Precision predictions for SUSY and GUT processes at hadron colliders

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Outline

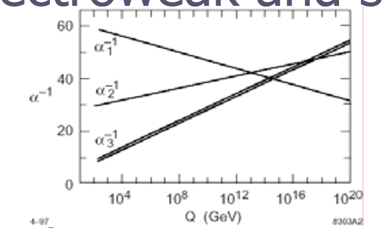
1. Beyond the Standard Model
   - Supersymmetry
   - Grand Unified Theories
2. Beyond fixed-order perturbation theory
   - Drell-Yan like processes at LO and NLO of (SUSY-) QCD
   - Transverse-momentum, threshold, and joint resummation
   - Parton showers in PYTHIA and MC@NLO
3. Numerical results and uncertainties
   - Transverse-momentum and invariant-mass spectra
   - Factorization- and renormalization-scale dependence
   - Parton-density uncertainties and non-perturbative effects
   - Resummation vs. Monte Carlo
   - Experimental issues
4. Summary

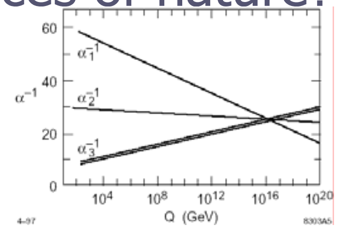
1 - Beyond the Standard Model

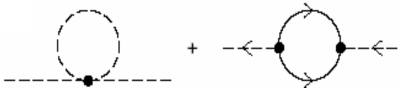
Beyond the Standard Model

- Where is the Higgs boson?
- Can we unify the electroweak and strong forces of nature?
- Is it possible to include gravity?
- Why is $m_h \ll m_{Pl.}$?
- Why is there so little anti-matter? Why is CP-symmetry broken?
- What is dark matter made of?

Supersymmetry

- Where is the Higgs boson?
  - SUSY-relation: $m_h < m_Z$ (at 2-loop level < 135 GeV)
- Can we unify the electroweak and strong forces of nature?
  - Standard Model: 

SUSY: 

- Is it possible to include gravity?
  - Local symmetry → supergravity → graviton, gravitino
- Why is $m_h \ll m_{Pl.}$?
  - No quadratic divergences, cancellation of 

- Why is there so little anti-matter? Why is CP-symmetry broken?
  - SUSY contains numerous complex phases → Phase in CKM matrix
- What is dark matter made of?
  - Stable LSP: Neutralino (mSUGRA, AMSB), gravitino (GMSB)

Minimal Supersymmetric Standard Model

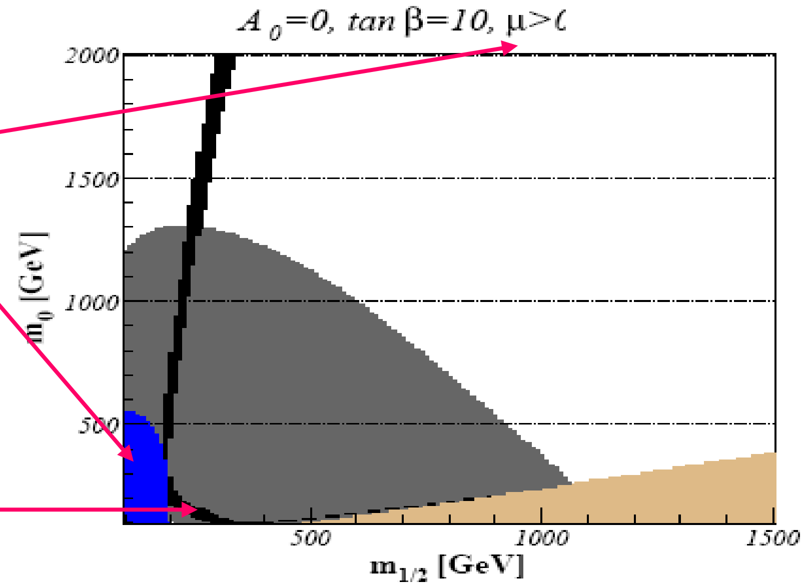
Names	Spin	P_R	Gauge Eigenstates	Mass Eigenstates
Higgs bosons	0	+1	$H_u^0\ H_d^0\ H_u^+\ H_d^-$	$h^0\ H^0\ A^0\ H^\pm$
squarks	0	−1	$\widetilde{u}_L\ \widetilde{u}_R\ \widetilde{d}_L\ \widetilde{d}_R$	(same)
			$\widetilde{s}_L\ \widetilde{s}_R\ \widetilde{c}_L\ \widetilde{c}_R$	(same)
			$\widetilde{t}_L\ \widetilde{t}_R\ \widetilde{b}_L\ \widetilde{b}_R$	$\widetilde{t}_1\ \widetilde{t}_2\ \widetilde{b}_1\ \widetilde{b}_2$
sleptons	0	−1	$\widetilde{e}_L\ \widetilde{e}_R\ \widetilde{\nu}_e$	(same)
			$\widetilde{\mu}_L\ \widetilde{\mu}_R\ \widetilde{\nu}_\mu$	(same)
			$\widetilde{\tau}_L\ \widetilde{\tau}_R\ \widetilde{\nu}_\tau$	$\widetilde{\tau}_1\ \widetilde{\tau}_2\ \widetilde{\nu}_\tau$
neutralinos	1/2	−1	$\widetilde{B}^0\ \widetilde{W}^0\ \widetilde{H}_u^0\ \widetilde{H}_d^0$	$\widetilde{N}_1\ \widetilde{N}_2\ \widetilde{N}_3\ \widetilde{N}_4$
charginos	1/2	−1	$\widetilde{W}^\pm\ \widetilde{H}_u^+\ \widetilde{H}_d^-$	$\widetilde{C}_1^\pm\ \widetilde{C}_2^\pm$
gluino	1/2	−1	$\widetilde{g}$	(same)
goldstino (gravitino)	1/2 (3/2)	−1	$\widetilde{G}$	(same)

SUSY-breaking and its parameter space

- No SUSY-particles observed $\rightarrow$ SUSY must be broken above M_W
  - Soft breaking in "hidden" sector (D- or F-terms)
  - Mediated to "visible" sector by some interaction
  - Avoids reintroduction of quadratic divergences
  - But: Even the MSSM has 105 masses, phases, and mixing angles
- Grand Unified Theories:
  - $M_1(M_{GUT}) = M_2(M_{GUT}) \rightarrow M_1(M_W) = 5/3 \tan^2 \theta_W M_2(M_W)$
- Minimal supergravity (mSUGRA):
  - $\tan \beta$, sign (μ); m_0, $m_{1/2}$, A_0
- Gauge mediation (GMSB):
  - $\tan \beta$, sign (μ); Λ, n_q, n_l, M_{mess}
- Anomaly mediation (AMSB):
  - $\tan \beta$, sign (μ); m_0, m_{aux}

Indirect constraints

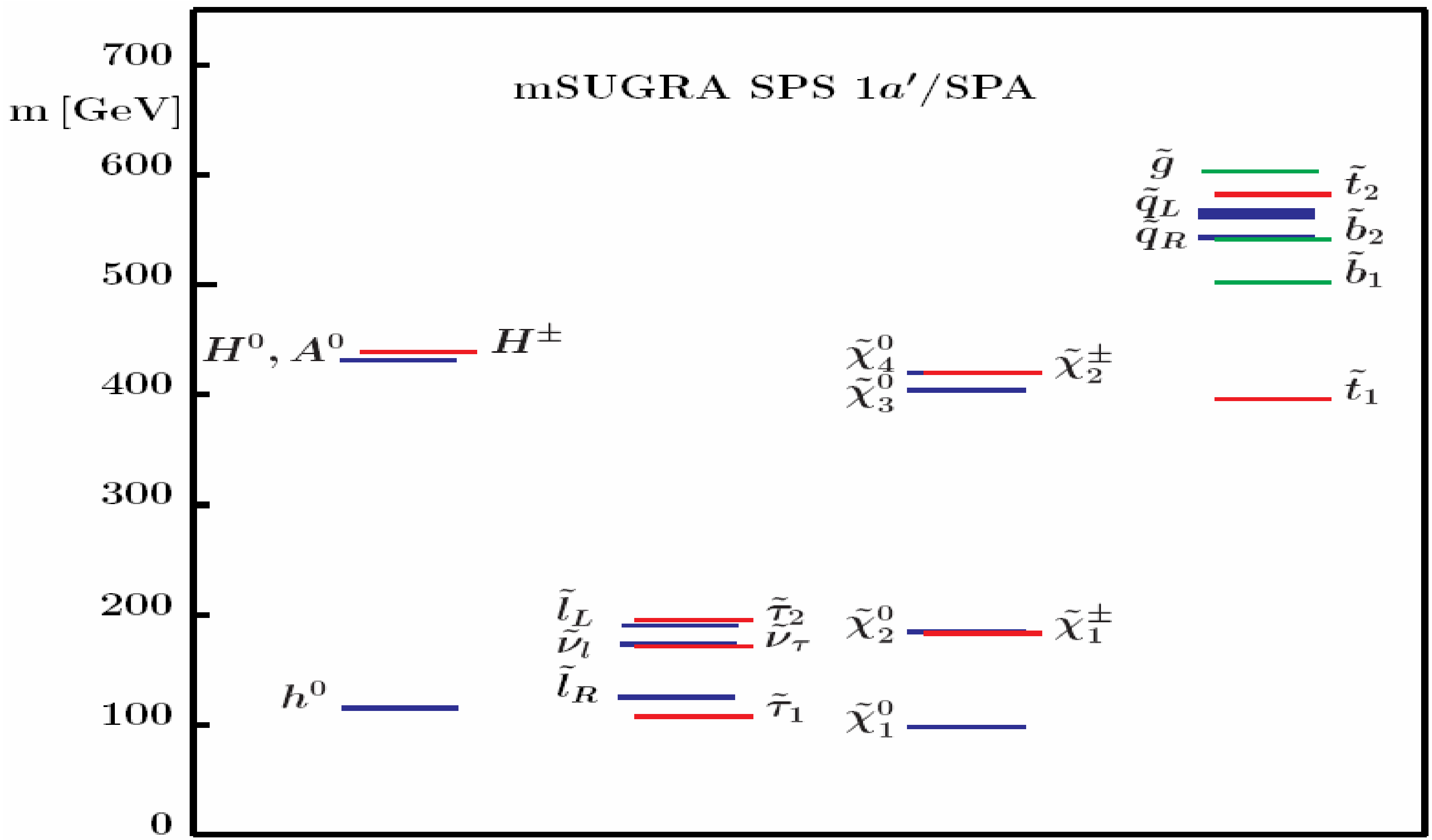
- Low-energy (FeynHiggs):
  - $\Delta a_\mu = (22 \pm 10) \times 10^{-10}$
    $\rightarrow$ wants μ>0
  - $\mathrm{BR}(b \rightarrow s\gamma) = (3.55 \pm 0.26) \times 10^{-4}$
    $\rightarrow$ excludes small m_0, $m_{1/2}$
  - $\Delta\rho = -\alpha T = 0.00102 \pm 0.00086$
    $\rightarrow$ forbids too large m_0, $m_{1/2}$
- Dark matter (DarkSUSY):
  - $0.094 < \Omega_{CDM} h^2 < 0.136$
    $\rightarrow$ three narrow regions
- Neutral kaons:
  - Δm_K, ε, ε'/ε
- B-/D-meson oscillations:
  - Δm_B, Δm_D
- Electric dipole moments ($\rightarrow$ILL):
  - d_n, d_e



	m_0 [GeV]	$m_{1/2}$ [GeV]	A_0 [GeV]	$\tan\beta$	$sign(\mu)$
A	700	200	0	10	1
B	100	400	0	10	1

[Bozzi, Fuks, Herrmann, MK, NPB 787 (2007) 1]

Typical minimal supergravity spectrum



Direct mass limits

particle		Condition	Lower limit (GeV/c^2)	Source
$\widetilde{\chi}_1^{\pm}$	gaugino	$M_{\widetilde{\nu}} > 200$ GeV/c^2	103	LEP 2
		$M_{\widetilde{\nu}} > M_{\widetilde{\chi}^{\pm}}$	85	LEP 2
		any $M_{\widetilde{\nu}}$	45	Z width
	Higgsino	$M_2 < 1$ TeV/c^2	99	LEP 2
	GMSB		150	DØ isolated photons
	RPV	$LL\overline{E}$ worst case	87	LEP 2
		$LQ\overline{D}$ $m_0 > 500$ GeV/c^2	88	LEP 2
$\widetilde{\chi}_1^0$	indirect	any $\tan\beta$, $M_{\widetilde{\nu}} > 500$ GeV/c^2	39	LEP 2
		any $\tan\beta$, any m_0	36	LEP 2
		any $\tan\beta$, any m_0, SUGRA Higgs	59	LEP 2 combined
	GMSB		93	LEP 2 combined
	RPV	$LL\overline{E}$ worst case	23	LEP 2
$\widetilde{e}_R$	$e\widetilde{\chi}_1^0$	$\Delta M > 10$ GeV/c^2	99	LEP 2 combined
$\widetilde{\mu}_R$	$\mu\widetilde{\chi}_1^0$	$\Delta M > 10$ GeV/c^2	95	LEP 2 combined
$\widetilde{\tau}_R$	$\tau\widetilde{\chi}_1^0$	$M_{\widetilde{\chi}_1^0} < 20$ GeV/c^2	80	LEP 2 combined
$\widetilde{\nu}$			43	Z width
$\widetilde{\mu}_R$, $\widetilde{\tau}_R$		stable	86	LEP 2 combined
$\widetilde{t}_1$	$c\widetilde{\chi}_1^0$	any $\theta_{\rm mix}$, $\Delta M > 10$ GeV/c^2	95	LEP 2 combined
		any $\theta_{\rm mix}$, $M_{\widetilde{\chi}_1^0} \sim \frac{1}{2} M_{\widetilde{t}}$	115	CDF
		any $\theta_{\rm mix}$ and any ΔM	59	ALEPH
	$b\ell\widetilde{\nu}$	any $\theta_{\rm mix}$, $\Delta M > 7$ GeV/c^2	96	LEP 2 combined
$\widetilde{g}$	any $M_{\widetilde{q}}$		195	CDF jets+$\not{E}_T$
$\widetilde{q}$	$M_{\widetilde{q}} = M_{\widetilde{g}}$		300	CDF jets+$\not{E}_T$

Grand Unified Theories (1)

- Green, Schwarz (1984):
  - 10-dim. string theories with $E_8 \times E_8$ or SO(32) symmetry anomaly-free
  - Only E_8 has chiral fermions (as does the Standard Model)
  - Compactification leads to E_6
- Symmetry breaking:
  - $E_6 \to SO(10) \times U(1)_\Psi \to SU(5) \times U(1)_X \times U(1)_\Psi$
  - Mixing: $Z'(\theta) = Z_\psi \cos\theta + Z_\chi \sin\theta$ and $Z''(\theta) = Z_\psi \sin\theta - Z_\chi \cos\theta$.
  - Choose $\theta = 90° \to Z' = Z_X$. Z_Ψ naturally acquires mass at higher scale
  - $SU(10) \to SU(5)$ at same scale as $SU(5) \to SU(3)_C \times SU(2)_L \times U(1)_Y$
  - Gauge couplings: $g_\chi = \sqrt{\frac{5}{3}} g' = \sqrt{\frac{5}{3}} g \tan\theta_W = \sqrt{\frac{5}{3}} \frac{e}{\sqrt{1-\sin^2\theta_W}}$
  - Photon protected by $U(1)_{em.}$, but Z' can mix with Z: $\tan^2\phi = \frac{m_Z^2 - m_1^2}{m_2^2 - m_Z^2}$
  - Large ratio $v_{10}/v_{SM} \to m_1 \simeq m_Z \ll m_2 \simeq m_{Z'}$
  - DELPHI: $\phi < 1.7$ mrad for $m_{Z'} = 440$ GeV

Grand Unified Theories (2)

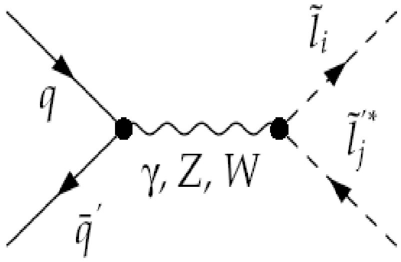
- Many more possibilities:
  - If $R_{GUT} > R_{SM} \rightarrow$ additional U(1)'s
- Models that survive precision electroweak data:
  - B – L
  - SO(10)
  - E_6 (extended Higgs sector + additional charged fermions)
- Parameterization (anomaly cancellation + gauge invariance):
  - M. Carena, A. Daleo, B. Dobrescu, T. Tait, PRD 70 (2004) 093009
- Search for Z'-bosons at hadron colliders:
  - Tevatron (run II): $M_{Z'} > 600 \ldots 900$ GeV (CDF coll.)
  - LHC (100 fb^{-1}): $M_{Z'} < 2.0 \ldots 5.0$ TeV (ATLAS coll.)
  - LHC (100 fb^{-1}): $M_{Z'} < 3.4 \ldots 4.3$ TeV (CMS coll.)
- Z_X couplings: $\mathcal{L} = \frac{g}{4\cos\theta_W}\bar{f}\gamma^\mu(v_f - a_f\gamma_5)fZ'_\mu$,

v_d	a_d	v_u	a_u	v_l	a_l	v_ν	a_ν
$2\sqrt{6}s_W/3$	$-\sqrt{6}s_W/3$	0	$\sqrt{6}s_W/3$	$-2\sqrt{6}s_W/3$	$-\sqrt{6}s_W/3$	$-\sqrt{6}s_W/2$	$-\sqrt{6}s_W/2$

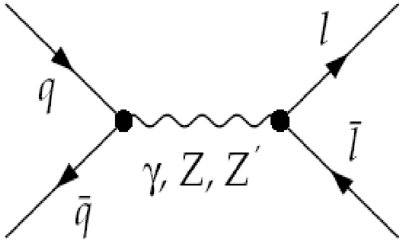
2 - Beyond fixed-order perturbation theory

Drell-Yan like processes at LO

- Slepton-pair production:
  - Sleptons are often light
    $\rightarrow$ Decay into SM lepton and $\not{E}_T$
  - Large background from WW/$t\bar{t}$ production
    $\rightarrow$ Importance of accurate theoretical predictions



- Z′-boson production:
  - Z′ $\rightarrow$ $l\bar{l}$ easily identifiable
    $\rightarrow$ Additional invariant-mass peak
  - Irreducible background from SM Drell-Yan
    $\rightarrow$ Importance of accurate theoretical predictions



Beyond fixed-order perturbation theory (1)

- Hadronic cross section:

$$\sigma_{gg} = \int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2 \; g(x_1, \mu_F^2) g(x_2, \mu_F^2) \hat{\sigma}_{gg}(\hat{s} = x_1 x_2 s)$$

- Large scale / scheme uncertainties at LO:
  - Factorization scale / scheme in parton densities $g(x,\mu_F^2)$
  - Renormalization scale / scheme in $\alpha_s(\mu_R^2)$
- Reduction of the dependence at NLO:
  - Virtual loop contributions
  - Real emission contributions
- Calculation at NNLO, ... [Hamberg, v. Neerven, Matsuura, 1991]
- Resummation to all orders

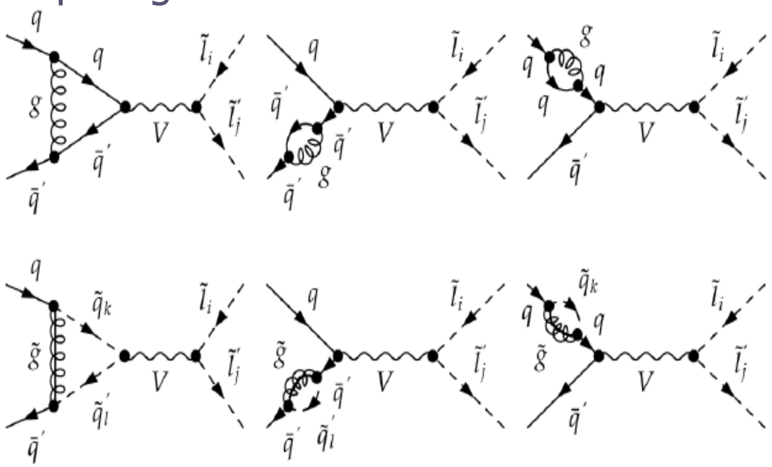
Beyond fixed-order perturbation theory (2)

- Main SUSY signal is $\not{E}_T$ (cf. W-bosons)
  - $\rightarrow$ Resummation of $\frac{\alpha_s^n}{q_T^2} \ln^m \frac{M^2}{q_T^2}$ terms when $q_T \rightarrow 0$
    [Catani, de Florian, Grazzini, 2001]
- SUSY- and GUT-particles are heavy (cf. top-quarks):
  - $\rightarrow$ Resummation of $\alpha_s^n \left(\frac{\ln^m(1-z)}{1-z}\right)_+$ terms when $z = M^2/s \rightarrow 1$
    [Krämer, Laenen, Spira, 1998]
- Both situations at the same time:
  - $\rightarrow$ Joined resummation ($m \leq 2n-1$)
    [Kulesza, Sterman, Vogelsang, 2002]

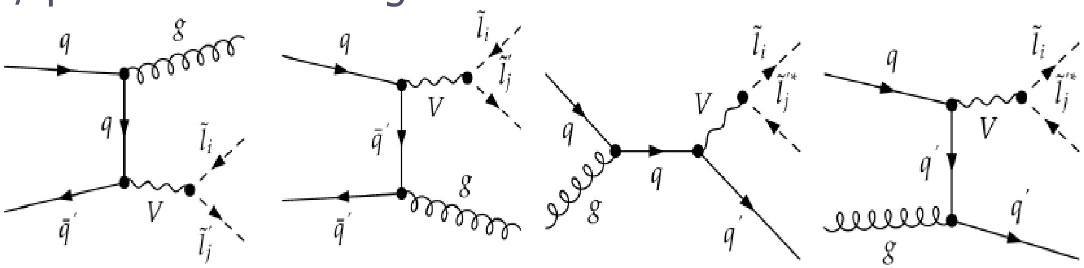
- R-parity conservation:
  - Extra SUSY-particles must be created in pairs
  - Only Standard Model particle radiation at $O(\alpha_s)$. Beyond:
  - SUSY-/GUT-particles are massive $\rightarrow$ no soft/collinear region

Drell-Yan like processes at NLO (1)

- (SUSY-) QCD one-loop diagrams:



- Real gluon/quark emission diagrams:



- Z′ production:
  - Same diagrams except for the SUSY loops
  - Sleptons are replaced by leptons

Drell-Yan like processes at NLO (2)

- Total cross section:

$$\sigma = \int_{(m_1+m_2)^2}^{S} dQ^2 \int_{\frac{Q^2}{S}}^{1} dx_A \int_{\frac{Q^2}{Sx_A}}^{1} dx_B \Big\{ \sum_{ij=q,\bar{q}} f_{i/A}(x_A,\mu_f) f_{j/B}(x_B,\mu_f) \frac{d\hat{\sigma}_{q\bar{q}}}{dQ^2}$$

$$+ \sum_{i=q,\bar{q}} \left[f_{i/A}(x_A,\mu_f) f_{g/B}(x_B,\mu_f) + f_{g/A}(x_A,\mu_f) f_{i/B}(x_B,\mu_f) \right] \frac{d\hat{\sigma}_{qg}}{dQ^2} \Big\},$$

- Quark-antiquark contribution (from Drell-Yan):

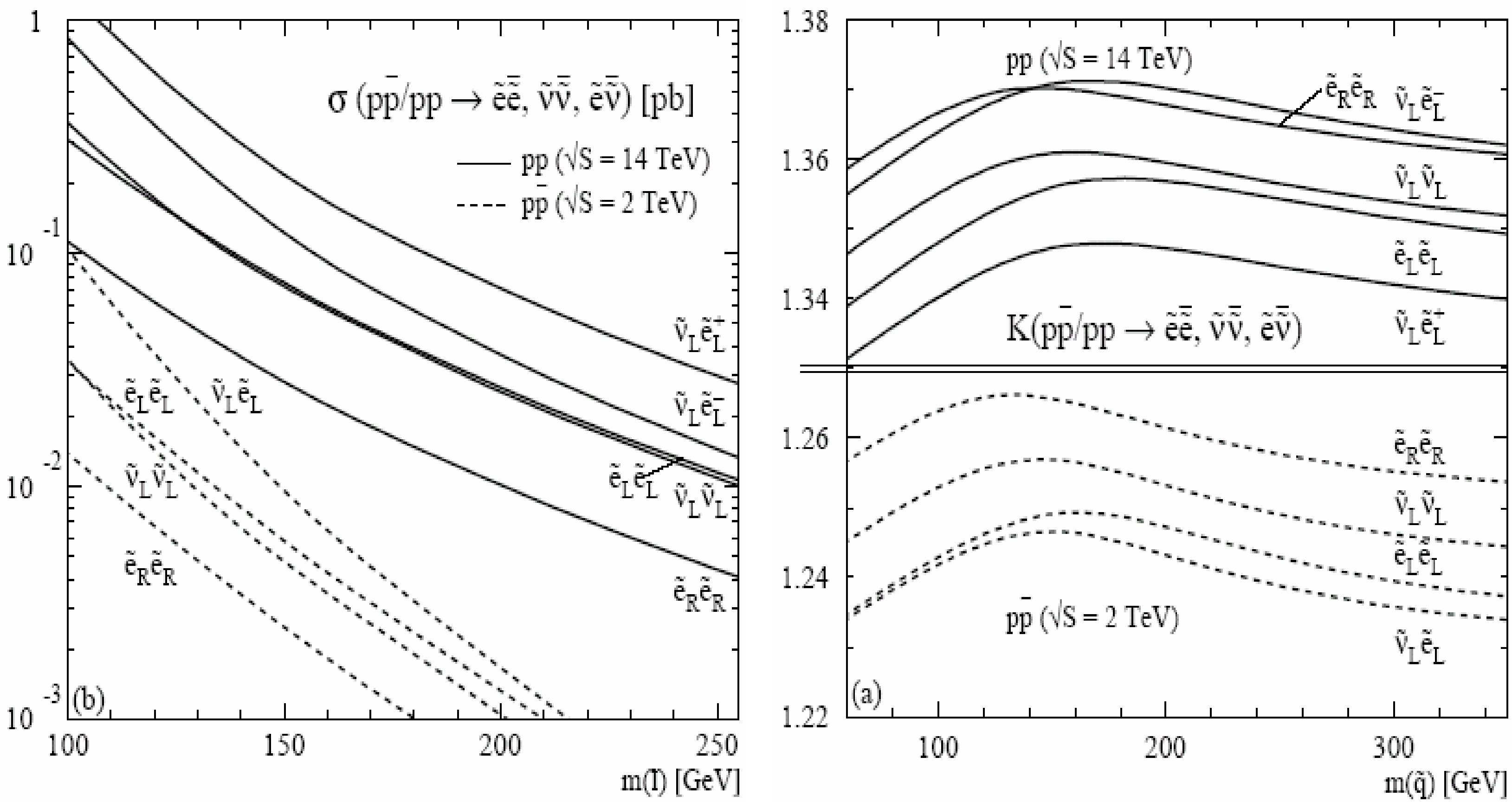
$$\frac{d\hat{\sigma}_{q\bar{q}}}{dQ^2} = \sigma_0 \Big\{ \delta(1-z) + \frac{\alpha_s(\mu_r)}{2\pi} \underbrace{\frac{4}{3}}_{C_F} \Big[4(1+z^2) \left(\frac{\ln(1-z)}{1-z} \right)_+ - 2\frac{(1+z^2)}{1-z} \ln z + \underbrace{\left(\frac{2\pi^2}{3} - 8 \right)}_{\mathcal{A}_q^{DY}} \delta(1-z) + \frac{3}{2} P_{qq}(z) \ln \frac{Q^2}{\mu_f^2} \Big] \Big\}$$

- Quark-gluon contribution (from Drell-Yan):

$$\frac{d\hat{\sigma}_{qg}}{dQ^2} = \sigma_0 \frac{\alpha_s(\mu_r)}{2\pi} \frac{1}{2} \left[\frac{3}{2} + z - \frac{3}{2} z^2 + 2 P_{qg}(z) \left(\ln \frac{(1-z)^2}{z} - 1 + \ln \frac{Q^2}{\mu_f^2} \right) \right]$$

[Baer, Harris, Reno, Phys. Rev. D 57 (1998) 5871]

Drell-Yan like processes at NLO (3)



[Beenakker, MK et al., Phys. Rev. Lett. 83 (1999) 3780]

Resummation formalism (1)

- Mellin transform:
  - Definition: $F(N) = \int_0^1 \mathrm{d}y\, y^{N-1} F(y)$
  - Logarithm: $\left(\frac{\ln(1-z)}{1-z}\right)_+ \quad \to \quad \ln^2 \overline{N} \quad \text{with} \quad \overline{N} = N \exp[\gamma_E]$
  - Invariant-mass spectrum:

$$\frac{\mathrm{d}\sigma^{(\mathrm{res})}}{\mathrm{d}M^2}(N) = \sum_{a,b} f_{a/h_1}(N+1)\, f_{b/h_2}(N+1)\, \hat{\sigma}_{ab}^{(\mathrm{res})}(N)$$

- Fourier transform:
  - Definition: $F(b) = \int_0^\infty \mathrm{d}x\, e^{-ibx}\, F(x)$
  - Logarithm: $\frac{1}{q_T^2} \ln \frac{M^2}{q_T^2} \quad \to \quad \ln \overline{b}^2 \quad \text{with} \quad \overline{b} = \frac{b\,M}{2} \exp[\gamma_E]$
  - Transverse-momentum spectrum:

$$\frac{\mathrm{d}^2\sigma^{(\mathrm{res})}}{\mathrm{d}M^2 \mathrm{d}q_T^2}(N, q_T) = \sum_{a,b} f_{a/h_1}(N+1)\, f_{b/h_2}(N+1) \int \frac{b}{2}\, \mathrm{d}b\, J_0(b\, q_T)\, \mathcal{W}_{ab}^F(N, b)$$

Resummation formalism (2)

- Resummed partonic cross sections:
  - $\hat{\sigma}^{(\mathrm{res})}_{ab}(N) = \sigma^{(LO)}\, \tilde{C}_{ab}(N) \exp\left\{\mathcal{G}(N)\right\}$
  - $\mathcal{W}^F_{ab}(N,b) = \mathcal{H}^F_{ab}(N) \exp\left\{\mathcal{G}(N,b)\right\}$
- Sudakov form factor: $\mathcal{G}(N,L) = L\, g^{(1)}(\alpha_s L) + \sum_{n=2}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^{n-2} g_N^{(n)}(\alpha_s L)$
  - Can be computed perturbatively as series in α_s L [L=ln(…)]
  - Is process-independent
  - Contains the singular soft-collinear radiation up to NLL order
- Coefficient functions $\tilde{C}$ and $\mathcal{H}^F$:
  - Can be computed perturbatively as series in α_s
  - Are process-dependent
  - Contain the regular terms in the limits N $\to \infty$ and b $\to \infty$

Resummation formalism (3)

- Logarithms:

	q_T	Joint	Threshold
$L = \ln(...)$	$\bar{b}^2 + 1$	$\bar{b} + \frac{\bar{N}}{1+\frac{\bar{b}}{4\bar{N}}}$	$\bar{N}$

- Transverse-momentum resummation:
  - Process-independent Sudakov form factor
  - Resummation only at large b, no artifacts at small b ("+1")
- Threshold resummation:
  - Consistent inclusion of collinear radiation in the C-function
- Joint resummation:
  - q_T and threshold resummation limits reproduced for large b and N
  - Process-independent Sudakov form factor
  - No subleading terms in perturbative expansions of $\sigma^{(res)}$ ($\eta=1/4$)

Resummation formalism (4)

- Three different kinematical regions:
  - Regular regions ($z \ll 1$, $q_T \gg 0$): Fixed-order (F.O.) reliable
  - Singular regions ($z \to 1$, $q_T \to 0$): Resummation (res) reliable
  - Intermediate regions: Both contributions needed
- Reorganization of the cross section:
  - Separate regular/singular terms, re-expand (exp) res. part
  - Matching procedure avoids double-counting:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}M^2}(\tau) = \frac{\mathrm{d}\sigma^{(\mathrm{F.O.})}}{\mathrm{d}M^2}(\tau) + \oint_{C_N} \frac{\mathrm{d}N}{2\pi i}\,\tau^{-N}\left[\frac{\mathrm{d}\sigma^{(\mathrm{res})}}{\mathrm{d}M^2}(N) - \frac{\mathrm{d}\sigma^{(\mathrm{exp})}}{\mathrm{d}M^2}(N)\right]$$

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}M^2\,\mathrm{d}q_T^2}(\tau, q_T) = \frac{\mathrm{d}^2\sigma^{(\mathrm{F.O.})}}{\mathrm{d}M^2\,\mathrm{d}q_T^2}(\tau, q_T) + \oint_{C_N} \frac{\mathrm{d}N}{2\pi i}\,\tau^{-N} \int \frac{b\mathrm{d}b}{2} J_0(q_T\, b)$$

$$\times \left[\frac{\mathrm{d}^2\sigma^{(\mathrm{res})}}{\mathrm{d}M^2\,\mathrm{d}q_T^2}(N, b) - \frac{\mathrm{d}^2\sigma^{(\mathrm{exp})}}{\mathrm{d}M^2\,\mathrm{d}q_T^2}(N, b)\right].$$

Parton showers in PYTHIA

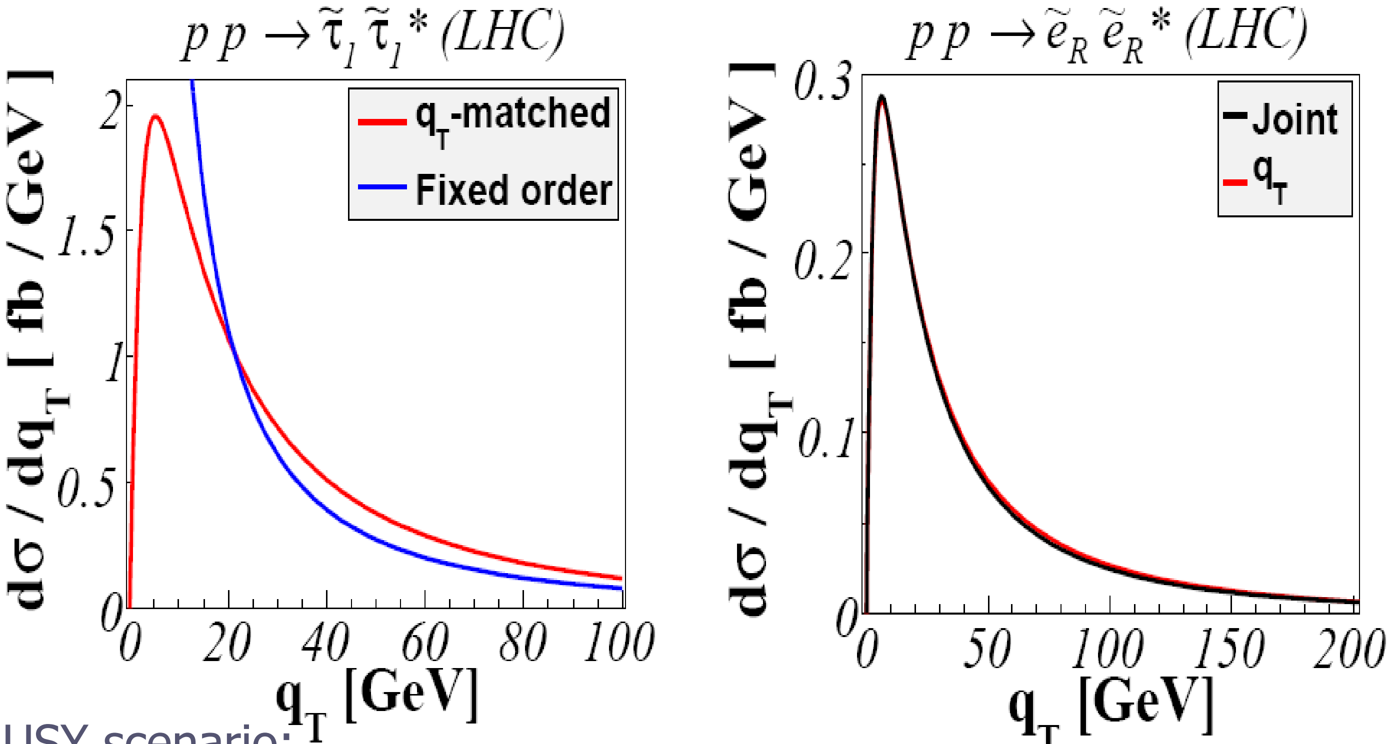
- LO matrix elements:
  - Parton branching: Sudakov form factor up to LL order
- Initial-state parton shower:
  - Hard scattering ($Q \approx M_{Z'}$) $\rightarrow$ preceding branchings ($Q_0 = 1$ GeV)
  - Power shower ($Q = \sqrt{S}$) $\rightarrow$ populate full phase space
- n+1 parton matrix elements [Miu, Sjöstrand (1999)]:
  - $R_{q\bar{q}\to(\gamma,Z,Z')g}(s,t) = \frac{(\mathrm{d}\sigma/\mathrm{d}t)_{\mathrm{ME}}}{(\mathrm{d}\sigma/\mathrm{d}t)_{\mathrm{PS}}} = \frac{\sum_{i=\gamma,Z,Z'}[t^2+u^2+2m_i^2 s]+\text{interference terms}}{\sum_{i=\gamma,Z,Z'}[s^2+m_i^4]+\text{interference terms}}$ $\in [½;1] \rightarrow$ reweight PS
  - $R_{qg\to Vq}(s,t) = \frac{(\mathrm{d}\sigma/\mathrm{d}t)_{\mathrm{ME}}}{(\mathrm{d}\sigma/\mathrm{d}t)_{\mathrm{PS}}} = \frac{\sum_{i=\gamma,Z,Z'}[s^2+u^2+2m_i^2 t]+\text{interference terms}}{\sum_{i=\gamma,Z,Z'}[(s-m_i^2)^2+m_i^4]+\text{interference terms}}$ $\in [1 ;3] \rightarrow$ preweight PS
- QED parton shower:
  - No significant difference observed
- Intrinsic $\langle k_T \rangle$ (below $Q_0 = 1$ GeV):
  - Not uniquely defined, non-perturbative regime $\rightarrow$ set $\langle k_T \rangle = 0$

Parton showers in MC@NLO

- Initial-state parton shower (HERWIG):
  - Coherent branchings $i \to jk$ with $z_j = E_j/E_i$ DGLAP-distributed
  - Emission angles $\theta_{jk}^2/2 \approx (p_j.p_k)/(E_jE_k)$ Sudakov-distributed up to LL
  - Backward evolution (Q_0= 2.5 GeV) $\to$ non-perturbative stage
  - Forced splitting of non-valence partons
- NLO matrix elements [Frixione, Webber (2002)]:
  - Born-like or standard events (S)
  - Hard emission events (H)
  - Add/subtract NLO part of Sudakov $\to$ $J_{(S,H)}$ finite and positive
  - Assign weight $w_i^{(S,H)} = \pm 1$
  - Total cross section: $\sigma_{\rm tot} = \sum_{i=1}^{N_{\rm tot}} w_i^{(S,H)} (J_H + J_S)/N_{\rm tot}$
- Implementation of Z' [Fuks, Klasen, Ledroit, Li, Morel (2007)]:
  - http://lpsc.in2p3.fr/klasen/software/

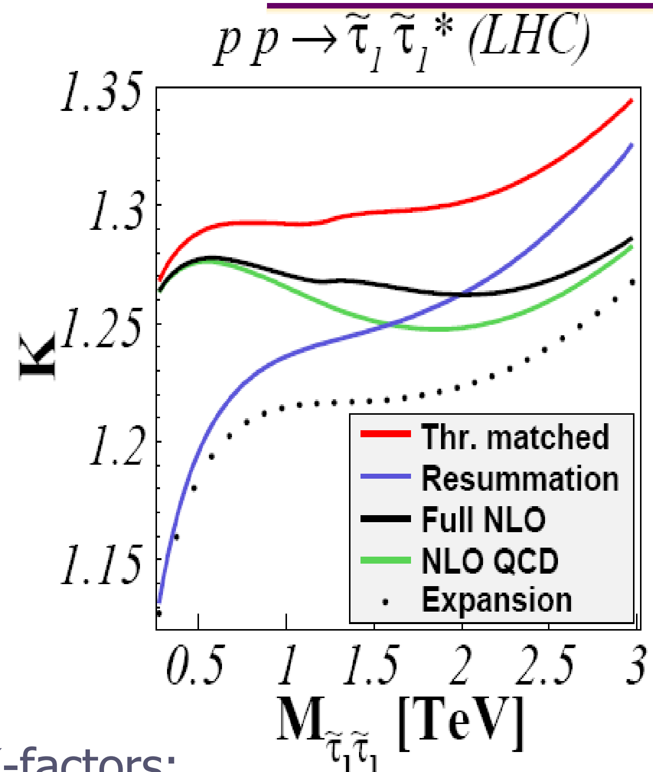
3 - Numerical results and uncertainties

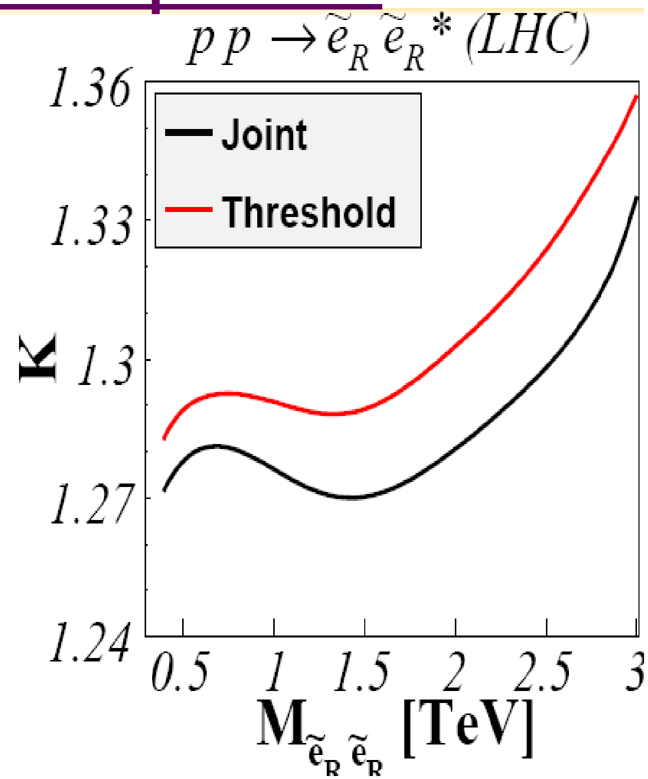
Transverse-momentum spectra



- SUSY scenario:
  - mSUGRA (SPS1a/BFHK-B, $m_{\tilde{l}} \approx$ 100-200 GeV)
- Resummation effects:
  - Finite cross section at small q_T, enhanced cross section at intermediate q_T
  - Very small difference in joint resummation from ln(N) at intermediate q_T

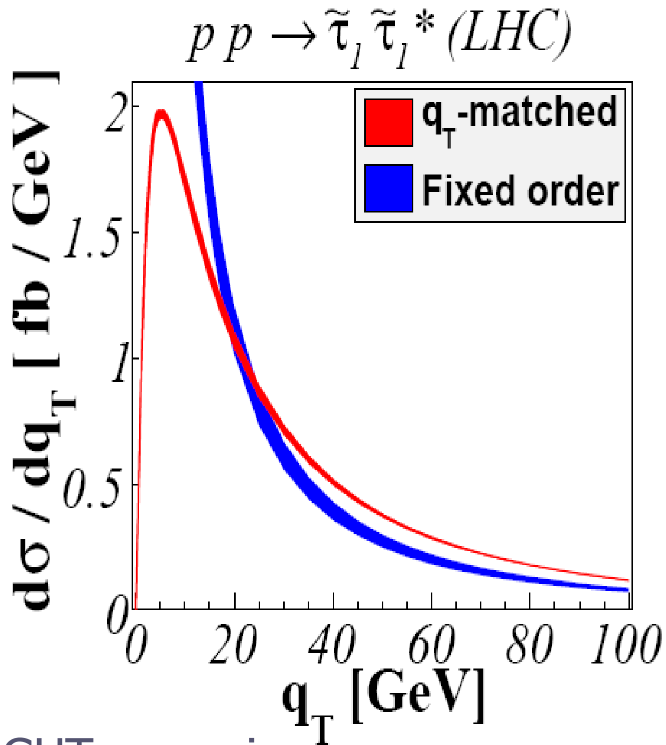
Invariant-mass spectra

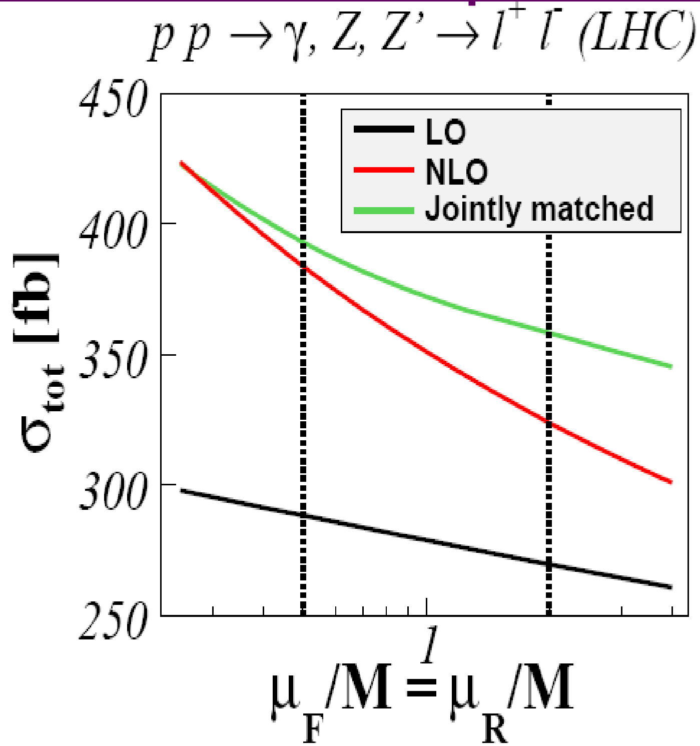




- K-factors:
  - $d\sigma^{(NLO/NLO+NLL)}/d\sigma^{(LO)}$, small effect from squark pair-production threshold
- Resummation effects:
  - At small (large) M, $d\sigma^{(res)} \approx d\sigma^{(exp)}$ ($d\sigma^{(F.O.)} \approx d\sigma^{(exp)}$) $\rightarrow$ F.O. (res) dominates
  - Larger difference in joint resummation from ln(b) at non-asymptotic N

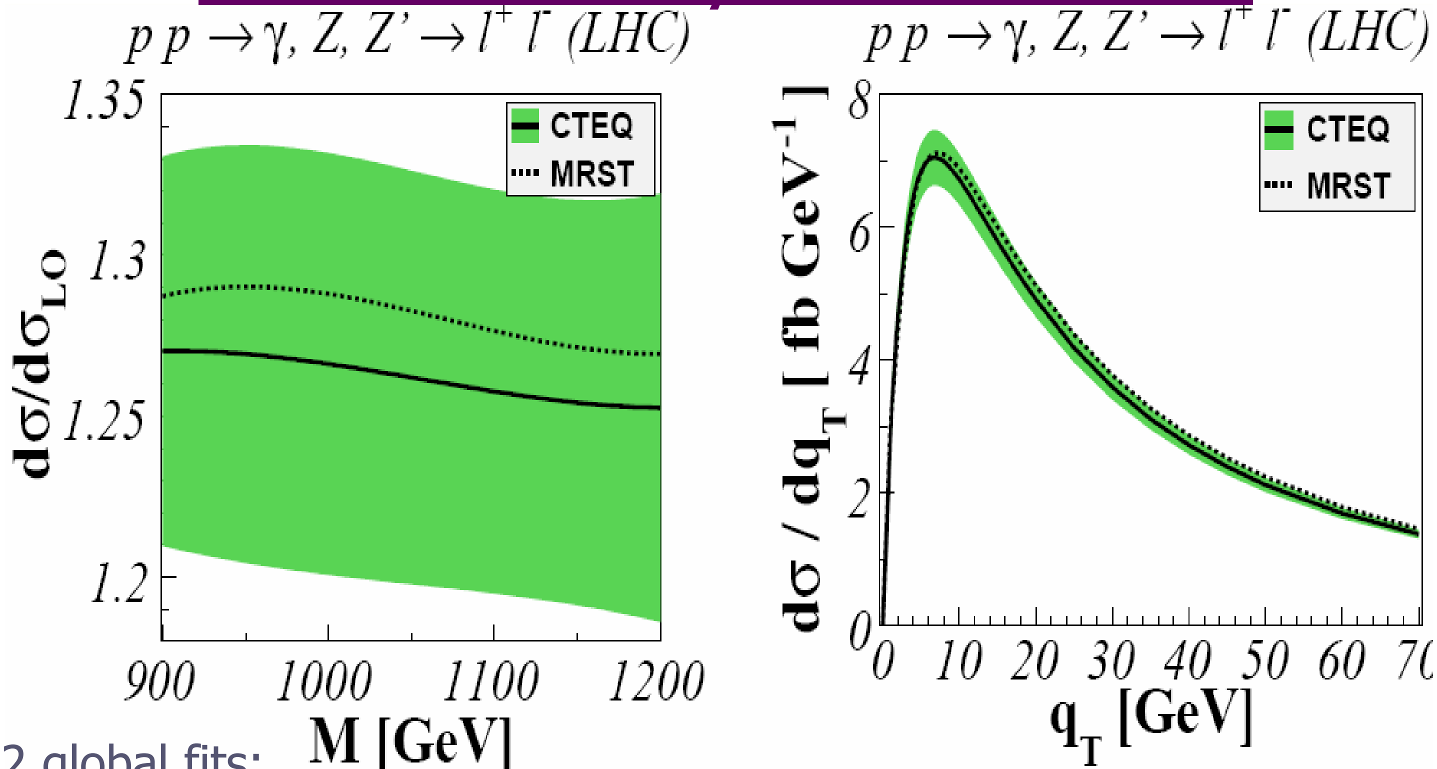
Factorization-/renormalization scale dependence





- GUT scenario:
  - $M_{Z'}$ = 1 TeV with $U(1)_X$ couplings
- Scale dependence:
  - q_T-spectrum (integrated over M): NLO (10%), NLO+NLL (5%)
  - σ_{tot} (900 GeV < M < 1200 GeV): LO (7%), NLO (17%), NLO+NLL (9%)

Parton-density uncertainties



- 2 global fits:
  - CTEQ6 vs. MRST: Different $d\sigma/dM$ ($\rightarrow$ x), similar $d\sigma/dq_T$ (MRST bit harder)
- Uncertainty band (CTEQ6):
  - Variations along 20 independent directions spanning 90% C.L. of fitted data
  - Uncertainty remains modest ($\approx$ 10%), similar to scale dependence

Non-perturbative effects (1)

- Fourier-transform:

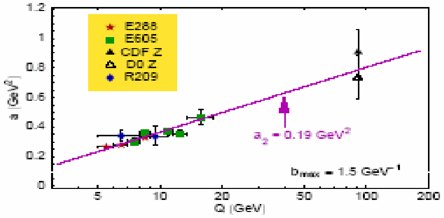
$$\Sigma_{ij}(N) = \sum_{a,b} \int_0^\infty b\,db \frac{q_T^2}{2} J_0(bq_T)\, C^F_{ca}\left(N, \alpha_S(b_0^2/b^2)\right) C^F_{\bar{c}b}\left(N, \alpha_S(b_0^2/b^2)\right)$$

$$\exp\left\{-\int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2}\left[A_c(\alpha_S(q^2)) \log\frac{Q^2}{q^2} + B^F_c(\alpha_S(q^2))\right] - \int_{b_0^2/b^2}^{\mu_F^2} \frac{dq^2}{q^2} (\gamma_{ai} + \gamma_{bj})(N, \alpha_S(q^2))\right\}$$

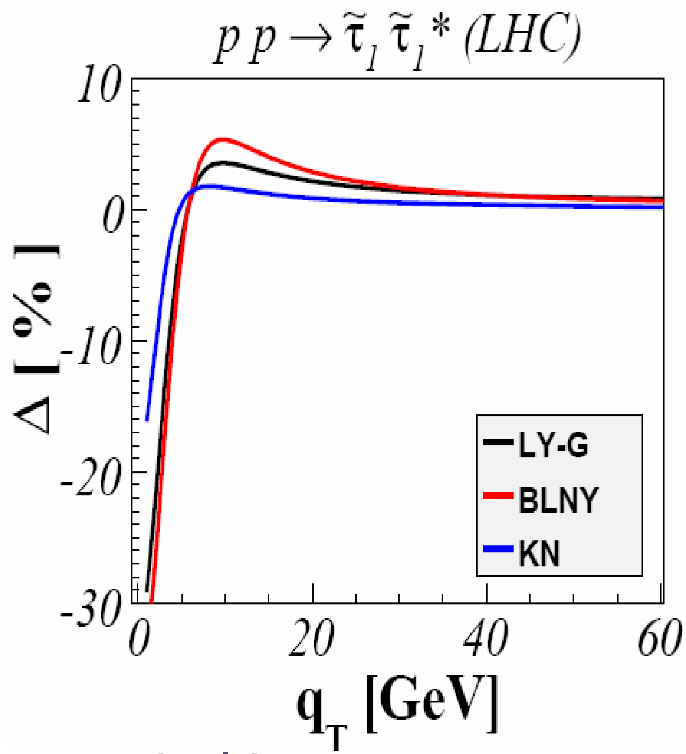
  - $b_{SP} \approx (\Lambda_{QCD}/Q)^p/\Lambda_{QCD}$, $p \approx 0.41$
  - $b \geq 1$ GeV^{-1} non-perturbative
  - CSS freeze b at $b_{max} \approx 0.5$ GeV^{-1}: $\widetilde{W}_{j\bar{k}}(b) = \widetilde{W}^{pert}_{j\bar{k}}(b_*)\widetilde{W}^{NP}_{j\bar{k}}(b)$, $b_* = \frac{b}{\sqrt{1+(b/b_{max})^2}}$
- Davies-Webber-Stirling [NPB 256 (1985) 413]:
  - $W^{NP}(b) = \exp[-b^2(g_1+g_2\log(Qb_{max}/2))]$ (Gauss.)

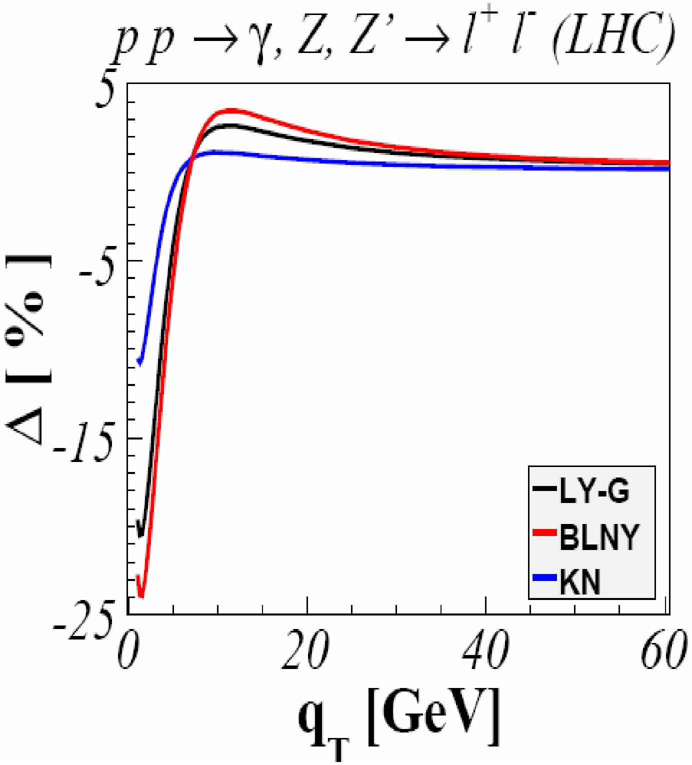
Parameter	DWS-G fit	LY-G fit	BLNY fit
g_1	0.016	0.02	0.21
g_2	0.54	0.55	0.68
g_3	0.00	-1.50	-0.60

- Ladinsky-Yuan [PRD 50 (1994) 4239]:
  - $W^{NP}(b) = \exp[-b^2(g_1+g_2\log(Qb_{max}/2))-b\, g_1g_3\log(100x_1x_2)]$
- Brock-Landry-Nadolsky-Yuan [PRD 67 (2003) 073016]:
  - $W^{NP}(b) = \exp[-b^2(g_1+g_2\log(Qb_{max}/2))-b^2g_1g_3\log(100x_1x_2)]$
- Konychev-Nadolsky [PLB 633 (2006) 710]:
  - Same as BLNY, but $b_{max} = 1.5$ GeV^{-1} (ext. pert.)
  - $g_1 = 0.201$, $g_2 = 0.184$, $g_3 = -0.129$



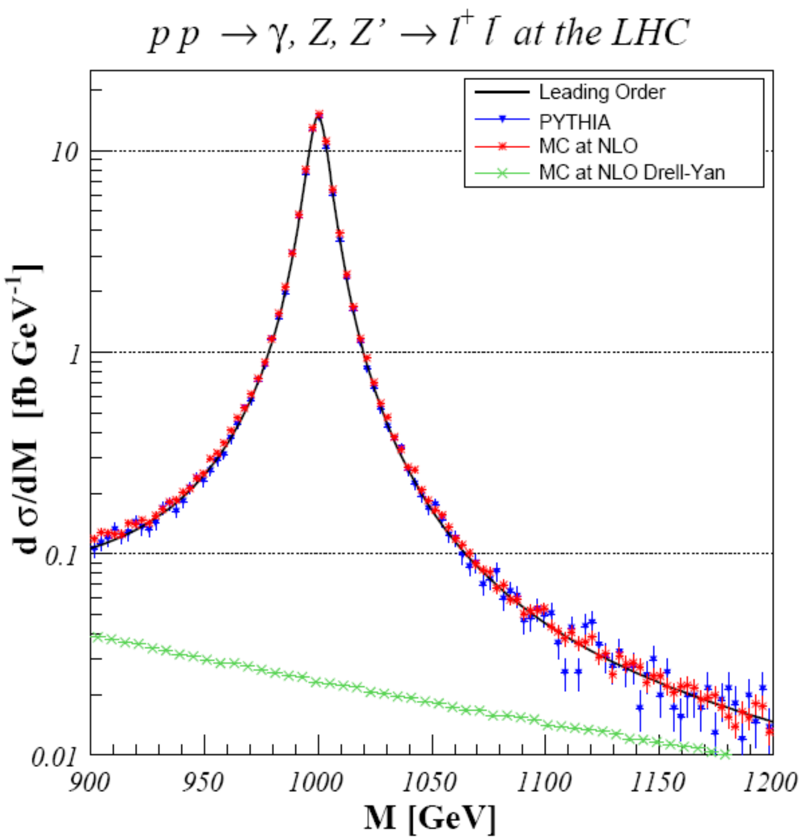
Non-perturbative effects (2)

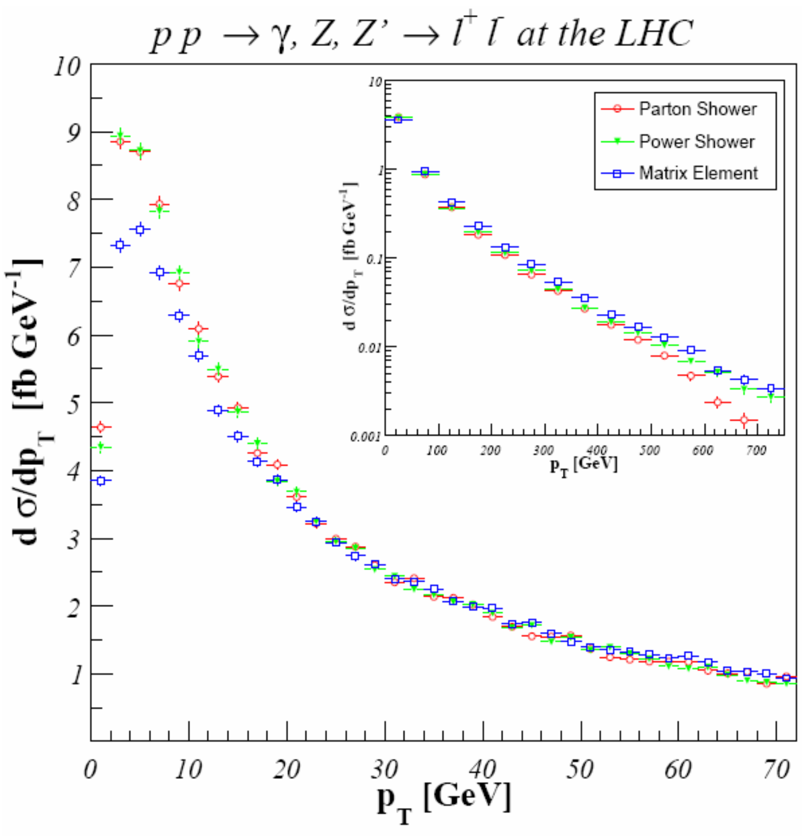




- Numerical importance:
  - Estimated using $\Delta = \dfrac{d\sigma^{(\text{res.+NP})} - d\sigma^{(\text{res.})}}{d\sigma^{(\text{res.})}}$
  - Under good control (< 5%) for sufficiently large q_T (> 5 GeV)

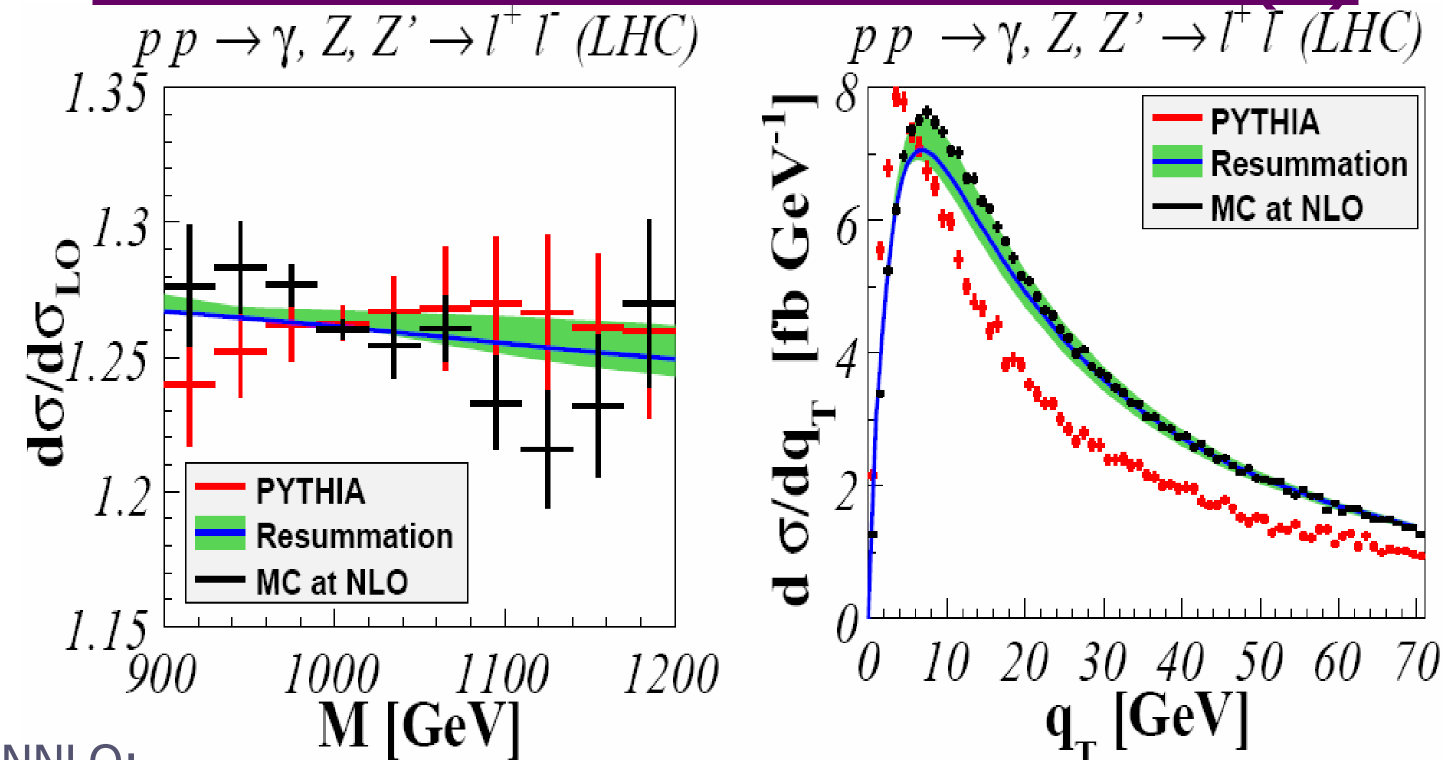
Resummation vs. Monte Carlo (1)





[Fuks, Ledroit, Li, Morel, Klasen, arXiv:0711.0749, NPB (in press)]

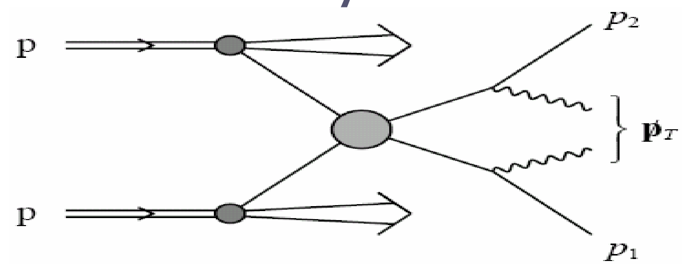
Resummation vs. Monte Carlo (2)

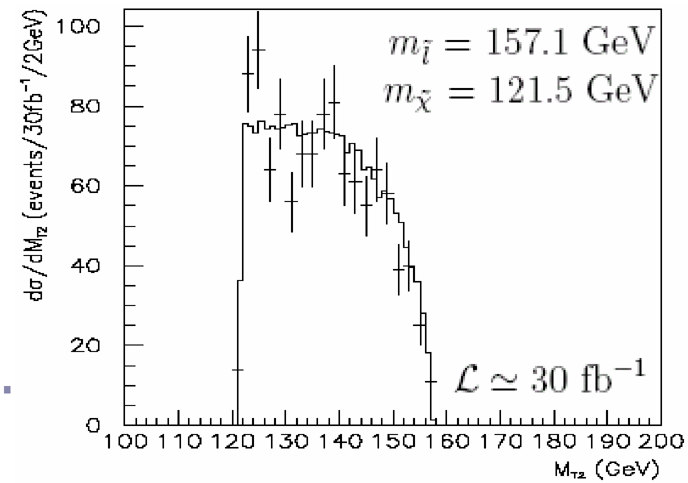


- NNLO:
  - Total cross section only, agrees well with NLO+NLL (not shown)
- Performance of Monte Carlo programs:
  - PYTHIA mass-spectrum renormalized by 1.26, q_T-spectrum too soft
  - MC@NLO (NLO+LL) agrees well with resummation (NLO+NNLO)

Experimental issues (1)

- Massive SUSY particle with dominant decay mode:
  - $pp \to X + \tilde{l}_R^+ \tilde{l}_R^- \to X + l^+ l^- \tilde{\chi}_1^0 \tilde{\chi}_1^0$
- Principal SUSY signal:
  - 2 unobserved massive LSPs $\to \not{p}_T$
- Transverse mass ($W^\pm$, UA1):
  - $m_T^2 = 2(E_T^e \not{E}_T - \mathbf{p}_T^e \cdot \not{\mathbf{p}}_T)$ with $m_T^2 \le m_W^2$
- "Stransverse mass":
  - $m_{\tilde{l}}^2 \ge m_T^2(\mathbf{p}_{Tl}, \mathbf{p}_{T\tilde{\chi}}) \equiv m_l^2 + m_{\tilde{\chi}}^2 + 2(E_{Tl}E_{T\tilde{\chi}} - \mathbf{p}_{Tl} \cdot \mathbf{p}_{T\tilde{\chi}})$
  - $m_{\tilde{l}}^2 \ge M_{T2}^2 \equiv \min_{\not{\mathbf{p}}_1 + \not{\mathbf{p}}_2 = \not{\mathbf{p}}_T} \left[\max\{m_T^2(\mathbf{p}_{Tl^-}, \not{\mathbf{p}}_1), m_T^2(\mathbf{p}_{Tl^+}, \not{\mathbf{p}}_2)\}\right]$
  - $\not{E}$ unknown $\to \not{p}_T$ not Lorentz-inv.
  - Minimize $m^2 = \not{p}_T'^2$ with $\not{E}_T = \sqrt{m^2 + \not{p}_T'^2}$

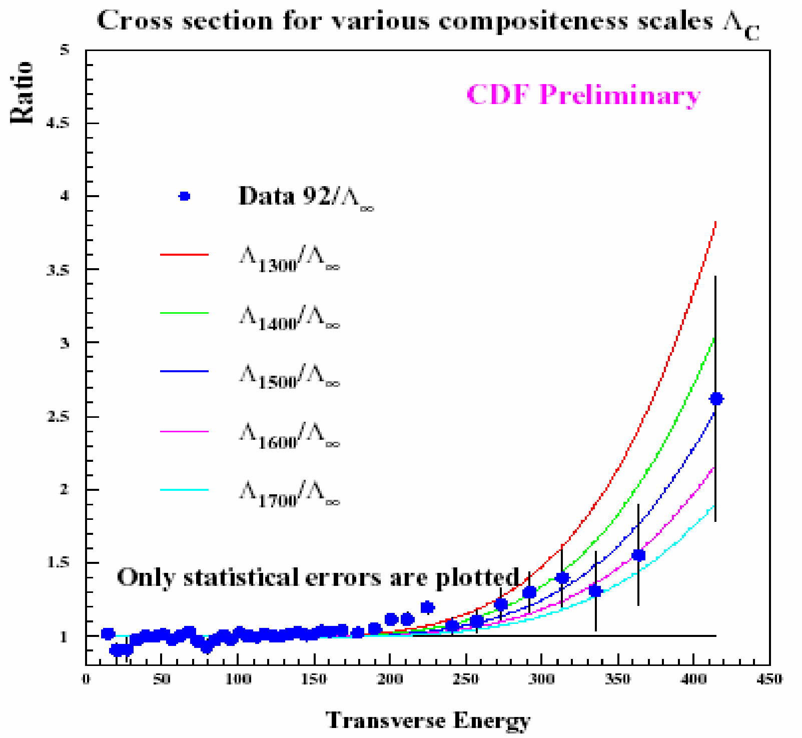




[Lester, Summers, Phys. Lett. B 463 (1999) 99]

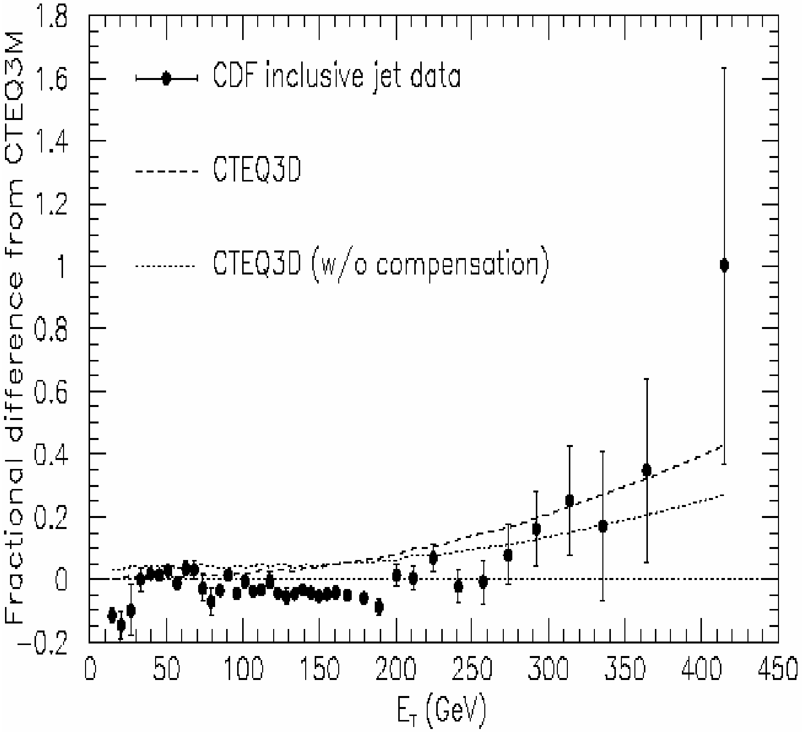
Experimental issues (2)

- Subquarks at the Tevatron?



[CDF Coll., PRL 77 (1996) 438]

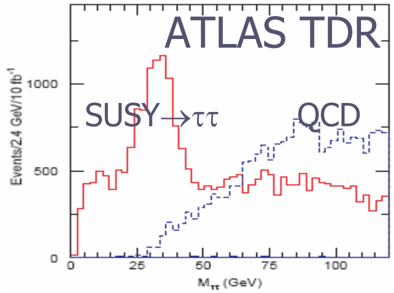
- Higher order effects?



[Klasen,Kramer, PLB386(1996)384]

Experimental issues (3)

- Calculation valid for all sleptons, but third generation particularly interesting $\rightarrow$ SUSY-breaking parameters
- τ-identification in ATLAS: [Hinchliffe, NPB PS 123 (2003) 229]
  - Requires significant q_T
  - Leptonic decays: $\not{q}_T$ + direct leptons $\rightarrow 100\times$
  - Hadronic decays: $\not{q}_T$ + QCD jets $\rightarrow 10\times$ larger
  - Low multiplicity / inv. mass, tuned in Z-decay



- τ-identification in CMS: [Gennai, NPB PS 123 (2003) 244]
  - Dedicated calorimeter trigger
  - Isolated tracks, matched to calorimeter jet axis
  - Reduces QCD background by factor 1000

4 - Summary

Summary

1. Beyond the Standard Model
   - SUSY: Broken → heavy particles ($\tilde{l}$) → $\not{E}_T$, (in-) direct constraints
   - GUTs: SO(10) → Z'-bosons → DY mass peak → early discovery
2. Beyond fixed-order perturbation theory
   - Drell-Yan like processes at LO and NLO of (SUSY-) QCD
   - Transverse-momentum, threshold and joint resummation
   - Parton showers in PYTHIA and MC@NLO
3. Numerical results and uncertainties
   - Resummation: NLO+NLL → precise, important far from crit. region
   - Monte Carlos : (N)LO+LL → easier implementation in exp. analysis
   - Uncertainties: Scales + PDFs $\lesssim \pm 10\%$, non-perturb. effects < 5%
4. Perspectives
   - Continue resummation program, experimental studies (sleptons, …)
   - Theory LHC France, France China Particle Physics Laboratory

Further reading

- Constraints:
  - mSUGRA: Bozzi, Fuks, Herrmann, MK, Nucl. Phys. B 787 (2007) 1
- Resummation:
  - q_T: Bozzi, Fuks, MK, Phys. Rev. D 74 (2006) 015001
  - Threshold: Bozzi, Fuks, MK, Nucl. Phys. B 777 (2007) 157
  - Joint: Bozzi, Fuks, MK, arXiv:0709.3057 (NPB, in press)
- MC@NLO:
  - Z′: Bozzi, Fuks, Ledroit, Li, MK, arXiv:0711.0749 (NPB in …), ATLAS note in preparation
  - Sleptons: In preparation (with CMS in Karlsruhe)
- Dark matter annihilation: Δm_b-resummation
  - A^0-funnel: Herrmann, MK, Phys. Rev. D 76 (2007) 117704
- Downloads:
  - http://lpsc.in2p3.fr/klasen/software
  - http://pheno.physik.uni-freiburg.de/~fuks/resum.html

LPSC
6) A NEW FORMALISM
Marcus Miller
Silver Rain

Backup slides

Dark matter annihiliation (1)

- Astrophysical evidence for dark matter:
  - Rotational spectra of spiral galaxies
  - Matter distribution after collision of two galaxy clusters
- Relic density (WMAP, 2σ):
  - $0.094 < \Omega_{CDM} h^2 < 0.136$ with $\Omega_{CDM} h^2 = m_{\tilde{\chi}_1^0} n_0 / \rho_c \propto \langle \sigma_{eff} v \rangle^{-1}$
  - Boltzmann eq.: $\frac{dn}{dt} = -3Hn - \langle \sigma_{\rm eff} v \rangle \left(n^2 - n_{eq}^2 \right)$ (DarkSUSY, micrOMEGAs)
- Supersymmetry:
  - P_R-conservation $\to$ LSP $\tilde{\chi}_1^0$ is stable $\to$ (co-)annihilation ($v \ll c$)
  - Large $\tan\beta$: $\tilde{\chi}_1^0 \tilde{\chi}_1^0 \to A^0 \to b\bar{b}$
  - LO cross section:

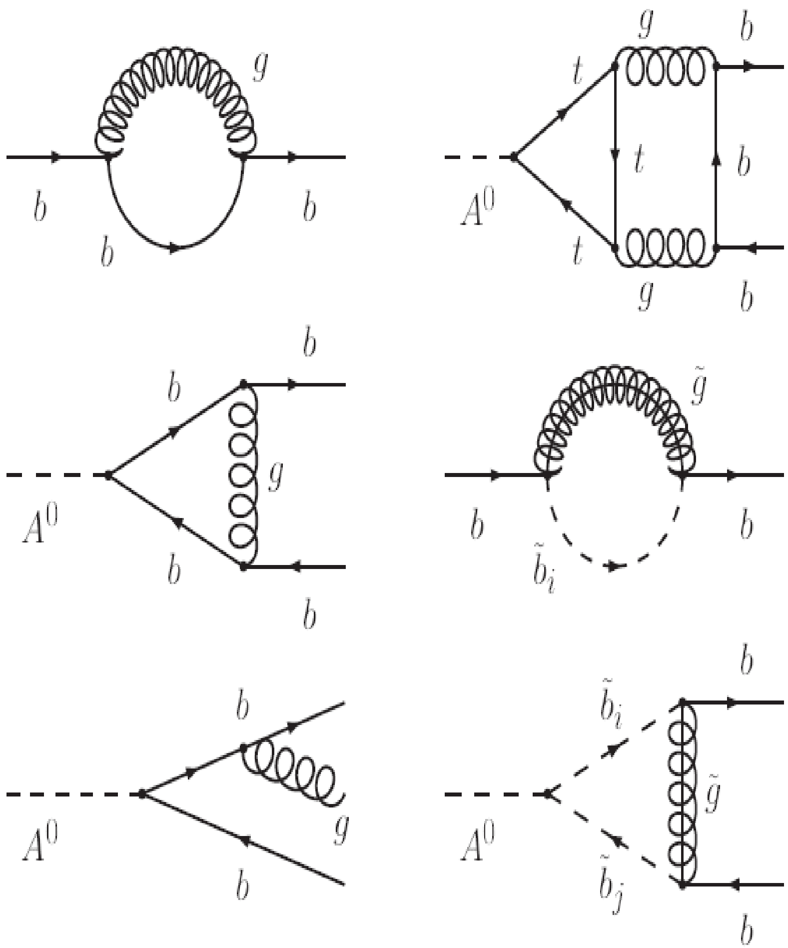
$$\sigma_{\rm LO}\, v = \frac{1}{2} \frac{\beta_b}{8\pi s} \frac{N_C\, g^2\, T_{A11}^2\, h_{Abb}^2\, s^2}{|s - m_A^2 + i m_A \Gamma_A|^2}$$ with $$h_{Abb} = -g m_b \tan\beta / (2 m_W)$$

Dark matter annihiliation (2)

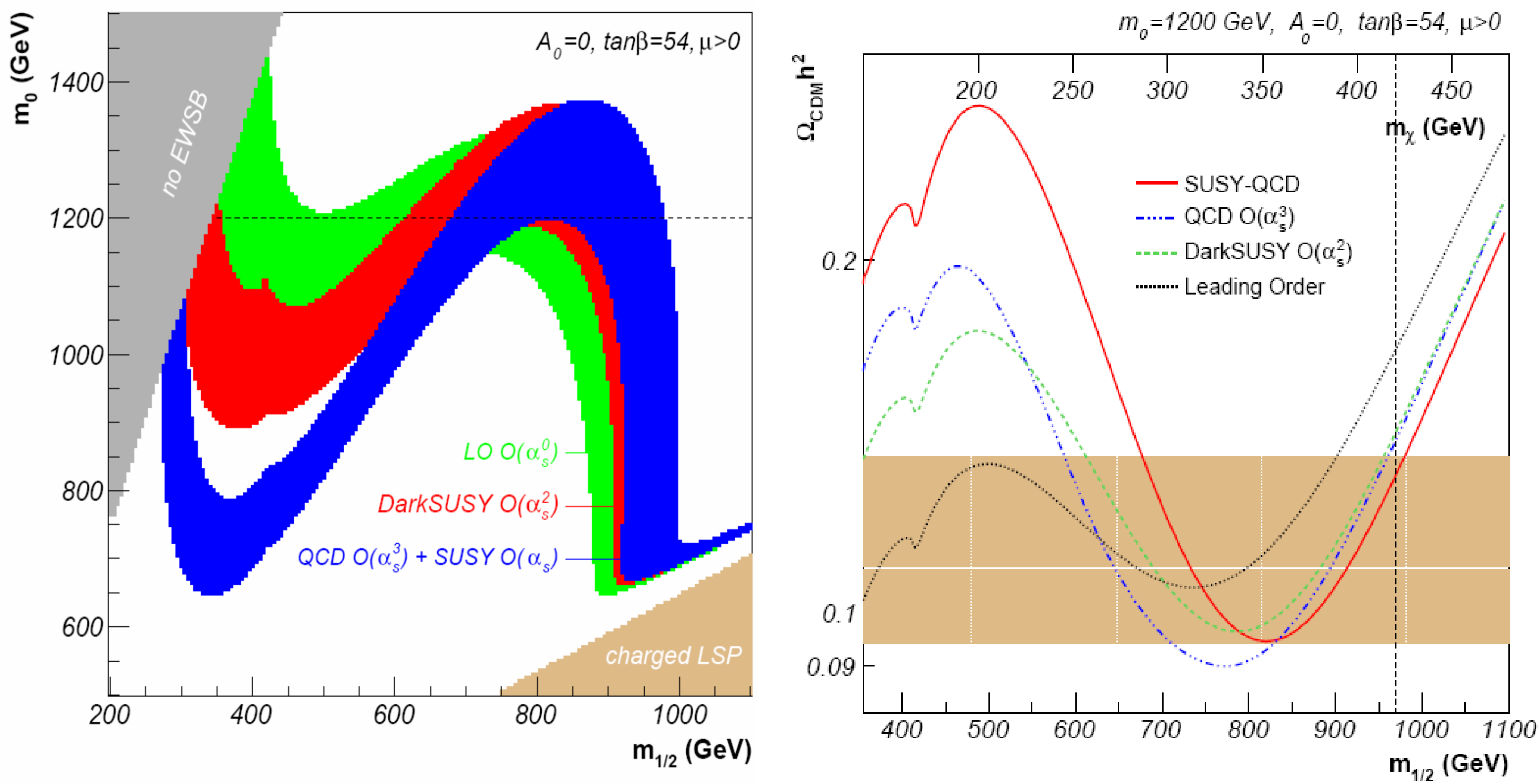
- QCD corrections:
  - At $m_b^2 \ll s$ ($\beta_b \to 1$):

    $$\Delta^{(1)}_{\rm QCD} \simeq \left(\frac{\alpha_s(s)}{\pi}\right) C_F \left[-\frac{3}{2}\log\frac{s}{m_b^2} + \frac{9}{4}\right]$$
  - Resummation: $m_b \to \overline{m}_b(s)$
  - Finite terms ($\overline{\rm MS}$ scheme):

    $$\Delta_{\rm QCD} = \left(\frac{\alpha_s(s)}{\pi}\right) C_F \frac{17}{4} + \left(\frac{\alpha_s(s)}{\pi}\right)^2 (35.94 - 1.36 n_f) + \left(\frac{\alpha_s(s)}{\pi}\right)^3 (164.14 - 25.76 n_f + 0.259 n_f^2)$$
- Top-loop corrections:
  - Separately gauge invariant
  - $$\Delta_{\rm top} = \frac{1}{\tan^2\beta}\left(\frac{\alpha_s(s)}{\pi}\right)^2 \left[\frac{23}{6} - \log\frac{s}{m_t^2} + \frac{1}{6}\log^2\frac{\bar{m}_b^2(s)}{s}\right]$$
- SUSY-QCD corrections:
  - $$\Delta m_b = \left(\frac{\alpha_s(s)}{\pi}\right) C_F \frac{m_{\tilde{g}}}{2}(A_b - \mu\tan\beta) I(m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2}, m^2_{\tilde{g}})$$
  - $$\Delta^{(\rm LE)}_{\rm SUSY} = \left(\frac{\alpha_s(s)}{\pi}\right) C_F \left(1 + \frac{1}{\tan^2\beta}\right) m_{\tilde{g}}\mu\tan\beta\, I(m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2}, m^2_{\tilde{g}})$$



Dark matter annihilation (3)



[Herrmann, Klasen, PRD 76 (2007) 117704]

Transverse-momentum resummation (1)

- Massive, color-less final state F, produced without q_T at LO
- Beyond LO: (Dominant) region of $q_T^2 \ll Q^2$ $\rightarrow$ $\alpha_S^n/q_T^2 \log^{2n-1} Q^2/q_T^2$
- Re-organized cross section: $\frac{d\sigma_F}{dQ^2\, dq_T^2\, d\phi} = \left[\frac{d\sigma_F}{dQ^2\, dq_T^2\, d\phi}\right]_{\rm res.} + \left[\frac{d\sigma_F}{dQ^2\, dq_T^2\, d\phi}\right]_{\rm fin.}$
- Resummed part: $\left[\frac{Q^2\, d\sigma_F}{dQ^2\, dq_T^2\, d\phi}\right]_{\rm res.} = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2 \int_0^\infty db\, \frac{b}{2}\, J_0(bq_T)\, f_{a/h_1}(x_1, b_0^2/b^2)\, f_{b/h_2}(x_2, b_0^2/b^2)\, S$
  - Coefficient functions: $\sum_c \int_0^1 dz_1 \int_0^1 dz_2\, C_{ca}^F(\alpha_S(b_0^2/b^2), z_1)\, C_{\bar{c}b}^F(\alpha_S(b_0^2/b^2), z_2)\, \delta(Q^2 - z_1 z_2 s)$
  - Sudakov form factor: $\exp\left\{-\int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2}\left[A_c(\alpha_S(q^2))\log\frac{Q^2}{q^2} + B_c(\alpha_S(q^2))\right]\right\} \frac{d\sigma_{c\bar{c}}^{(LO)F}}{d\phi}$
- Coefficient functions $A(\alpha_S)$, $B(\alpha_S)$, $C(\alpha_S)$:
  - Compare order-by-order with known fixed-order result
  - Extract from known IR-behavior of tree-level / 1-loop amplitudes
  - Work in moment-space (Davies, Stirling): $\Sigma(N) = \int_0^{1-2q_T/Q} dz\, z^N \frac{Q^2 q_T^2}{d\sigma_0/d\phi} \frac{d\sigma}{dq_T^2 dQ^2 d\phi}$
- Finite part: Subtract truncated expansion of $\sigma_{\rm res.}$ $\rightarrow$ $\frac{d\hat{\sigma}_{ab}^{(\rm fin.)}}{dq_T^2} = \left[\frac{d\hat{\sigma}_{ab}}{dq_T^2}\right]_{\rm f.o.} - \left[\frac{d\hat{\sigma}_{ab}^{(\rm res.)}}{dq_T^2}\right]_{\rm f.o.}$

[De Florian, Grazzini, Nucl. Phys. B 616 (2001) 247]

Transverse-momentum resummation (2)

- Resummed part (moment-space):
  - PDFs evaluated at μ_F^2: $\Sigma(N) = \sum_{i,j} f_{i/h_1}(N,\mu_F^2) f_{j/h_2}(N,\mu_F^2)\, \Sigma_{ij}(N)$
  - Partonic cross section:

$$\Sigma_{ij}(N) = \sum_{a,b} \int_0^\infty b\, db \frac{q_T^2}{2} J_0(b q_T)\, C^F_{ca}\left(N, \alpha_S\left(b_0^2/b^2\right)\right) C^F_{\bar{c}b}\left(N, \alpha_S\left(b_0^2/b^2\right)\right)$$
$$\exp\left\{ -\int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2}\left[A_c(\alpha_S(q^2)) \log\frac{Q^2}{q^2} + B_c^F(\alpha_S(q^2))\right] - \int_{b_0^2/b^2}^{\mu_F^2} \frac{dq^2}{q^2} \left(\gamma_{ai}+\gamma_{bj}\right)(N, \alpha_S(q^2)) \right\}$$

  - 1st -order expansion: $\Sigma_{c\bar{c}}^{(1)}(N) = A_c^{(1)} \log\frac{Q^2}{q_T^2} + B_c^{(1)} + 2\gamma_{cc}^{(1)}(N)$
  - 2nd-order expansion:

$$\Sigma_{c\bar{c}}^{(2)}(N) = \log^3\frac{Q^2}{q_T^2}\left[-\frac{1}{2}\left(A_c^{(1)}\right)^2\right] + \log^2\frac{Q^2}{q_T^2}\left[-\frac{3}{2}\left(B_c^{(1)} + 2\gamma_{cc}^{(1)}(N)\right)A_c^{(1)} + \beta_0 A_c^{(1)}\right]$$
$$+\log\frac{Q^2}{q_T^2}\left[A_c^{(2)} + \beta_0\left(B_c^{(1)} + 2\gamma_{cc}^{(1)}(N)\right) - \left(B_c^{(1)} + 2\gamma_{cc}^{(1)}(N)\right)^2 + 2A_c^{(1)} C_{cc}^{(1)F}(N) - 2\sum_{j\neq c}\gamma_{cj}^{(1)}(N)\gamma_{jc}^{(1)}(N)\right]$$
$$+B_c^{(2)F} + 2\gamma_{cc}^{(2)}(N) + 2\left(B_c^{(1)} + 2\gamma_{cc}^{(1)}(N)\right)C_{cc}^{(1)F}(N) + 2\zeta(3)\left(A_c^{(1)}\right)^2 - 2\beta_0 C_{cc}^{(1)F}(N) + 2\sum_{j\neq c}\left[C_{cj}^{(1)F}(N)\gamma_{jc}^{(1)}(N)\right]$$

- LL: $A_q^{(1)} = 2C_F$ $A^{(1,2)}$ universal, same as in threshold resummation
- NLL: $A_q^{(2)} = 2C_F K$ with $K = \left(\frac{67}{18} - \frac{\pi^2}{6}\right) C_A - \frac{10}{9} n_f T_R$, $B_q^{(1)} = -3C_F$ (universal),
  $C_{ab}^{(1)F}(z) = -\hat{P}^\epsilon_{ab}(z) + \delta_{ab}\,\delta(1-z)\left(C_a\frac{\pi^2}{6} + \boxed{\frac{1}{2}\mathcal{A}_a^F(\phi)}\right)$ with $\boxed{\mathcal{A}_q^{DY} = C_F\left(-8 + \frac{2}{3}\pi^2\right)}$
- NNLL: $A_{q,}^{(3)}$ only num. known [Vogt], $B_a^{(2)F} = -2\,\gamma_a^{(2)} + \beta_0\left(\frac{2}{3}C_a\pi^2 + \boxed{\mathcal{A}_a^F(\phi)}\right)$, $C_q^{(2)F}$

Transverse-momentum resummation (3)

- Modified coefficient function $B(\alpha_S)$:
  - $\widetilde{B}_N(\alpha_S) = B(\alpha_S) + 2\beta(\alpha_S)\,\frac{d\ln C_N(\alpha_S)}{d\ln\alpha_S} + 2\gamma_N(\alpha_S)$
  - Process-independent
- Universal "Sudakov" form factor:
  - $\mathcal{G}_N(\alpha_S(\mu_R^2), L; M^2/\mu_R^2, M^2/Q^2) = -\int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2}\left[A(\alpha_S(q^2))\,\ln\frac{M^2}{q^2} + \widetilde{B}_N(\alpha_S(q^2))\right]$
- "Resummation scale": $\ln M^2 b^2 = \ln Q^2 b^2 + \ln M^2/Q^2$
  - Arbitrary separation of coefficient functions and form factor
  - Expansion parameter: $L \equiv \ln\frac{Q^2 b^2}{b_0^2}$
  - Modified exp. param.: $\widetilde{L} \equiv \ln\left(\frac{Q^2 b^2}{b_0^2} + 1\right)$ $\to$ 0, when Qb « 1
  - Removes resummation in large-q_T region: $\exp\{\mathcal{G}(\alpha_S, \widetilde{L})\} \to 1$

    [Bozzi, Catani, De Florian, Grazzini, NPB 737 (2006) 73]

Threshold resummation

- Perturbative cross section:
  - For q_T-resummation $\rightarrow$ need only real emission
  - For threshold resummation $\rightarrow$ need also virtual corrections
  - Now include squark mixing $\rightarrow$ $\sigma_{q\bar{q}'}^{(1;\mathrm{SUSY})}\left(z, M^2; \frac{M^2}{\mu_F^2}, \frac{M^2}{\mu_R^2}\right) = \frac{\alpha^2 \pi C_F \beta^3}{36 M^2}\left[f_W \frac{\left|L_{W\tilde{l}_i\tilde{\nu}_l}\right|^2}{32 x_W^2 (1-x_W)^2 (1-m_W^2/M^2)^2}\right]\delta(1-z)$
- Resummation procedure:
  - Threshold logarithms: $\lambda = [\beta_0 \alpha_s \ln \bar{N}]/\pi$
  - Resummed cross section: $\hat{\sigma}_{ab}^{(\mathrm{RES})}(N, \alpha_s) = \sigma_0^{(\prime)} C_{ab}(\alpha_s) \exp\left[S(N, \alpha_s)\right]$
  - Exponential form factor: $S(N, \alpha_s) = g_1(\lambda) \ln \bar{N} + g_2(\lambda)$
  - LL: $g_1(\lambda) = \frac{A_1}{\beta_0 \lambda}[2\lambda + (1-2\lambda)\ln(1-2\lambda)]$ and
  - NLL: $g_2(\lambda) = \frac{A_1\beta_1}{\beta_0^3}\left[2\lambda + \ln(1-2\lambda) + \frac{1}{2}\ln^2(1-2\lambda)\right] - \frac{A_2}{\beta_0^2}[2\lambda + \ln(1-2\lambda)] + \frac{A_1}{\beta_0}[2\lambda + \ln(1-2\lambda)]\ln\frac{M^2}{\mu_R^2} - \frac{2A_1\lambda}{\beta_0}\ln\frac{M^2}{\mu_F^2},$

Joint resummation

- Initial-state soft-gluon radiation:
  - q_T *and* threshold logarithms
  - Reproduce LL and NLL in limits $\bar{b} \to \infty$ *and* $\bar{N} \to \infty$
  - Joint resummation: $\chi(\bar{b},\bar{N}) = \bar{b} + \frac{\bar{N}}{1+\eta\,\bar{b}/\bar{N}}$ with $\bar{b} \equiv b\,M\,e^{\gamma_E}/2$ and $\bar{N} \equiv N e^{\gamma_E}$

    [Laenen, Sterman, Vogelsang, PRD 63 (2001) 114018]
  - Choose η = ¼ → no sizeable subleading terms
- Resummed cross section:
  - $$\frac{d^2\sigma^{(\mathrm{res})}}{dM^2\,dp_T^2}(N,b) = \sum_{a,b,c} f_{a/h_a}(N+1;\mu_F)\, f_{b/h_b}(N+1;\mu_F)\,\hat{\sigma}^{(0)}_{c\bar{c}}\exp\left[\mathcal{G}_c(N,b;\alpha_s,\mu_R)\right]\left[\delta_{ca}\delta_{\bar{c}b} + \sum_{n=1}^{\infty}\left(\frac{\alpha_s(\mu_R)}{\pi}\right)^n \mathcal{H}^{(n)}_{ab\to c\bar{c}}\Big(N;\mu_R,\mu_F\Big)\right]$$
  - Regular part: $\mathcal{H}^{(1)}_{ab\to c\bar{c}}\Big(N;\mu_R,\mu_F\Big) = \delta_{ca}\delta_{\bar{c}b}\,H^{(1)}_{c\bar{c}}(\mu_R) + \delta_{ca}\,C^{(1)}_{\bar{c}/b}(N) + \delta_{\bar{c}b}\,C^{(1)}_{c/a}(N) + \Big(\delta_{ca}\gamma^{(1)}_{\bar{c}/b}(N) + \delta_{\bar{c}b}\gamma^{(1)}_{c/a}(N)\Big)\ln\frac{M^2}{\mu_F^2}$
  - DY scheme: $H^{(1)}_{c\bar{c}}(\mu_R) \equiv 0,\quad C^{(1)}_{q/q}(N) = \frac{2}{3\,N\,(N+1)} + \frac{\pi^2-8}{3}$, and $C^{(1)}_{q/g}(N) = \frac{1}{2\,(N+1)\,(N+2)}$
  - Eikonal factor:

    $$\mathcal{G}_c(N,b;\alpha_s,\mu_R) = g_c^{(1)}(\lambda)\,\ln\chi + g_c^{(2)}(\lambda;\mu_R)$$