Theory challenges for the LHC

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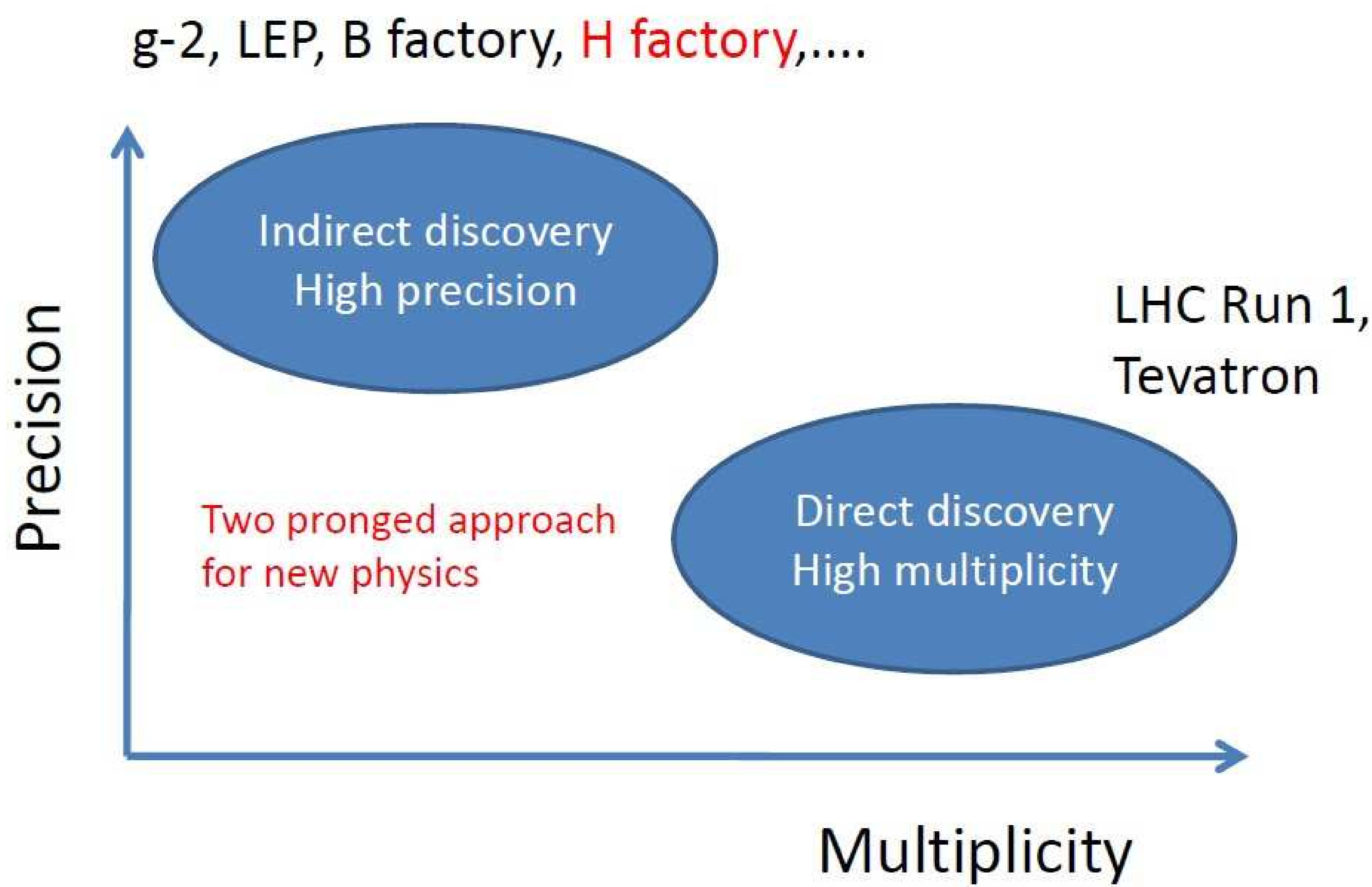
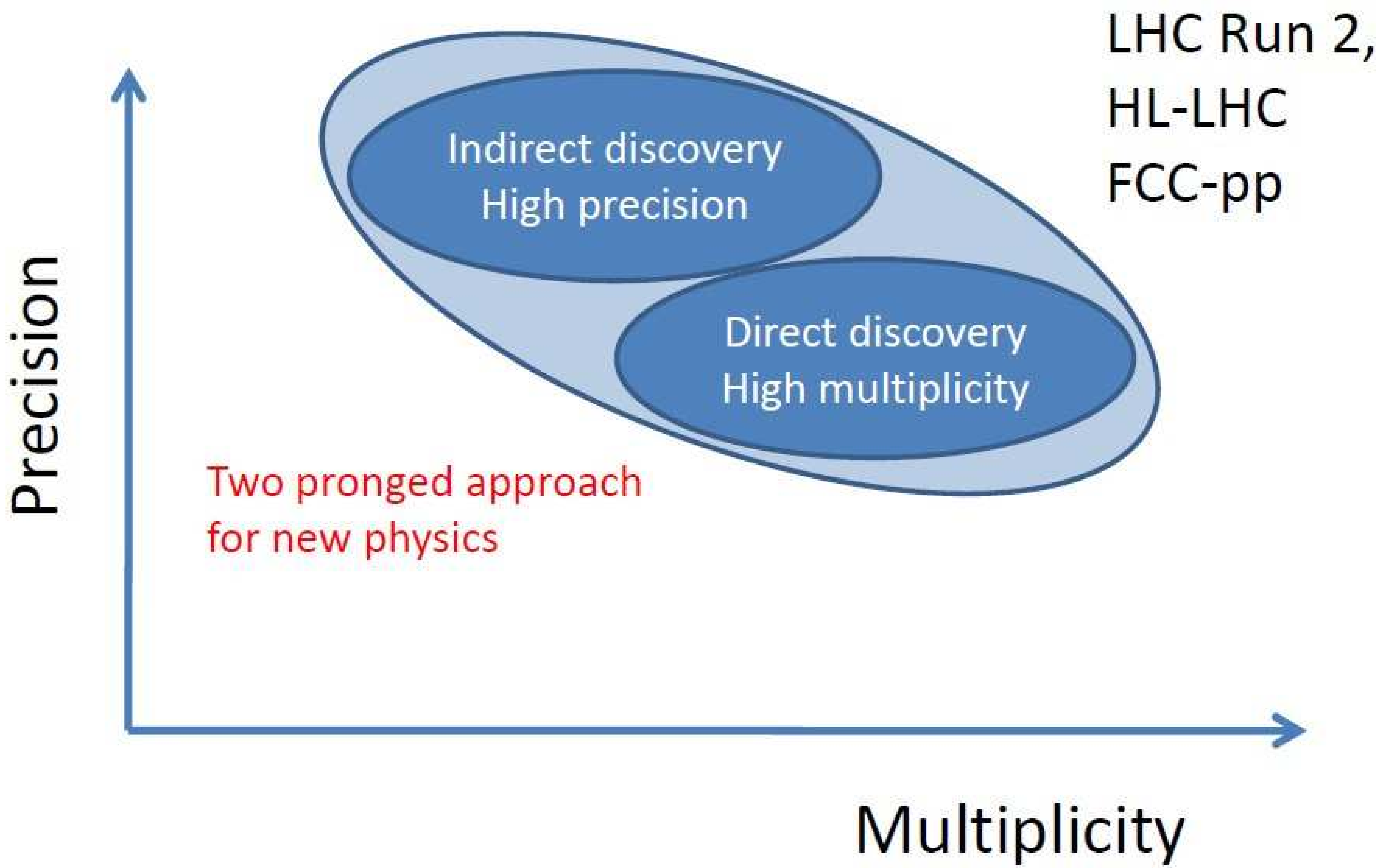
The Task for Experimental Particle Physics



The Task for Experimental Particle Physics

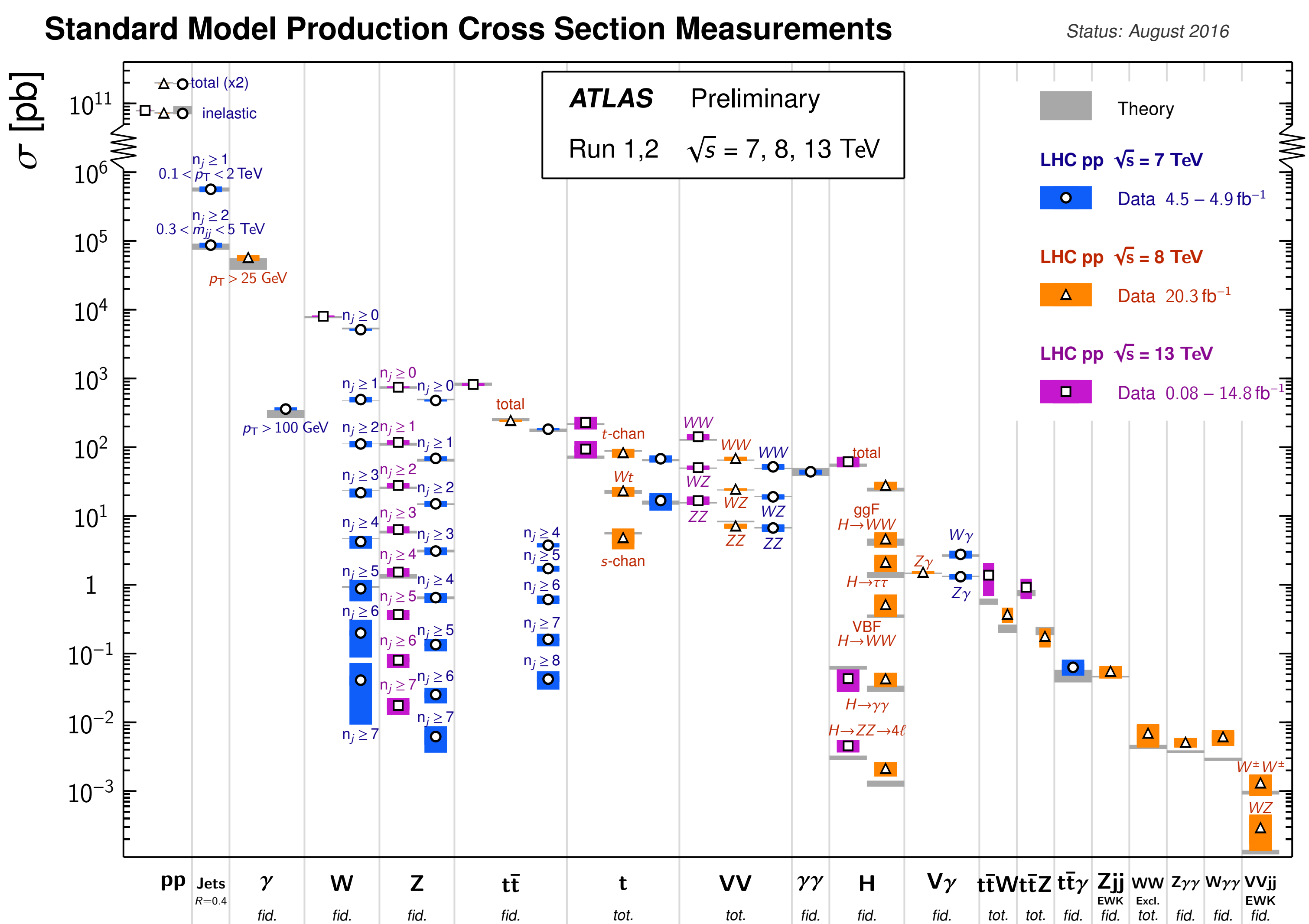


The challenge from the LHC

- ✓ Everything (signals, backgrounds, luminosity measurement) involves QCD
- ✓ Strong coupling is not small: $\alpha_s(M_Z) \sim 0.12$ and running is important
  - ➠ events have high multiplicity of hard partons
  - ➠ each hard parton fragments into a cluster of collimated particles jet
  - ➠ higher order perturbative corrections can be large
  - ➠ theoretical uncertainties can be large
- ✓ Processes can involve multiple energy scales: e.g. p_T^W and M_W
  - ➠ may need resummation of large logarithms
- ✓ Parton/hadron transition introduces further issues, but for suitable (infrared safe) observables these effects can be minimised
  - ➠ importance of infrared safe jet definition
  - ➠ accurate modelling of underlying event, hadronisation, ...

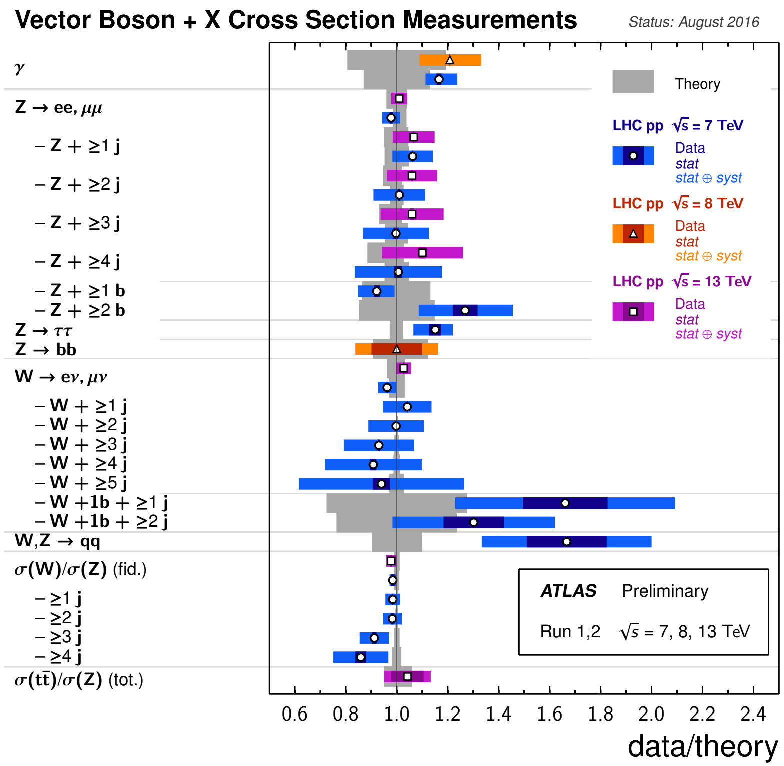
✓ ✓ Nevertheless, excellent agreement between theory and experiment over a wide range of observables

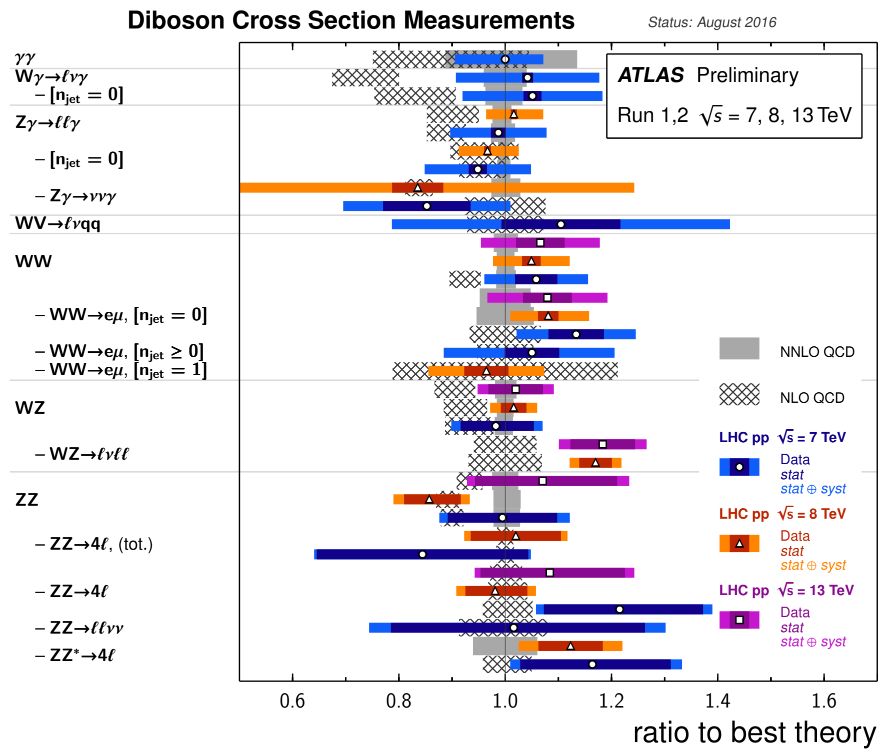
Cross Sections at the LHC



excellent agreement between theory and experiment over a wide range of observables

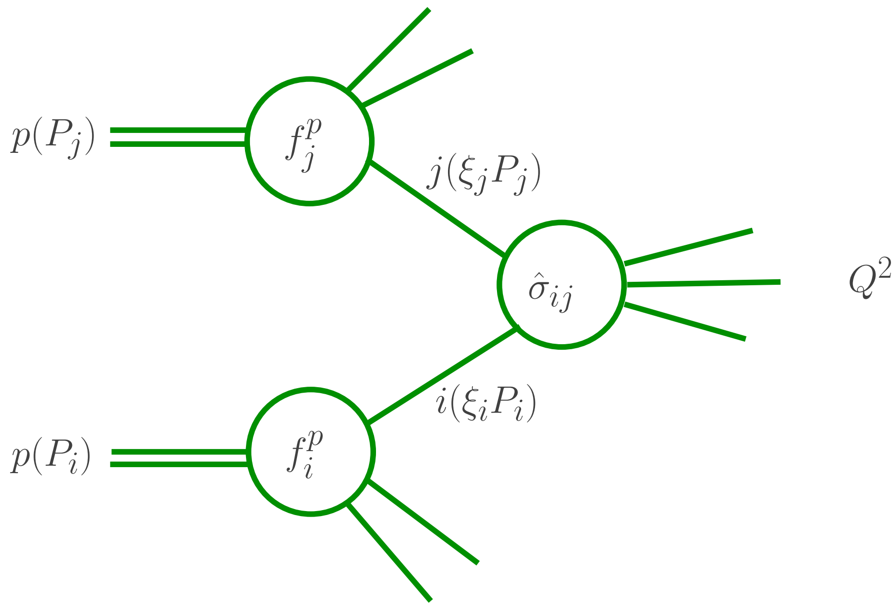
Discrepancies with data?





No BSM discovered yet. . . but plenty of BNLO

Theoretical Framework



$$\sigma(Q^2) = \int \sum_{i,j} d\hat{\sigma}_{ij}(\alpha_s(\mu_R), \mu_R^2/Q^2, \mu_F^2/Q^2) \otimes f_i^p(\mu_F) \otimes f_j^p(\mu_F) \qquad \left[+\mathcal{O}\left(\frac{1}{Q^2}\right)\right]$$

- ✓ partonic cross sections $d\hat{\sigma}_{ij}$
- ✓ running coupling $\alpha_s(\mu_R)$
- ✓ parton distributions $f_i(x, \mu_F)$
- ✓ renormalization/factorization scale μ_R, μ_F
- ✓ jet algorithm + parton shower + hadronisation model + underlying event + ...

Theoretical Uncertainties

- Missing Higher Order corrections (MHO)
  - truncation of the perturbative series
  - often estimated by scale variation - renormalisation/factorisation
  - ✓ systematically improvable by inclusion of higher orders
  - ✓ systematically improvable by resummation of large logs
- Uncertainties in input parameters
  - parton distributions
  - masses, e.g., m_W, m_h, $[m_t]$
  - couplings, e.g., $\alpha_s(M_Z)$
  - ✓ systematically improvable by better description of benchmark processes
- Uncertainties in parton/hadron transition
  - fragmentation (parton shower)
  - ✓ systematically improvable by matching/merging with higher orders
  - (✓) improvable by estimation of non-perturbative effects
  - hadronisation (model)
  - underlying event (tunes)

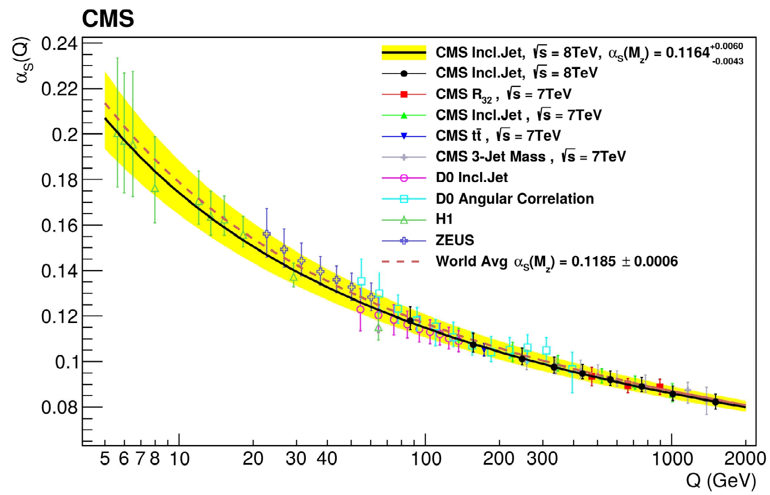
Goal: Reduce theory uncertainties by a factor of two compared to where we are now in next decade

The strong coupling

World Average

Year	$\alpha_s(M_Z)$
2008	0.1176 ± 0.0009
2012	0.1184 ± 0.0007
2014	0.1185 ± 0.0006
2016	0.1181 ± 0.0011

- ✓ Average of wide variety of measurements
  - ✓ τ-decays
  - ✓ e^+e^- annihilation
  - ✓ Z resonance fits
  - ✓ DIS
  - ✓ Lattice
- ✓ Generally stable to choice of measurements



- ✓ Impressive demonstration of running of α_s past O(1 TeV)
- ✓ …but some outlier values from global PDF fits, e.g.,
  $\alpha_s(M_Z) \sim 0.1136 \pm 0.0004$ JR14
  $\alpha_s(M_Z) \sim 0.1132 \pm 0.0011$ ABM14
  $\alpha_s(M_Z) \sim 0.1147 \pm 0.0008$ ABMP16
- ➠ Still need to understand uncertainty and make more precise determination

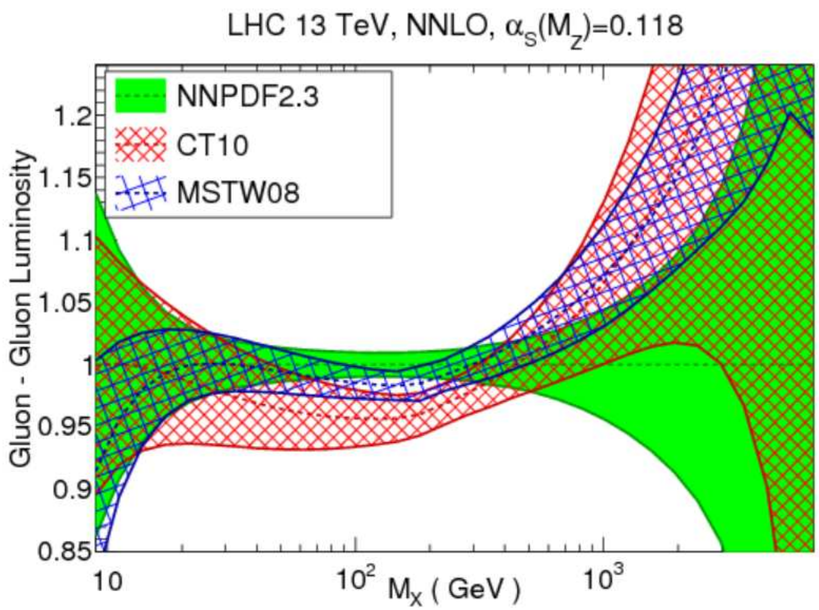
1% on α_s ➠ n% on process of $\mathcal{O}(\alpha_s^n)$

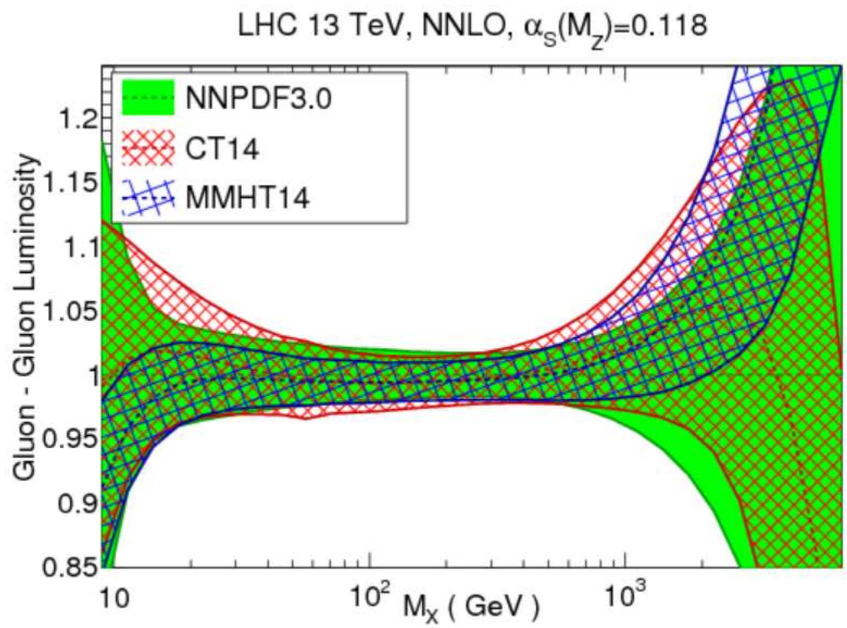
Parton Distribution Functions

All fits NNLO

Set	DIS	DY	jets	LHC	errors
MMHT14	✓	✓	✓	✓	hessian
CT14	✓	✓	✓	✓	hessian
NNPDF3.0	✓	✓	✓	✓	Monte Carlo
HeraPDF2.0	✓	✘	✘	✘	hessian
ABM14 (ABMP16)	✓	✓	✓	✘ (✓)	hessian
JR14	✓	✓	✓	✘	hessian

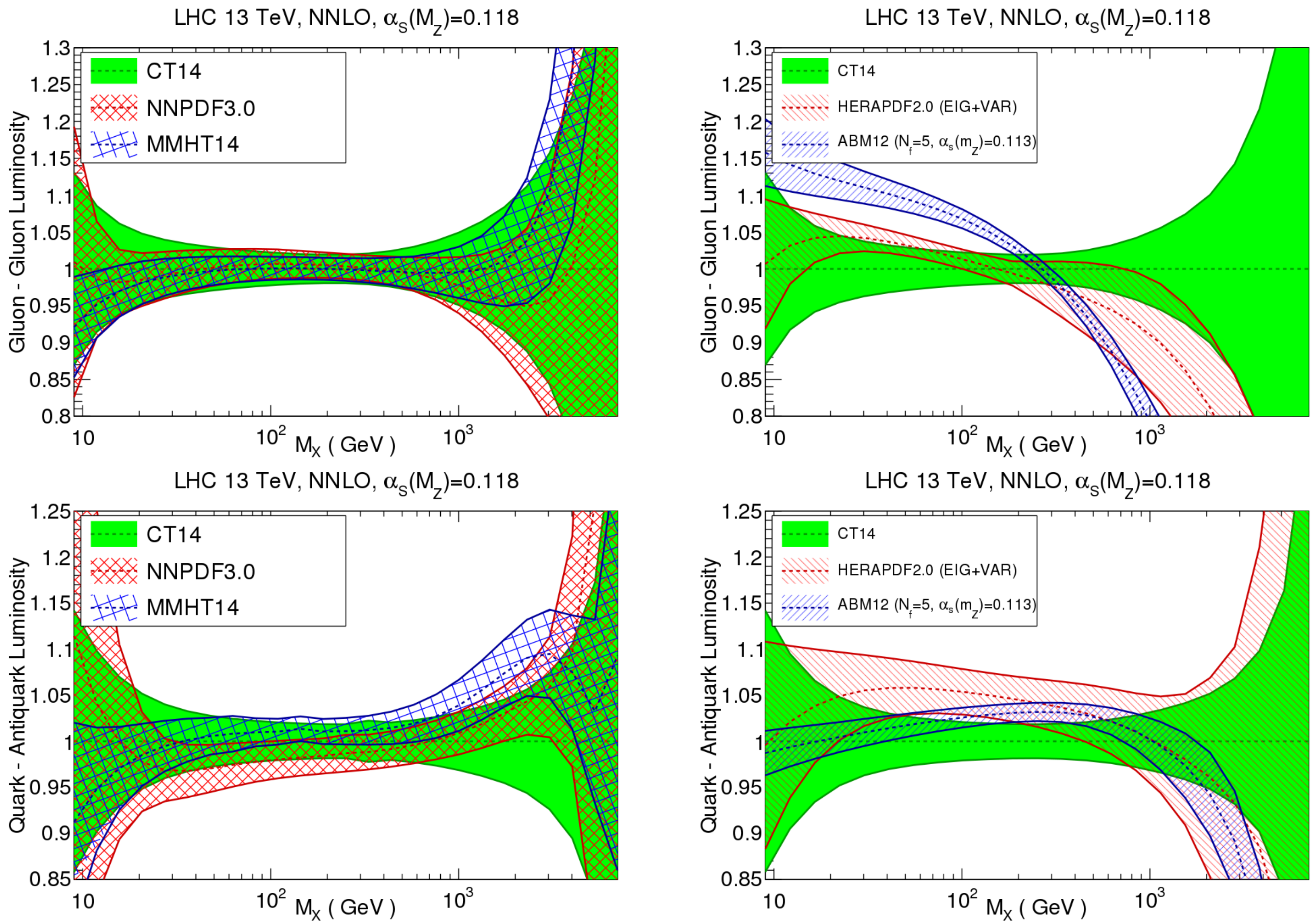
✓ Clear reduction in gluon-gluon luminosity for $M_X \sim 125$ GeV





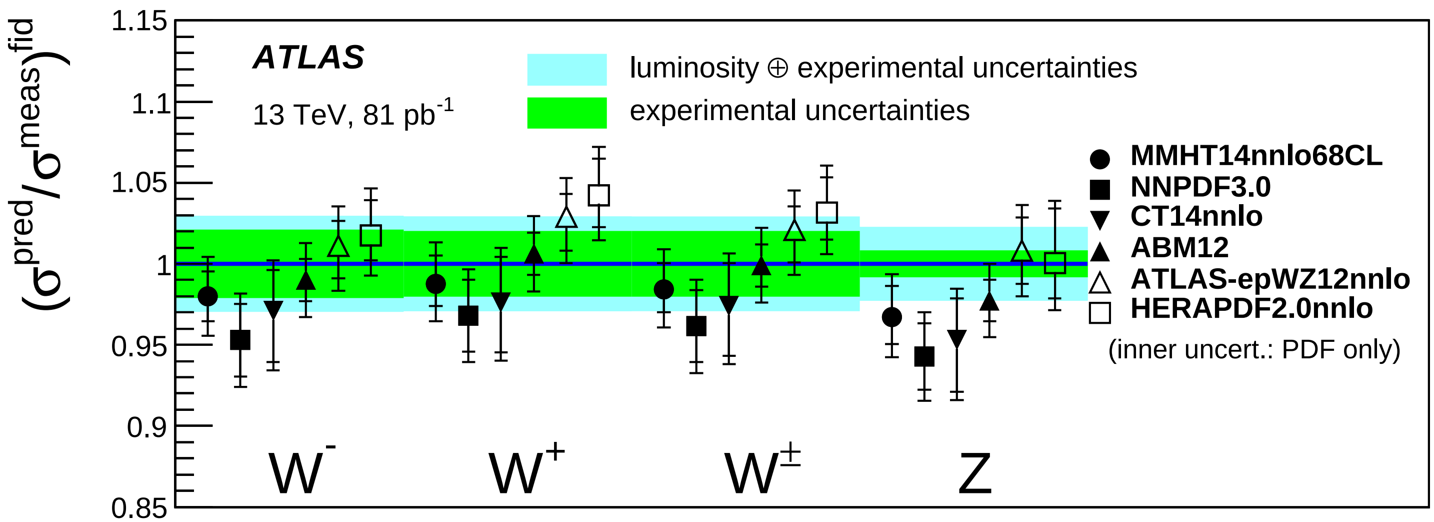
✓ . . . with commensurate reduction in uncertainty on Higgs cross section

Parton Distribution Functions



but still differences of opinion

Parton Distribution Functions



and disagreements even for the best measured cross sections

sensitivity to inputs into the PDF fits

✓ strange content of proton

✓ mass of charm quark

Partonic cross sections

$$\hat{\sigma} \sim \alpha_s^n \left(\hat{\sigma}^{LO} + \left(\frac{\alpha_s}{2\pi}\right) \hat{\sigma}^{NLO}_{QCD} \quad + \quad \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}^{NNLO}_{QCD} + \left(\frac{\alpha_s}{2\pi}\right)^3 \hat{\sigma}^{N3LO}_{QCD} + \ldots \right.$$

$$+ \quad \left(\frac{\alpha_W}{2\pi}\right) \hat{\sigma}^{NLO}_{EW} + \left(\frac{\alpha_W}{2\pi}\right) \left(\frac{\alpha_s}{2\pi}\right) \hat{\sigma}^{NNLO}_{QCD\mathrm{x}EW} \cdots$$

NLO QCD

- ✓ NLO QCD is the current state of the art

NNLO QCD

- ✓ provides the first serious estimate of the theoretical uncertainty
- ✓ rapid development with many new results in past couple of years

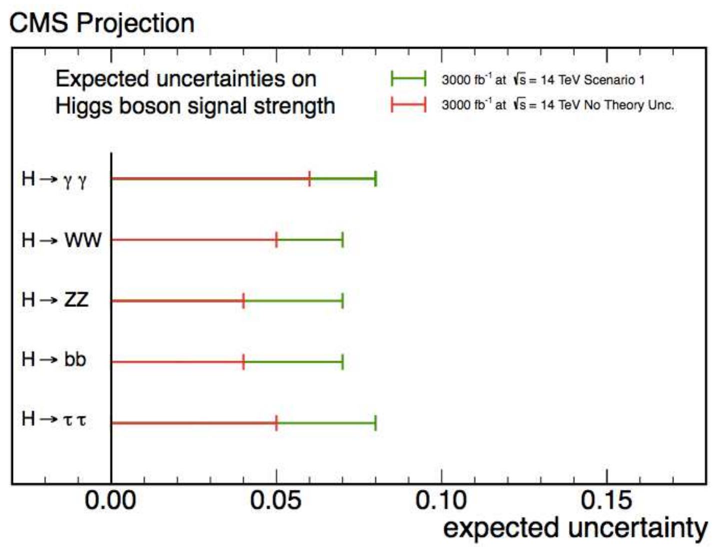
NLO EW

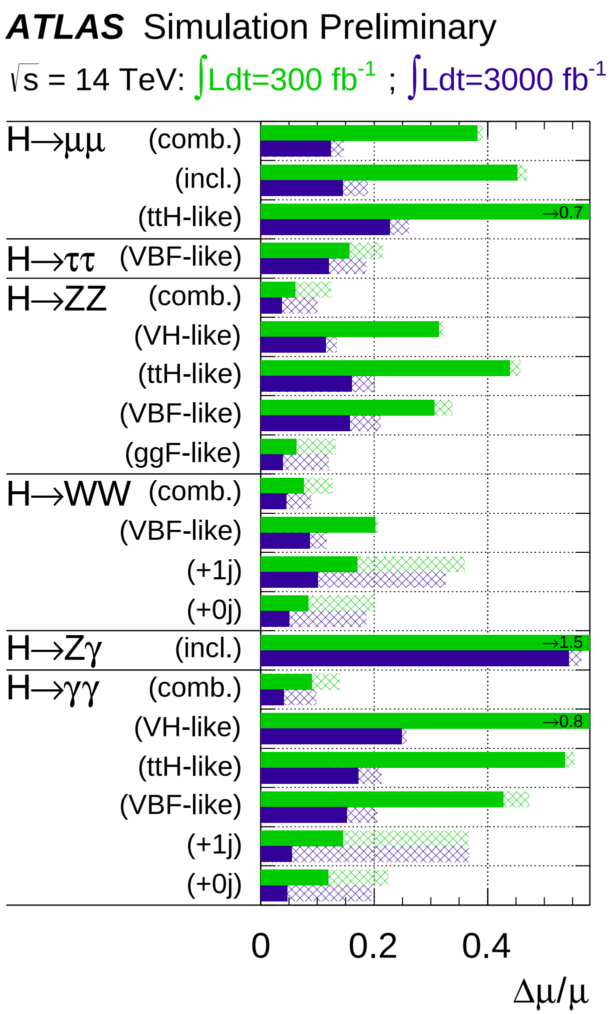
- ✓ naively similar size to NNLO QCD
- ✓ particularly important at high energies/p_T and near resonances

N3LO QCD

- ✓ recent landmark results for Higgs production

Motivation for more accurate theoretical calculations





✓ Theory uncertainty has big impact on quality of measurement

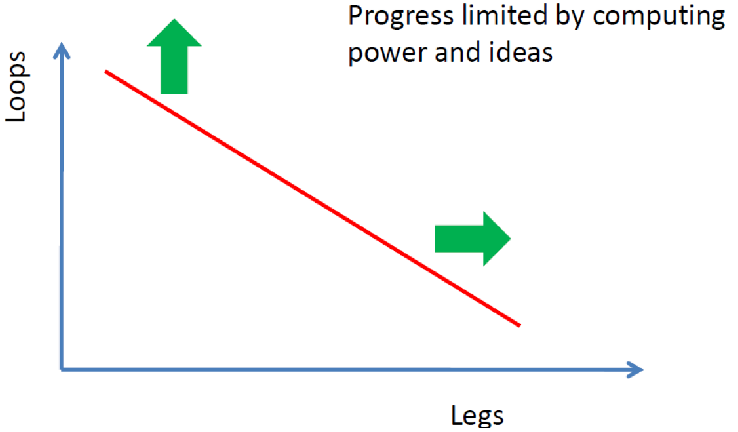
➠ Revised wishlist of theoretical predictions for

- Higgs processes
- Processes with vector bosons
- Processes with top or jets

Les Houches 2015,
arXiv:1605.04692

What is the hold up?

Rough idea of complexity of process $\sim$ #Loops + #Legs (+ #Scales)



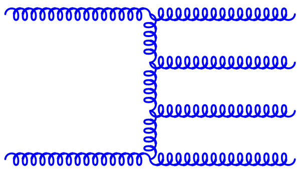
- loop integrals are ultraviolet/infrared divergent
- complicated by extra mass/energy scales
- loop integrals often unknown
  - ✓ completely solved at NLO
- real (tree) contributions are infrared divergent
- isolating divergences complicated
  - ✓ completely solved at NLO
- currently far from automation
  - ✓ mostly solved at NLO

Current standard: NLO

Anatomy of a Higher Order calculation

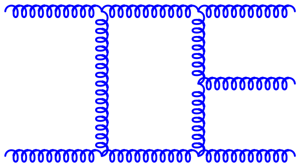
e.g. pp to JJ at NNLO

✓ double real radiation matrix elements $\mathrm{d}\hat{\sigma}^{RR}_{NNLO}$



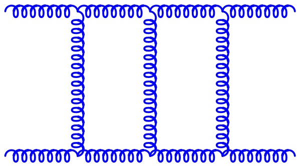
- implicit poles from double unresolved emission

✓ single radiation one-loop matrix elements $\mathrm{d}\hat{\sigma}^{RV}_{NNLO}$



- explicit infrared poles from loop integral
- implicit poles from soft/collinear emission

✓ two-loop matrix elements $\mathrm{d}\hat{\sigma}^{VV}_{NNLO}$



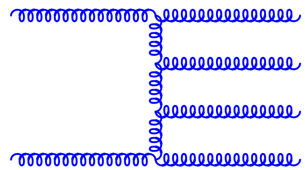
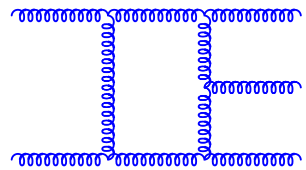
- explicit infrared poles from loop integral

$$\mathrm{d}\hat{\sigma}_{NNLO} \sim \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\hat{\sigma}^{RR}_{NNLO} + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\hat{\sigma}^{RV}_{NNLO} + \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\hat{\sigma}^{VV}_{NNLO}$$

Anatomy of a Higher Order calculation

e.g. pp to JJ at NNLO

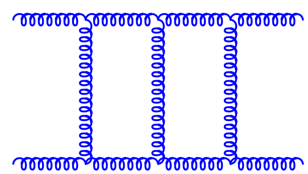
- ✓ Double real and real-virtual contributions used in NLO calculation of X+1 jet

  Can exploit NLO automation

  . . . but needs to be evaluated in regions of phase space where extra jet is not resolved

- ✚ Two loop amplitudes - very limited set known



  . . . currently far from automation

- ✚ Method for cancelling explicit and implicit IR poles - overlapping divergences

  . . . currently not automated

IR cancellation at NNLO

✓ The aim is to recast the NNLO cross section in the form

$$\begin{aligned} \mathrm{d}\hat{\sigma}_{NNLO} &= \int_{\mathrm{d}\Phi_{m+2}} \left[\mathrm{d}\hat{\sigma}^{RR}_{NNLO} - \mathrm{d}\hat{\sigma}^{S}_{NNLO}\right] \\ &+ \int_{\mathrm{d}\Phi_{m+1}} \left[\mathrm{d}\hat{\sigma}^{RV}_{NNLO} - \mathrm{d}\hat{\sigma}^{T}_{NNLO}\right] \\ &+ \int_{\mathrm{d}\Phi_{m}} \left[\mathrm{d}\hat{\sigma}^{VV}_{NNLO} - \mathrm{d}\hat{\sigma}^{U}_{NNLO}\right] \end{aligned}$$

where the terms in each of the square brackets is finite, well behaved in the infrared singular regions and can be evaluated numerically.

✚ Much more complicated cancellations between the double-real, real-virtual and double virtual contributions

✚ intricate overlapping divergences

NNLO - IR cancellation schemes

Unlike at NLO, we do not have a fully general NNLO IR cancellation scheme

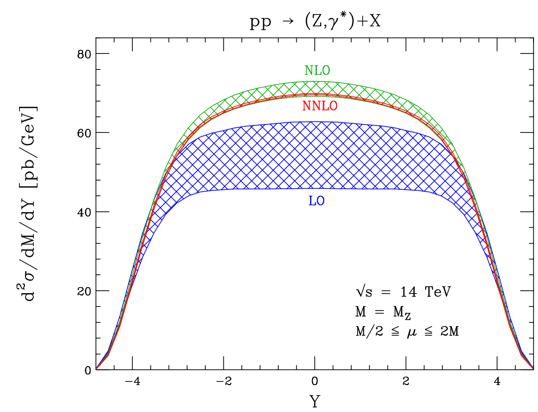
- Antenna subtraction — Gehrmann, Gehrmann-De Ridder, NG (05)
- Colourful subtraction — Del Duca, Somogyi, Trocsanyi (05)
- q_T subtraction — Catani, Grazzini (07)
- STRIPPER (sector subtraction) — Czakon (10); Boughezal et al (11)
  Czakon, Heymes (14)
- N-jettiness subtraction — Boughezal, Focke, Liu, Petriello (15)
  Gaunt, Stahlhofen, Tackmann, Walsh (15)
- Projection to Born — Cacciari, Dreyer, Karlberg, Salam, Zanderighi (15)

Each method has its advantages and disadvantages

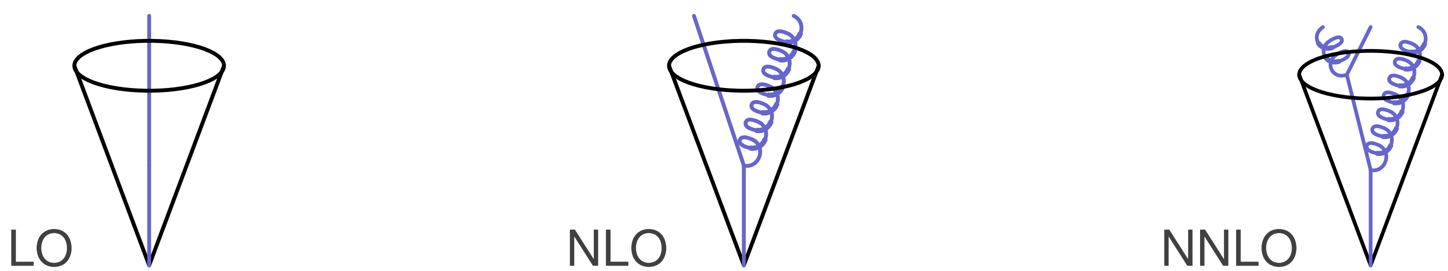
	Analytic	FS colour	IS colour	Azimuthal	Approach
Antenna	✓	✓	✓	✘	Subtraction
Colourful	✓	✓	✘	✓	Subtraction
q_T	✓	✘ (✓)	✓	—	Slicing
STRIPPER	✘	✓	✓	✓	Subtraction
N-jettiness	✓	✓	✓	—	Slicing
P2B	✓	✓	✓	—	Subtraction

What to expect from NNLO (1)

- ✓ Reduced renormalisation scale dependence



- ✓ Better able to judge convergence of perturbation series
- ✓ Fiducial (parton level) cross sections. Fully differential, so that experimental cuts can be applied directly
- ✓ Event has more partons in the final state so perturbation theory can start to reconstruct the shower
  - ➠ better matching of jet algorithm between theory and experiment

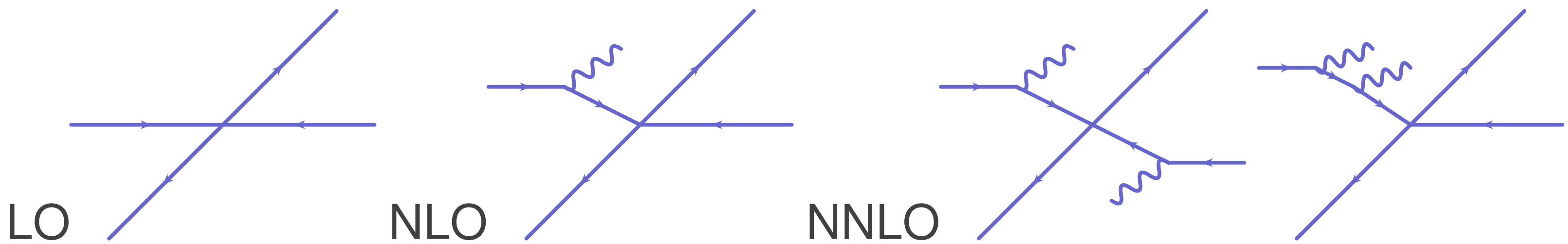


What to expect from NNLO (2)

- ✓ All channels present at NNLO

LO	NLO	NNLO
gg	gg, qg	gg, qg, qq
$q\bar{q}$	$q\bar{q}$, qg	$q\bar{q}$, qg, gg

- ✓ Better description of transverse momentum of final state due to double radiation off initial state



  - ✓ At LO, final state has no transverse momentum
  - ✓ Single hard radiation gives final state transverse momentum, even if no additional jet
  - ✓ Double radiation on one side, or single radiation of each incoming particle gives more complicated transverse momentum to final state

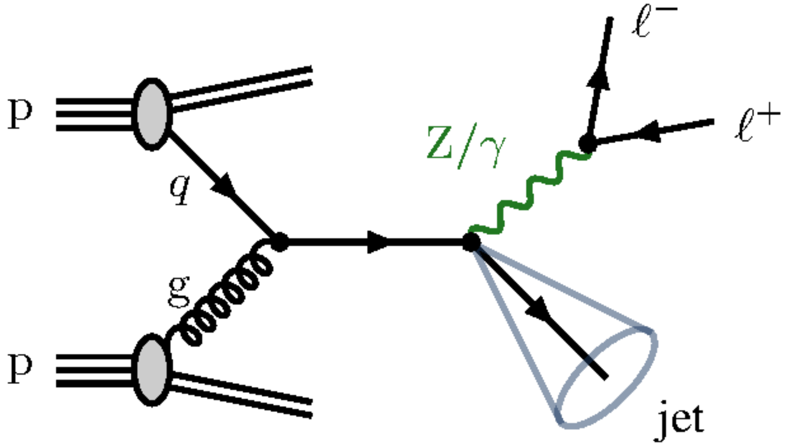
NNLOJET

X. Chen, J. Cruz-Martinez, J. Currie, A. Gehrmann-De Ridder, T. Gehrmann, NG, A. Huss, M. Jaquier, T. Morgan, J. Niehues, J. Pires

Implementing NNLO corrections using Antenna subtraction including decays for

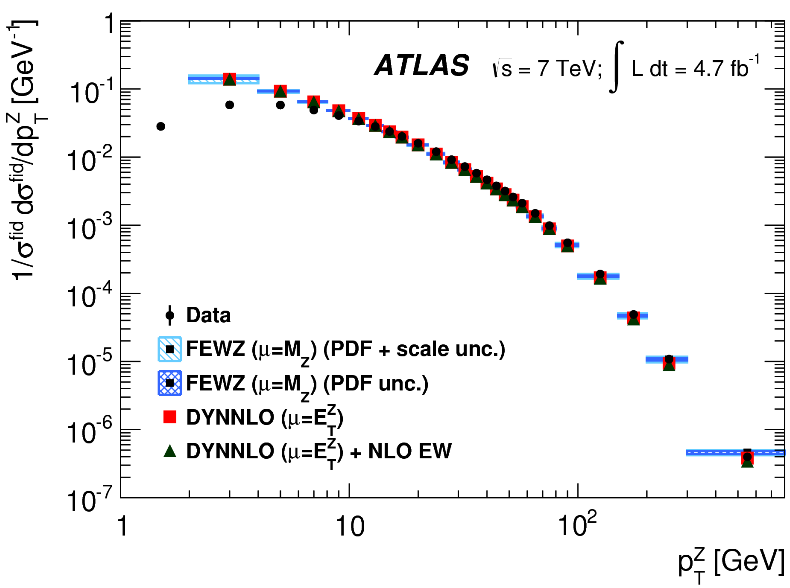
- ✓ $pp \to H, W, Z$
- ✓ $pp \to H + J$ — 1408.5325, 1607.08817
- ✓ $pp \to Z + J$ — 1507.02850, 1605.04295, 1610.01843
- ✓ $pp \to JJ$ — 1301.7310, 1310.3993, 1611.01460
- ✓ $ep \to JJ + (J)$ — 1606.03991
- ✓ $\ldots$

Inclusive p_T spectrum of Z



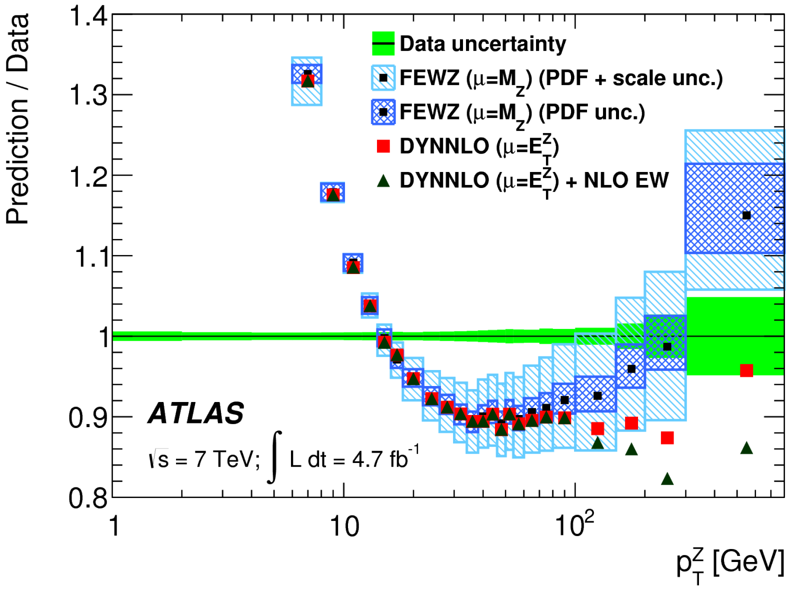
$pp \to Z/\gamma^* \to \ell^+\ell^- + X$

- large cross section
- clean leptonic signature



- fully inclusive wrt QCD radiation
- only reconstruct ℓ^+, ℓ^- so clean and precise measurement
- potential to constrain gluon PDFs

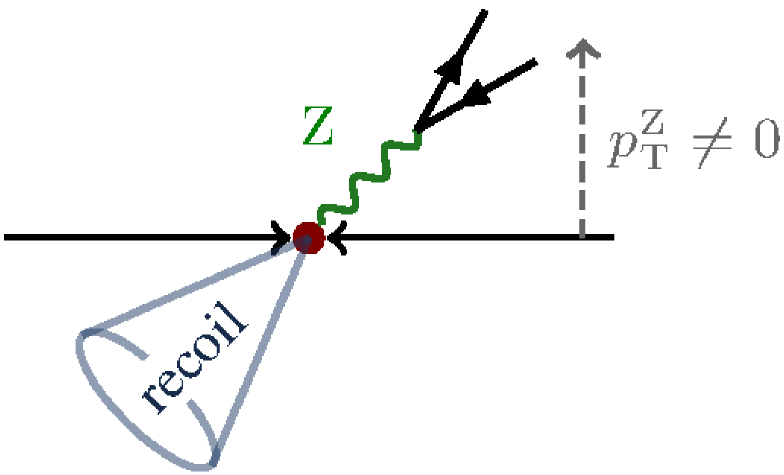
Inclusive p_T spectrum of Z



✚ low $p_T^Z \leq 10$ GeV, resummation required

✚ $p_T^Z \geq 20$ GeV, fixed order prediction about 10% below data

✘ *Very precise measurement of Z p_T poses problems to theory*,
D. Froidevaux, HiggsTools School



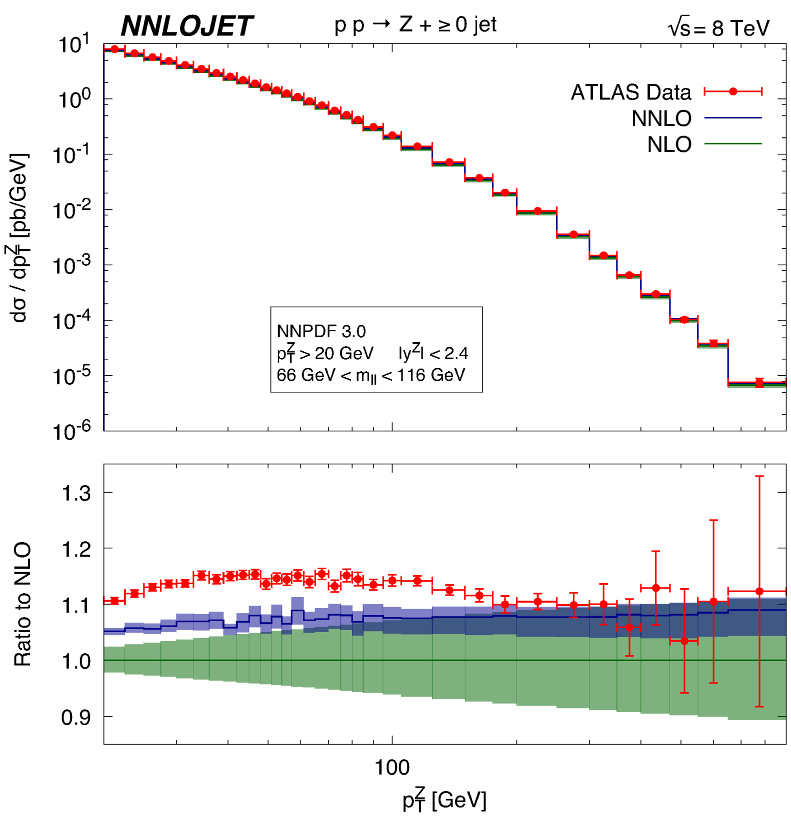
FEWZ/DYNNLO are Z + 0 jet @ NNLO

✘ Only NLO accurate in this distribution

✓ Requiring recoil means Z + 1 jet @ NNLO required

Inclusive p_T spectrum of Z

(1)
$$\left.\frac{\mathrm{d}\hat{\sigma}}{dp_T^Z}\right|_{p_T^Z>20\ \mathrm{GeV}} \equiv \frac{\mathrm{d}\hat{\sigma}_{LO}^{ZJ}}{dp_T^Z} + \frac{\mathrm{d}\hat{\sigma}_{NLO}^{ZJ}}{dp_T^Z} + \frac{\mathrm{d}\hat{\sigma}_{NNLO}^{ZJ}}{dp_T^Z}$$

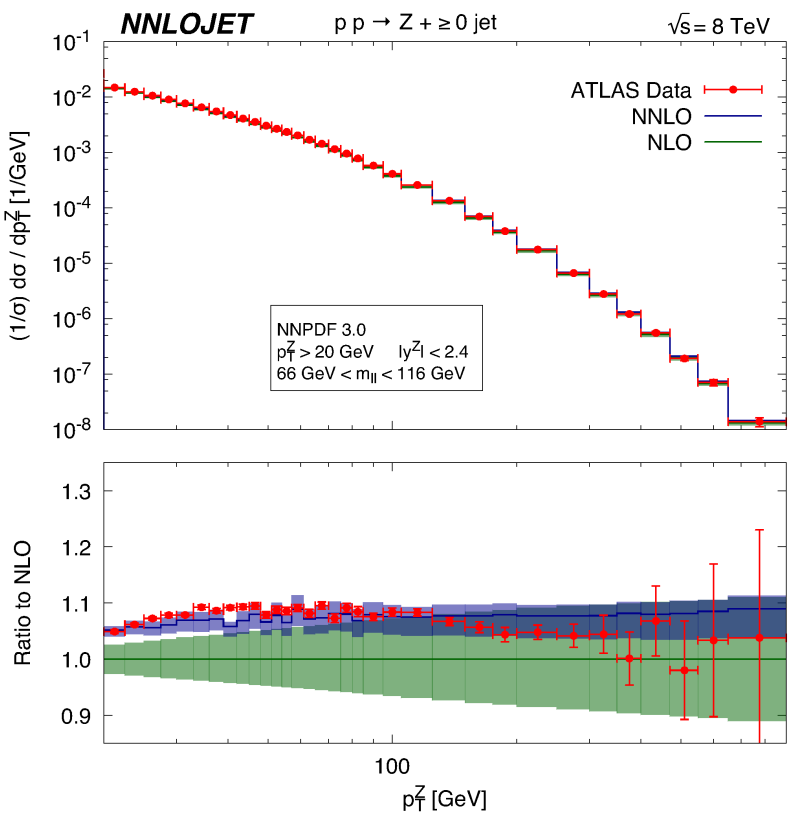


- ✓ NLO corrections $\sim 40 - 60\%$
- ✓ significant reduction of scale uncertainties NLO $\rightarrow$ NNLO
- ✓ NNLO corrections relatively flat $\sim 4 - 8\%$
- ✓ improved agreement, but not enough
- ✓ Note that for $66\ \mathrm{GeV} < m_{\ell\ell} < 116\ \mathrm{GeV}$

$$\sigma_{\mathrm{exp}} = 537.1 \pm 0.45\% \pm 2.8\%\ \mathrm{pb}$$

$$\sigma_{\mathrm{NNLO}} = 507.9^{+2.4}_{-0.7}\ \mathrm{pb}$$

Normalised Z p_T spectrum



$$\frac{1}{\sigma}\cdot\frac{\mathrm{d}\hat{\sigma}}{dp_T^Z}\Bigg|_{p_T^Z > 20\ \mathrm{GeV}}$$

with

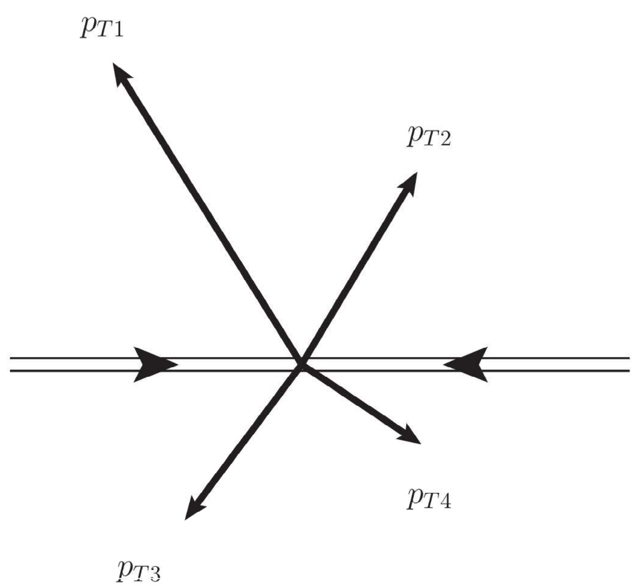
$$\sigma = \int_0^\infty \frac{\mathrm{d}\hat{\sigma}}{dp_T^Z} dp_T^Z \equiv \sigma_{LO}^Z + \sigma_{NLO}^Z + \sigma_{NNLO}^Z.$$

- ✓ Much improved agreement
- ✓ luminosity uncertainty cancels
- ✓ dependence on EW parameters reduced
- ✓ dependence on PDFs reduced ➠ study

Single Jet Inclusive Distribution

Currie, NG, Pires (16)

✓ Classic jet observable



✓ Every jet in the event enters in the distribution

✓ Expect sensitivity to PDFs

✓ ... and to α_s

✓ All sub-processes included – gg, gq, $q\bar{q}$, qq etc

✓ in leading colour approximation i.e. all $\alpha_s^2 N^2$, $\alpha_s^2 N N_F$, $\alpha_s^2 N_F^2$ contributions relative to Born

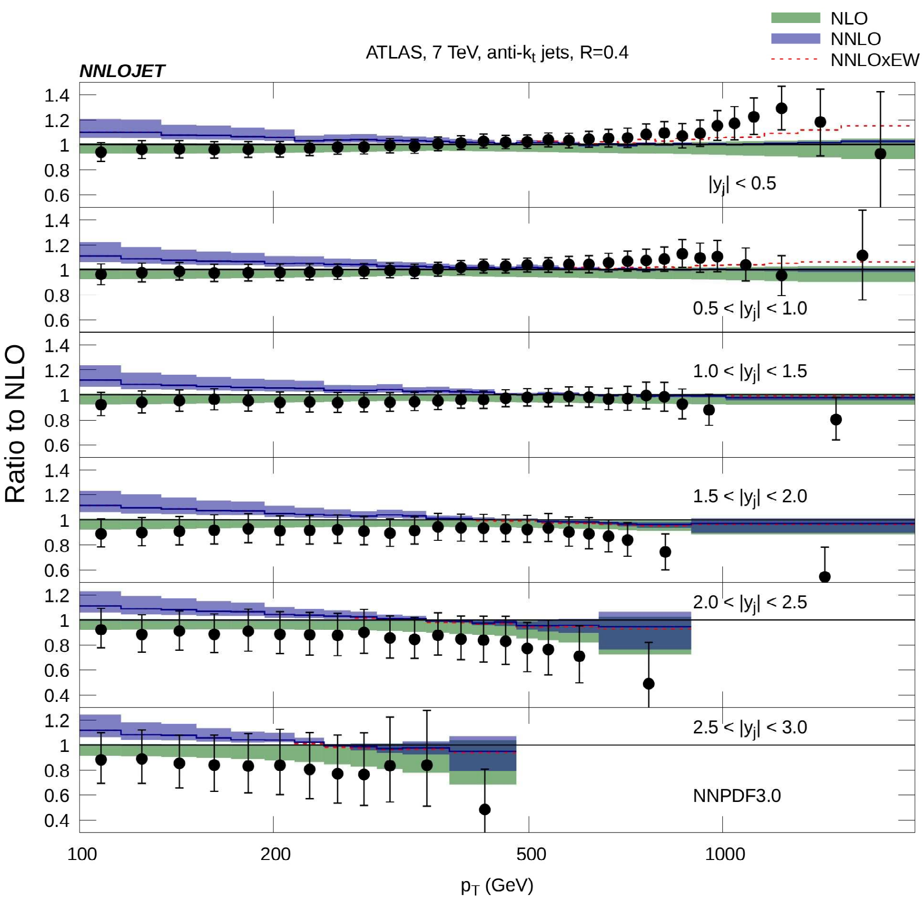
✘ missing corrections O(1), N_F/N, $1/N^2$, N_F/N^3, $1/N^4$

✓ expect to be less than 10% of the NNLO correction (as at NLO)

Single Jet Inclusive Distribution – R=0.4

Currie, NG, Pires (16)

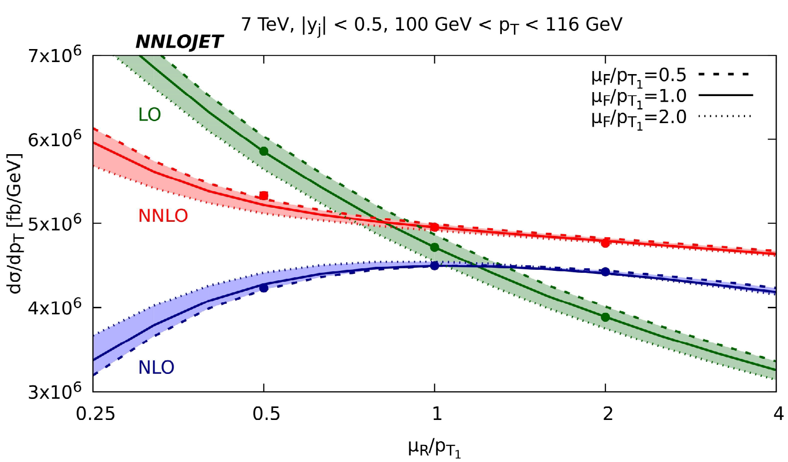
- ✓ ATLAS 7 TeV data, 4.5 fb^{-1}
  JHEP02(2015)153
  JHEP09(2015)141 (Erratum)
- ✓ anti-k_T algorithm with $R = 0.4$
- ✓ six rapidity slices,
  0 – 0.5, 0.5 – 1.0, 1.0 – 1.5, 1.5 – 2.0,
  2.0 – 2.5, 2.5 – 3.0
- ✓ NNPDF3.0_NNLO PDFs
- ✓ negligible NP corrections

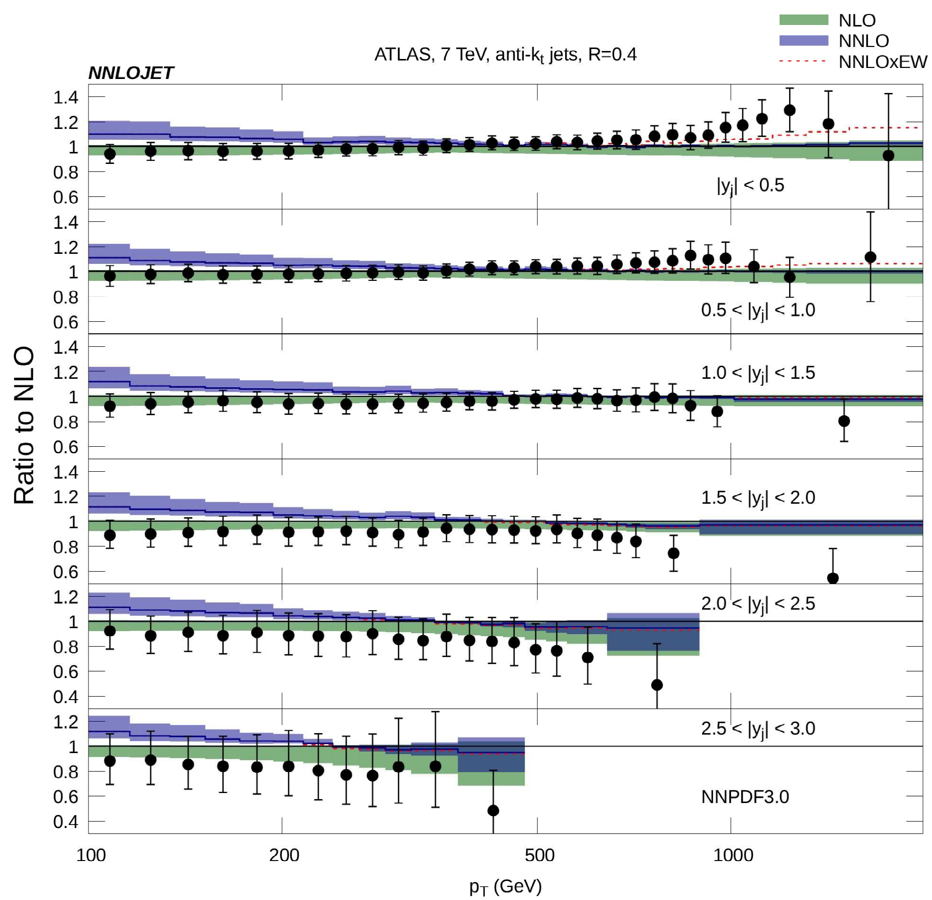


Single Jet Inclusive Distribution – R=0.4

Currie, NG, Pires (16)

- ✓ NLO describes the data pretty well
- ✓ NLO has relatively small scale dependence
  - – because the central scale choice lies close to the turning point in the scale variation plot
- ✓ NNLO effects around 10% at low p_T and small at high p_T





Single Jet Inclusive Distribution – R=0.4

Currie, NG, Pires (16)

✓ To evaluate effect of higher orders, it is often convenient to use K factors e.g.

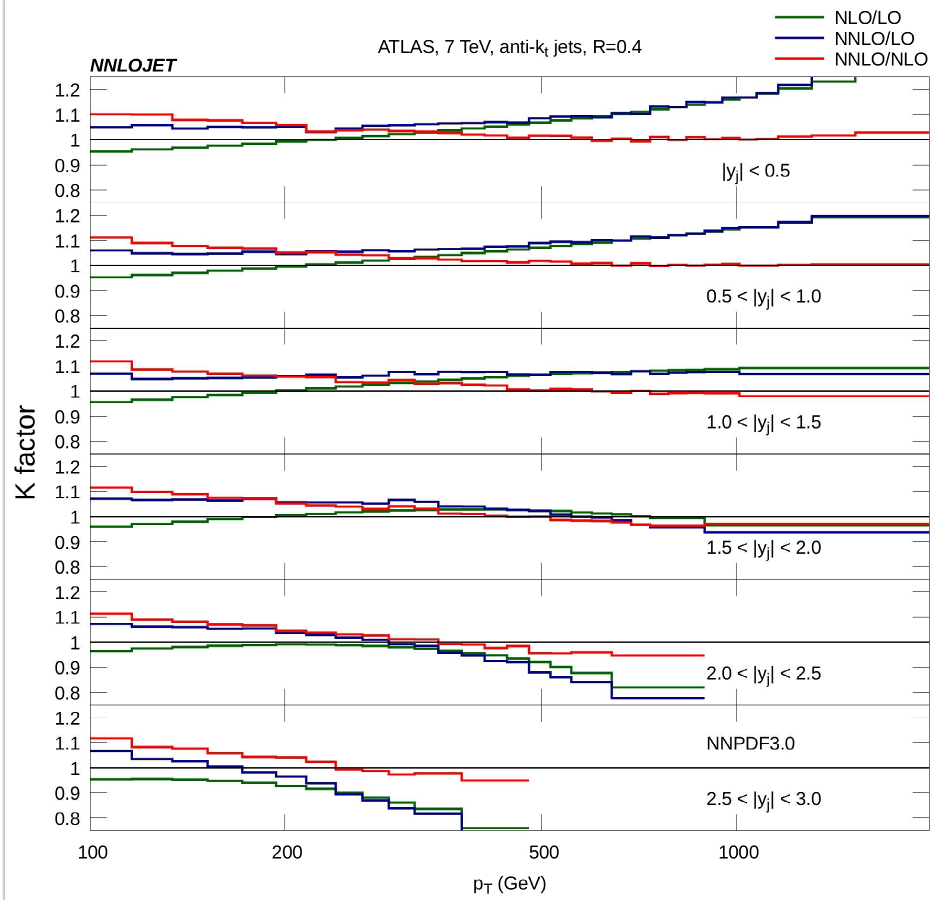
$$K = \frac{\mathrm{d}\sigma^{NNLO}/dp_T}{\mathrm{d}\sigma^{NLO}/dp_T}$$

✓ Same PDFs used for LO, NLO, NNLO

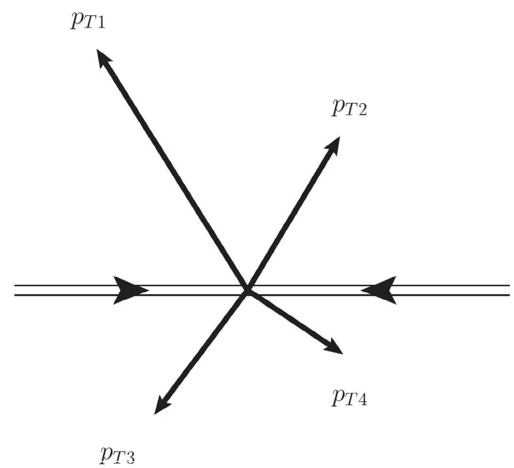
✘ Can argue that should use LO PDF for LO prediction, NLO PDF for NLO prediction.

➠ Change to K is a higher order effect.

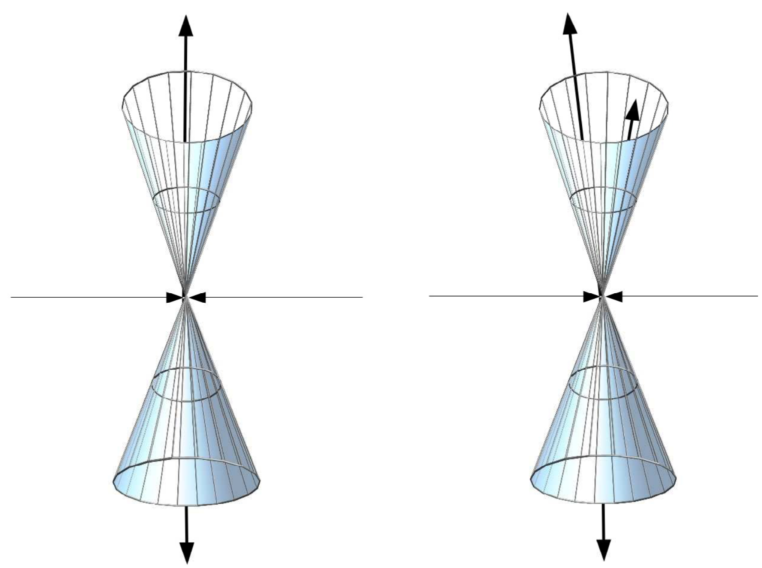
➠ This changes the K factor, by changing the more uncertain denominator



Scale Choice



- ✓ no fixed hard scale for jet production
- ✓ two widely used scale choices
  - ➠ leading jet p_T (p_{T1})
  - ➠ individual jet p_T (p_T)



- ✓ different scale changes PDF and α_s
- ✓ no difference for back-to-back jet configurations (only arises at higher orders)

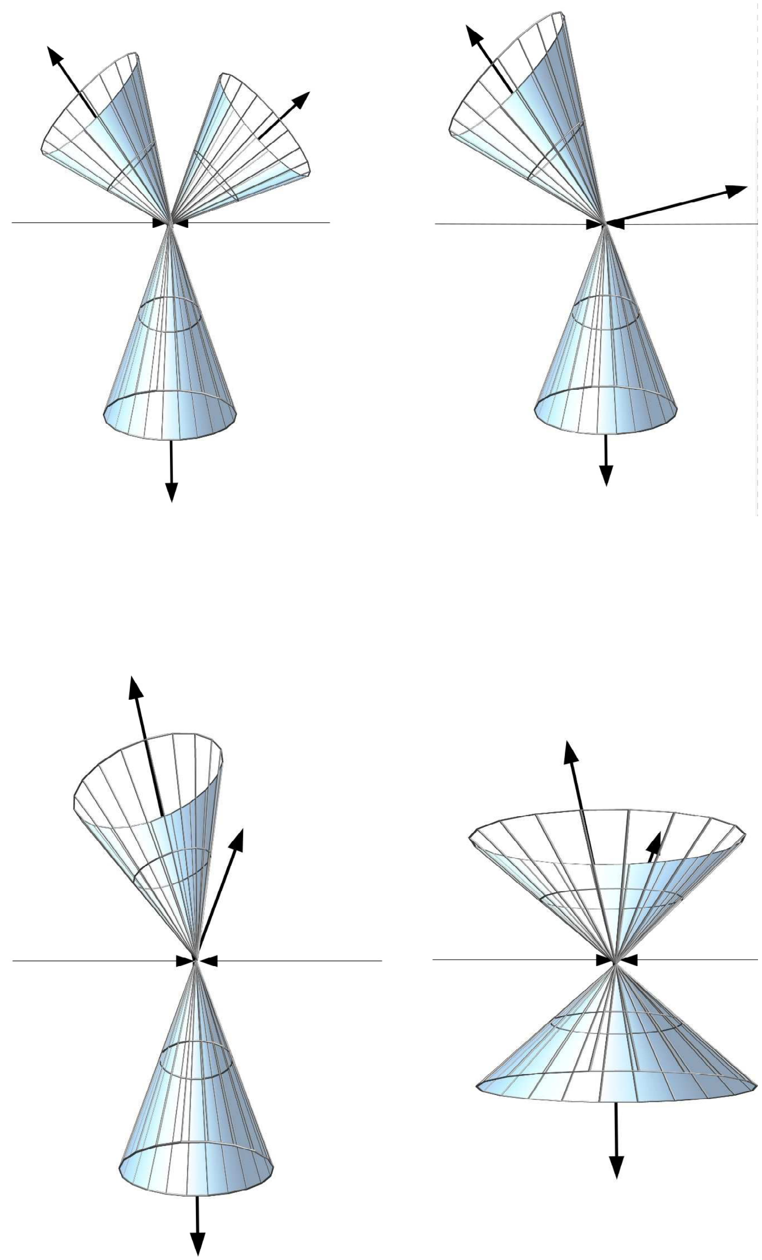
Scale Choice

At NLO, $p_T \neq p_{T1}$ for

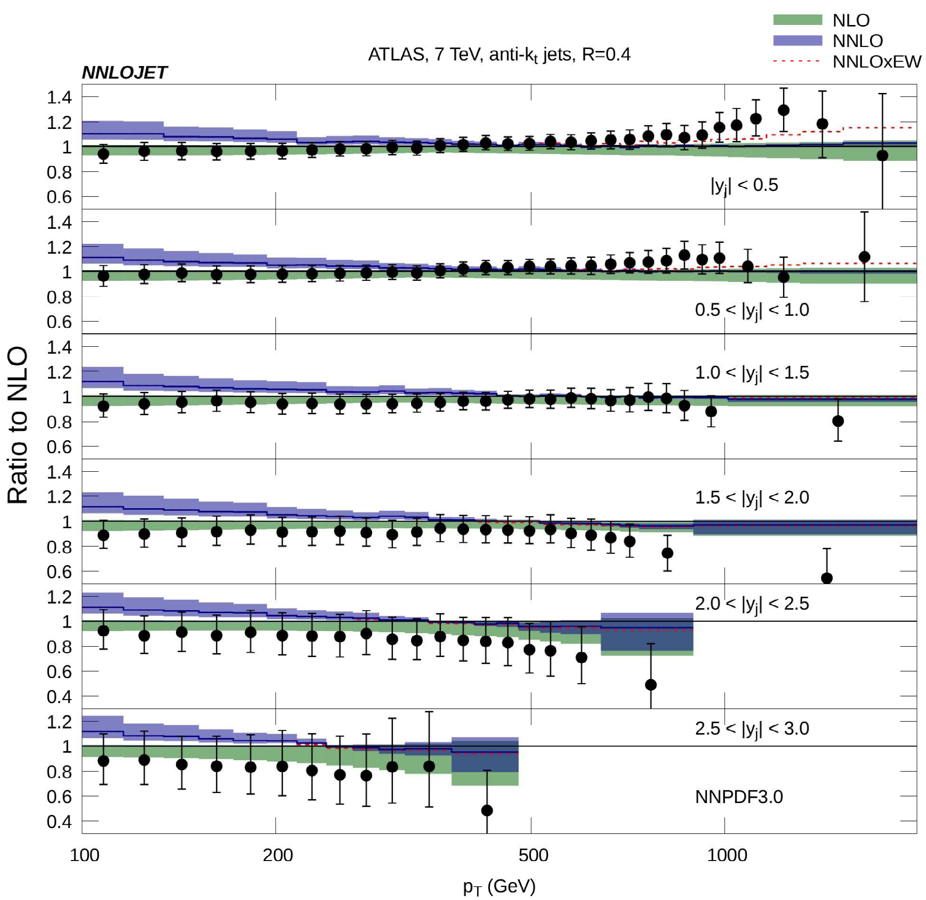
- ✓ 3-jet rate (small effect)
- ✓ 2-jet rate (3rd parton falls outside jet)

Changing R has an effect on the cross section, but also on the scale choice:

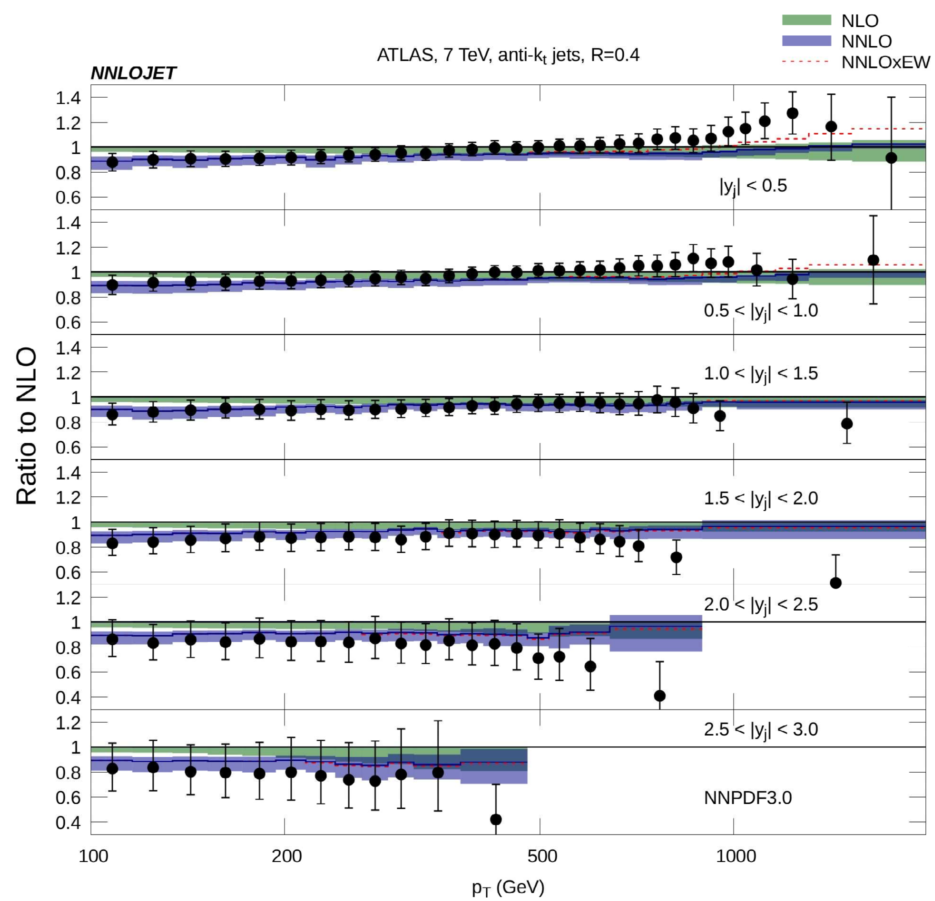
- ✓ introduces spurious R-dependence in scale choice
- ✓ p_{T1} scale has no R-dependence at NLO, unlike p_T
- ✓ at NNLO p_{T1} scale depends on R in some four-parton configurations



Single Jet Inclusive Distribution – R=0.4



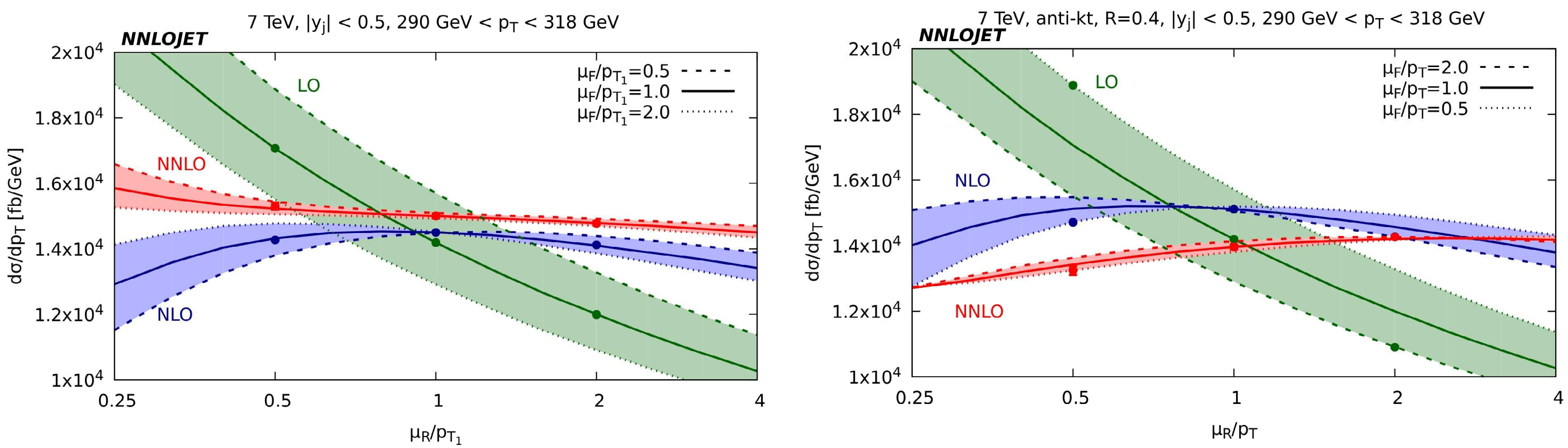
$$\mu_R = \mu_F = p_{T1}$$



$$\mu_R = \mu_F = p_T$$

- ✘ Quite different behaviour!
- ✓ NLO with $\mu = p_{T1}$ describes $R = 0.4$ data quite well
- ✓ NNLO with $\mu = p_T$ describes $R = 0.4$ data quite well

Single Jet Inclusive Distribution – R=0.4



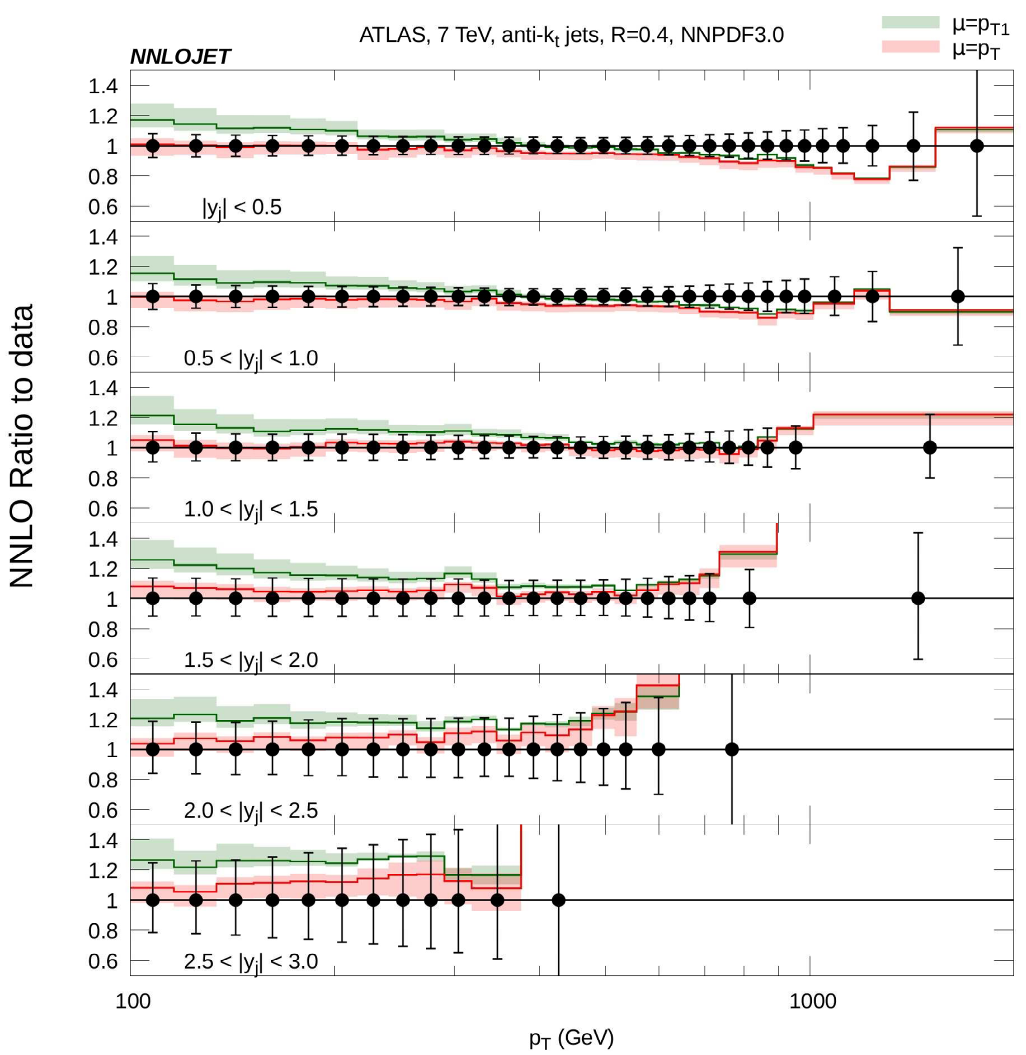
$\mu_R = \mu_F = p_{T1}$ $\qquad$ $\mu_R = \mu_F = p_T$

✘ Quite different behaviour!

➠ scale uncertainty much smaller than difference between scale choices

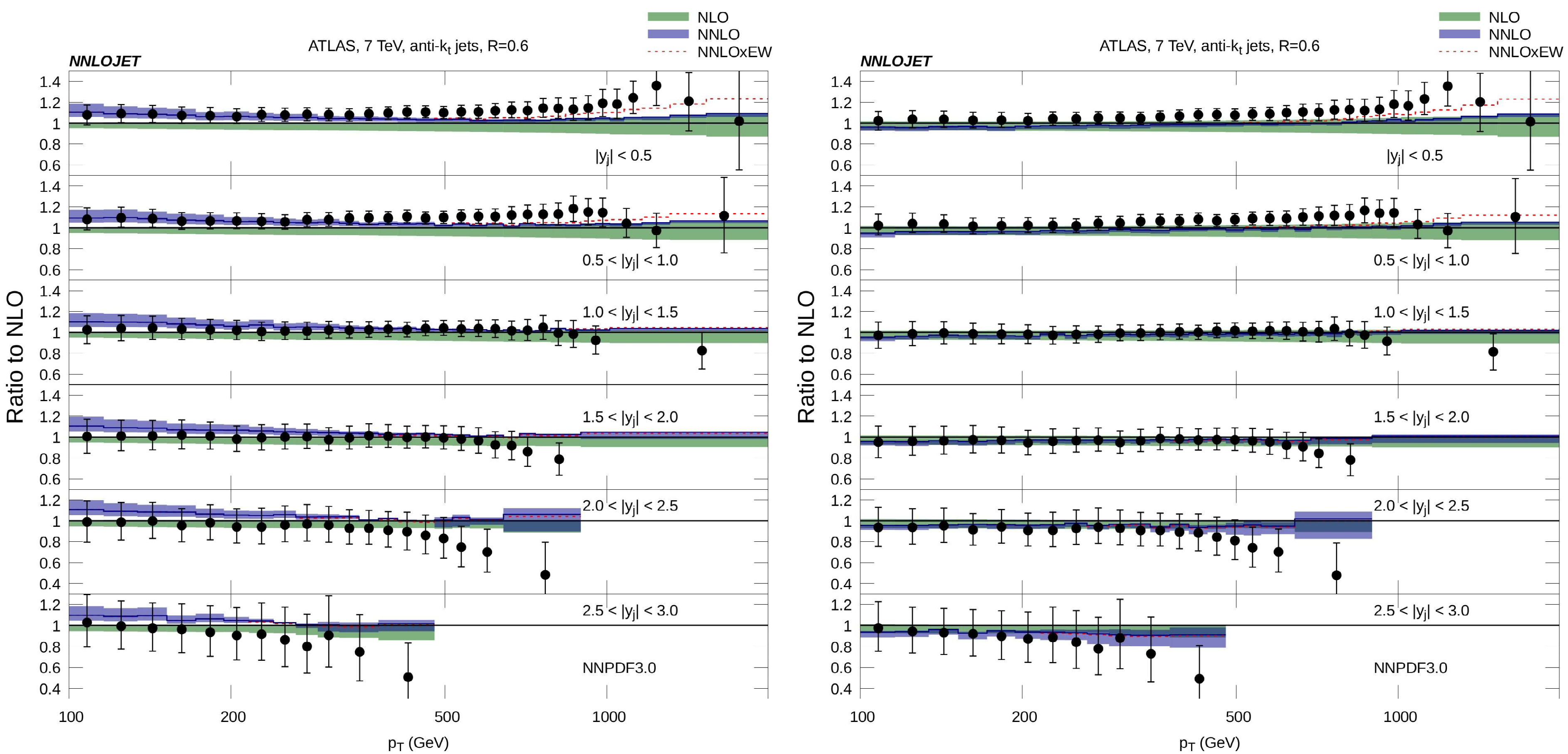
➠ explore alternative scale choices

Single Jet Inclusive Distribution – R=0.4



✘ Scale uncertainty is smaller than the uncertainty in choosing p_T or p_{T1}

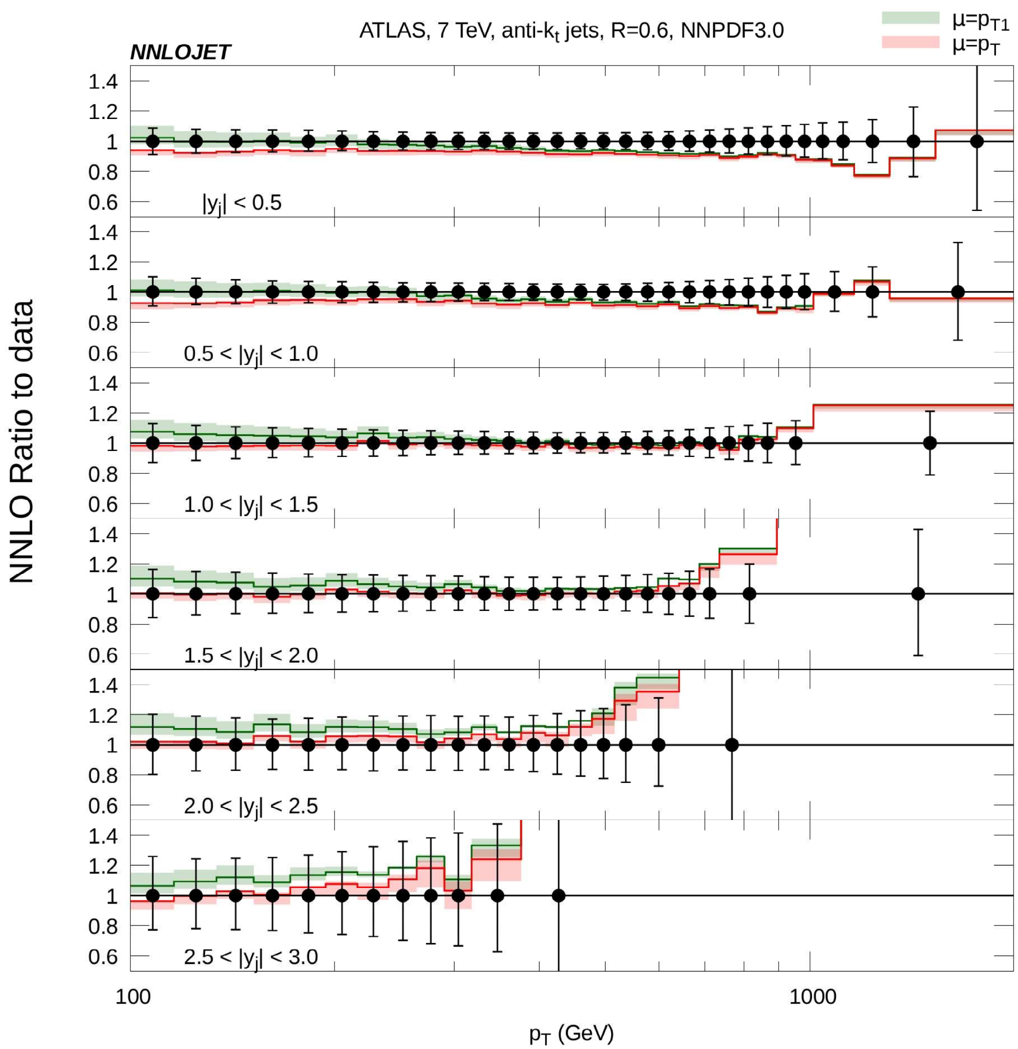
Single Jet Inclusive Distribution – R=0.6



$$\mu_R = \mu_F = p_{T1} \qquad \mu_R = \mu_F = p_T$$

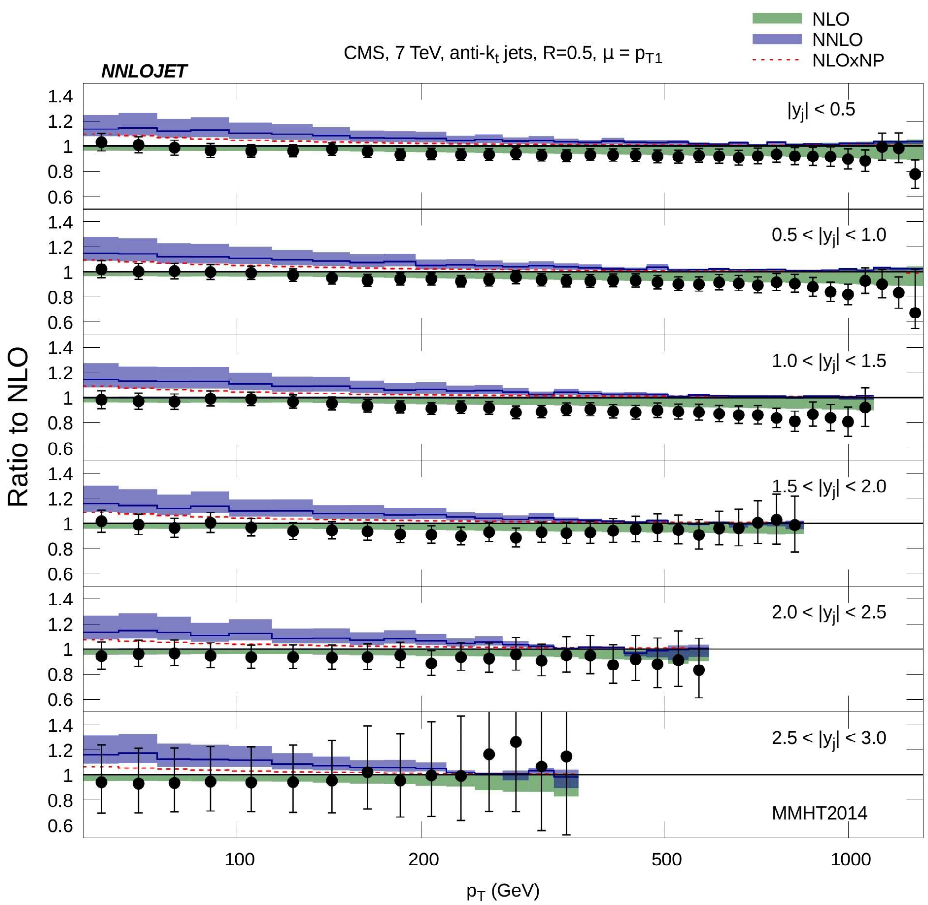
- ✘ Quite different behaviour!
- ✓ NLO with $\mu = p_T$ describes $R = 0.6$ data quite well
- ✓ NNLO with $\mu = p_{T1}$ describes $R = 0.6$ data quite well

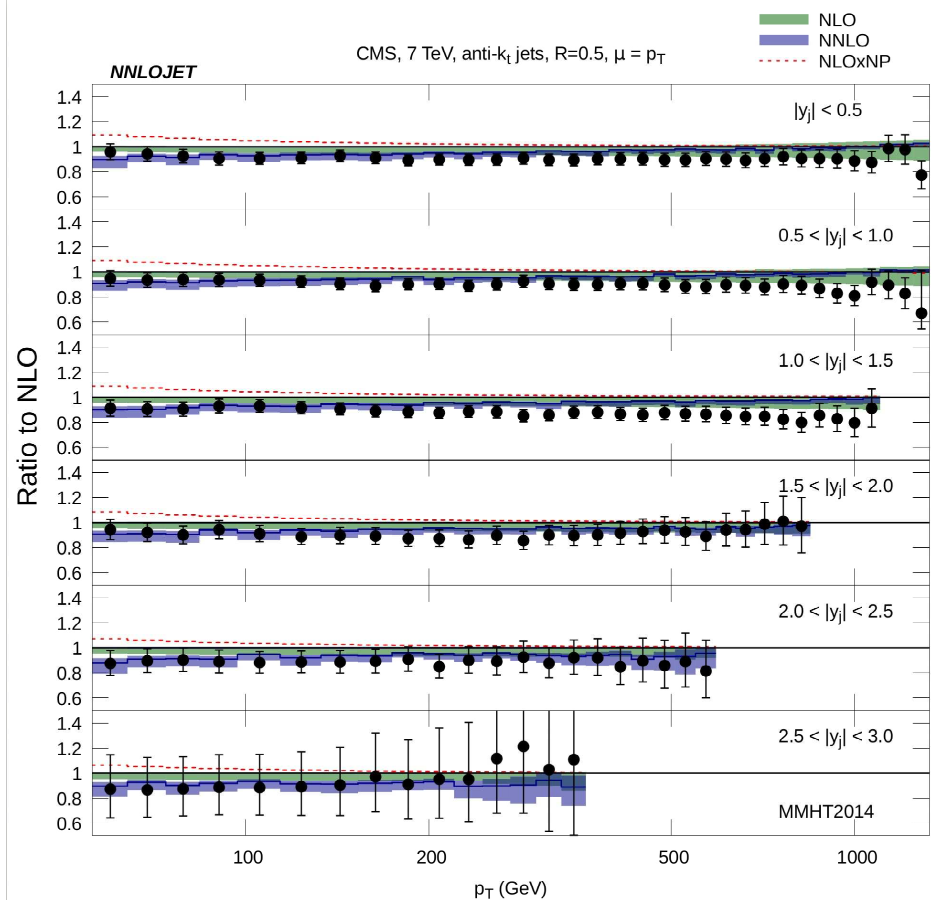
Single Jet Inclusive Distribution – R=0.6



- ✘ Scale uncertainty is smaller than the uncertainty in choosing p_T or p_{T1}

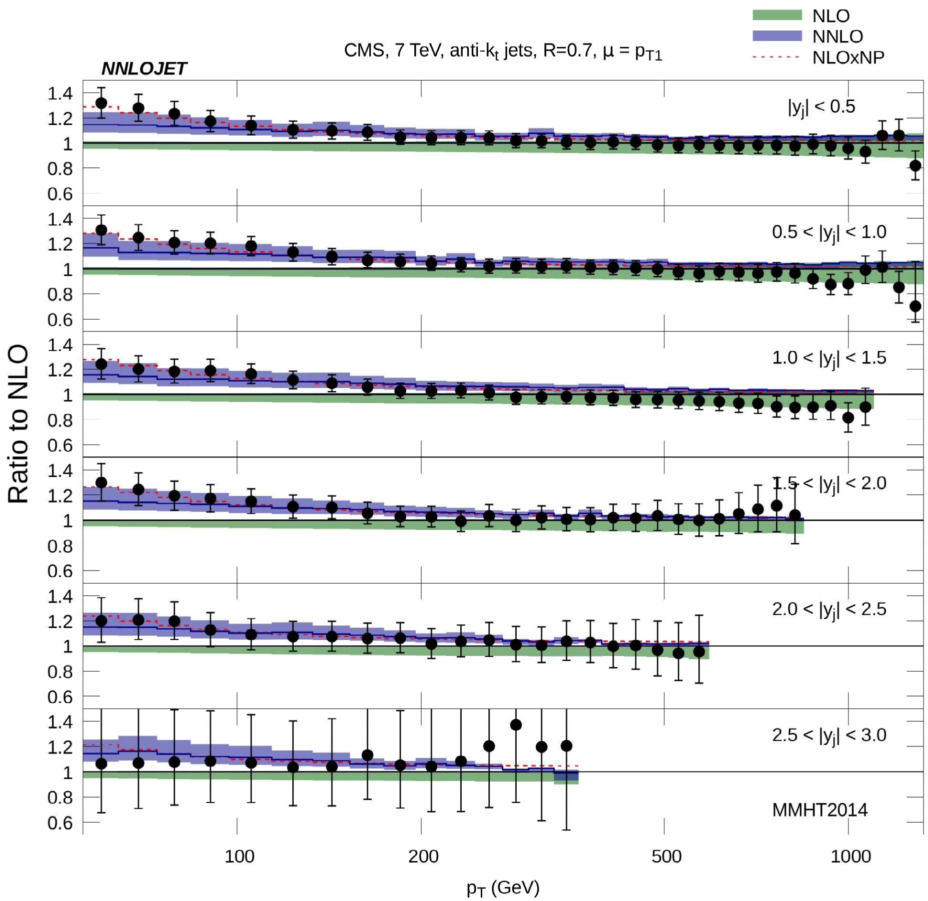
Single Jet Inclusive Distribution – R=0.5

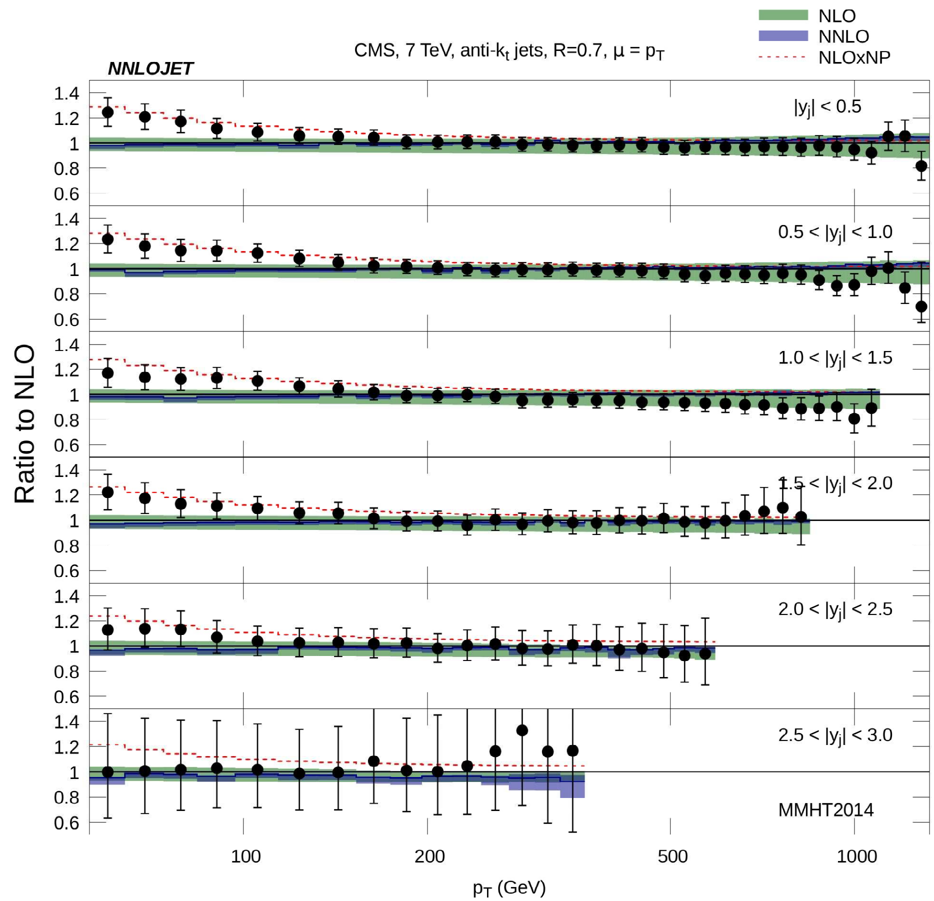




- ✓ CMS 7 TeV data
- ✘ increasing NP corrections with smaller jet p_T

Single Jet Inclusive Distribution – R=0.7





- ✓ CMS 7 TeV data
- ✘ increasing NP corrections with increasing cone size

CPU cost

✓ Standalone production run with fixed $\sqrt{s}$, fixed R, fixed PDF, three scale variation for $\mu = p_{T1}$ and $\mu = p_T$ (Warmup $\sim$ 1-2%)

Job Type	No. Jobs	Runtime/Job (hr)	Total Runtime
LO	200	0.5	100
NLO-V	500	1.5	750
NLO-R	500	2	1000
NNLO-VV	600	20	12000
NNLO-RV	2500	50	125000
NNLO-RRa	3500	50	175000
NNLO-RRb	2000	20	40000
			353850

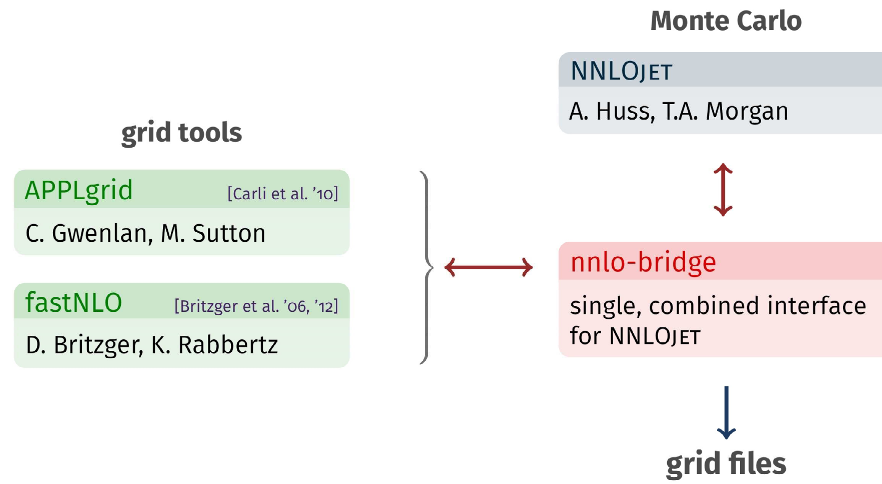
✓ because LO is independent of R and $p_T = p_{T1}$ to obtain different cone sizes/different scales can do a (much cheaper) NLO 3-jet calculation

$$\frac{d\sigma^{NNLO}(R_2)}{dp_T} = \frac{d\sigma^{NNLO}(R_1)}{dp_T} + \left(\frac{d\sigma^{R}(R_2)}{dp_T} - \frac{d\sigma^{R}(R_1)}{dp_T}\right)$$
$$+ \left(\frac{d\sigma^{RV}(R_2)}{dp_T} - \frac{d\sigma^{RV}(R_1)}{dp_T}\right) + \left(\frac{d\sigma^{RR}(R_2)}{dp_T} - \frac{d\sigma^{RR}(R_1)}{dp_T}\right)$$

APPLfast-NNLO interface

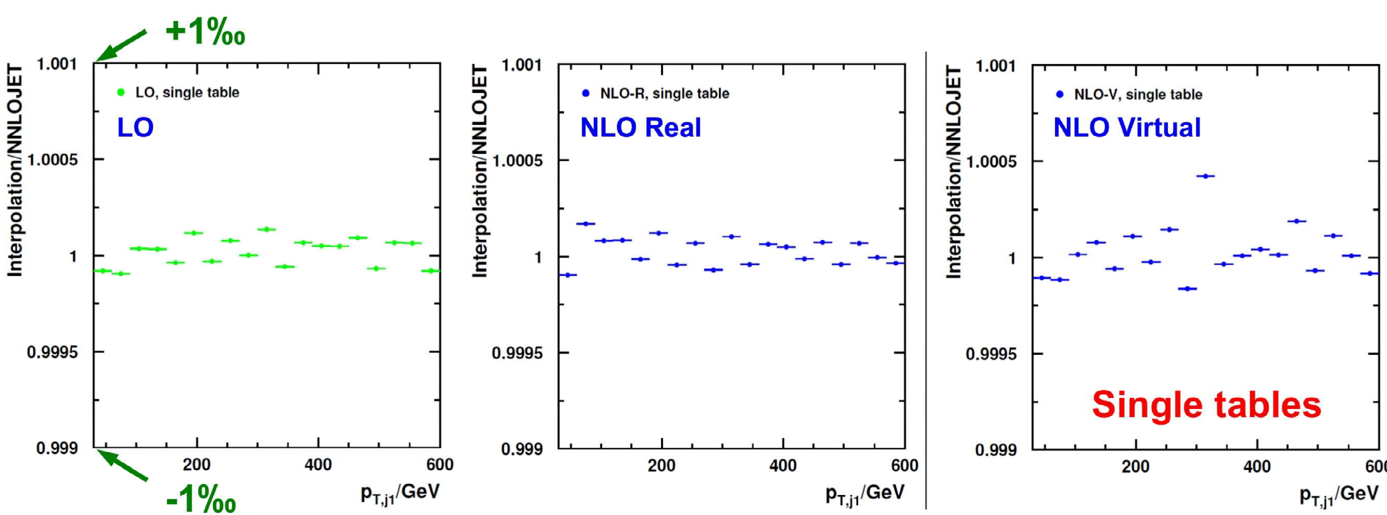
NNLOJET + D. Britzger, C. Gwenlan, M. Sutton, K. Rabbertz

- ✓ write out grid in x_1, x_2, Q^2
- ✓ swap out PDFs and α_s later at virtually no aditional cost
- ✓ file size $\mathcal{O}(10 - 100MB)$
- ✘ need to fix binning beforehand



generic interface ∀ processes available in NNLOJET

Z+jet approximation test for LO, NLO-R, and NLO-V
Agreement at subpermille level



APPLfast-NNLO interface

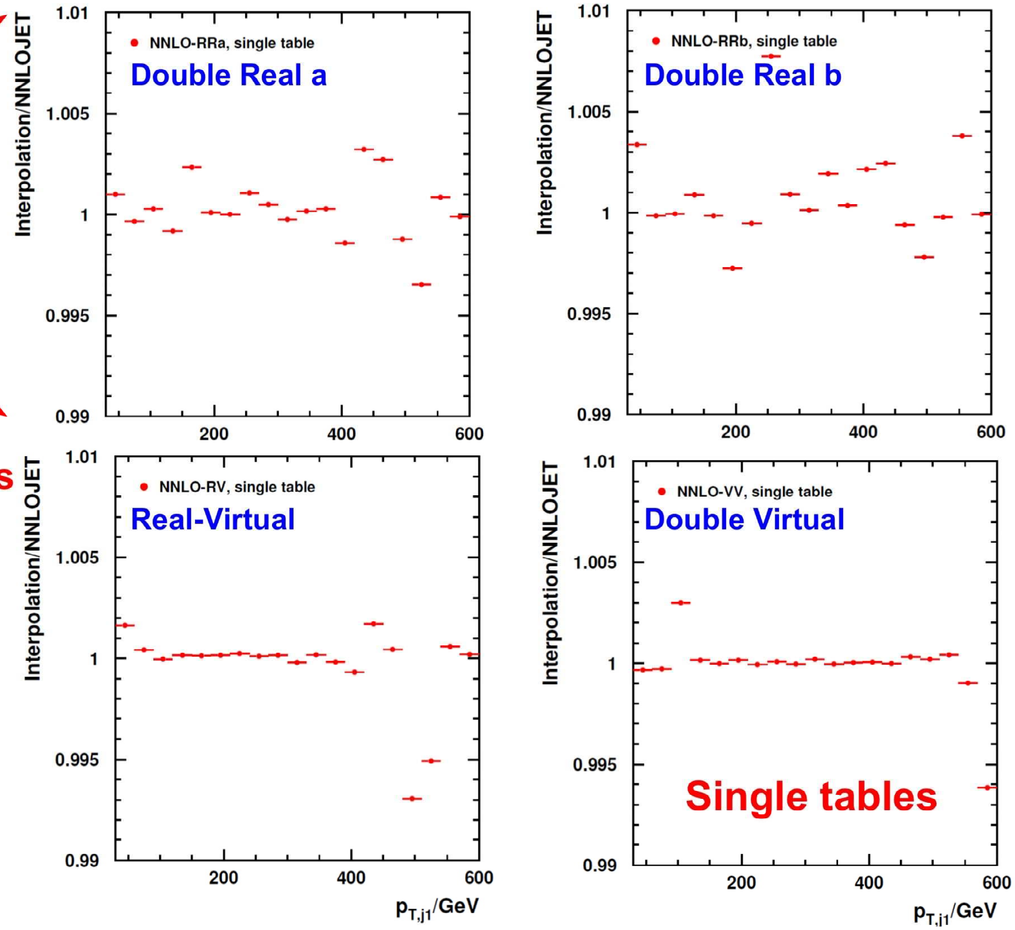
+1%

Z+jet approximation
test for NNLO parts

Agreement at
subpercent level

-1%

Some impact of fluctuations
visible
→ to be dealt with in
combination procedure.



(a/b indicates a technical
phase space separation
for this process)

- ✘ Still some work to do to combine interpolation grids
- ✓ But bridge code is working and expect new NNLO grids in 2017

Rabbertz, PDF4LHC 7 March 2017

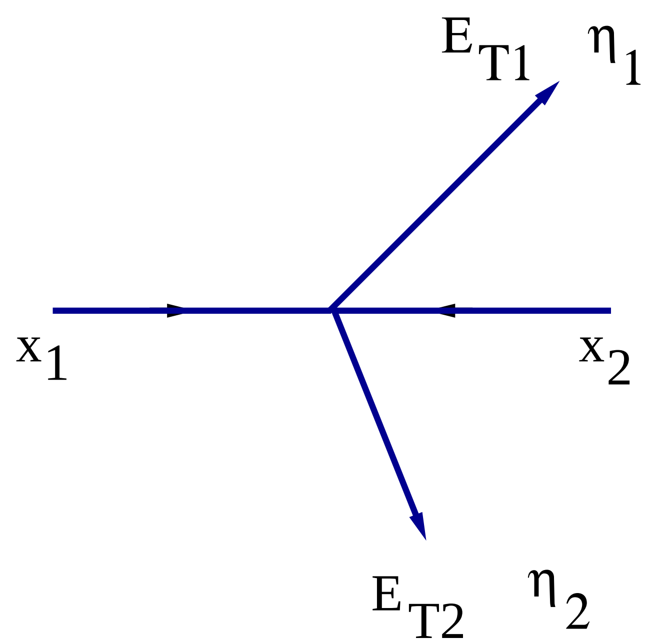
Maximising the impact of NNLO calculations

Triple differential form for a $2 \to 2$ cross section

$$\frac{d^3\sigma}{dE_T d\eta_1 d\eta_2} = \frac{1}{8\pi} \sum_{ij} x_1 f_i(x_1,\mu_F)\, x_2 f_j(x_2,\mu_F)\, \frac{\alpha_s^2(\mu_R)}{E_T^3} \frac{|\mathcal{M}_{ij}(\eta^*)|^2}{\cosh^4 \eta^*}$$

✓ Direct link between observables E_T, η_1, η_2 and momentum fractions/parton luminosities

$$x_1 = \frac{E_T}{\sqrt{s}} \left(\exp(\eta_1) + \exp(\eta_2)\right),$$
$$x_2 = \frac{E_T}{\sqrt{s}} \left(\exp(-\eta_1) + \exp(-\eta_2)\right)$$

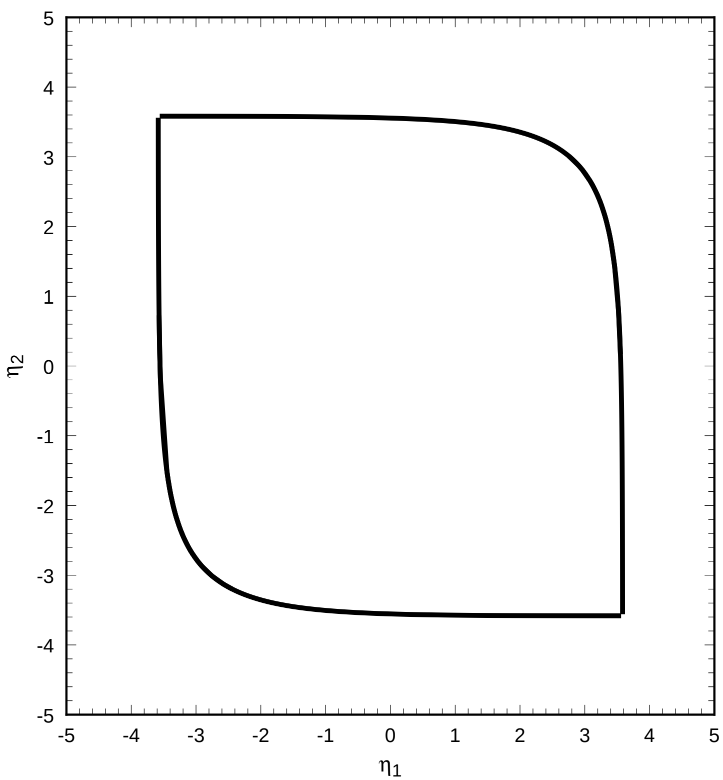


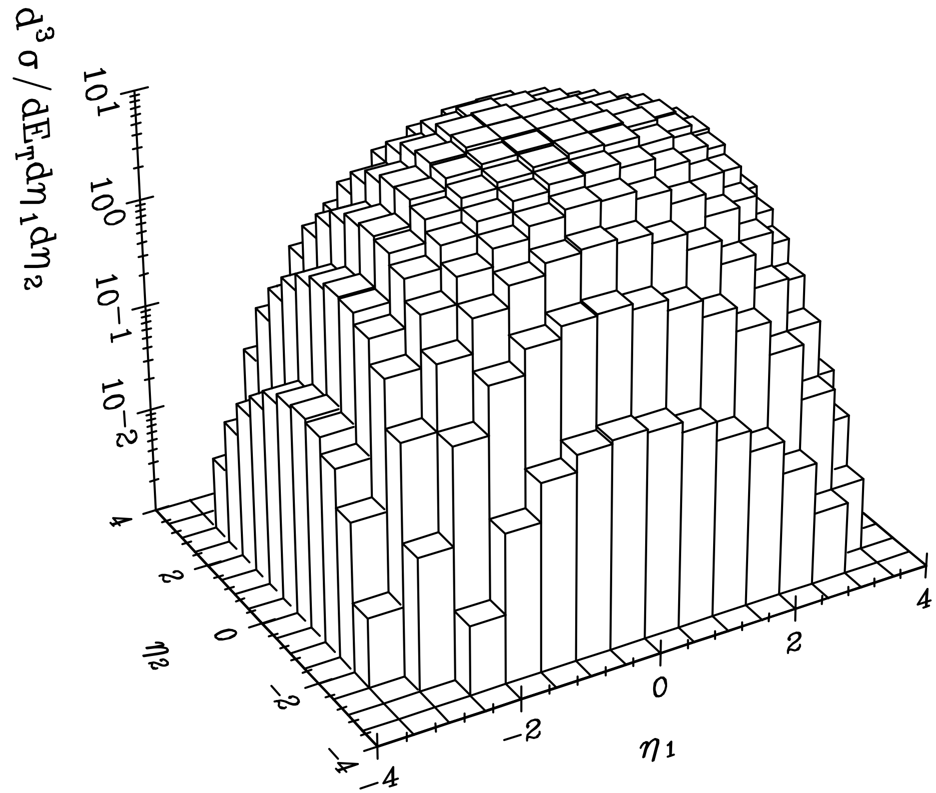
✓ and matrix elements that only depend on

$$\eta^* = \frac{1}{2}(\eta_1 - \eta_2)$$

Triple differential distribution

✓ Range of x_1 and x_2 fixed allowed LO phase space for jets $E_T \sim 200$ GeV at $\sqrt{s} = 7$ TeV





✓ Shape of distribution can be understood by looking at parton luminosities and matrix elements (in for example the single effective subprocess approximation)

Giele, NG, Kosower, hep-ph/9412338

Phase space considerations

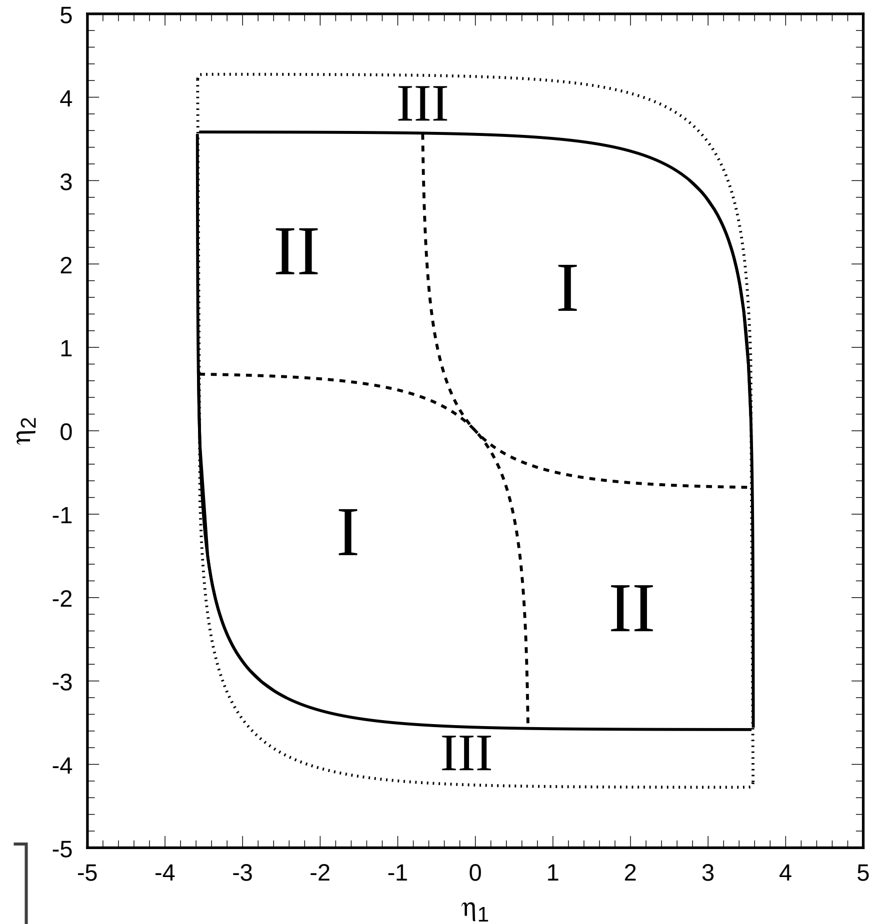
✓ Phase space boundary fixed when one or more parton fractions $\to 1$.

I $\eta_1 > 0$ and $\eta_2 > 0$ OR $\eta_1 < 0$ and $\eta_2 < 0$
➠ one x_1 or x_2 is less than x_T
- small x

II $\eta_1 > 0$ and $\eta_2 < 0$ OR $\eta_1 < 0$ and $\eta_2 > 0$
➠ both x_1 and x_2 are bigger than x_T
- large x

III growth of phase space at NLO (if $E_{T1} > E_{T2}$)

$$\left[x_T^2 < x_1 x_2 < 1 \qquad \text{and} \qquad x_T = 2E_T/\sqrt{s} \right]$$



Measuring PDF's at the LHC?

Should be goal of LHC to be as self sufficient as possible!

Study triple differential distribution for as many $2 \to 2$ processes as possible!

- ✓ Medium and large x gluon and quarks
  - ✓ $pp \to$ di-jets — dominated by gg scattering
  - ✓ $pp \to \gamma$ + jet — dominated by qg scattering
  - ✓ $pp \to \gamma\gamma$ — dominated by $q\bar{q}$ scattering
- ✓ Light flavours and flavour separation at medium and small x
  - ✓ Low mass Drell-Yan
  - ✓ W lepton asymmetry
  - ✓ $pp \to Z+$jet
- ✓ Strangeness and heavy flavours
  - ✓ $pp \to W^{\pm} + c$ — probes $s, \ \bar{s}$ distributions
  - ✓ $pp \to Z + c$ — probes c distribution
  - ✓ $pp \to Z + b$ — probes b distribution

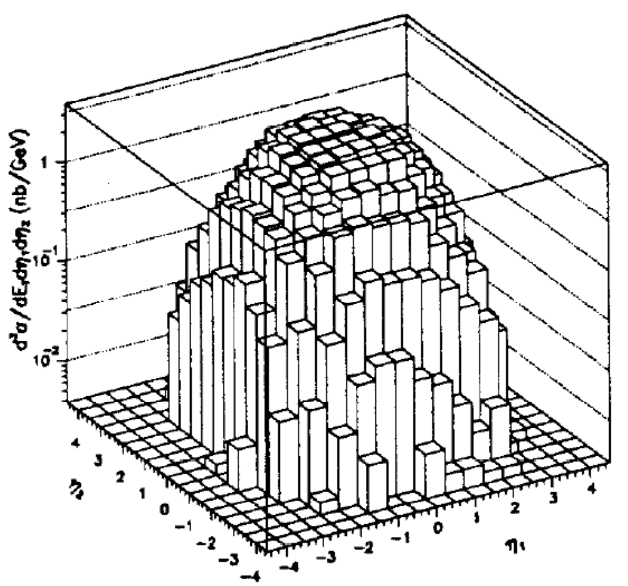
Measurements of strong coupling

- ✓ With incredible jet energy resolution, the LHC can do better!!
- ✓ by simultaneously fitting the parton density functions and strong coupling
- ✓ If the systematic errors can be understood, the way to do this is via the triple differential cross section

Giele, NG, Yu, hep-ph/9506442

- ✓ and add NNLO $W^{\pm}$+jet, Z+jet, γ+jet calculations (with flavour tagging) as they become available

D0 preliminary, 1994



Summary - Where are we now?

- ✓ First high precision N3LO calculations available

  could help reduce Missing Higher Order uncertainty by a factor of two

- ✓ Substantial and rapid progress in NNLO
  - ✚ many new calculations available
  - ➠ improved descriptions of experimental data
  - — codes typically require significant CPU resource
  - ✓ NNLO is emerging as standard for benchmark processes and could lead to improved pdfs etc.

    could help reduce theory uncertainty due to inputs by a factor of two

- ✓ NNLO automation?
  - as we gain analytical and numerical experience with NNLO calculations, can we further exploit the developments at NLO
  - automation of two-loop contributions?
  - automation of infrared subtraction terms?

- ✓ Is there a better way of estimating the theoretical uncertainties?

Summary - NNLOJET

- ✓ `NNLOJET` is able to make a range of fully differential NNLO predictions for fiducial cross sections that can be compared directly with data
- ✓ Z+jet
  - ✚ inclusive p_T^Z spectrum predicted to NNLO accuracy for $p_T^Z > p_{T,\mathrm{cut}}^Z$
  - ✚ observe a reduction of the scale uncertainty and an improvement in the theory vs. data comparison
  - ✚ Normalised distributions show excellent agreement between data and NNLO
- ✓ dijet
  - ✚ single jet inclusive p_T spectrum predicted to NNLO accuracy
  - — no obvious improvement in the theory vs. data comparison (R)
  - — difference between common scale choices p_T and p_{T1} larger than scale uncertainty

Work in progress:

- ✓ Including other processes, e.g W+jet, other Higgs decays, flavour tagged jets
- ✓ Studying potential of data to constrain PDF sets and interface to `APPLfast-NNLO`

Back up slides

Slicing v Subtraction example

$$V = \frac{F(0)}{\epsilon}, \qquad\qquad R = \int_0^1 dx \frac{F(x)}{x^{1+\epsilon}}$$

Slicing

$$\begin{aligned} \sigma &= V + R \\ &= \frac{F(0)}{\epsilon} \\ &+ \int_0^X dx \frac{F(0)}{x^{1+\epsilon}} + \int_X^1 dx \frac{F(x)}{x} \\ &= F(0)\ln(X) + \int_X^1 dx \frac{F(x)}{x} \end{aligned}$$

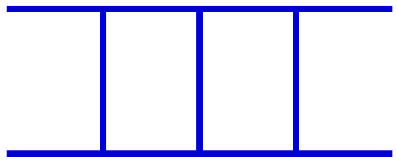
- ✓ Approximation made for $x < X$
- ✓ X should be small, but not so small that numerical errors dominate
- ✓ q_T and N-jettiness schemes related to soft-collinear resummation

Subtraction

$$\begin{aligned} \sigma &= V + R \\ &= \frac{F(0)}{\epsilon} + \int_0^1 dx \frac{S(x)}{x^{1+\epsilon}} \\ &+ \int_0^1 dx \left[\frac{F(x)}{x^{1+\epsilon}} - \frac{S(x))}{x^{1+\epsilon}} \right] \\ &= \text{finite} + \int_0^1 dx \left[\frac{F(x) - S(x)}{x} \right] \end{aligned}$$

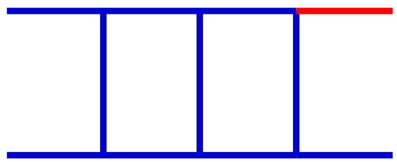
- ✓ $S(x) \to F(0)$ as $x \to 0$
- ✓ integral of $S(x)$ must be computed
- ✓ antenna, STRIPPER, ColorFul, P2B all subtraction schemes

Two Loop Master Integrals - analytic

✓ 

Smirnov (99); Smirnov, Tausk (99)

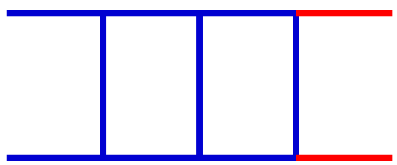
➠ enables $pp \to \gamma\gamma,\, \gamma J,\, JJ$

✓ 

Gehrmann and Remiddi (00,01,02)

➠ enables $pp \to WJ,\, ZJ,\, HJ,\, W\gamma,\, Z\gamma,$

$e^+e^- \to JJJ,\, ep \to JJ(+J)$

✓ 

Gehrmann, Tancredi, Weihs (13);

Gehrmann, von Manteuffel, Tancredi, Weihs (14);

Caola, Henn, Melnikov, Smirnov (14);

Papadopoulos, Tommasini, Wever (14)

➠ enables $pp \to WW,\, ZZ,\, WZ,\, HH$

✓ now intensive work towards two-loop five point integrals

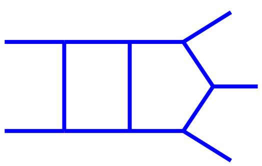
Two Loop Master Integrals - analytic

✓ Basis functions for two-loop pentagon graphs with massless internal propagators known - Goncharov Polylogs

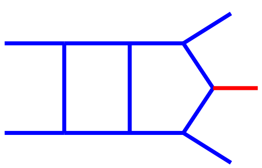
$$G(a_n, a_{n-1}, \ldots, a_1, t) = \int_0^t \frac{dt}{t_n - a_n} G(a_{n-1}, \ldots, a_1, t_n)$$

✓ Canonical (Henn) basis for evaluating integral as series in ϵ

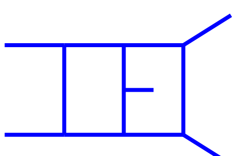
$$\partial_x \vec{f} = \epsilon \hat{A}_x(x, y, z, \ldots) \vec{f}$$

✓  Gehrmann, Henn, Lo Presti (15); Papadopoulos, Tomassini, Wever (15)

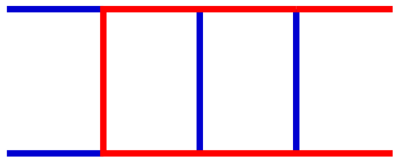
➠ enables $pp \to JJJ, \gamma\gamma J, \gamma\gamma\gamma$

✓  Papadopoulos, Tomassini, Wever (15)

➠ enables $pp \to VJJ, HJJ$

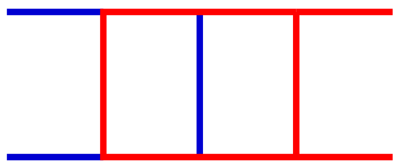
✘  nonplanar graphs still unknown

Two Loop Master Integrals - numeric

✓ 

Czakon (07); Bonciani, Ferroglia, Gehrmann, Studerus (09)

➠ enables $pp \to t\bar{t}$

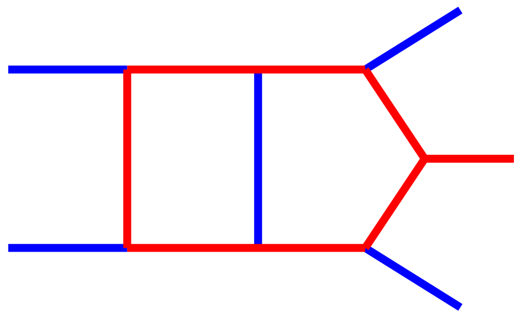
✓ 

Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke (16)

➠ enables $pp \to HH$ at NLO with massive top loop

✓ now intensive work including additional scales

Two Loop Master Integrals - numeric

- ✓ Integrals with massive propagators much more complicated, new types of (elliptic) functions needing input from mathematics Tancredi, Remiddi (16); Adams, Bogner, Weinzierl (15,16)
- ✓  e.g. Higgs plus Jet production via massive quark loop
  - ✓ First results as one-fold (elliptic) integrals Bonciani et al (16)
  - ✓ Light quark effects Melnikov et al (16)

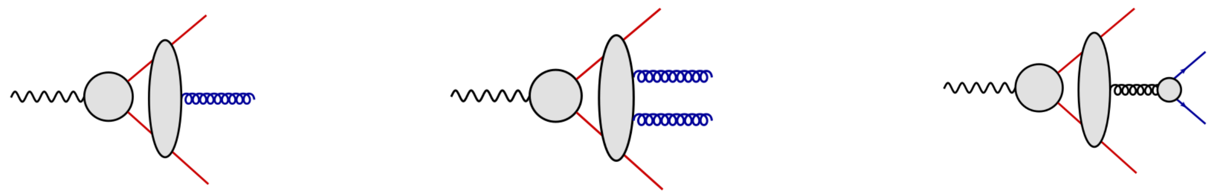
Antenna subtraction at NNLO

✓ Antenna subtraction exploits the fact that matrix elements already possess the intricate overlapping divergences

$$X_3^0(i,j,k) \sim \frac{|\mathcal{M}_3^0(i,j,k)|^2}{|\mathcal{M}_2^0(I,K)|^2}, \qquad X_4^0(i,j,k,l) \sim \frac{|\mathcal{M}_4^0(i,j,k,l)|^2}{|\mathcal{M}_2^0(I,L)|^2}$$

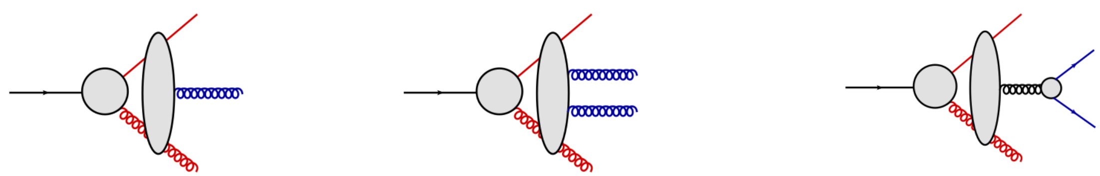
Quark-antiquark:

$\gamma^* \to q\bar{q} + \cdots$



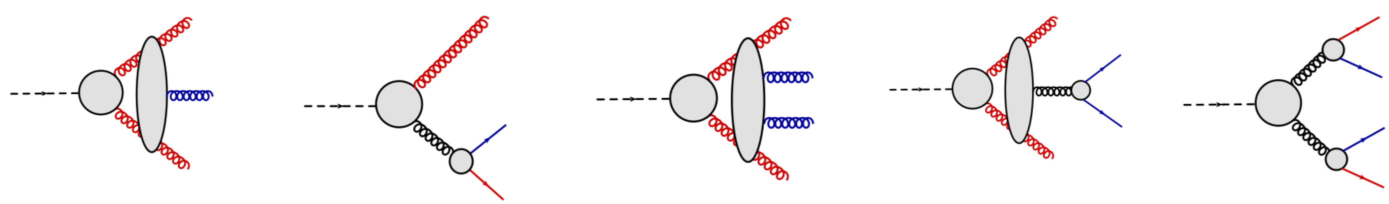
Quark-gluon:

$\tilde{\chi}^0 \to \tilde{g}g + \cdots$



Gluon-gluon:

$H \to gg + \cdots$



✓ plus mappings $i+j+k \to I+J$, $i+j+k+l \to I+L$

Antenna subtraction at NNLO

✓ Antenna mimics all singularities of QCD

$$\begin{tikzcd}
 & A_4^0(1_q, 2_g, 3_g, 4_{\bar{q}}) \arrow[dl, "1||2||3"'] \arrow[d, "2{,}3\sim 0"] \arrow[dr, "3||4 + 2\sim 0"] & \\
P_{qgg}^0 & S_{1234}^0 & S_{1;234}^0
\end{tikzcd}$$

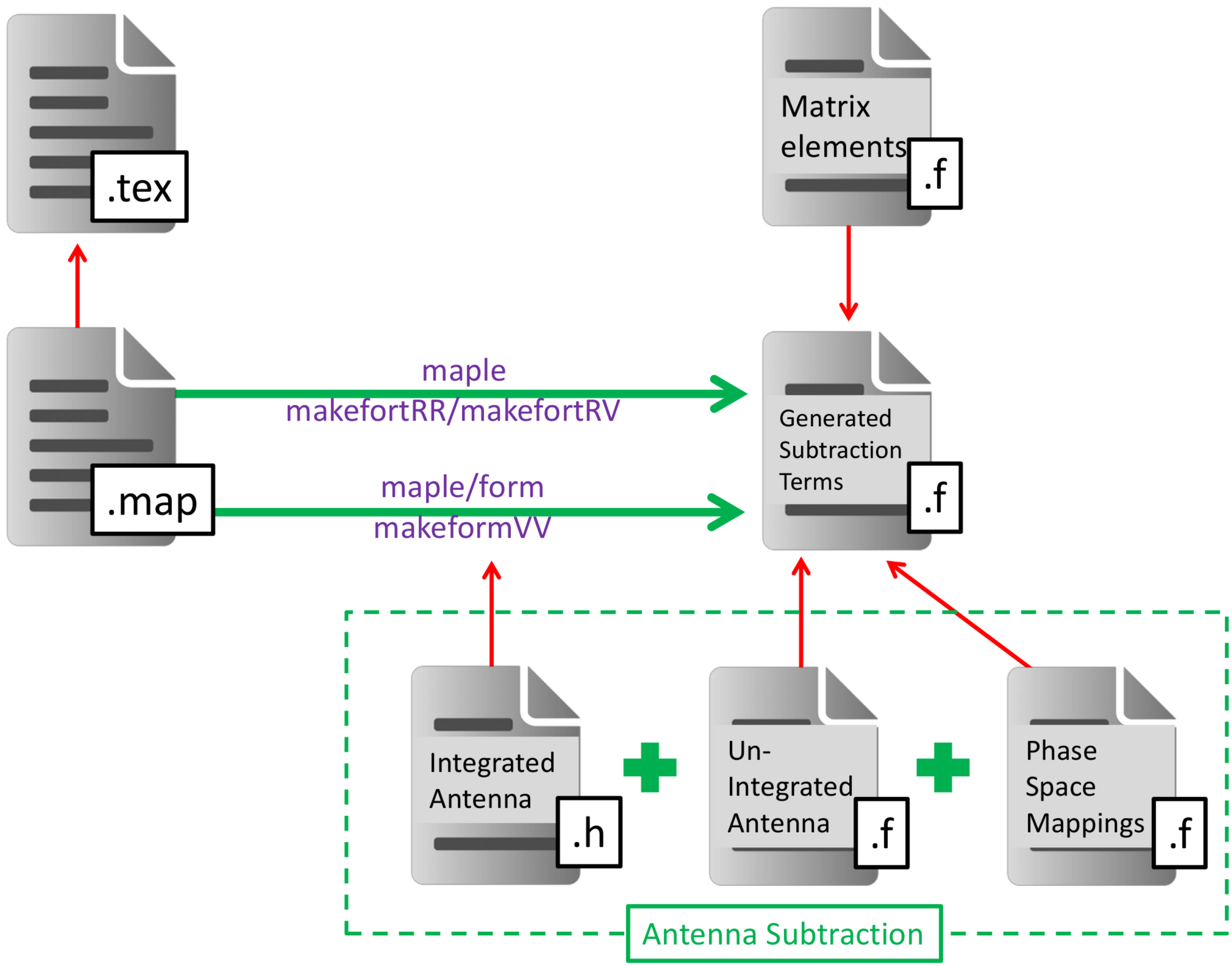
✓ Phase space map smoothly interpolates momenta for reduced matrix element between limits

$$\begin{aligned}
(\widetilde{123}) &= xp_1 + r_1 p_2 + r_2 p_3 + z p_4 \\
(\widetilde{234}) &= (1-x)p_1 + (1-r_1)p_2 + (1-r_2)p_3 + (1-z)p_4
\end{aligned}$$

Antenna subtraction at NNLO

- ✓ All unintegrated antennae available
  - ✓✓ Final-Final — Gehrmann-De Ridder, Gehrmann, NG, (05)
  - ✓✓ Initial-Final — Daleo, Gehrmann, Maitre, (07)
  - ✓✓ Initial-Initial — Daleo, Gehrmann, Maitre, (07)
    NG, Pires, (10)
- ✓ All antennae analytically integrated
  - ✓✓ Final-Final — Gehrmann-De Ridder, Gehrmann, NG, (05)
  - ✓✓ Initial-Final — Daleo, Gehrmann-De Ridder, Gehrmann, Luisoni, (10)
  - ✓✓ Initial-Initial — Gehrmann, Monni, (11)
    Boughezal, Gehrmann-De Ridder, Ritzmann, (11)
    Gehrmann, Ritzmann, (12)
- ✚ Laurent expansion in ϵ

Automatically generating the code (1)



Maple script: RR example


```
+F40a(i,j,k,l)*A4g0(1,2,[i,j,k],[j,k,l])
-f30FF(i,j,k)*f30FF([i,j],[j,k],l)
*A4g0(1,2,[[i,j],[j,k]],[[j,k],l])
...
```

$$+\,F_4^{0,a}(i,j,k,l)\,A_4^0(1,2,(\widetilde{ijk}),(\widetilde{jkl}))$$

$$-\,f_3^0(i,j,k)\,f_3^0((\widetilde{ij}),(\widetilde{jk}),l)\,A_4^0(1,2,[(\widetilde{ij}),(\widetilde{jk})],(\widetilde{(\widetilde{jk})l}))$$

...

- ✓ X_4^0, X_3^0 (and X_3^1 in RV) are unintegrated antennae
- ✓ `[i,j,k]` or $(\widetilde{ijk})$ are mapped momenta

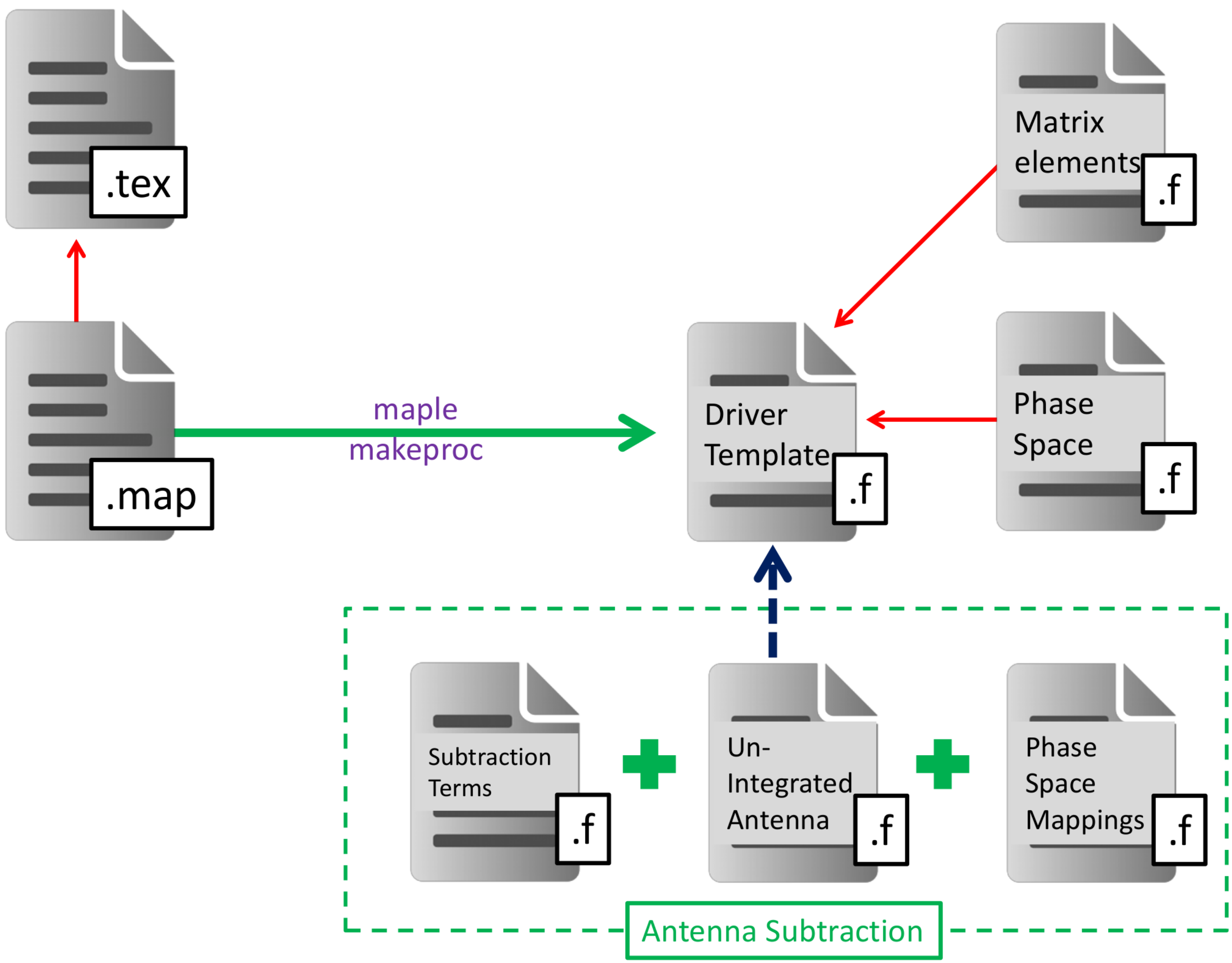
Maple script: VV example


```
-(+1/2*calgF40FI(2,3)
+1/2*calgF31FI(2,3)
+b0/e*1/2*QQ(s23)*calgF30FI(2,3)
-b0/e*1/2*calgF30FI(2,3)
-1/2*calgF30FI(2,3)*1/2*calgF30FI(2,3)
-1/2*gamma2gg(z2)
+b0/e*1/2*gamma1gg(z2)
)*A4g0(1,2,3,4)
...
```

✓ $\mathcal{X}_4^0$, $\mathcal{X}_3^0$ and $\mathcal{X}_3^1$ are integrated antennae

$$
\begin{aligned}
+\Bigg[\quad & - \frac{1}{2}\,\mathcal{F}^0_{4,g}(s_{23}) \\
& - \frac{1}{2}\,\mathcal{F}^1_{3,g}(s_{23}) \\
& - \frac{b_0}{2\epsilon}\left(\frac{s_{23}}{\mu_R^2}\right)^{-\epsilon} \mathcal{F}^0_{3,g}(s_{23}) \\
& + \frac{b_0}{2\epsilon}\,\mathcal{F}^0_{3,g}(s_{23}) \\
& + \frac{1}{4}\,\mathcal{F}^0_{3,g}(s_{23}) \otimes \mathcal{F}^0_{3,g}(s_{23}) \\
& + \frac{1}{2}\,\Gamma^{(2)}_{gg}(z_2)
\end{aligned}
$$

Automatically generating the code (2)



Maple script to produce driver template


```
R:=[
[A5g0,[g,g,g,g,g],1],
[B3g0,[qb,g,g,g,q],1/nc],
...
]:
```

$$d\sigma_{gg}^{R} = \mathcal{N}_{LO}\left(\frac{\alpha_s N}{2\pi}\right)\Bigg[$$

$$+2\,\frac{1}{3!}\left(\sum_{12} \mathtt{A5g0}(1,2,3,4,5) - \mathtt{ggA5g0SNLO}(1,2,3,4,5)\right)$$

$$+\frac{N_F}{N}\left(\sum_{6} \mathtt{B3g0}(3,1,2,4,5) - \mathtt{ggB3g0SNLO}(3,1,2,4,5)\right)$$

$$\ldots\Bigg]$$

Checks

✓ Analytic pole cancellations for RV, VV

$$\text{Poles}\left(d\sigma^{RV} - d\sigma^{T}\right) = 0$$

$$\text{Poles}\left(d\sigma^{VV} - d\sigma^{U}\right) = 0$$

```
09:26:35  ...maple/process/Z
$ form autoqgB1g2ZgtoqU.frm
FORM 4.1 (Mar 13 2014) 64-bits
    #-

   poles = 0;

  6.58 sec out of 6.64 sec
```

✓ Unresolved limits for RR, RV

$$d\sigma^{S} \longrightarrow d\sigma^{RR}$$

$$d\sigma^{T} \longrightarrow d\sigma^{RV}$$

$q\bar{q} \to Z + g_3\ g_4\ g_5$ (g_3 soft & $g_4 \parallel \bar{q}$)