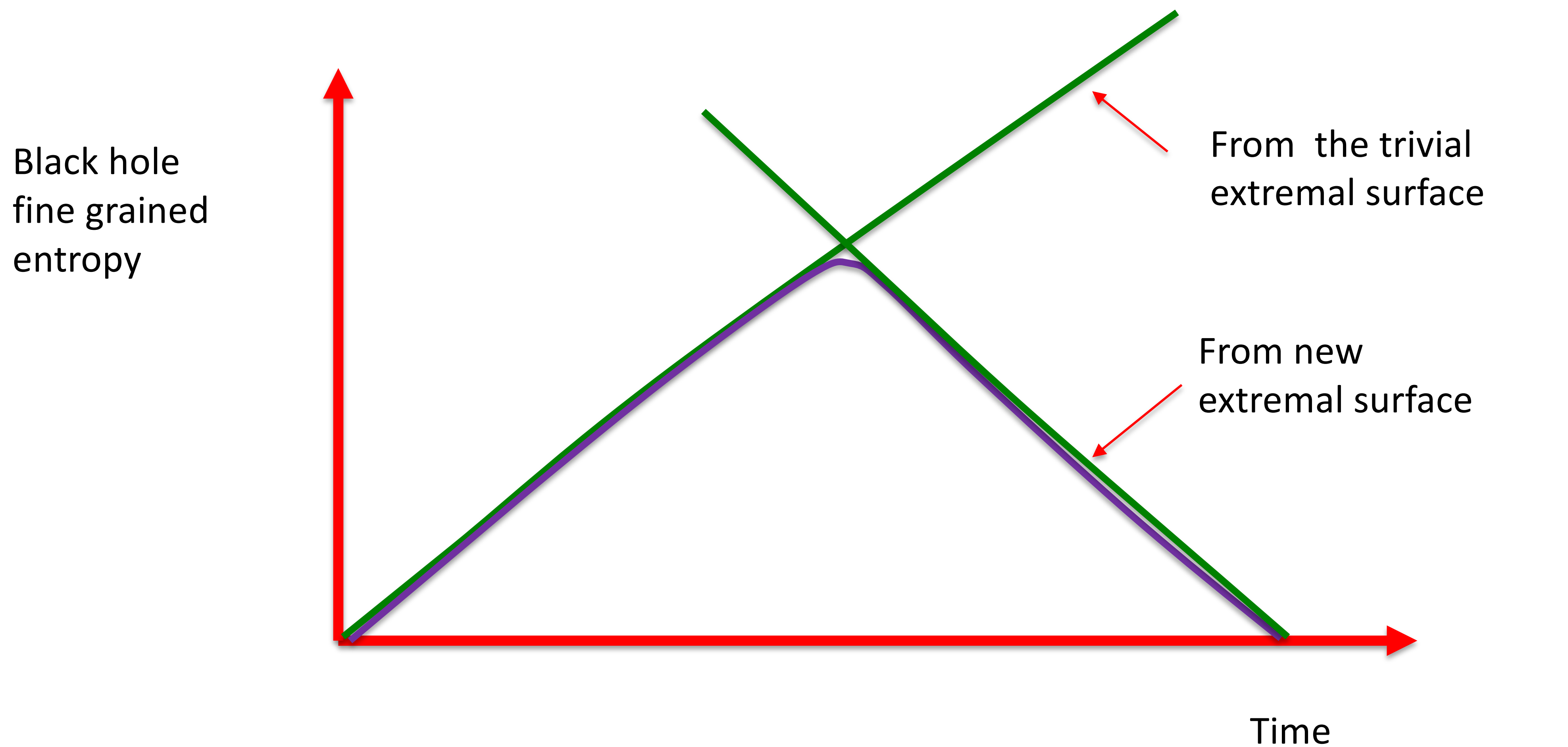
Two implications for old black holes :

1) Fine grained entropy is close to the old black hole entropy.
2) The quantum system describing the black hole describes only a portion of the interior

Fine grained entropy of the black hole

(of the quantum system describing the black hole)

Almheiri, Engelhardt, Marolf, Maxfield ; Penington 2019

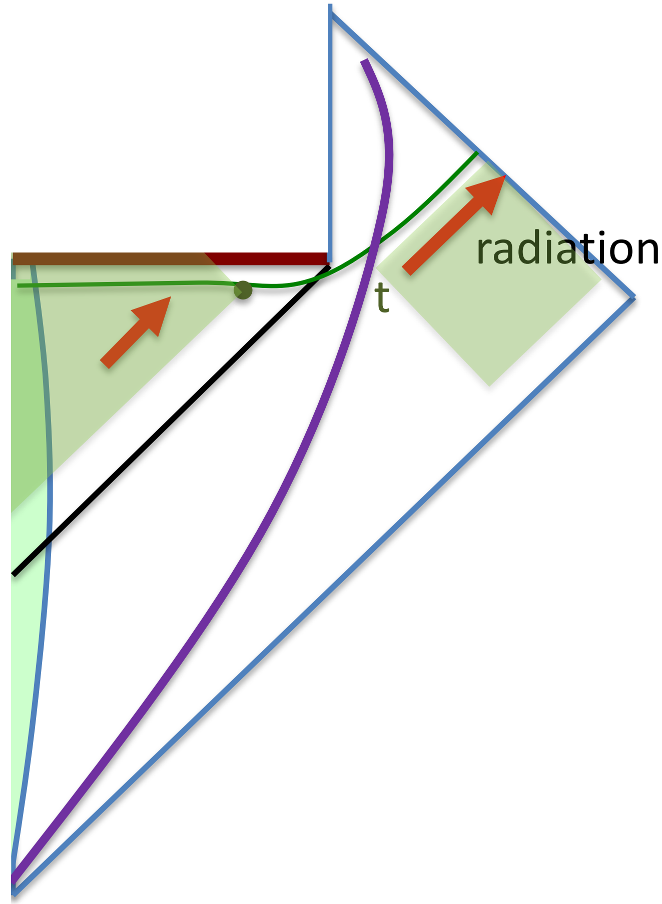


What about the entropy of the radiation ?

Entropy of radiation?

- The radiation appears to be in a mixed state.
- Why?
- Because it was entangled with the fields that were inside the black hole.

- How do we know this?
- Through the evolution of gravity.
- We should use the gravity rules to compute the fine grained entropy.



Naïve entropy of the radiation

Correct entropy of the radiation

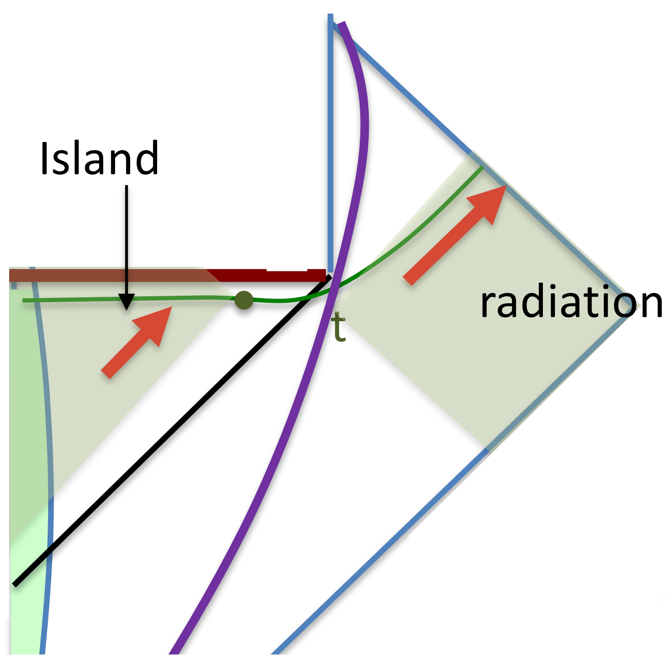
New rule

$$S(\mathbf{Rad}) = \min \left[S(\mathrm{Rad} \cup \mathrm{Islands}) + \frac{\mathrm{Area(Islands)}}{4G_N} \right]$$

Exact entropy

Entropy in the effective field theory description

Penington
Almheiri, Mahajan, JM, Zhao



- Is this just an ``accounting trick’’?

- It is the accounting rule that gravity instructs us to use!.

- This ``accounting rule’’ was derived before it was applied to this problem.

Derivation of the gravitational fine grained entropy formula

It can be derived using a reasoning similar to the Gibbons Hawking euclidean black hole argument.

Use a ``replica trick'' where we introduce n copies of the system and continue near n =1.

$$Z_n = Tr[\rho^n] \longrightarrow S = (1 - n\partial_n) \log Z_n |_{n=1}$$

This introduces a small conical defect angle, a kind of cosmic string. Minimizing the action we minimize over the area.

Lewkowycz, JM; Faulkner Lewkowycz JM; Dong, Lewkowycz.

In the case of ``islands'' the geometry contains wormholes connecting the n copies. ``Replica wormholes''.

Two classical contributions to the gravitational path integral.

One gives the Hawking answer and the other the Page answer.

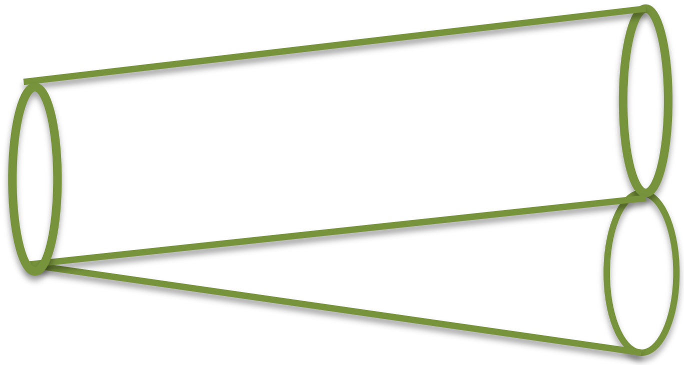
The Page one dominates at late times.

It is a Hawking/Page transition.

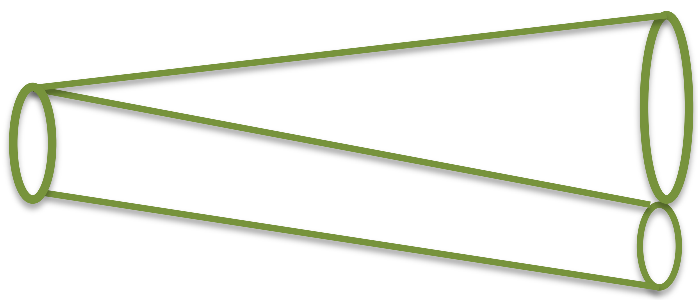
Penington, Shenker, Stanford, Yang ; Almheiri, Hartman, J.M., Shaghoulian, Taj, 2019

Two topologies for black holes

Euclidean
Black hole



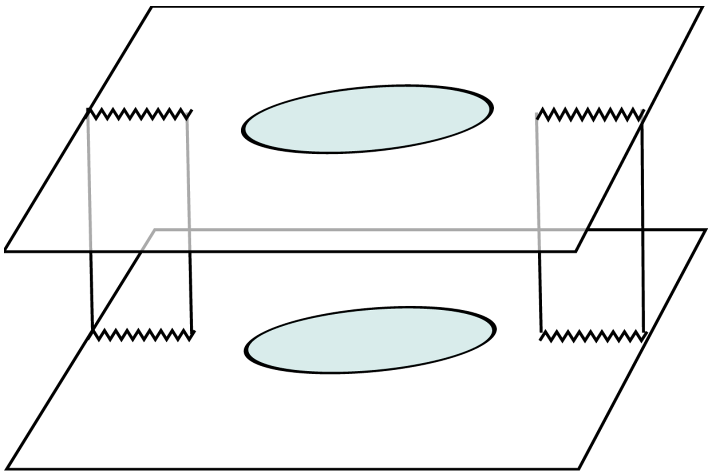
Flat space



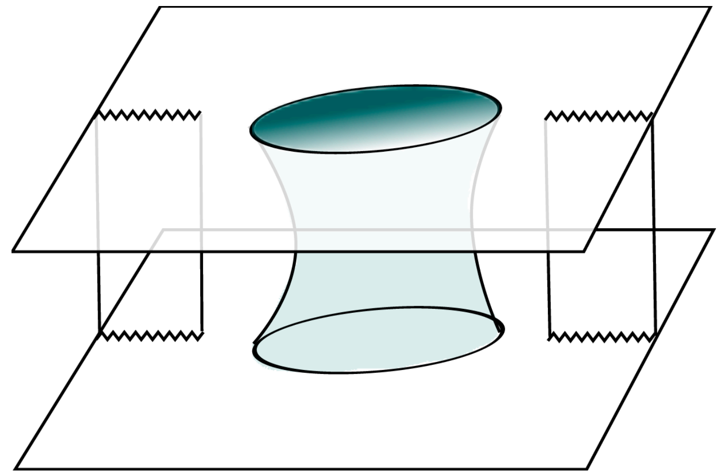
Hawking-Page transition

Hawking
Page

Replica wormholes: n=2



Solution that gives
Hawking's result



Replica wormhole, giving the
Page answer when it dominates
at late times.

Conclusions

- We reviewed thermodynamic black hole entropy
- We described the fine grained gravitational entropy formula.
- We applied it to the computation of the entropy of radiation.

- A lot of what we discussed was derived by thinking about aspects of AdS/CFT, which itself was derived using string theory.
- But you only need gravity as an effective theory to apply these formulas.

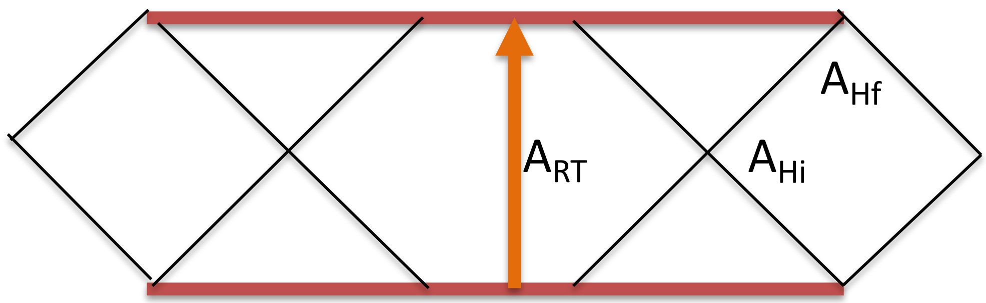
I am in awe of how clever gravity is !

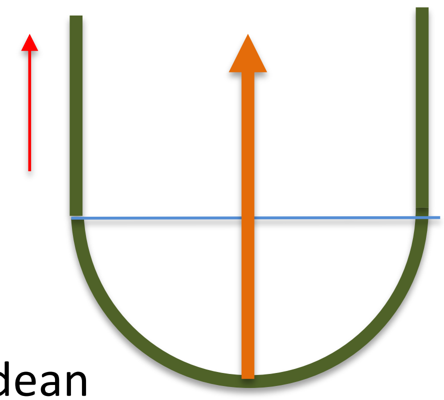
Future

- More explicit picture for the microstates.
- What further lessons is this teaching us about the interior? The singularity ?
- A hundred years ago a black holes were a mathematical curiosity and today they are an observational reality.
- Hopefully, the same will happen with the aspects of black holes we have been discussing!

Thank you !

Longer wormhole





$$A_{RT} < A_{Hi} < A_{Hf}$$

Minimal (extremal) area