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JÜLICH FORSCHUNGSZENTRUM

Life on Earth – an Accident ?

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The anthropic principle

THE ANTHROPIC PRINCIPLE

- so **many** parameters in the Standard Model, the landscape of string theory, ...

⇒ The anthropic principle:

"The observed values of all physical and cosmological quantities are not equally probable but they take on values restricted by the requirement that there exist sites where carbon-based life can evolve and by the requirements that the Universe be old enough for it to have already done so."

Carter 1974, Barrow & Tippler 1988, ...

⇒ can this be tested? / have physical consequences?

- Ex. 1: "Anthropic bound on the cosmological constant" Weinberg (1987) [743 cites]
- Ex. 2: "The anthropic string theory landscape" Susskind (2003) [916 cites]

A PRIME EXAMPLE for the ANTHROPIC PRINCIPLE

- Hoyle (1953):

Prediction of an excited level in carbon-12 to allow for a sufficient production of heavy elements (^{12}C, ^{16}O,...) in stars

- was later heralded as a prime example for the AP:

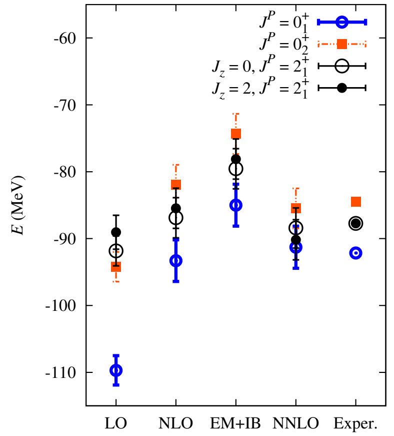
"As far as we know, this is the only genuine anthropic principle prediction"
Carr & Rees 1989

"In 1953 Hoyle made an anthropic prediction on an excited state – 'level of life' – for carbon production in stars"
Linde 2007

"A prototype example of this kind of anthropic reasoning was provided by Fred Hoyle's observation of the triple alpha process..."
Carter 2006

The RELEVANT QUESTION

Date: Sat, 25 Dec 2010 20:03:42 -0600
From: Steven Weinberg ⟨weinberg@zippy.ph.utexas.edu⟩
To: Ulf-G. Meissner ⟨meissner@hiskp.uni-bonn.de⟩
Subject: Re: Hoyle state in 12C



Dear Professor Meissner,

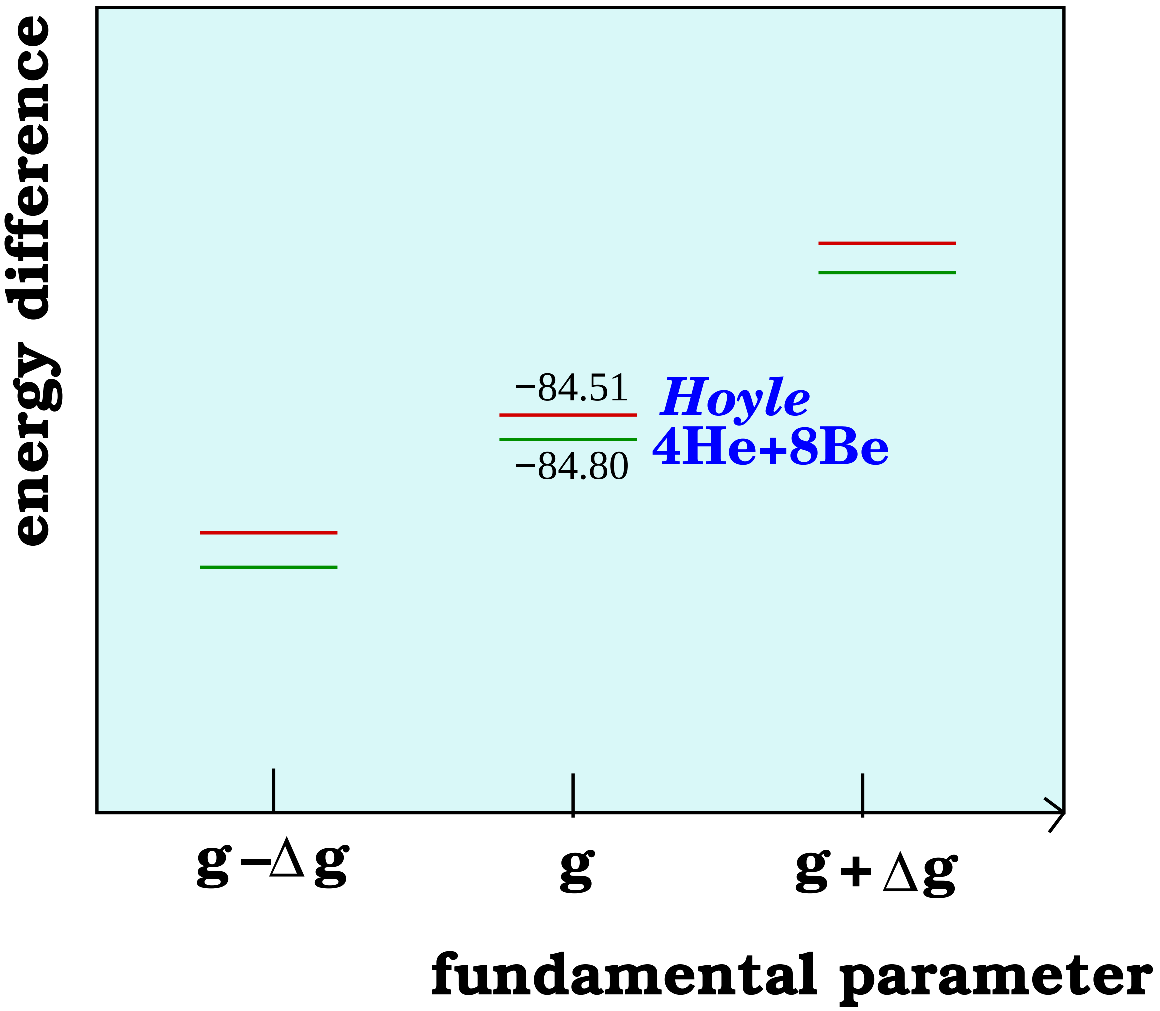
Thanks for the colorful graph. It makes a nice Christmas card. But I have a detailed question. Suppose you calculate not only the energy of the Hoyle state in C12, but also of the ground states of He4 and Be8. How sensitive is the result that the energy of the Hoyle state is near the sum of the rest energies of He4 and Be8 to the parameters of the theory? I ask because I suspect that for a pretty broad range of parameters, the Hoyle state can be well represented as a nearly bound state of Be8 and He4.

All best,
Steve Weinberg

- How does the Hoyle state move relative to the 4He+8Be threshold, if we change the fundamental parameters of QCD+QED?
- not possible in nature, *but on a high-performance computer!*

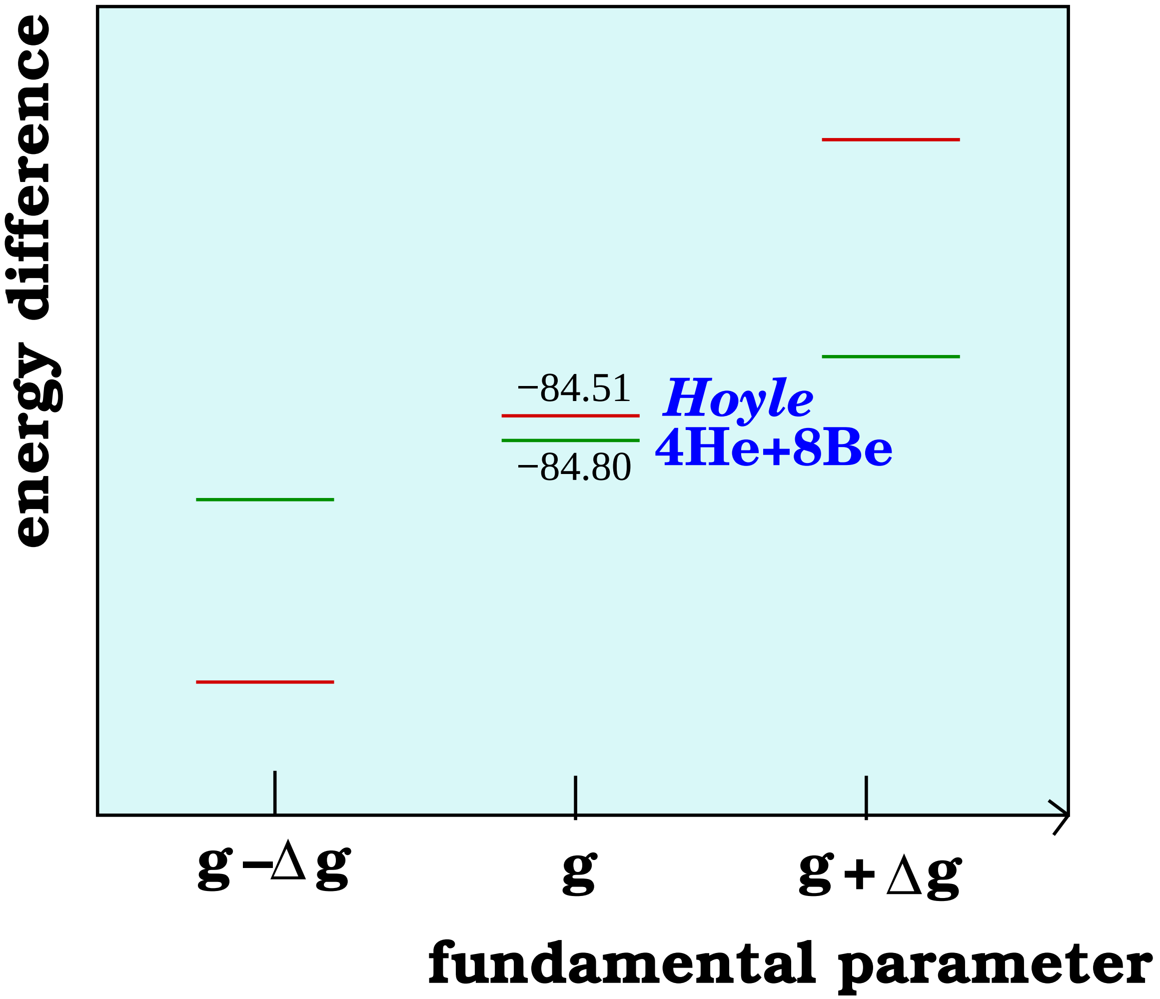
The NON-ANTHROPIC SCENARIO

- Weinberg's assumption: The Hoyle state stays close to the 4He+8Be threshold



The ANTHROPIC SCENARIO

- The AP strikes back: The Hoyle state moves away from the 4He+8Be threshold



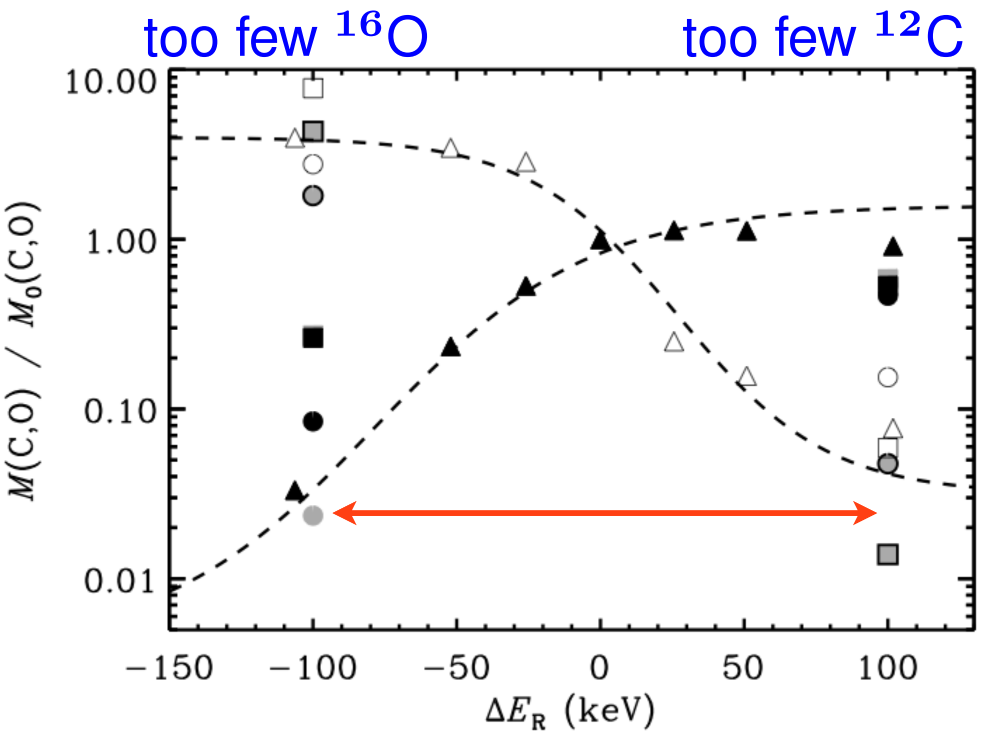
EARLIER STUDIES of the ANTHROPIC PRINCIPLE

- rate of the 3α-process: $r_{3\alpha} \sim \Gamma_\gamma \exp\left(-\frac{\Delta E_{h+b}}{kT}\right)$

$$\Delta E_{h+b} = E^\star_{12} - 3E_\alpha = 379.47(18)\ \text{keV}$$

- how much can ΔE_{h+b} be changed so that there is still enough ^{12}C and ^{16}O?

$\Rightarrow$ $\boxed{|\Delta E_{h+b}| \lesssim 100\ \text{keV}}$



Oberhummer et al., Science **289** (2000) 88

Csoto et al., Nucl. Phys. A **688** (2001) 560

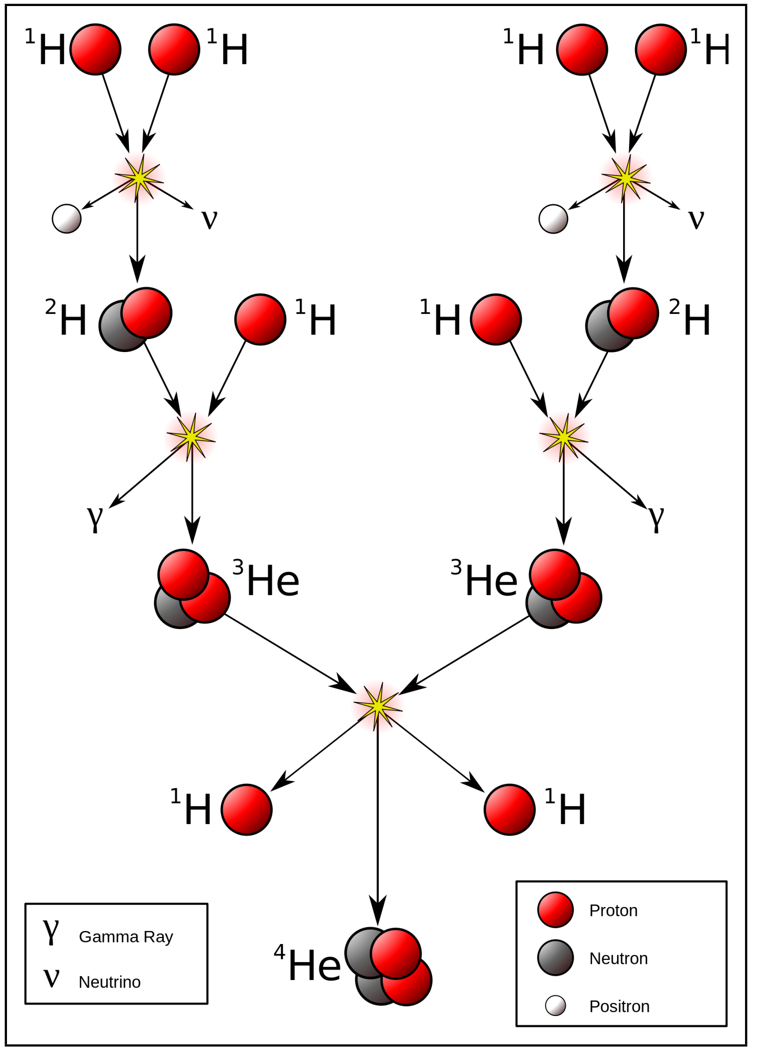
Schlattl et al., Astrophys. Space Sci. **291** (2004) 27

[Livio et al., Nature **340** (1989) 281]

Definition of the physics problem

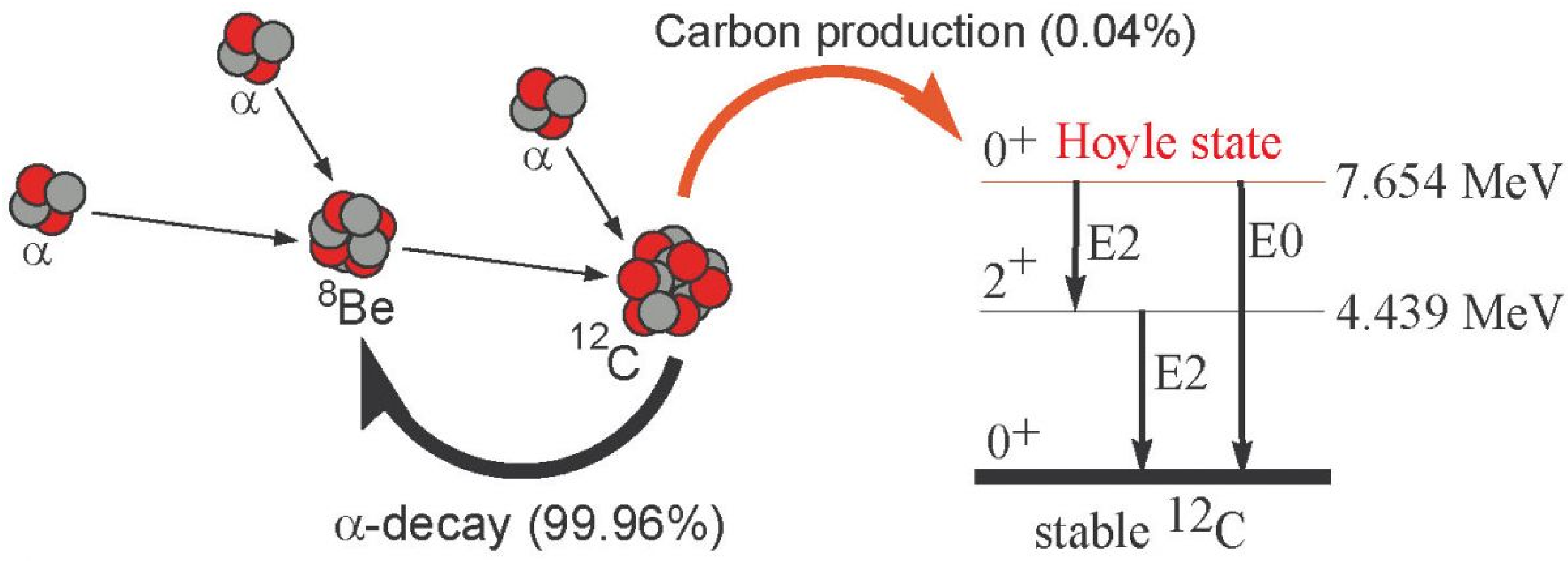
ELEMENT GENERATION

- Elements are generated in the Big Bang & in stars through the **fusion** of protons & nuclei [pp chain or CNO-cycle]
- All is simple until ^{4}He
- Only elements up to Be are produced in the Big Bang [BBNucleosynthesis]
- **Life-essential** elements like ^{12}C and ^{16}O are generated in hot, old stars (triple-alpha reaction, see later)
- Note also that nuclei make up the visible matter in the Universe



[from Wikipedia]

THE TRIPLE-ALPHA PROCESS → MOVIE



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- the ^{8}Be nucleus is instable, long lifetime → 3 alphas must meet
- the Hoyle state sits just above the continuum threshold
  → most of the excited carbon nuclei decay
  (about 4 out of 10000 decays produce stable carbon)
- carbon is further turned into oxygen but w/o a resonant condition

⇒a triple wonder !

EMERGENCE of STRUCTURE in QCD

- The strong interactions are described by QCD:

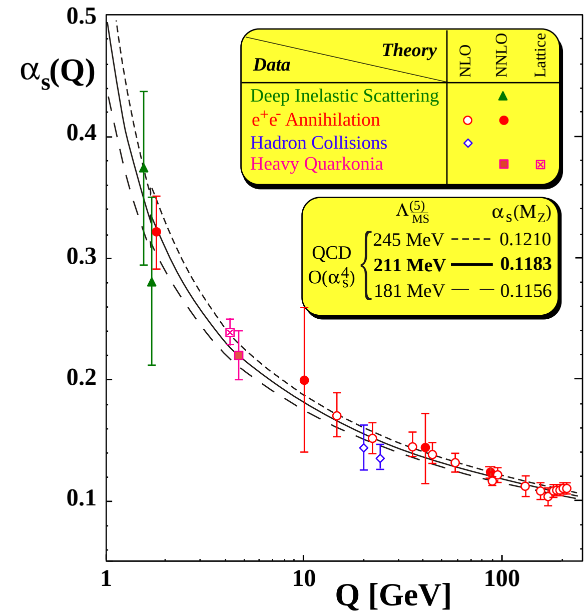
$$\mathcal{L} = -\frac{1}{4g^2} G_{\mu\nu} G^{\mu\nu} + \sum_{f=u,d,s,c,b,t} \bar{q}_f \left(i\gamma_\mu D^\mu - m_f \right) q_f + \ldots$$

- **up** and **down** quarks are very light, a few MeV

- Quarks and gluons are confined within **hadrons**

- Protons and neutrons form **atomic nuclei**

⇒ This requires the inclusion of electromagnetism described by QED with $\alpha_{\rm EM} \simeq 1/137$



[from S. Bethge]

So how sensitive are these strongly interacting composites to variations of the fundamental parameters of QCD+QED?
or: how accidental is life on Earth?

Quark mass dependence of the nuclear forces

Berengut, Epelbaum, Flambaum, Hanhart, UGM, Nebreda, Pelaez,
Phys. Rev. D **87** (2013) 085018

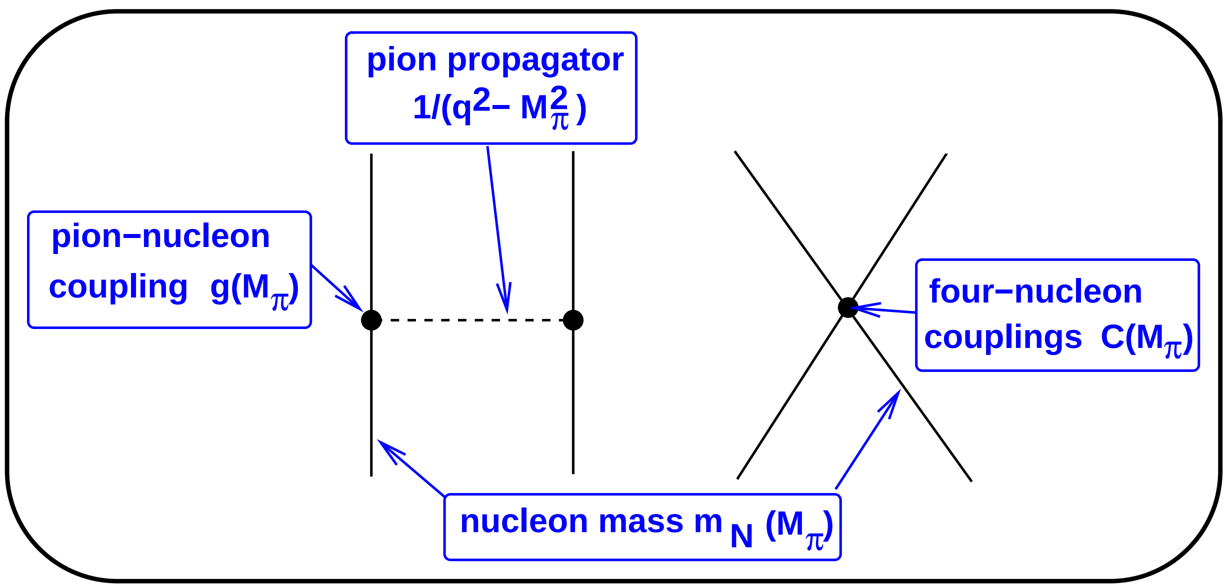
INGREDIENTS

- Nuclear forces are given by chiral EFT based on Weinberg's power counting

Weinberg 1991

$\Rightarrow$ Pion-exchange contributions and short-distance multi-N operators

- graphical representation of the quark mass dependence of the LO potential



- always use the Gell-Mann–Oakes–Renner relation: $M^2_{\pi^\pm} \sim (m_u + m_d)$

- fulfilled to better than 94% in QCD

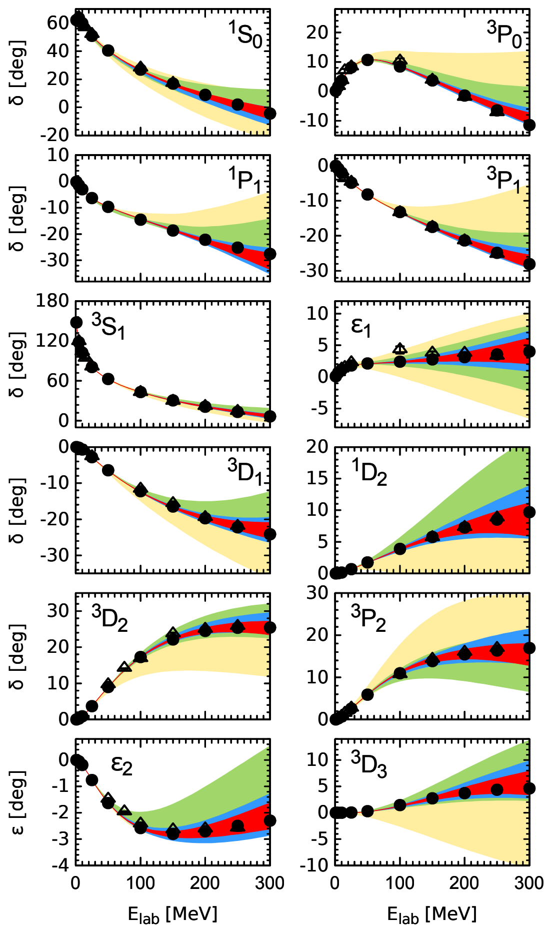
Colangelo, Gasser, Leutwyler 2001

CHIRAL NUCLEAR EFT: RESULTS

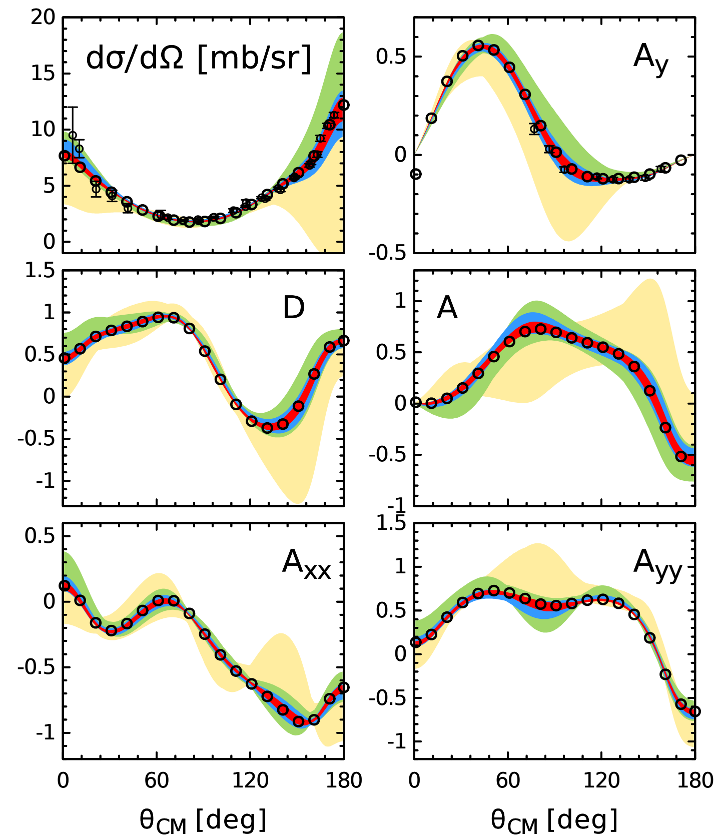
- Expansion to fifth order in the chiral expansion [Weinberg's power counting]

  Epelbaum, Krebs, UGM, EPJA **51** (2015) 53; Phys. Rev. Lett. **115** (2015) 122301

- phase shifts



- np scattering at 200 MeV



NLO
N2LO
N3LO
N4LO

QUARK MASS DEPENDENCE of HADRON MASSES etc

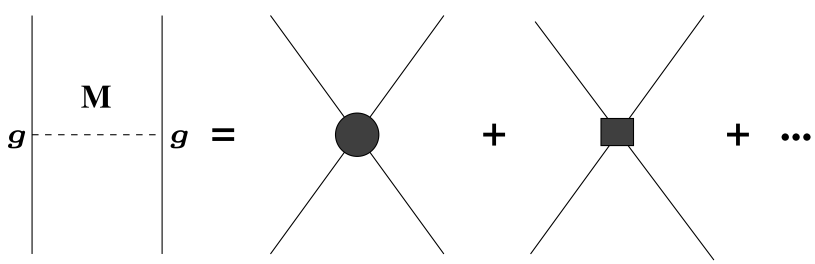
- Quark mass dependence of hadron properties:

$$\frac{\delta O_H}{\delta m_f} \equiv K_H^f \frac{O_H}{m_f}\,, \quad f = u, d, s$$

- Pion and nucleon properties from lattice QCD combined with CHPT

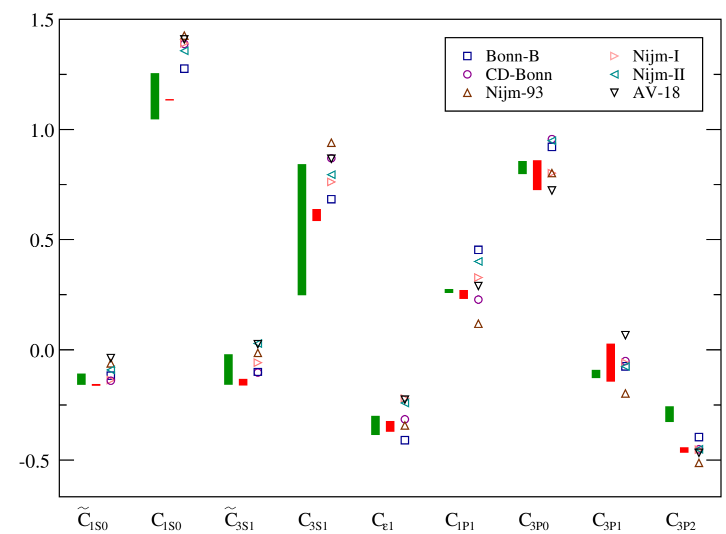
- Contact interactions modeled by heavy meson exchanges + unitarized CHPT

Epelbaum, UGM, Glöckle, Elster (2002) — Hanhart, Pelaez, Rios (2008)



$$\frac{g^2}{t - M^2} = -\frac{g^2}{M^2} - \frac{g^2\,t}{M^4} + \ldots$$

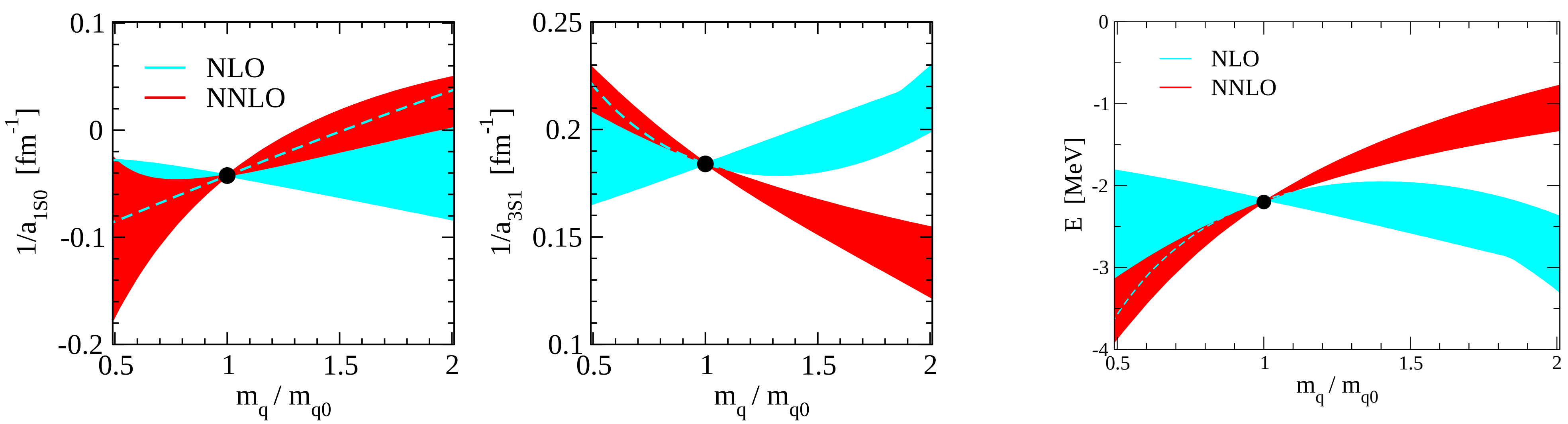
$\hookrightarrow$



RESULTS for the NN SYSTEM

- Putting pieces together for the two-nucleon system:

$$K^q_{a,1S0} = 2.3^{+1.9}_{-1.8}, \quad K^q_{a,3S1} = 0.32^{+0.17}_{-0.18}, \quad K^q_{\mathrm{B(deut)}} = -0.86^{+0.45}_{-0.50}$$



- Extends and improves earlier work based on EFTs and models

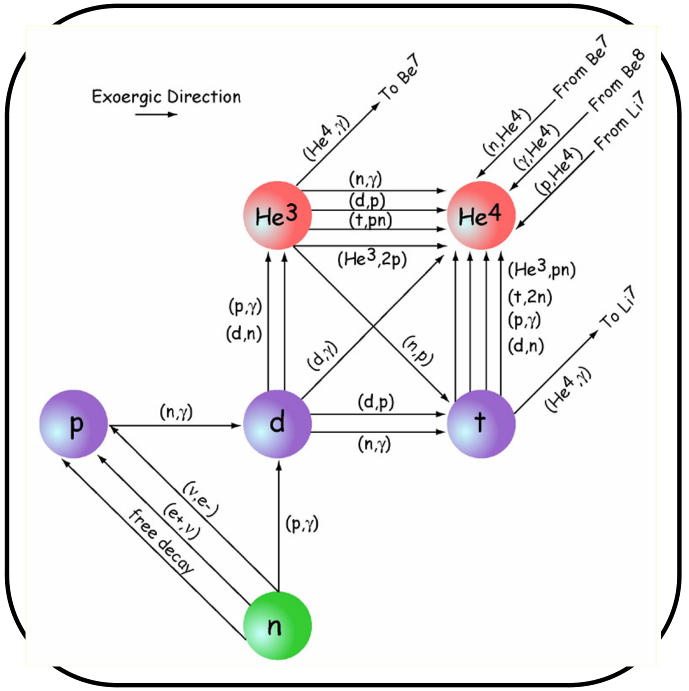
Beane, Savage (2003), Epelbaum, UGM, Glöckle (2003), Mondejar, Soto (2007), Flambaum, Wiringa (2007), Bedaque, Luu, Platter (2011) [BLP], . . .

- connection to lattice QCD results $\longrightarrow$ outlook

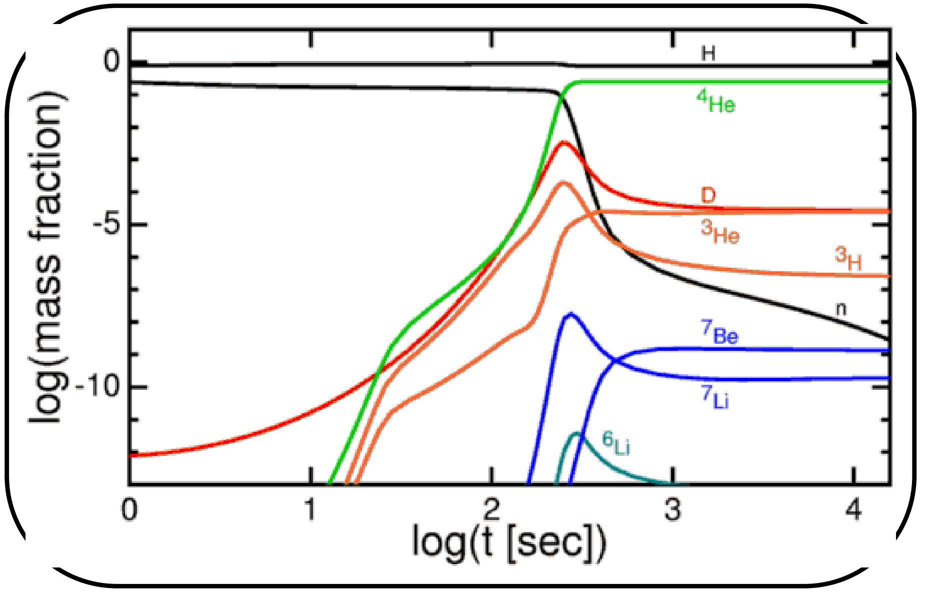
Impact on BBN

Berengut, Epelbaum, Flambaum, Hanhart, UGM, Nebreda, Pelaez,
Phys. Rev. D **87** (2013) 085018

BBN NETWORK & ELEMENT ADUNDANCES



from Cococubed.com



from Burles, Nollett & Turner

- consider element generation up to ^{7}Li, ^{7}Be
- how does this network / the abundances of the elements change under variations of the quark masses?

$\Rightarrow$ calculate BBN response matrix of primordial abundances Y_a at fixed baryon/photon ratio [first in the isospin limit]

LIMITS for the QUARK MASS VARIATION

- Average of [deut/H] and ^{4}He(Y_p):

$$\frac{\delta m_q}{m_q} = 0.02 \pm 0.04$$

- in contrast to earlier studies, we provide reliable error estimates (EFT)
- but: BLP find a stronger constraint due to the neutron life time (affects $Y(^4\mathrm{He})$)
- re-evaluate this under the model-independent assumption that *all* quark & lepton masses vary with the **Higgs VEV** v (CHPT w/ virtual photons)

$\Rightarrow$ results are dominated by the ^{4}He abundance:

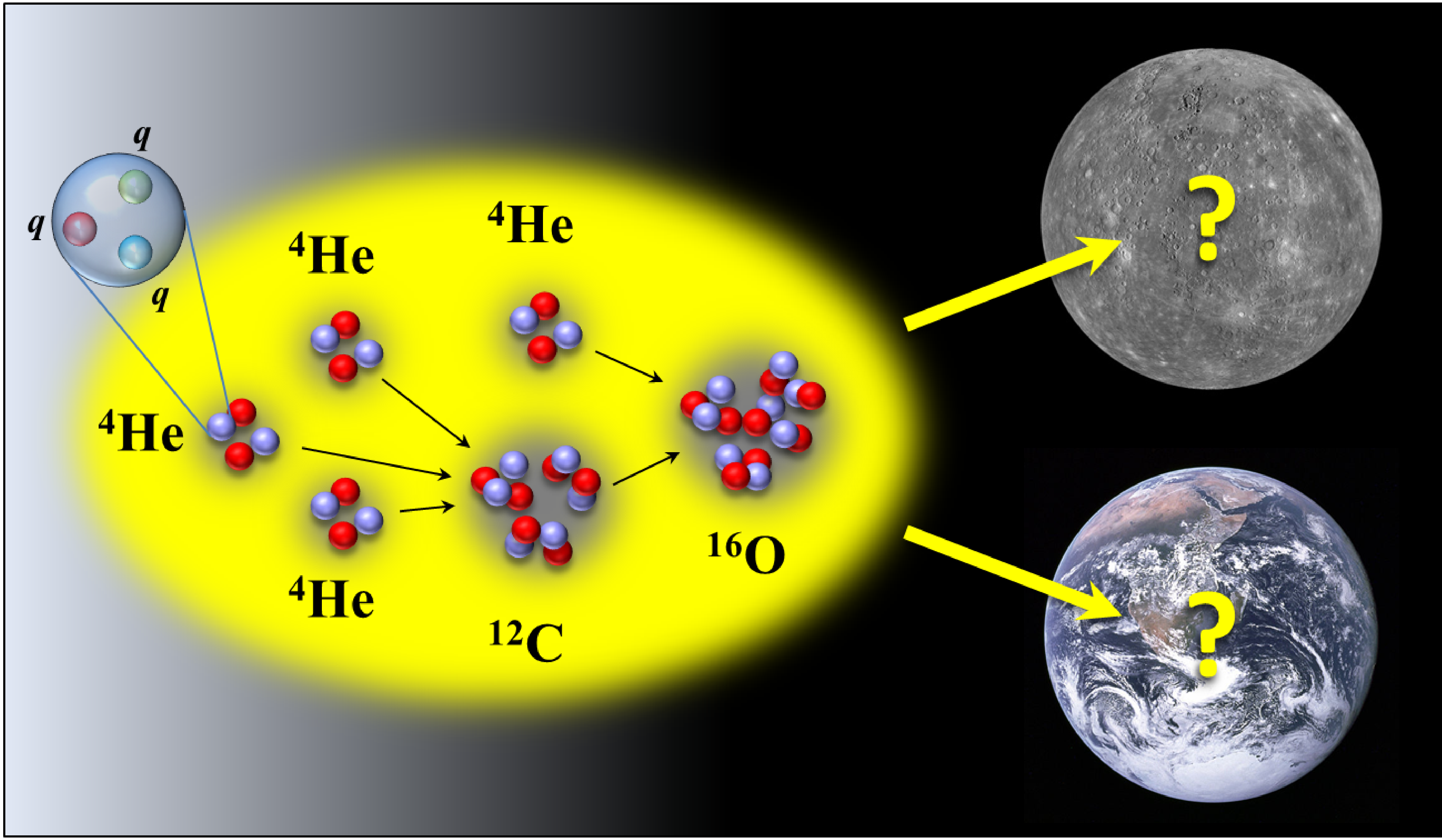
$$\left|\frac{\delta v}{v}\right| = \left|\frac{\delta m_q}{m_q}\right| \leq 0.9\%$$

The fate of carbon-based life as a function of the quark mass

Epelbaum, Krebs, Lähde, Lee, UGM
Phys. Rev. Lett. **110** (2013) 112502
Eur. Phys. J. **A 48**:82 (2013)

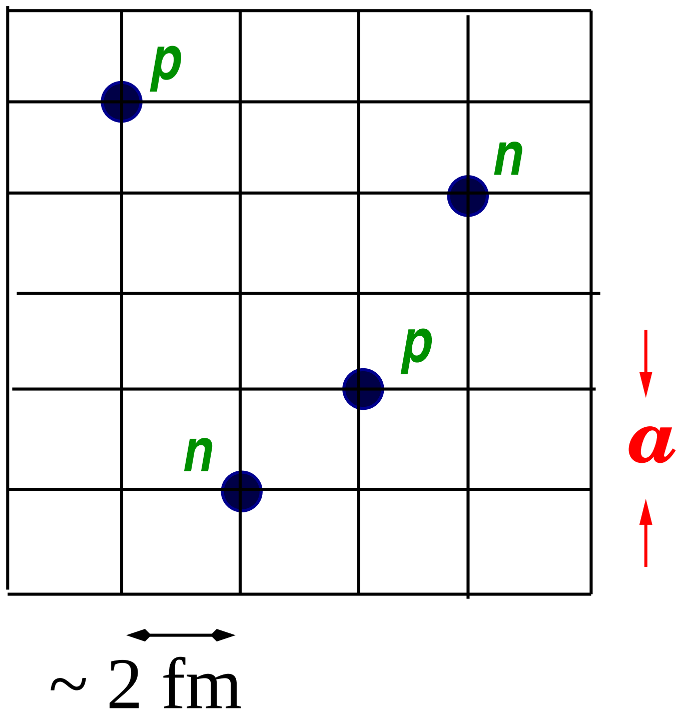
FINE-TUNING of FUNDAMENTAL PARAMETERS

Fig. courtesy Dean Lee



THE TOOL: NUCLEAR LATTICE SIMULATIONS

Frank, Brockmann (1992), Koonin, Müller, Seki, van Kolck (2000) , Lee, Schäfer (2004), . . .
Borasoy, Krebs, Lee, UGM, Nucl. Phys. **A768** (2006) 179; Borasoy, Epelbaum, Krebs, Lee, UGM, Eur. Phys. J. **A31** (2007) 105



- *new method* to tackle the nuclear many-body problem
- discretize space-time $V = L_s \times L_s \times L_s \times L_t$:
  nucleons are point-like fields on the sites
- discretized chiral potential w/ pion exchanges
  and contact interactions + Coulomb

$\rightarrow$ see Epelbaum, Hammer, UGM, Rev. Mod. Phys. **81** (2009) 1773

- typical lattice parameters

$$\Lambda = \frac{\pi}{a} \simeq 300 \text{ MeV [UV cutoff]}$$

- strong suppression of sign oscillations due to approximate Wigner SU(4) symmetry

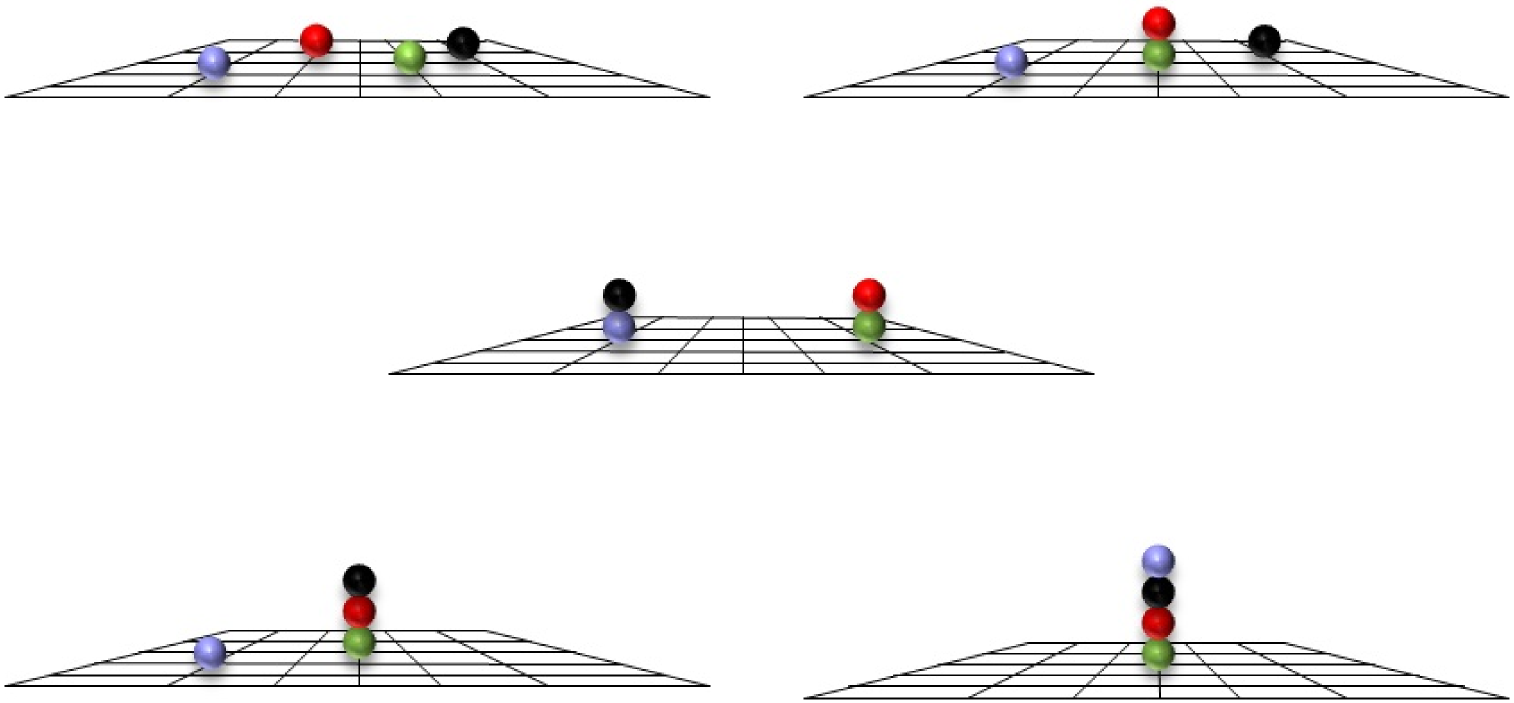
J. W. Chen, D. Lee and T. Schäfer, Phys. Rev. Lett. **93** (2004) 242302, T. Lähde et al., Eur. Phys. J. A**51** (2015) 92

- physics independent of the lattice spacing for $a = 1 \ldots 2$ fm

J. Alarcon et al., EPJA **53** (2017) 83

- hybrid Monte Carlo & transfer matrix (similar to LQCD)

CONFIGURATIONS



⇒ all *possible* configurations are sampled

⇒ *clustering* emerges *naturally*

COMPUTATIONAL EQUIPMENT

- Present = JUQUEEN (BlueGene/Q)
- Future = JURON (IBM-NVIDIA pilot)

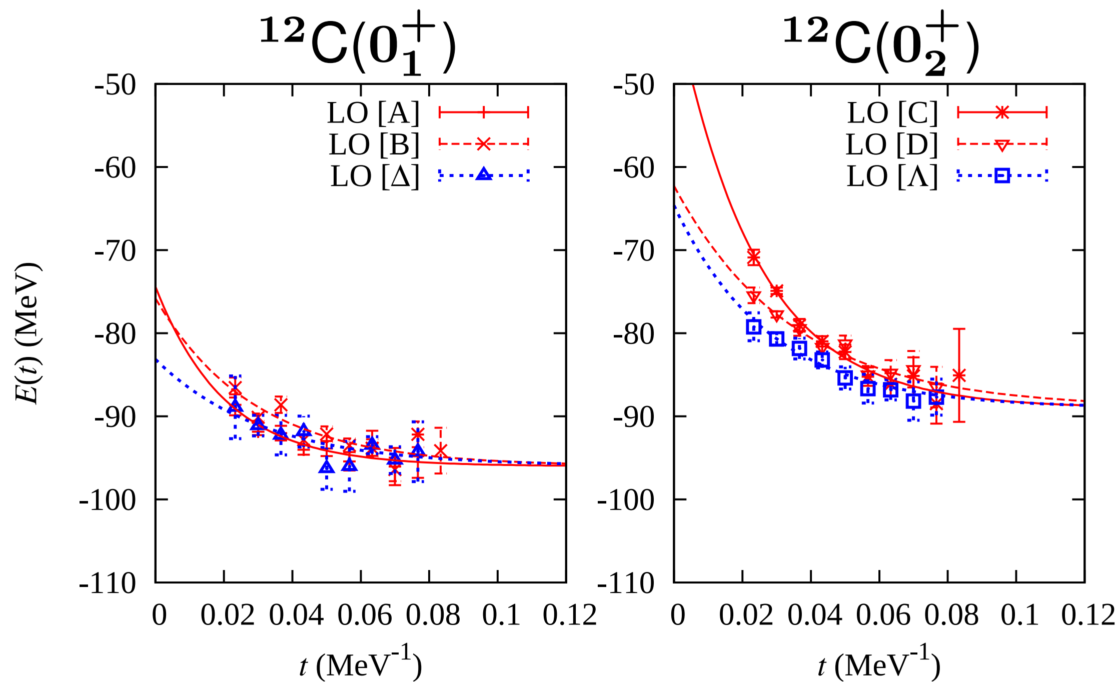
6 Pflops

FIXING PARAMETERS and FIRST RESULTS

Epelbaum, Krebs, Lee, UGM, Phys. Rev. Lett. **104** (2010) 142501; Eur. Phys. J. A **45** (2010) 335; . . .

- some groundstate energies and differences [NNLO, 11+2 LECs]

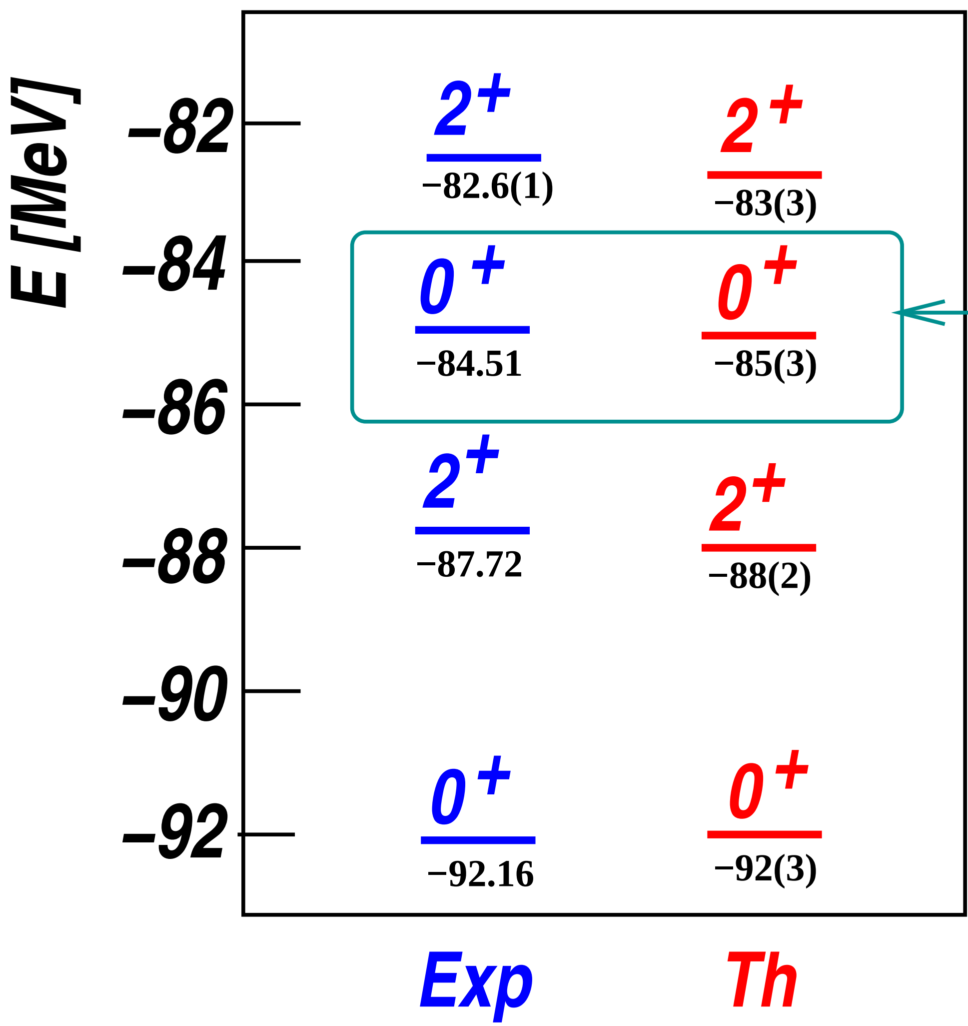
	E [MeV]	NLEFT	Exp.
old algorithm	^{3}He - ^{3}H	0.78(5)	0.76
	^{4}He	−28.3(6)	−28.3
	^{8}Be	−55(2)	−56.5
	^{12}C	−92(3)	−92.2
new algorithm	^{16}O	−131(1)	−127.6
	^{20}Ne	−166(1)	−160.6
	^{24}Mg	−198(2)	−198.3
	^{28}Si	−234(3)	−236.5



- promising results ⇒ uncertainties down to the 1% level
- excited states more difficult ⇒ projection MC method + triangulation

The SPECTRUM of CARBON-12

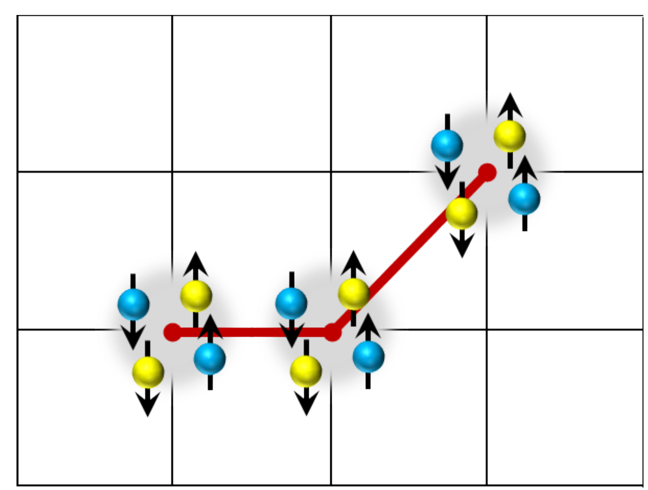
- After $8 \cdot 10^6$ hrs JUGENE/JUQUEEN (and “some” human work)



Hoyle

⇒ First ab initio calculation of the Hoyle state ✓

Structure of the Hoyle state:



Epelbaum, Krebs, Lee, UGM, Phys. Rev. Lett. 106 (2011) 192501

Epelbaum, Krebs, Lähde, Lee, UGM, Phys. Rev. Lett. 109 (2012) 252501

FINE-TUNING: MONTE-CARLO ANALYSIS

Epelbaum, Krebs, Lähde, Lee, UGM, PRL **110** (2013) 112502

- consider first QCD only → calculate $\partial \Delta E / \partial M_\pi$
- relevant quantities (energy *differences*)

$$\boxed{\Delta E_h \equiv E_{12}^* - E_8 - E_4, \quad \Delta E_b \equiv E_8 - 2E_4}$$

- energy differences depend on parameters of QCD (LO analysis)

$$\boxed{E_i = E_i\left(M_\pi^{\rm OPE}, m_N(M_\pi), g_{\pi N}(M_\pi), C_0(M_\pi), C_I(M_\pi)\right)}$$

$$g_{\pi N} \equiv g_A/(2F_\pi)$$

- remember: $M_{\pi^\pm}^2 \sim (m_u + m_d)$ Gell-Mann, Oakes, Renner (1968)

⇒ quark mass dependence ≡ pion mass dependence

PION MASS VARIATIONS

- consider pion mass changes as *small perturbations*

$$\left.\frac{\partial E_i}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} = \left.\frac{\partial E_i}{\partial M_\pi^{\rm OPE}}\right|_{M_\pi^{\rm phys}} + x_1 \left.\frac{\partial E_i}{\partial m_N}\right|_{m_N^{\rm phys}} + x_2 \left.\frac{\partial E_i}{\partial g_{\pi N}}\right|_{g_{\pi N}^{\rm phys}} + x_3 \left.\frac{\partial E_i}{\partial C_0}\right|_{C_0^{\rm phys}} + x_4 \left.\frac{\partial E_i}{\partial C_I}\right|_{C_I^{\rm phys}}$$

with

$$x_1 \equiv \left.\frac{\partial m_N}{\partial M_\pi}\right|_{M_\pi^{\rm phys}}, \quad x_2 \equiv \left.\frac{\partial g_{\pi N}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}}, \quad x_3 \equiv \left.\frac{\partial C_0}{\partial M_\pi}\right|_{M_\pi^{\rm phys}}, \quad x_4 \equiv \left.\frac{\partial C_I}{\partial M_\pi}\right|_{M_\pi^{\rm phys}}$$

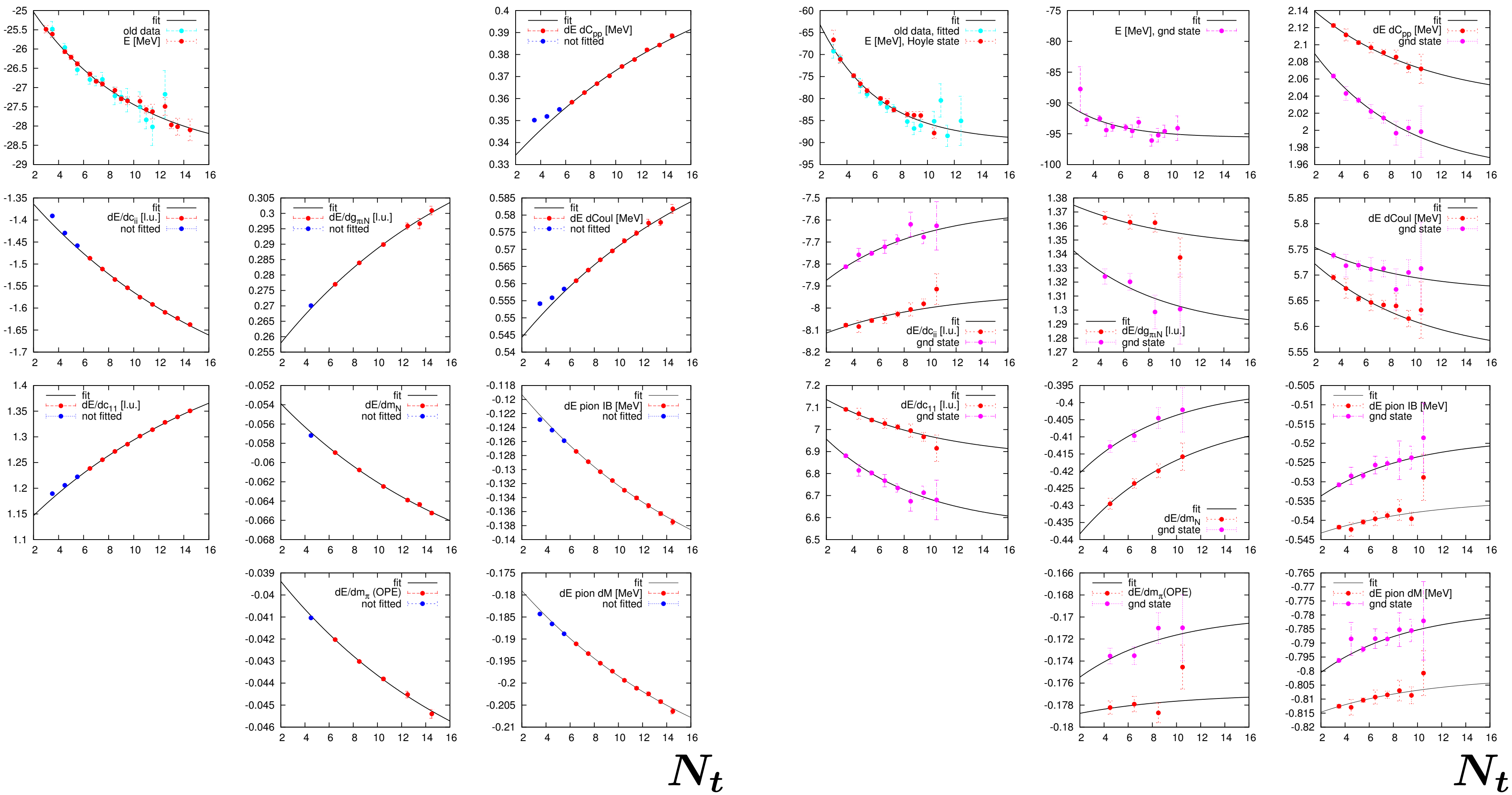
⇒ problem reduces to the calculation of the various derivatives using AFQMC and the determination of the x_i

- x_1 and x_2 can be obtained from LQCD plus CHPT
- x_3 and x_4 can be obtained from two-body scattering and its M_π-dependence

AFQMC RESUTS for the DERIVATIVES

- ^{4}He
- ^{12}C$(0_1^+, 0_2^+)$

$$E(N_t) = E(\infty) + \text{const}\ \exp(-N_t/\tau)$$



DETERMINATION of the x_i

- x_1 from the quark mass expansion of the nucleon mass:

$$\boxed{x_1 \simeq 0.8 \pm 0.2}$$

- x_2 from the quark mass expansion of the pion decay constant and the nucleon axial-vector constant:

$$\boxed{x_2 \simeq -0.056 \ldots 0.008}$$

- x_3 and x_4 can be obtained from a two-nucleon scattering analysis

$\Rightarrow$ while this can straightforwardly be computed, we prefer to use a representation that substitutes x_3 and x_4 by:

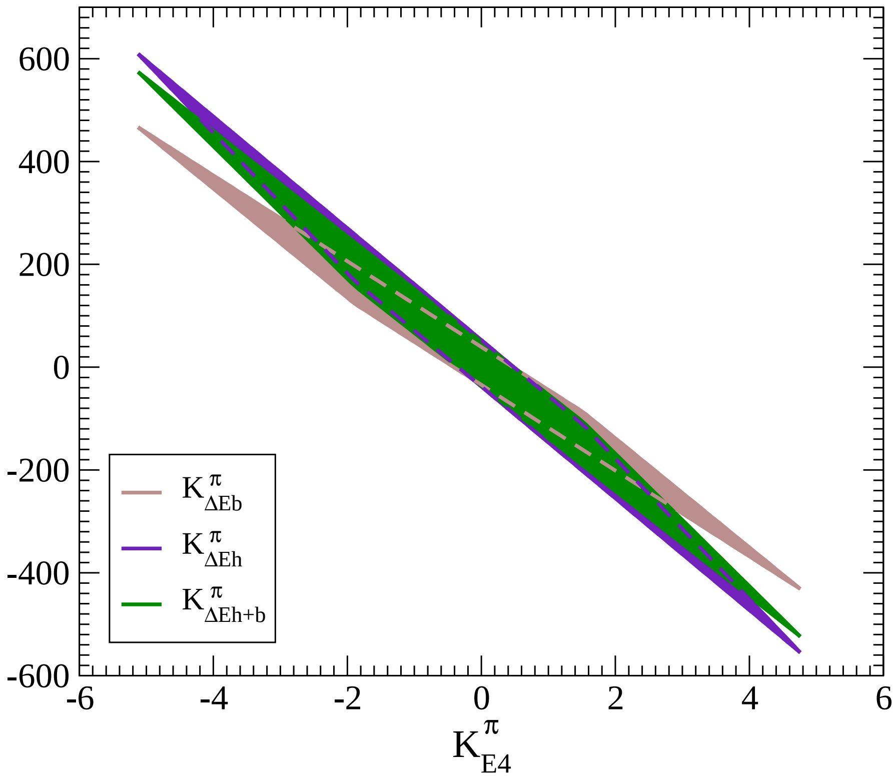
$$\left.\frac{\partial a_s^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}}, \qquad \left.\frac{\partial a_t^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}}$$

$\Rightarrow$ we are ready to study the pertinent energy differences

CORRELATIONS

- map $C_{0,I}(M_\pi)$ onto $\bar{A}_{s,t} \equiv \partial a_{s,t}^{-1}/\partial M_\pi\big|_{M_\pi^{\rm phys}}$ [singlet/triplet scatt. length]

- vary the derivatives $\bar{A}_{s,t} \equiv \partial a_{s,t}^{-1}/\partial M_\pi\big|_{M_\pi^{\rm phys}}$ within $-1, \ldots, +1$:



$$\Delta E_b = E(^8\mathrm{Be}) - 2E(^4\mathrm{He})$$

$$\Delta E_h = E(^{12}\mathrm{C}^*) - E(^8\mathrm{Be}) - E(^4\mathrm{He})$$

$$\Delta E_{h+b} = E(^{12}\mathrm{C}^*) - 3E(^4\mathrm{He})$$

$$\boxed{\frac{\partial O_H}{\partial M_\pi} = K_H^\pi \frac{O_H}{M_\pi}}$$

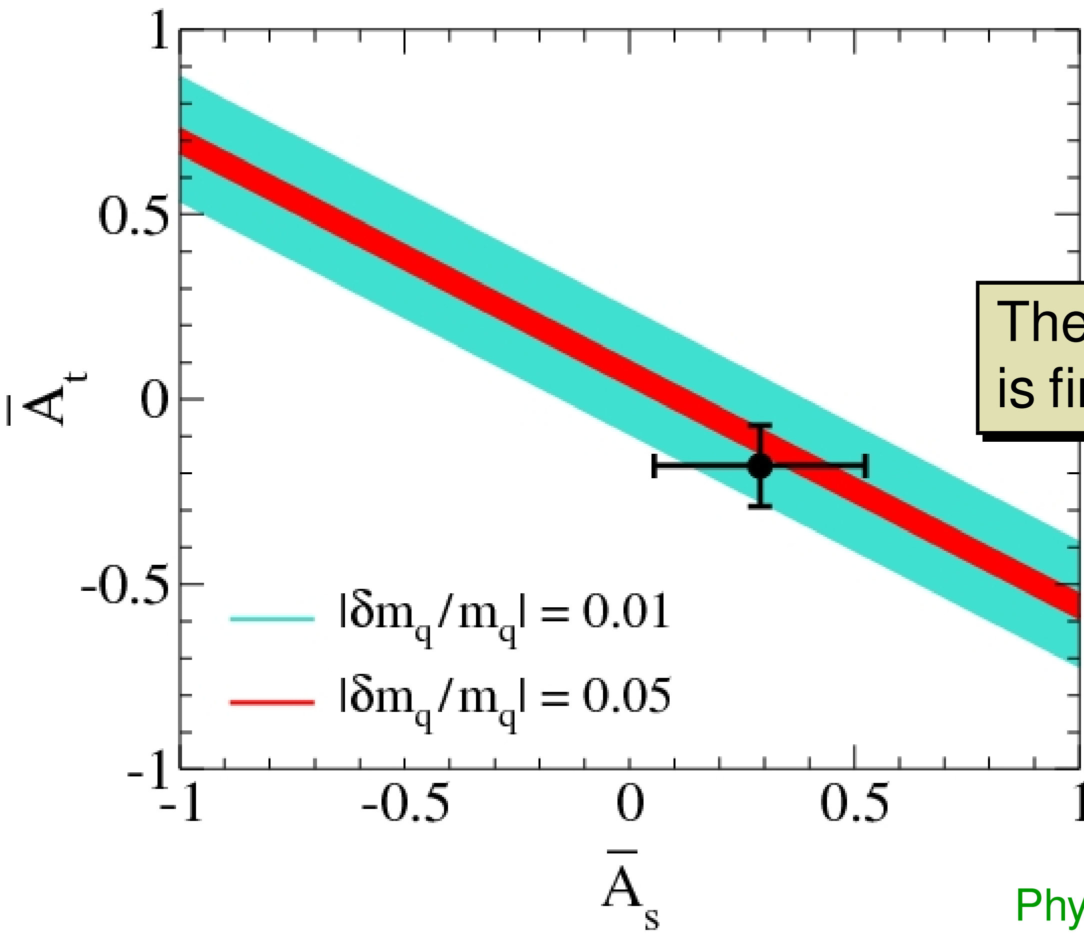
- all fine-tunings in the triple-alpha process are *correlated*, as speculated

Weinberg (2000)

THE END-OF-THE-WORLD PLOT

- $|\delta(\Delta E_{h+b})| < 100$ keV

$$\rightarrow \left| \left(0.571(14)\bar{A}_s + 0.934(11)\bar{A}_t - 0.069(6) \right) \frac{\delta m_q}{m_q} \right| < 0.0015$$



$$\bar{A}_{s,t} \equiv \left. \frac{\partial a_{s,t}^{-1}}{\partial M_\pi} \right|_{M_\pi^{\rm phys}}$$

The light quark mass is fine-tuned to $\simeq 2-3\,\%$

Similarly: $\alpha_{\rm EM}$ is fine-tuned to $\simeq 2.5\%$

Berengut et al., Phys. Rev. **D 87** (2013) 085018

SUMMARY & OUTLOOK

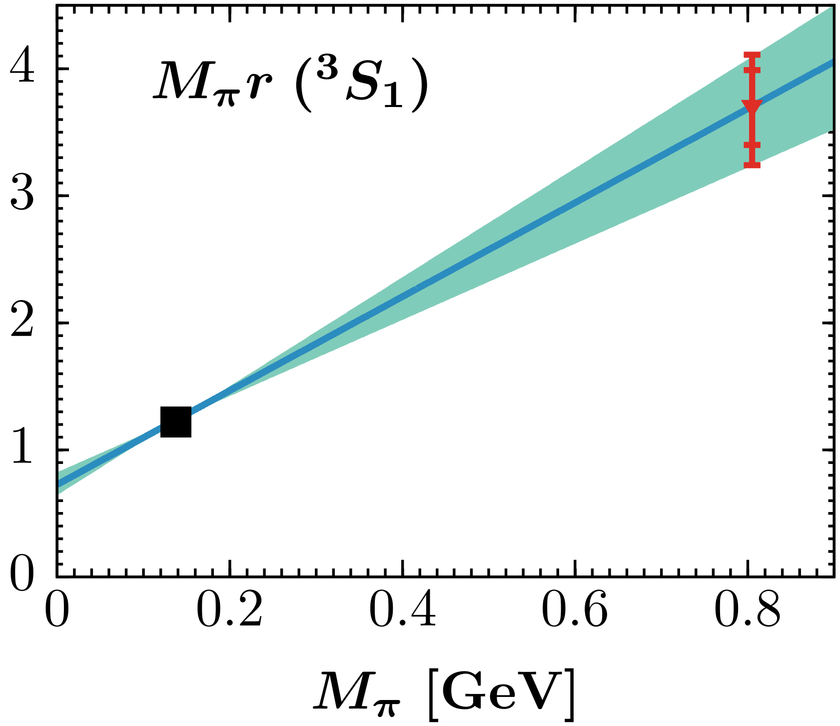
- Chiral nuclear EFT: best approach to nuclear forces and few-body systems
- Study of the nuclear force as a function of the quark masses
  $\rightarrow$ pion-exchanges straightforward, contact interactions require modeling
  $\hookrightarrow$ first connection to LQCD
- Impact on BBN: incl. the neutron lifetime (all masses $\sim v$): $|\delta m_q/m_q| \leq \mathbf{0.9\%}$
- Nuclear lattice simulations as a new quantum many-body approach
  $\rightarrow$ allow to vary the parameters of QCD+QED
  $\rightarrow$ investigate changes in nuclear properties
  $\rightarrow$ scattering in reach $\hookrightarrow$ $\alpha - \alpha$ scattering
- Fine-tuning of $m_{\rm quark}$ and $\alpha_{\rm EM}$ $\rightarrow$ viability of carbon-oxygen based life
  $\Rightarrow$ changes in $m_{\rm quark}$ of about 2-3 % and in $\alpha_{\rm EM}$ of about 2.5% are allowed
  $\Rightarrow$ LQCD required to reduce the uncertainties! $\hookrightarrow$ challenge!

$\Rightarrow$ conditions for life are fine-tuned

NEW RESULTS for the NN SYSTEM

- Various lattice QCD results for binding energies and scattering parameters
- combine with low-energy theorems for the modified ERE

Baru et al., Phys. Rev. C **92** (2015) 014001, Phys .Rev. C **94** (2016) 014001



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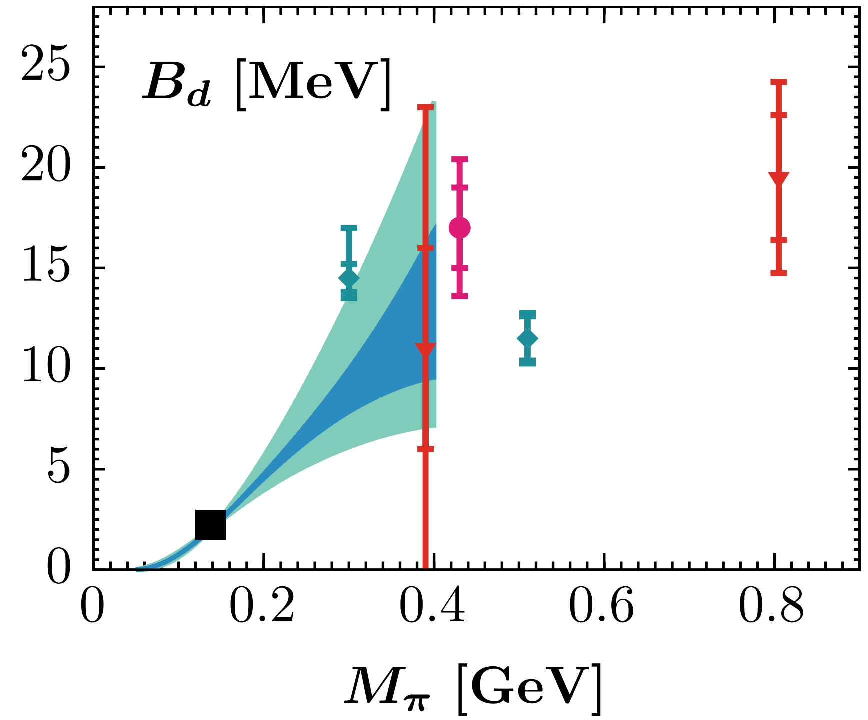


NPLQCD, Phys. Rev. C88 (2013) 024003
NPLQCD, Phys. Rev. D92 (2015) 114512
Yamazaki et al., Phys. Rev. D92 (2015) 014501

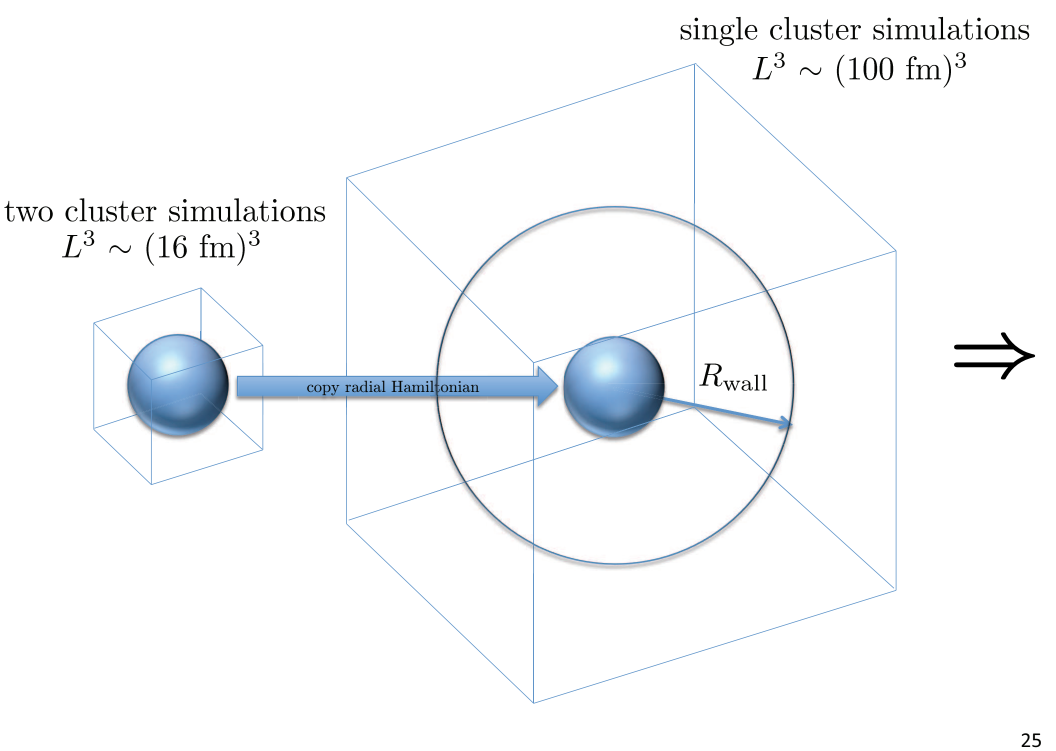
◇ Yamazaki et al.
▽ NPLQCD 2013
○ NPLQCD 2015

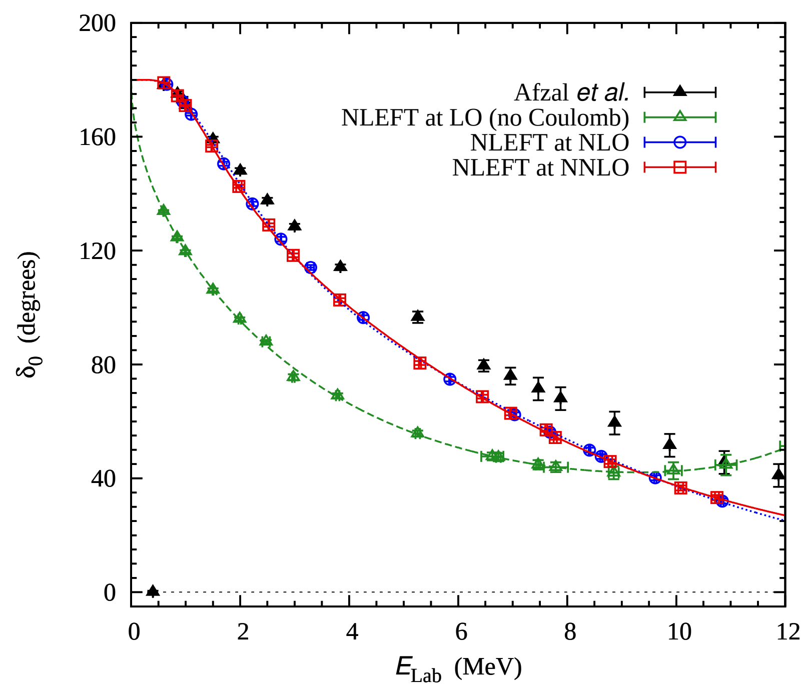
AB INITIO CALCULATION of α-α SCATTERING

- use lattice MC to construct an ab-initio cluster (adiabatic) Hamiltonian

- Use adiabatic Hamiltonian to compute scattering/reaction amplitudes

Elhatisari et al., Nature **528** (2015) 111





- D-wave equally well described

SPARES

PION EXCHANGE CONTRIBUTIONS

- Work to NNLO, need quark mass dependence of M_π, F_π, m_N, g_A

$\Rightarrow$ using lattice + CHPT gives: $K^q_{M_\pi} = 0.494^{+0.009}_{-0.013}$, $K^q_{F_\pi} = 0.048 \pm 0.012$

$$K^q_{m_N} = 0.048^{+0.002}_{-0.006}$$

- situation for g_A not quite clear

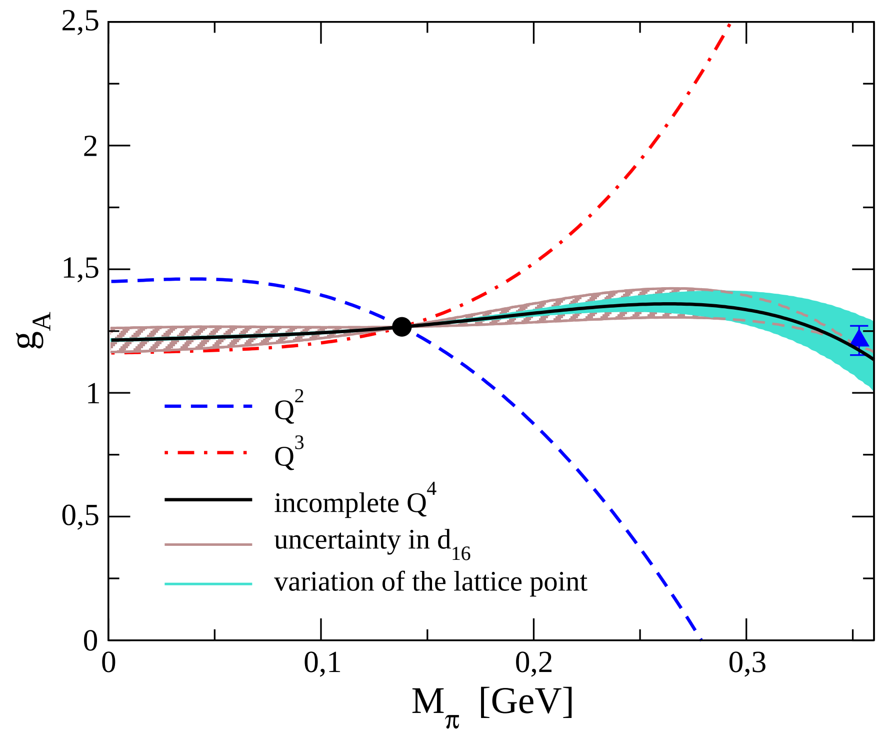
  LQCD data show little quark mass dep.

  chiral expansion converges slowly

  two-loop representation might suffice
  to make contact with flat LQCD data
  Bernard, UGM (2006)

  $\rightarrow$ use a simplified two-loop representation

  $\rightarrow$ fixes quark mass dep. of $V_{1\pi} + V_{2\pi}$



QUARK MASS DEP. of the SHORT-DISTANCE TERMS

- Consider a typical OBEP with $M = \sigma, \rho, \omega, \delta, \eta$
- Quark mass dependence of the sigma and rho from unitarized CHPT

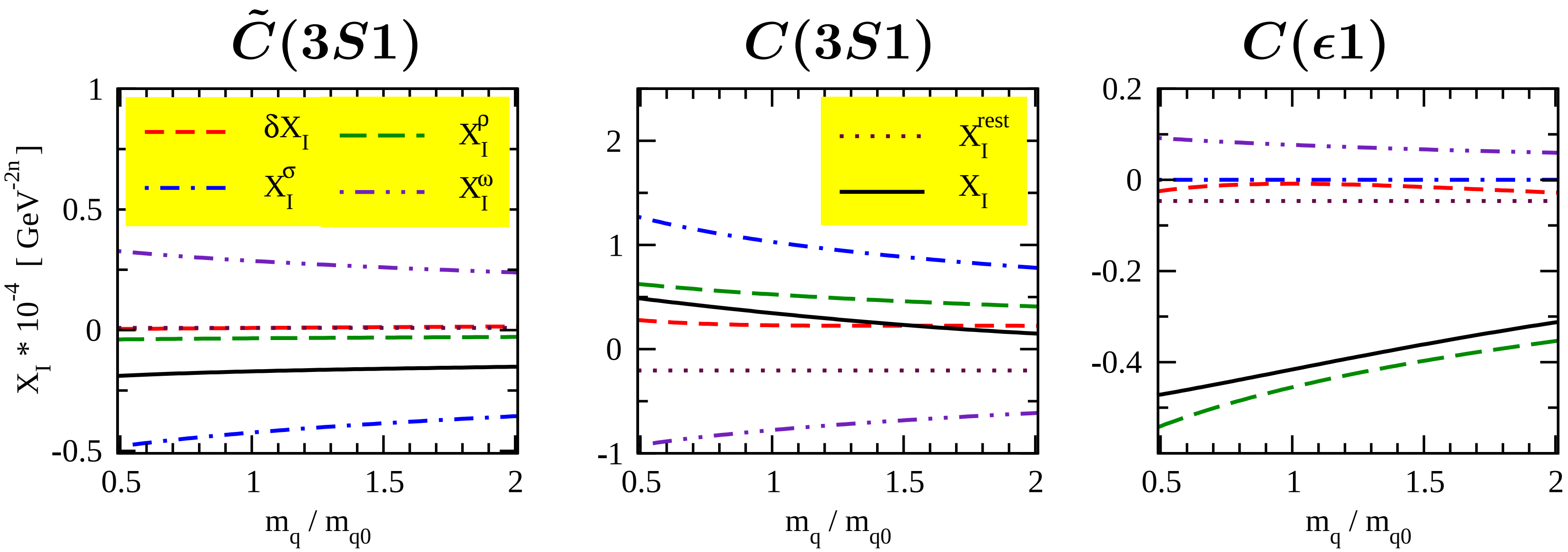
Hanhart, Pelaez, Rios (2008)

$\Rightarrow$ $K^q_{M_\sigma} = 0.081 \pm 0.007, \quad K^q_{M_\rho} = 0.058 \pm 0.002$

$\Rightarrow$ couplings appear quark mass independent (requires refinement in the future)

- assume a) that $K^q_\omega = K^q_\rho$ and b) neglect dep. of δ, η

$\Rightarrow$



A SHORT HISTORY of the HOYLE STATE

- Heavy element generation in massive stars: triple-α process

Bethe 1938, Öpik 1952, Salpeter 1952, Hoyle 1954, . . .

$$^{4}\text{He} + {}^{4}\text{He} \rightleftharpoons {}^{8}\text{Be}$$
$$^{8}\text{Be} + {}^{4}\text{He} \rightleftharpoons {}^{12}\text{C}^{*} \rightarrow {}^{12}\text{C} + \gamma$$
$$^{12}\text{C} + {}^{4}\text{He} \rightleftharpoons {}^{16}\text{O} + \gamma$$

- Hoyle's contribution: calculation of relative abundances of ^{4}He, ^{12}C and ^{16}0
  $\Rightarrow$ need a resonance close to the ^{8}Be + ^{4}He threshold at $E_R = 0.35$ MeV
  $\Rightarrow$ this corresponds to a $J^P = 0^+$ excited state 7.7 MeV above the g.s.

- a corresponding state was experimentally confirmed at Caltech at
  $E - E(\text{g.s.}) = 7.653 \pm 0.008$ MeV

Dunbar et al. 1953, Cook et al. 1957

- still on-going experimental activity, e.g. EM transitions at SDALINAC

M. Chernykh et al., Phys. Rev. Lett. 98 (2007) 032501

- and how about theory ? $\rightarrow$ this talk

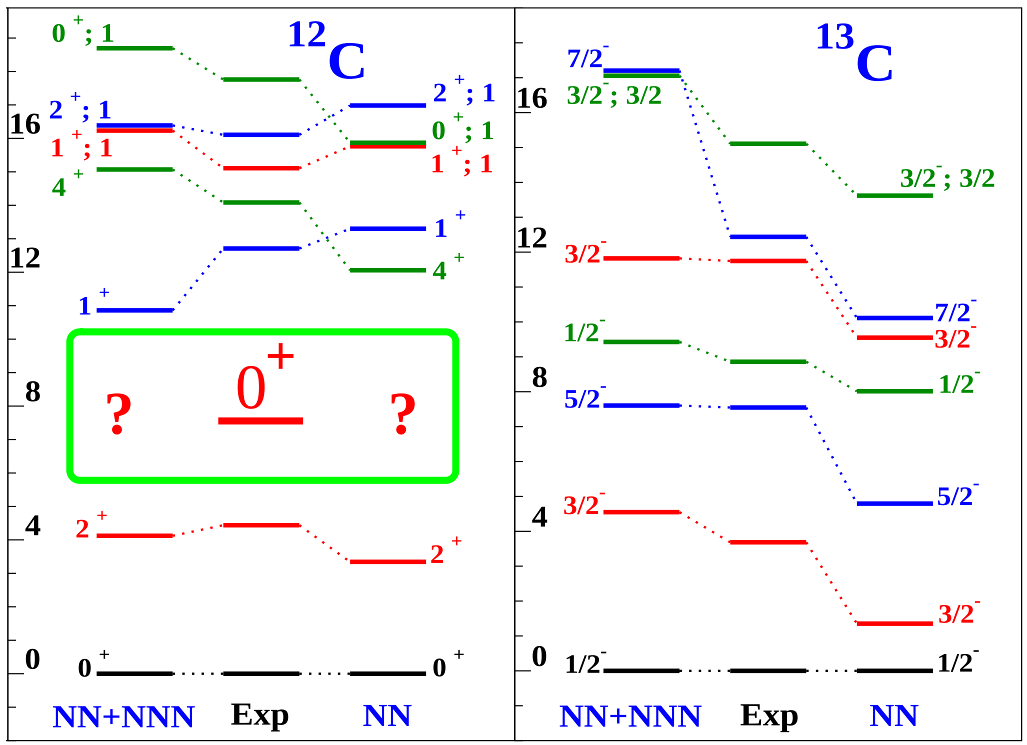
- side remark: NOT driven by anthropic considerations

H. Kragh, Arch. Hist. Exact Sci. 64 (2010) 721

AN ENIGMA for NUCLEAR THEORY

- Ab initio calculation in the no-core shell model: $\approx 10^7$ CPU hrs on JAGUAR

P. Navratil et al., Phys. Rev. Lett. **99** (2007) 042501; R. Roth et al., Phys. Rev. Lett. **107** (2011) 072501



$\Rightarrow$ excellent description, but no trace of the Hoyle state

QUARK MASS VARIATIONS of HEAVIER NUCLEI

- In BBN, we also need the variation of ^{3}He and ^{4}He. All other BEs are kept fixed.
- use the method of BLP: Bedaque, Luu, Platter, PRC 83 (2011) 045803

$$K^q_{^A\mathrm{He}} = K^q_{a,\,1S0} K^{a,\,1S0}_{^A\mathrm{He}} + K^q_{\mathrm{deut}} K^{\mathrm{deut}}_{^A\mathrm{He}}\ , \quad A = 3,4$$

with

$$K^{a,\,1S0}_{^3\mathrm{He}} = 0.12 \pm 0.01\ , \quad K^{\mathrm{deut}}_{^3\mathrm{He}} = 1.41 \pm 0.01$$

$$K^{a,\,1S0}_{^4\mathrm{He}} = 0.037 \pm 0.011\ , \quad K^{\mathrm{deut}}_{^4\mathrm{He}} = 0.74 \pm 0.22$$

so that

$$\Rightarrow \boxed{K^q_{^3\mathrm{He}} = -0.94 \pm 0.75, \quad K^q_{^4\mathrm{He}} = -0.55 \pm 0.42}$$

$\Rightarrow$ calculate BBN response matrix of primordial abundances Y_a
at fixed baryon/photon ratio [first in the isospin limit]

RESULTS for HEAVIER NUCLEI

- calculate BBN response matrix of primordial abundances Y_a
  at fixed baryon/photon ratio :

$$\frac{\delta \ln Y_a}{\delta \ln m_q} = \sum_{X_i} \frac{\partial \ln Y_a}{\partial \ln X_i} K^q_{X_i}$$

$\Rightarrow$

X	d	^{3}He	^{4}He	^{6}Li	^{7}Li
a_s	−0.39	0.17	0.01	−0.38	2.64
$B_{\rm deut}$	−2.91	−2.08	0.67	−6.57	9.44
$B_{\rm trit}$	−0.27	−2.36	0.01	−0.26	−3.84
$B_{^3{\rm He}}$	−2.38	3.85	0.01	−5.72	−8.27
$B_{^4{\rm He}}$	−0.03	−0.84	0.00	−69.8	−57.4
$B_{^6{\rm Li}}$	0.00	0.00	0.00	78.9	0.00
$B_{^7{\rm Li}}$	0.03	0.01	0.00	0.02	−25.1
$B_{^7{\rm Be}}$	0.00	0.00	0.00	0.00	99.1
τ	0.41	0.14	0.72	1.36	0.43

updated Kawano code

Kawano, FERMILAB-Pub-92/04-A

RESULTS

- putting pieces together:

$$\left.\frac{\partial \Delta E_h}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} = -0.455(35)\left.\frac{\partial a_s^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} - 0.744(24)\left.\frac{\partial a_t^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} + 0.056(10)$$

$$\left.\frac{\partial \Delta E_b}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} = -0.117(34)\left.\frac{\partial a_s^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} - 0.189(24)\left.\frac{\partial a_t^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} + 0.012(9)$$

- x_1 and x_2 only affect the small constant terms
- also calculated the shifts of the individual energies (not shown here)

INTERPRETATION

- $(\partial\Delta E_h/\partial M_\pi)/(\partial\Delta E_b/\partial M_\pi) \simeq 4$
  $\Rightarrow$ ΔE_h and ΔE_b cannot be independently fine-tuned

- Within error bars, $\partial\Delta E_h/\partial M_\pi$ & $\partial\Delta E_b/\partial M_\pi$ appear unaffected by the choice of x_1 and x_2 $\to$ indication for α-clustering

- the triple alpha process is controlled by :

$$\Delta E_{h+b} \equiv \Delta E_h + \Delta E_b = E_{12}^\star - 3E_4$$

$$\left.\frac{\partial\Delta E_{h+b}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} = -0.571(14)\left.\frac{\partial a_s^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} - 0.934(11)\left.\frac{\partial a_t^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} + 0.069(6)$$

$\Rightarrow$ quark mass dependence of the scattering lengths discussed earlier