Deep-inelastic scattering
– *from HERA to LHC* –

Sven-Olaf Moch

Sven-Olaf.Moch@desy.de

DESY, Zeuthen

– Festkolloquium für Fridger Schrempp, Hamburg, February 19, 2007 –

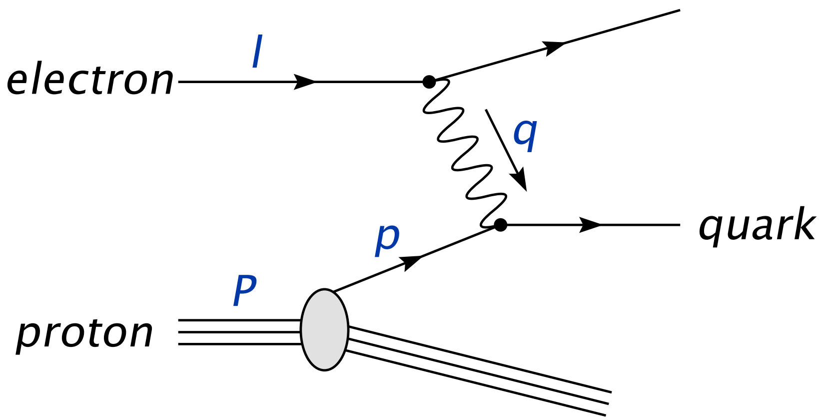
Proton colliders

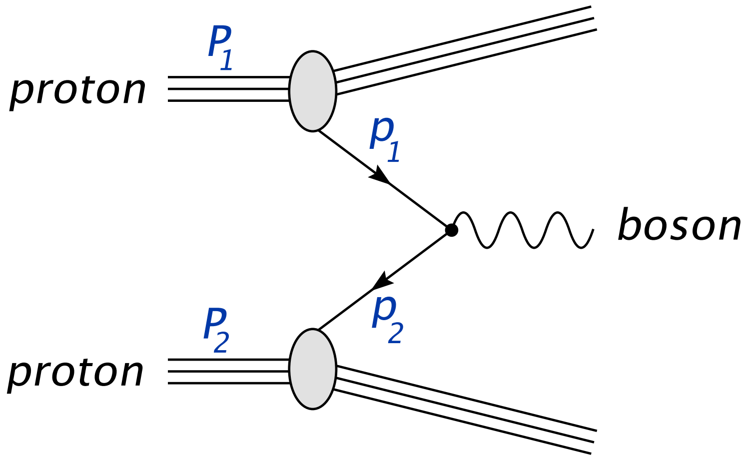
- HERA: deep structure of proton at highest Q^2 and smallest x
- LHC: Higgs boson search at highest energies $\sqrt{S} = 14$TeV

- Quantum Chromodynamics (QCD) ubiquitous at proton colliders
  - reliable understanding essential for precision and discovery physics

Perturbative QCD at colliders

- Hard scattering at hadron colliders
  - constituent partons from incoming hadrons interact at short distance (large momentum transfer Q^2)





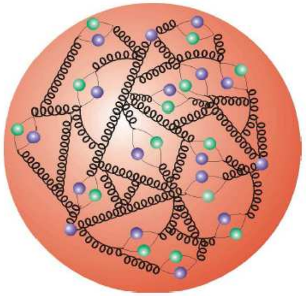
- QCD factorization
  - separate sensitivity to dynamics from different scales
- **DIS**: photon momentum $Q^2 = -q^2$, Bjorken's $x = Q^2/(2p \cdot q)$
- **pp**: energy $Q^2 = 2p_1 \cdot p_2$, ratio to W/Z-boson mass $x = M^2_{W/Z}/Q^2$

Hard scattering cross section

- QCD factorization

$$\sigma_{had} = \begin{cases} \sum_i f_i \otimes \hat{\sigma}_{\gamma i \to X} \left(\alpha_s(\mu^2), Q^2, \mu^2 \right) & \textbf{DIS} \\ \sum_{ij} f_i \otimes f_j \otimes \hat{\sigma}_{ij \to X} \left(\alpha_s(\mu^2), Q^2, \mu^2 \right) & \textbf{pp} \end{cases}$$

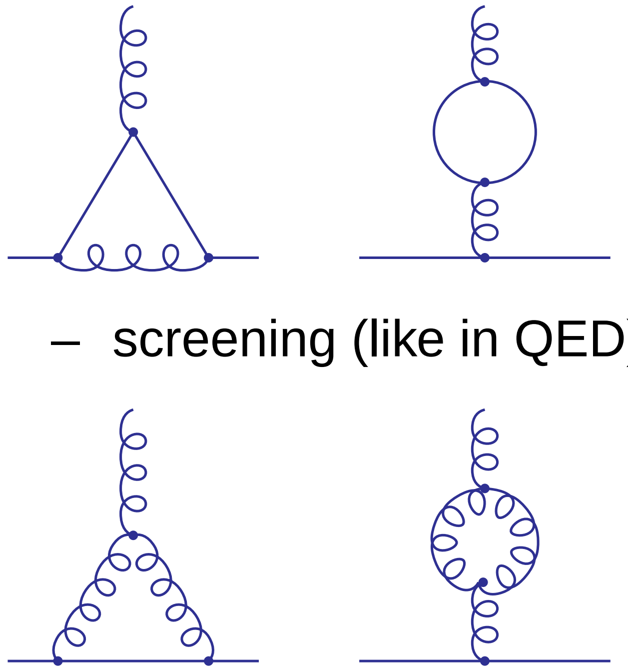
- Parton cross sections $\hat{\sigma}_{\gamma i \to X}$ and $\hat{\sigma}_{ij \to X}$ calculable pertubatively in powers of α_s
  - short distance interaction of constituent partons
- Parton distributions $f_i \to$ parton luminosity
  - proton: very complicated multi-particle bound state
  - colliders: wide-band beams of quarks and gluons



- Standard approach to uncertainties in theoretical predictions
  - variation of factorization scale μ: $\dfrac{d}{d \ln \mu^2} \sigma_{had} = \mathcal{O}(\alpha_s^{l+1})$

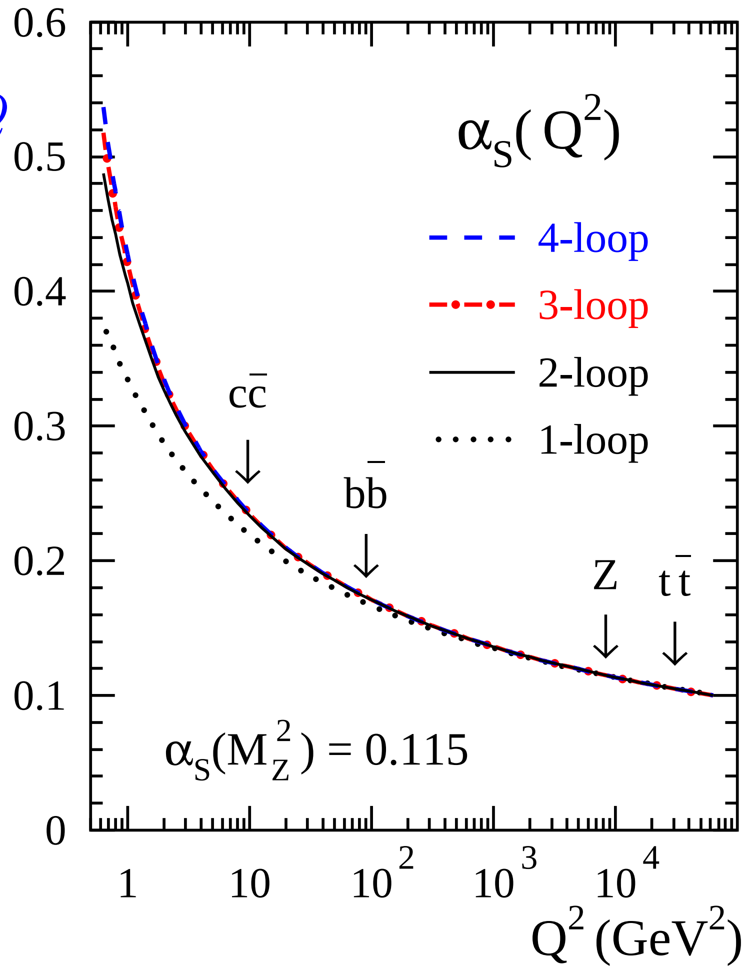
Asymptotic freedom of QCD

- Effective coupling constant α_s depends on resolution, momentum scale Q



  – screening (like in QED)

  – anti-screening (color charge of g)



- At large scales: application of perturbation theory (but $\alpha_s \gg \alpha_{\text{QED}}$)

Approaches to the calculation of σ_{had}

- LO (leading order)
  - Automated tree level calculations in Standard Model, MSSM, . . . (Madgraph,Alpgen, CompHEP, . . .)
  - LO + parton shower
  - String inspired techniques
- NLO (next-to-leading order)
  - Analytical (or numerical) calculations of diagrams yield parton level Monte Carlos (NLOJET++, MCFM, . . .)
  - NLO + parton shower (MC@NLO, VINCIA)
- NNLO (next-to-next-to-leading order)
  - selected results known (mostly inclusive kinematics)
- N^{3}LO (next-to-next-to-next-to-leading order)
  - very few . . .

Approaches to the calculation of σ_{had}

easy

- LO (leading order)
  - Automated tree level calculations in Standard Model, MSSM, . . . (Madgraph,Alpgen, CompHEP, . . .)
  - LO + parton shower
  - String inspired techniques
- NLO (next-to-leading order)
  - Analytical (or numerical) calculations of diagrams yield parton level Monte Carlos (NLOJET++, MCFM, . . .)
  - NLO + parton shower (MC@NLO, VINCIA)
- NNLO (next-to-next-to-leading order)
  - selected results known (mostly inclusive kinematics)
- N^3LO (next-to-next-to-next-to-leading order)
  - very few . . .

difficult

Approaches to the calculation of σ_{had}

easy

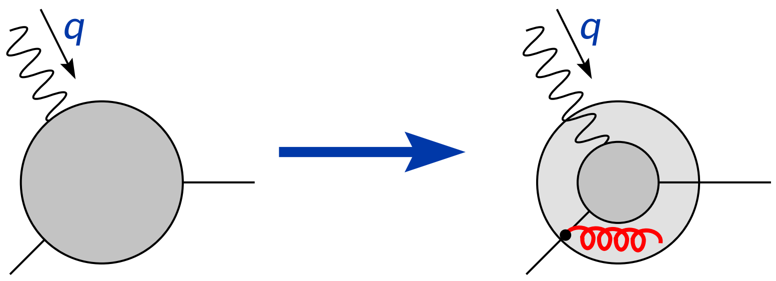
- LO (leading order)
  - Automated tree level calculations in Standard Model, MSSM, . . . (Madgraph,Alpgen, CompHEP, . . .)
  - LO + parton shower
  - String inspired techniques
- NLO (next-to-leading order)
  - Analytical (or numerical) calculations of diagrams yield parton level Monte Carlos (NLOJET++, MCFM, . . .)
  - NLO + parton shower (MC@NLO, VINCIA)
- NNLO (next-to-next-to-leading order)
  - selected results known (mostly inclusive kinematics)

← level of talk here

- N^3LO (next-to-next-to-next-to-leading order)
  - very few . . .

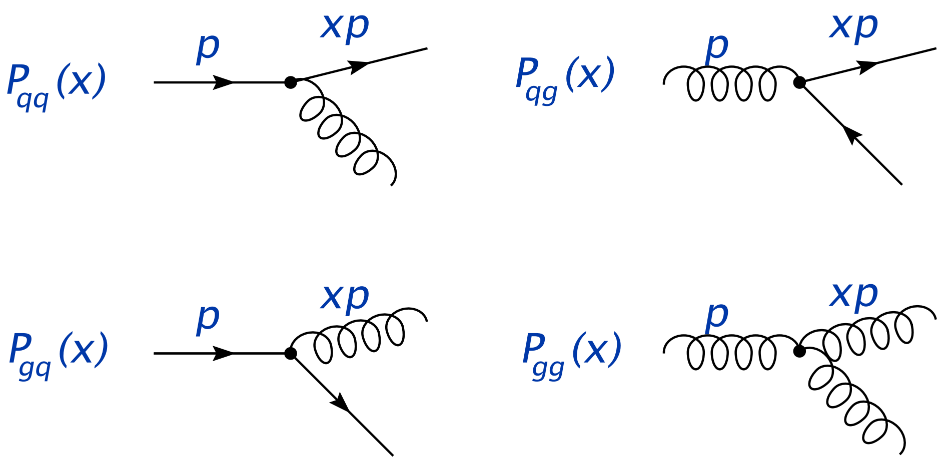
difficult

Parton luminosity

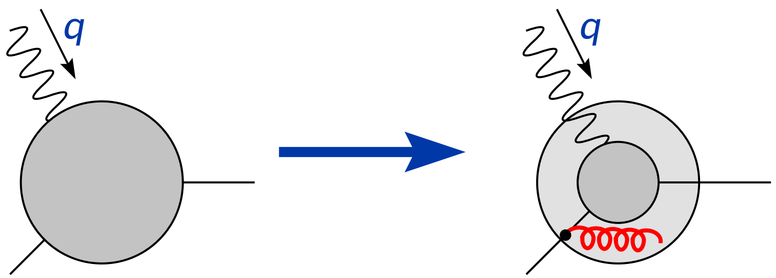


- Proton in resolution $1/Q \longrightarrow$ sensitive to lower momentum partons

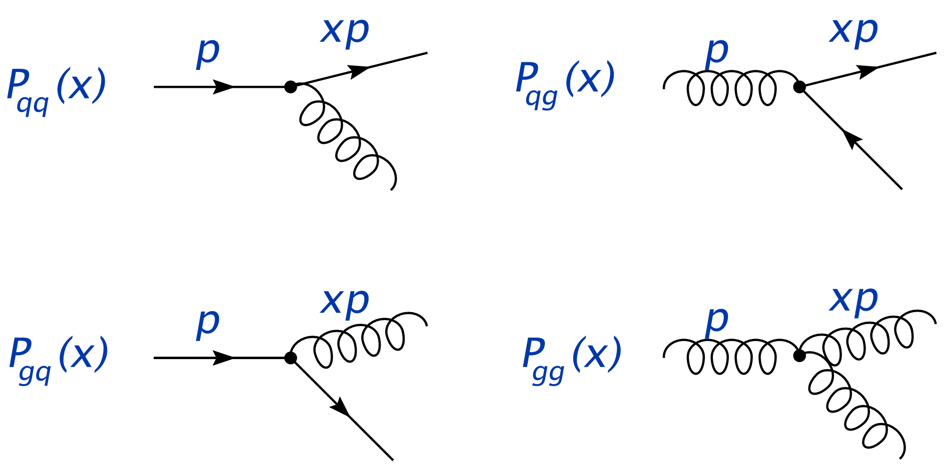
- Feynman diagrams in leading order



Parton luminosity



- Feynman diagrams in leading order



- Proton in resolution $1/Q \longrightarrow$ sensitive to lower momentum partons

- Evolution equations for parton distributions f_i
  - predictions from fits to reference processes (universality)

$$\frac{d}{d \ln \mu^2} f_i(x, \mu^2) = \sum_k \left[P_{ik}(\alpha_s(\mu^2)) \otimes f_k(\mu^2) \right](x)$$

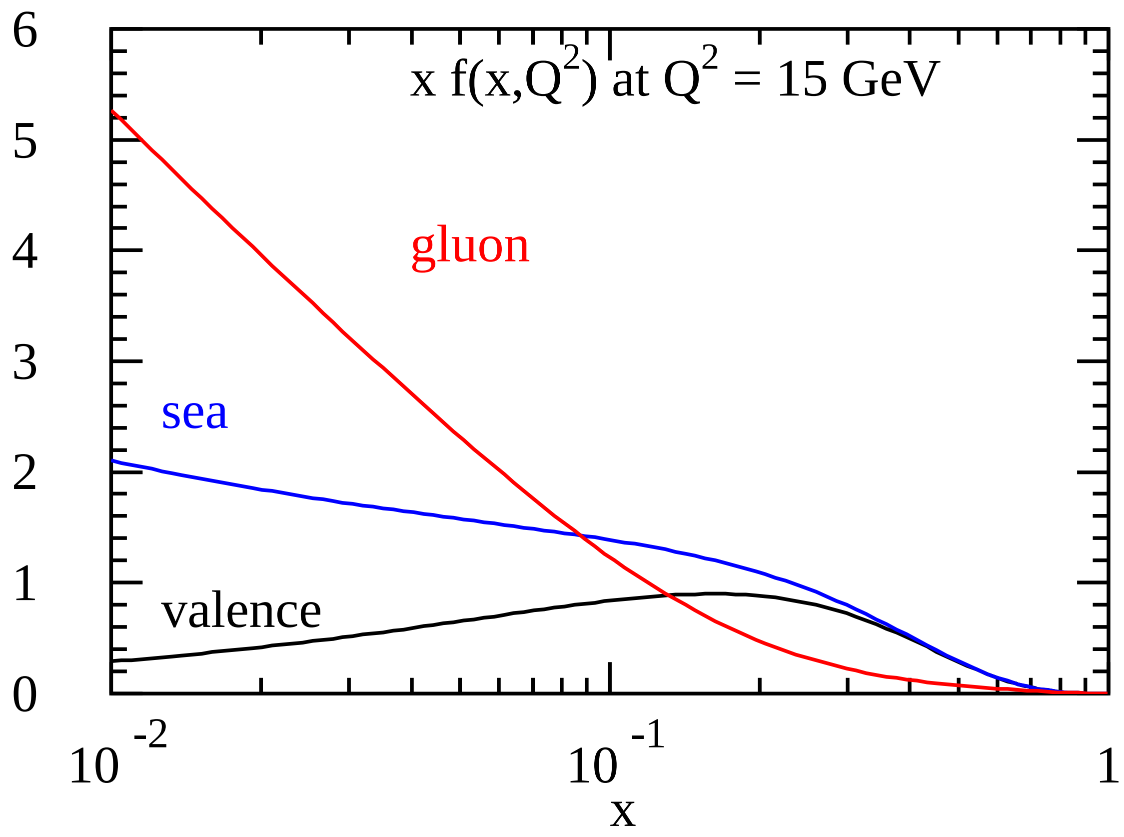
- Splitting functions P

$$P = \underbrace{\alpha_s \, P^{(0)} + \alpha_s^2 \, P^{(1)}} + \alpha_s^3 \, P^{(2)} + \ldots$$

NLO: standard approximation (large uncertainties)

Parton distributions in proton

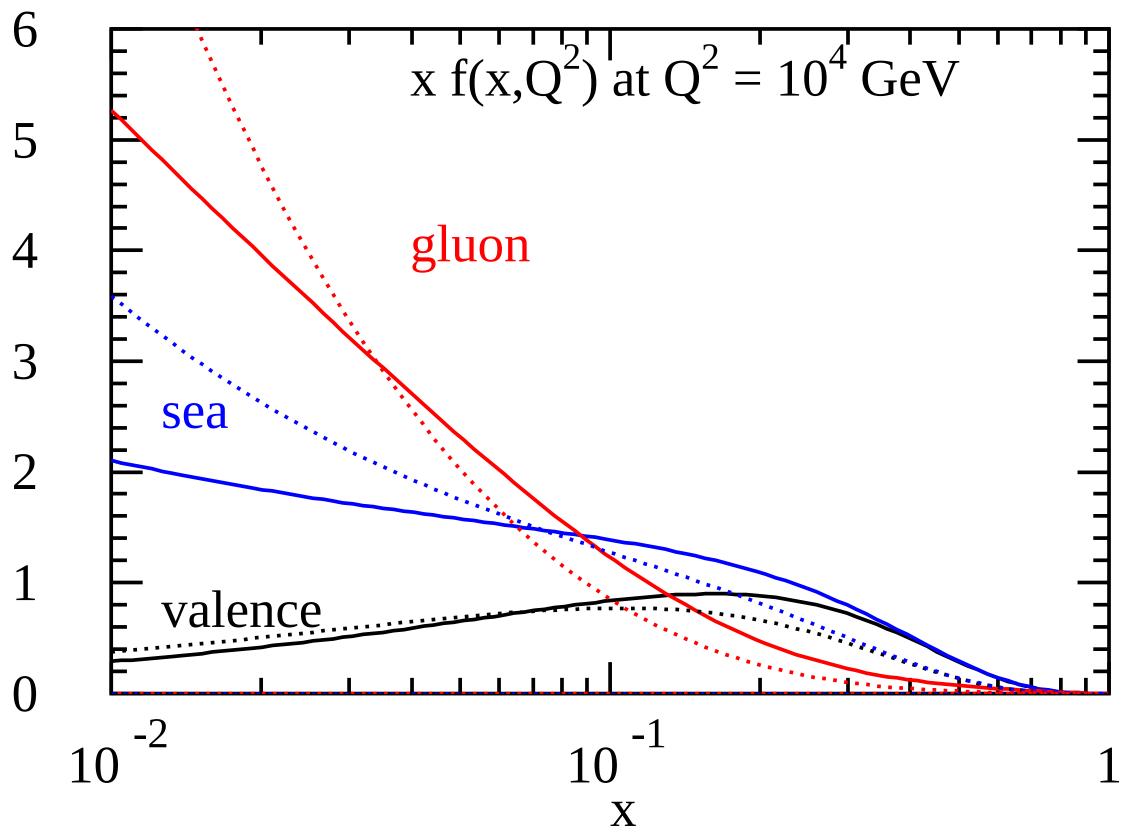
- Valence $q - \bar{q}$ (additive quantum numbers) sea (part with $q + \bar{q}$)



- Parameterization (bulk of data from deep-inelastic scattering)
  - structure function $F_2 \longrightarrow$ quark distribution
  - scale evolution (perturbative QCD) $\longrightarrow$ gluon distribution

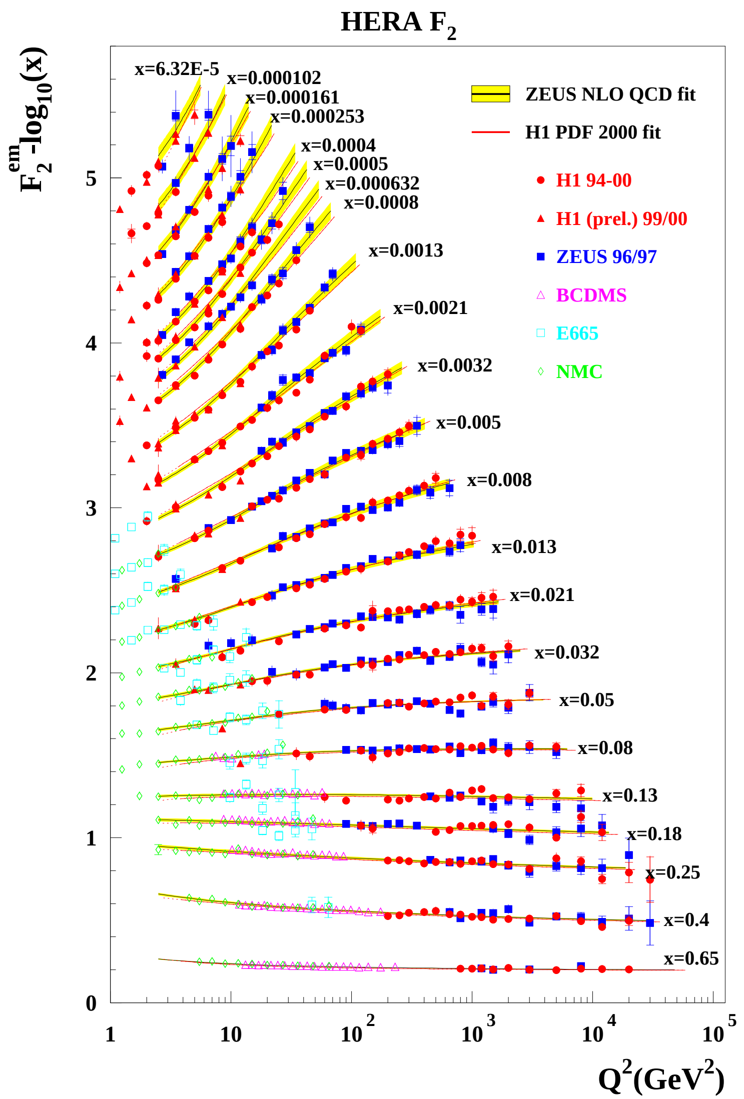
Parton distributions in proton

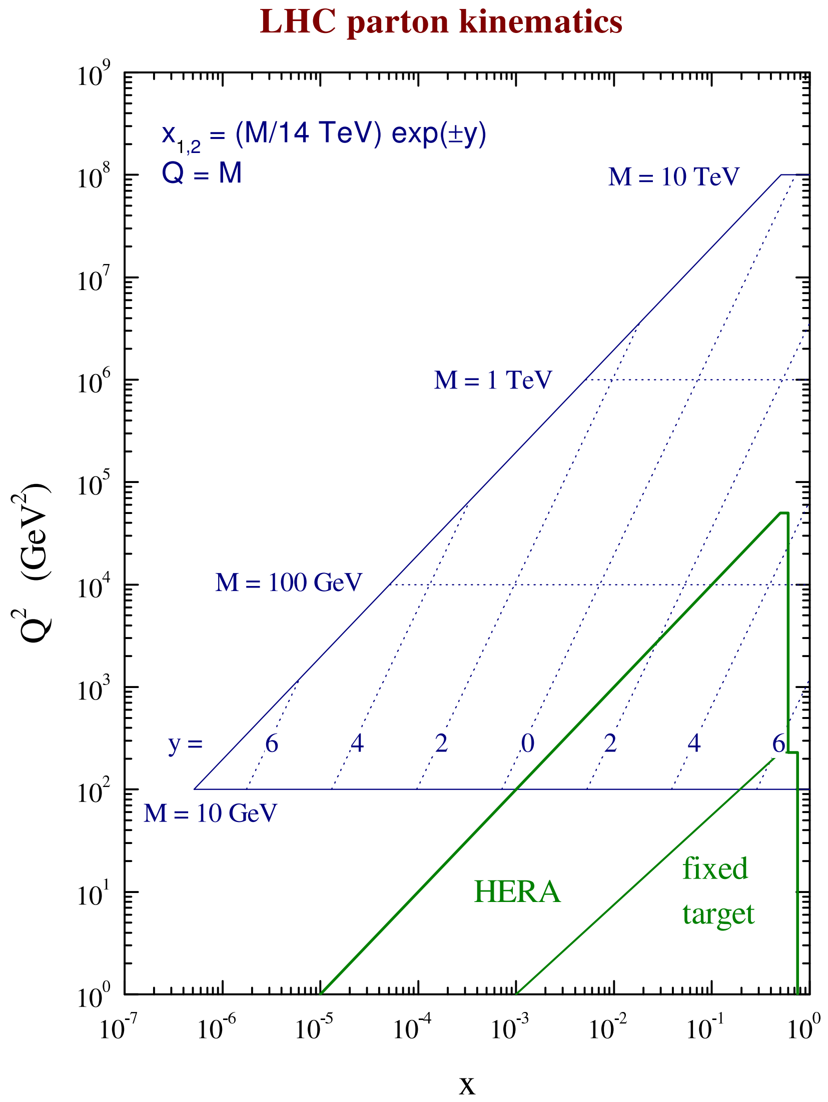
- Valence $q - \bar{q}$ (additive quantum numbers) sea (part with $q + \bar{q}$)



- Parameterization (bulk of data from deep-inelastic scattering)
  - structure function $F_2 \longrightarrow$ quark distribution
  - scale evolution (perturbative QCD) $\longrightarrow$ gluon distribution

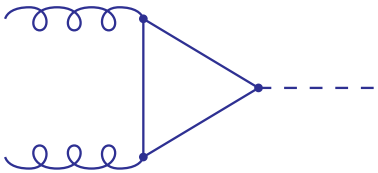
PDFs from HERA to LHC



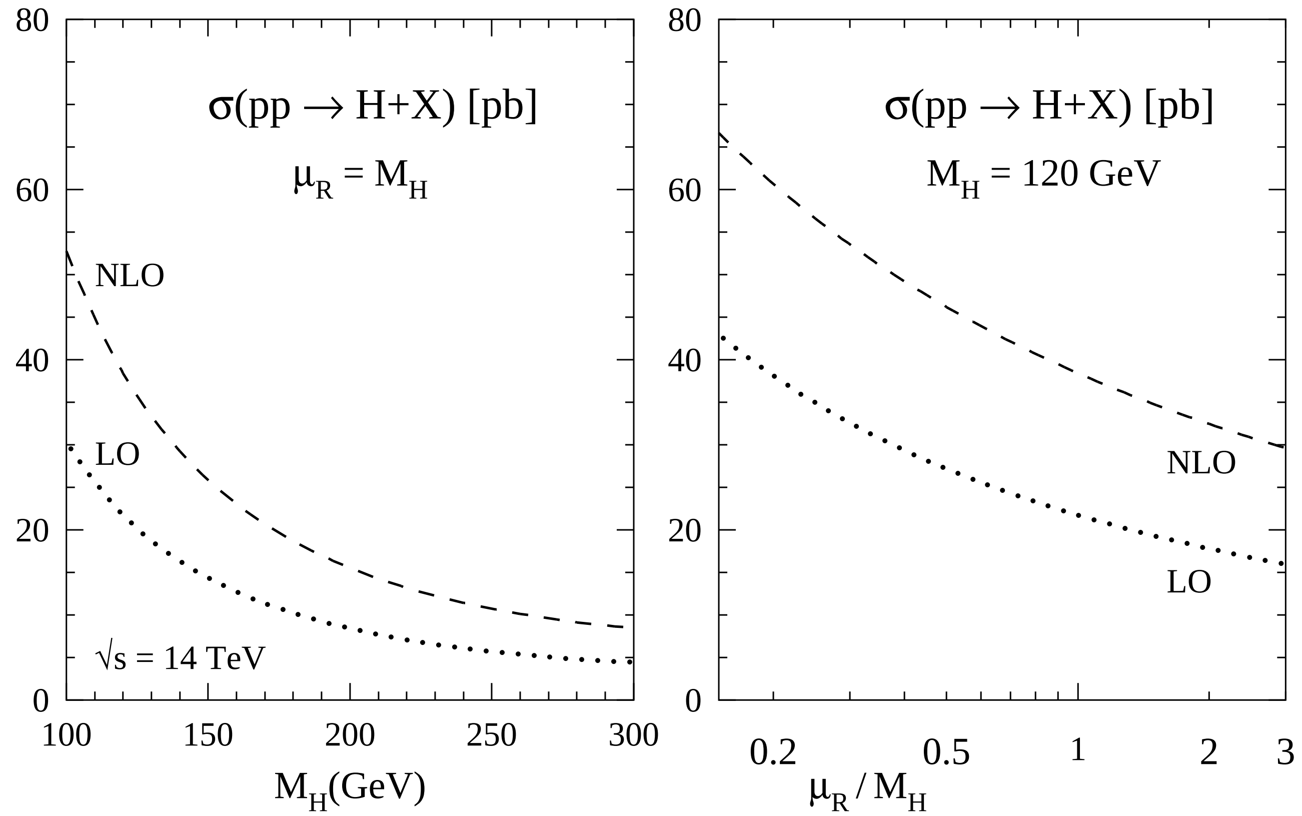


- **HERA → LHC:** scale evolution in Q^2 over three orders of magnitude

Higgs boson production at LHC (I)



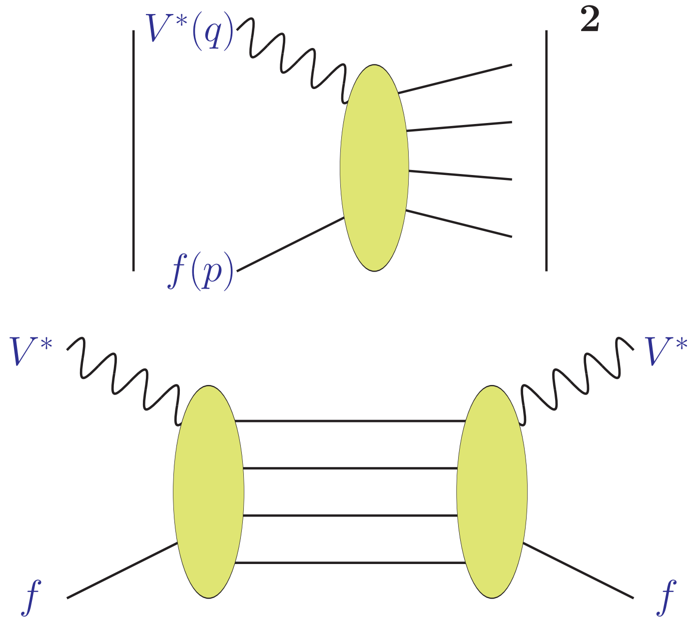
- Dominant channel $gg \to H + X$ via top-quark loop



- Estimate of uncertainty: apparent convergence, variation of scale μ
- NLO approximation insufficient for reliable predictions

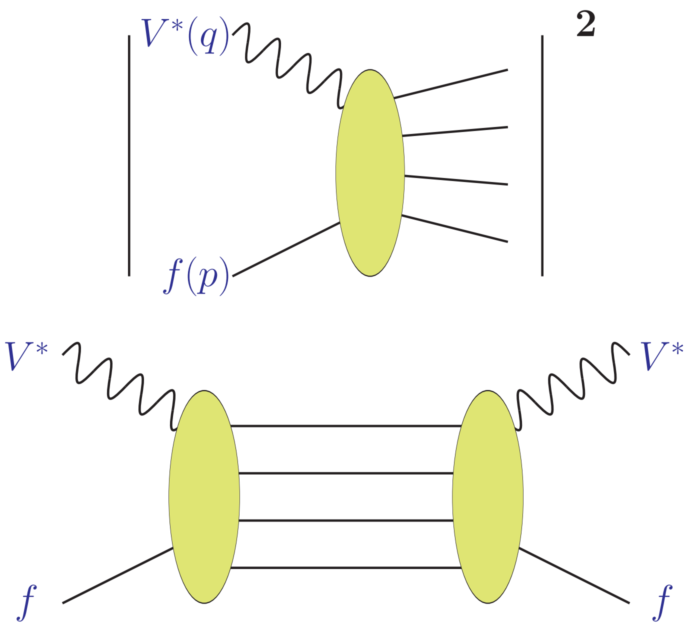
Our calculation in deep-inelastic scattering

- **"Loop technology"**: optical theorem
  total cross section $\longleftrightarrow$ imaginary part of Compton amplitude



Our calculation in deep-inelastic scattering

- **"Loop technology"**: optical theorem
  total cross section $\longleftrightarrow$ imaginary part of Compton amplitude



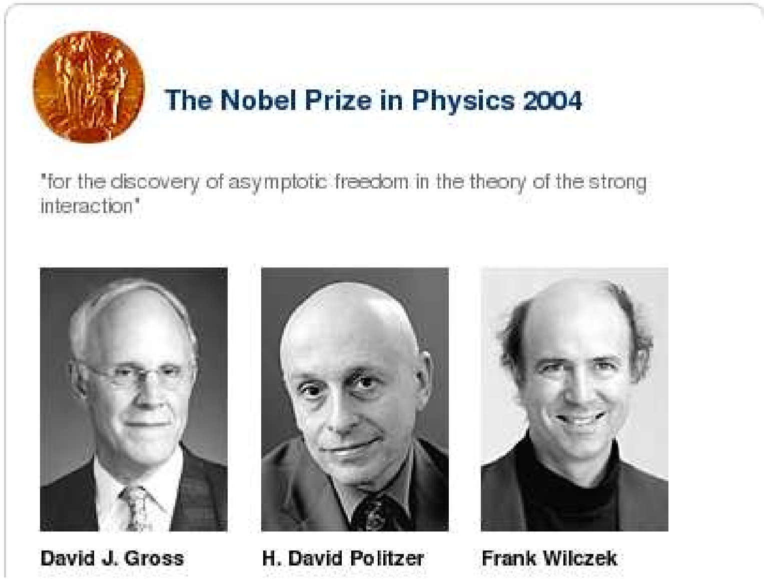
	tree	1-loop	2-loop	3-loop
qγ	1	3	25	359
gγ		2	17	345
hγ			2	56
qW	1	3	32	589
qϕ		1	23	696
gϕ	1	8	218	6378
hϕ		1	33	1184
sum	3	18	350	9607

- more than 10 FTE years and a few CPU years
  - computer algebra updates: FORM $\rightarrow$ FORM 3.1 $\rightarrow$ FORM 3.2 $\rightarrow \ldots$
  - $> 10^5$ tabulated symbolic integrals (> 3GB)

Splitting functions for a quarter of a century

$$\begin{aligned}
P_{\mathrm{ns}}^{(0)}(x) &= C_F\,(2p_{\mathrm{qq}}(x) + 3\delta(1-x))\\
P_{\mathrm{ps}}^{(0)}(x) &= 0\\
P_{\mathrm{qg}}^{(0)}(x) &= 2n_f\,p_{\mathrm{qg}}(x)\\
P_{\mathrm{gq}}^{(0)}(x) &= 2C_F\,p_{\mathrm{gq}}(x)\\
P_{\mathrm{gg}}^{(0)}(x) &= C_A\Big(4p_{\mathrm{gg}}(x) + \frac{11}{3}\delta(1-x)\Big) - \frac{2}{3}n_f\,\delta(1-x)
\end{aligned}$$

1973



$$\begin{aligned}
P_{\mathrm{ns}}^{(1)+}(x) &= 4C_AC_F\Big(p_{\mathrm{qq}}(x)\Big[\frac{67}{18} - \zeta_2 + \frac{11}{6}\mathrm{H}_0 + \mathrm{H}_{0,0}\Big] + p_{\mathrm{qq}}(-x)\Big[\zeta_2 + 2\mathrm{H}_{-1,0} - \mathrm{H}_{0,0}\Big]\\
&\quad + \frac{14}{3}(1-x) + \delta(1-x)\Big[\frac{17}{24} + \frac{11}{3}\zeta_2 - 3\zeta_3\Big]\Big) - 4C_Fn_f\Big(p_{\mathrm{qq}}(x)\Big[\frac{5}{9} + \frac{1}{3}\mathrm{H}_0\Big] + \frac{2}{3}(1-x)\\
&\quad + \delta(1-x)\Big[\frac{1}{12} + \frac{2}{3}\zeta_2\Big]\Big) + 4{C_F}^2\Big(2p_{\mathrm{qq}}(x)\Big[\mathrm{H}_{1,0} - \frac{3}{4}\mathrm{H}_0 + \mathrm{H}_2\Big] - 2p_{\mathrm{qq}}(-x)\Big[\zeta_2 + 2\mathrm{H}_{-1,0}\\
&\quad - \mathrm{H}_{0,0}\Big] - (1-x)\Big[1 - \frac{3}{2}\mathrm{H}_0\Big] - \mathrm{H}_0 - (1+x)\mathrm{H}_{0,0} + \delta(1-x)\Big[\frac{3}{8} - 3\zeta_2 + 6\zeta_3\Big]\Big)\\
P_{\mathrm{ns}}^{(1)-}(x) &= P_{\mathrm{ns}}^{(1)+}(x) + 16C_F\Big(C_F - \frac{C_A}{2}\Big)\Big(p_{\mathrm{qq}}(-x)\Big[\zeta_2 + 2\mathrm{H}_{-1,0} - \mathrm{H}_{0,0}\Big] - 2(1-x)\\
&\quad - (1+x)\mathrm{H}_0\Big)
\end{aligned}$$

$$P_{\mathrm{ps}}^{(1)}(x) = 4C_Fn_f\Big(\frac{20}{9}\frac{1}{x} - 2 + 6x - 4\mathrm{H}_0 + x^2\Big[\frac{8}{3}\mathrm{H}_0 - \frac{56}{9}\Big] + (1+x)\Big[5\mathrm{H}_0 - 2\mathrm{H}_{0,0}\Big]\Big)$$

$$\begin{aligned}
P_{\mathrm{qg}}^{(1)}(x) &= 4C_An_f\Big(\frac{20}{9}\frac{1}{x} - 2 + 25x - 2p_{\mathrm{qg}}(-x)\mathrm{H}_{-1,0} - 2p_{\mathrm{qg}}(x)\mathrm{H}_{1,1} + x^2\Big[\frac{44}{3}\mathrm{H}_0 - \frac{218}{9}\Big]\\
&+4(1-x)\Big[\mathrm{H}_{0,0} - 2\mathrm{H}_0 + x\mathrm{H}_1\Big] - 4\zeta_2x - 6\mathrm{H}_{0,0} + 9\mathrm{H}_0\Big) + 4C_Fn_f\Big(2p_{\mathrm{qg}}(x)\Big[\mathrm{H}_{1,0} + \mathrm{H}_{1,1} + \mathrm{H}_2\\
&-\zeta_2\Big] + 4x^2\Big[\mathrm{H}_0 + \mathrm{H}_{0,0} + \frac{5}{2}\Big] + 2(1-x)\Big[\mathrm{H}_0 + \mathrm{H}_{0,0} - 2x\mathrm{H}_1 + \frac{29}{4}\Big] - \frac{15}{2} - \mathrm{H}_{0,0} - \frac{1}{2}\mathrm{H}_0\Big)
\end{aligned}$$

$$\begin{aligned}
P_{\mathrm{gq}}^{(1)}(x) &= 4C_AC_F\Big(\frac{1}{x} + 2p_{\mathrm{gq}}(x)\Big[\mathrm{H}_{1,0} + \mathrm{H}_{1,1} + \mathrm{H}_2 - \frac{11}{6}\mathrm{H}_1\Big] - x^2\Big[\frac{8}{3}\mathrm{H}_0 - \frac{44}{9}\Big] + 4\zeta_2 - 2\\
&-7\mathrm{H}_0 + 2\mathrm{H}_{0,0} - 2\mathrm{H}_1x + (1+x)\Big[2\mathrm{H}_{0,0} - 5\mathrm{H}_0 + \frac{37}{9}\Big] - 2p_{\mathrm{gq}}(-x)\mathrm{H}_{-1,0}\Big) - 4C_Fn_f\Big(\frac{2}{3}x\\
&-p_{\mathrm{gq}}(x)\Big[\frac{2}{3}\mathrm{H}_1 - \frac{10}{9}\Big]\Big) + 4{C_F}^2\Big(p_{\mathrm{gq}}(x)\Big[3\mathrm{H}_1 - 2\mathrm{H}_{1,1}\Big] + (1+x)\Big[\mathrm{H}_{0,0} - \frac{7}{2} + \frac{7}{2}\mathrm{H}_0\Big] - 3\mathrm{H}_{0,0}\\
&+1 - \frac{3}{2}\mathrm{H}_0 + 2\mathrm{H}_1x\Big)
\end{aligned}$$

$$\begin{aligned}
P_{\mathrm{gg}}^{(1)}(x) &= 4C_An_f\Big(1 - x - \frac{10}{9}p_{\mathrm{gg}}(x) - \frac{13}{9}\Big(\frac{1}{x} - x^2\Big) - \frac{2}{3}(1+x)\mathrm{H}_0 - \frac{2}{3}\delta(1-x)\Big) + 4{C_A}^2\Big(27\\
&+(1+x)\Big[\frac{11}{3}\mathrm{H}_0 + 8\mathrm{H}_{0,0} - \frac{27}{2}\Big] + 2p_{\mathrm{gg}}(-x)\Big[\mathrm{H}_{0,0} - 2\mathrm{H}_{-1,0} - \zeta_2\Big] - \frac{67}{9}\Big(\frac{1}{x} - x^2\Big) - 12\mathrm{H}_0\\
&-\frac{44}{3}x^2\mathrm{H}_0 + 2p_{\mathrm{gg}}(x)\Big[\frac{67}{18} - \zeta_2 + \mathrm{H}_{0,0} + 2\mathrm{H}_{1,0} + 2\mathrm{H}_2\Big] + \delta(1-x)\Big[\frac{8}{3} + 3\zeta_3\Big]\Big) + 4C_Fn_f\Big(2\mathrm{H}_0\\
&+\frac{2}{3}\frac{1}{x} + \frac{10}{3}x^2 - 12 + (1+x)\Big[4 - 5\mathrm{H}_0 - 2\mathrm{H}_{0,0}\Big] - \frac{1}{2}\delta(1-x)\Big)\,.
\end{aligned}$$

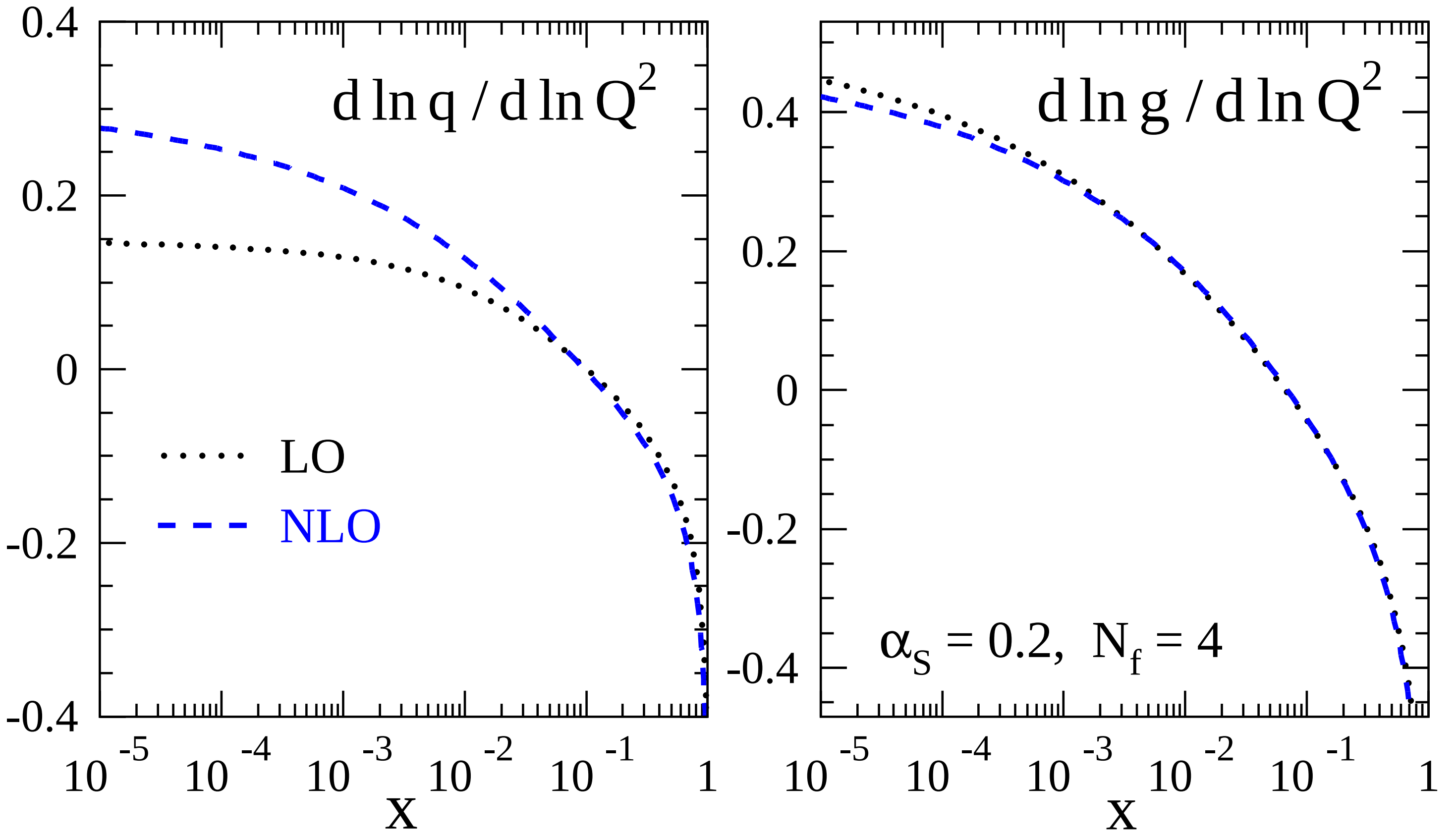
1980

NNLO splitting functions S.M. Vermaseren, Vogt

2004

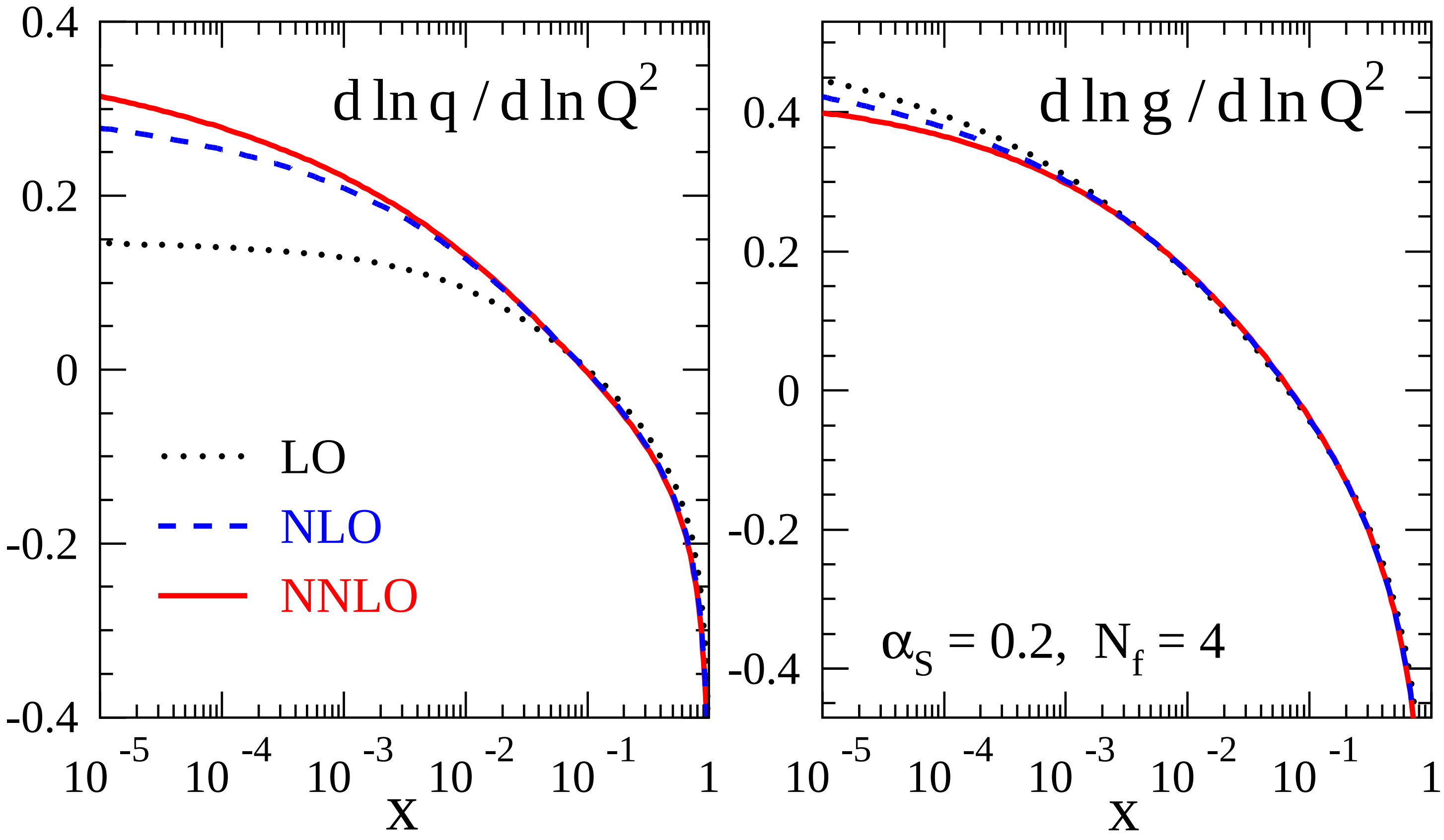
Perturbative stability of evolution

- Scale derivatives of quark and gluon distributions at $Q^2 \approx 30$ GeV^2



Perturbative stability of evolution

- Scale derivatives of quark and gluon distributions at $Q^2 \approx 30$ GeV^2

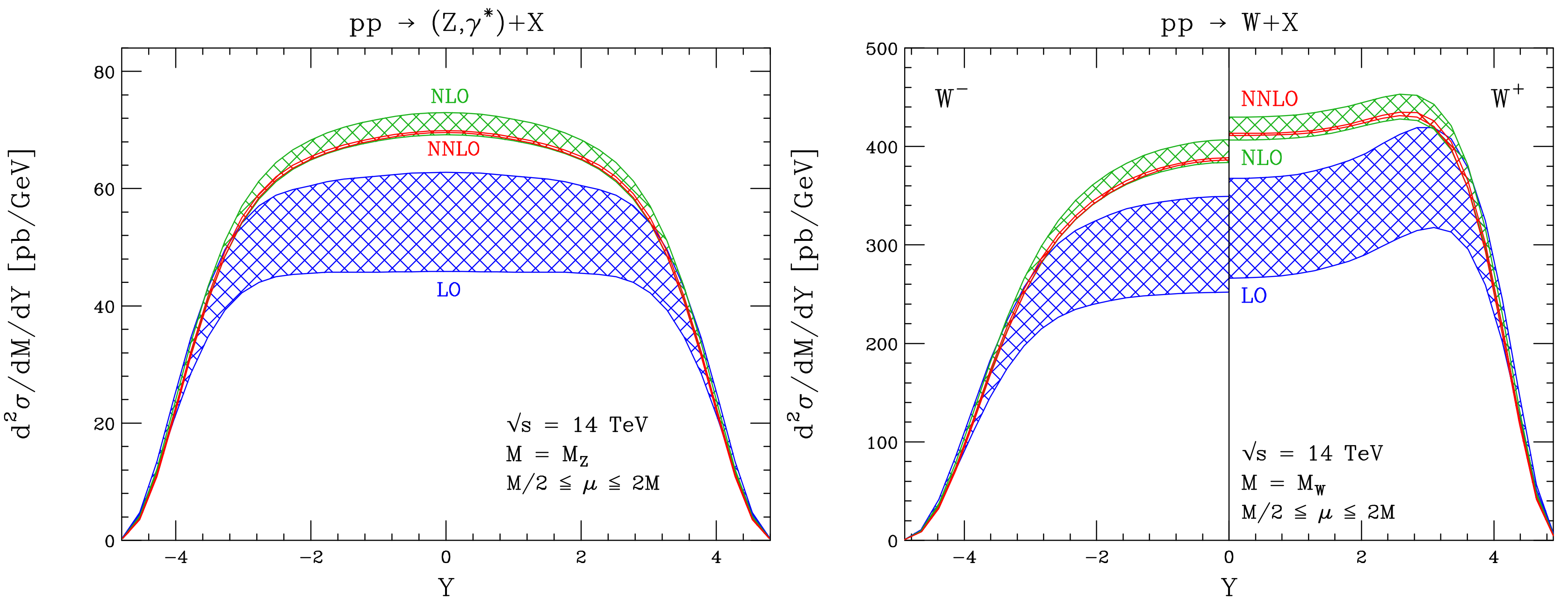


- Expansion very stable except for very small momenta $x \lesssim 10^{-4}$

Impact on precision of LHC predictions

- $W^{\pm}, Z$-boson rapidity distribution (scale variation $\frac{m_{W,Z}}{2} \leq \mu \leq 2m_{W,Z}$)

  Anastasiou, Petriello, Melnikov '05



- NNLO QCD theoretical uncertainties (renormalization / factorization scale) at level of 1% Dissertori et al. '05
  - one of the few cross sections known to NNLO in pQCD
- "Standard candle" process for parton luminosity

Updates of PDFs (exp)

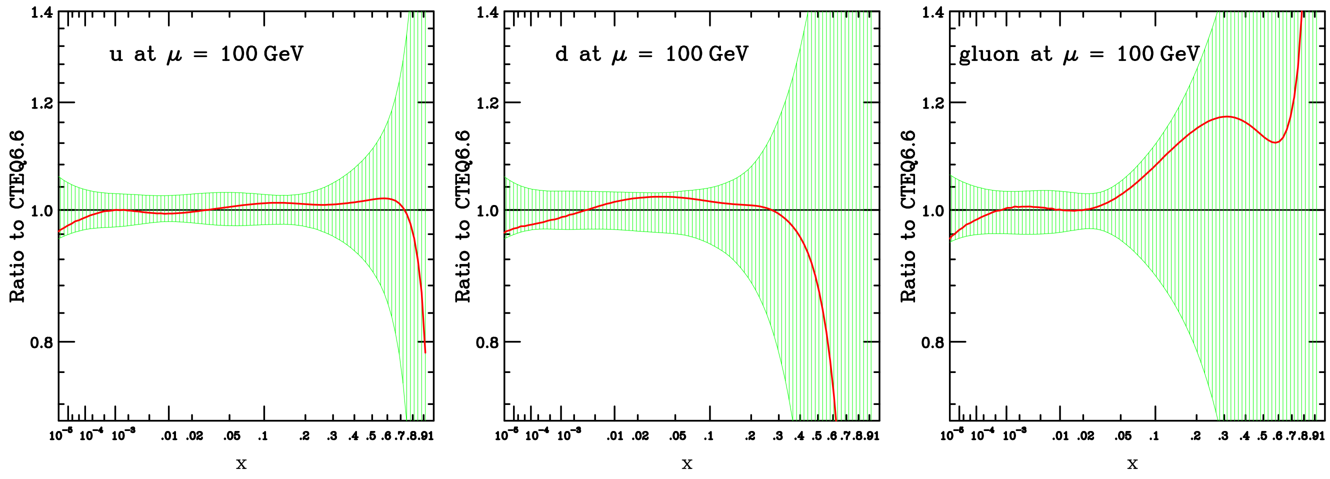
- New experimental data
  - results from neutrino-nucleon DIS for strange quark PDFs ($s \neq \bar{s}$)
- Uncertainty on $\bar{u}$, $\bar{d}$ doubles from 1.5% to 3% at $Q^2 \simeq M_W^2$ MSTW '07
  - s, $\bar{s}$ feed into F_2 NC DIS constraint $4/9(u + \bar{u}) + 1/9(d + \bar{d} + s + \bar{s})$

Updates of PDFs (exp)

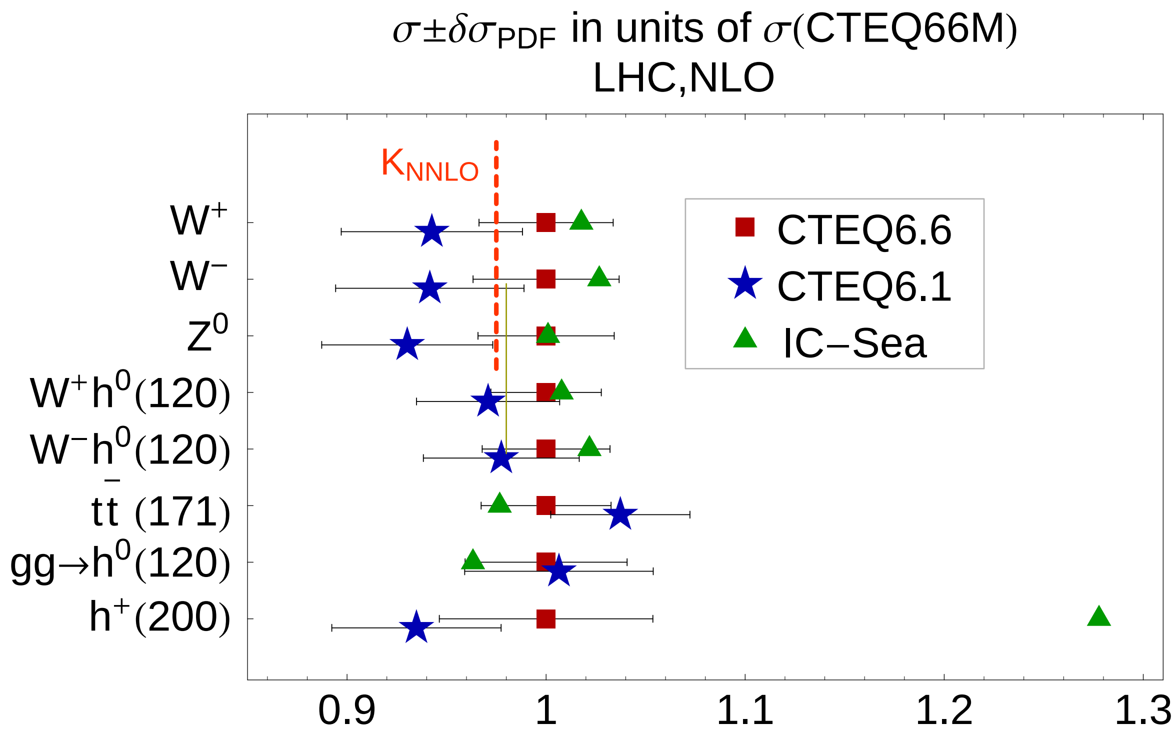
- New experimental data
  - results from neutrino-nucleon DIS for strange quark PDFs ($s \neq \bar{s}$)
- Uncertainty on $\bar{u}$, $\bar{d}$ doubles from 1.5% to 3% at $Q^2 \simeq M_W^2$ MSTW '07
  - s, $\bar{s}$ feed into F_2 NC DIS constraint $4/9(u+\bar{u}) + 1/9(d+\bar{d}+s+\bar{s})$

Updates of PDFs (th)

- Improved heavy quark (charm) threshold
  - matching consistent with QCD factorization CTEQ '08
- Significant changes due to larger light flavor PDFs



Cross sections at LHC

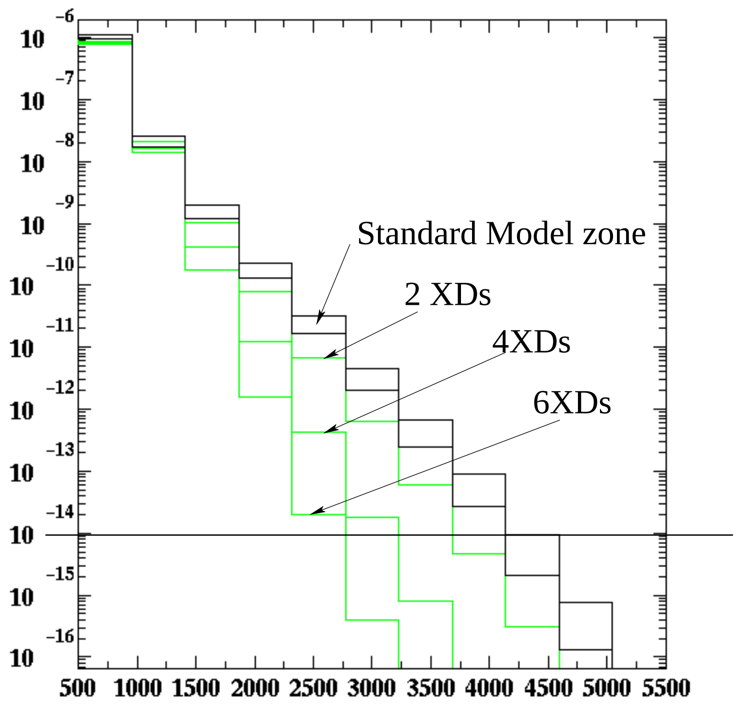


- Predictions for $W^{\pm}/Z$ cross sections at LHC shift by 8% between PDF sets CTEQ6.6 and CTEQ6.1 (improved theory!)
  - sensitivity to PDFs in the $x \sim 10^{-3}$ range
- $W^{\pm}/Z$-ratio golden calibration measurement

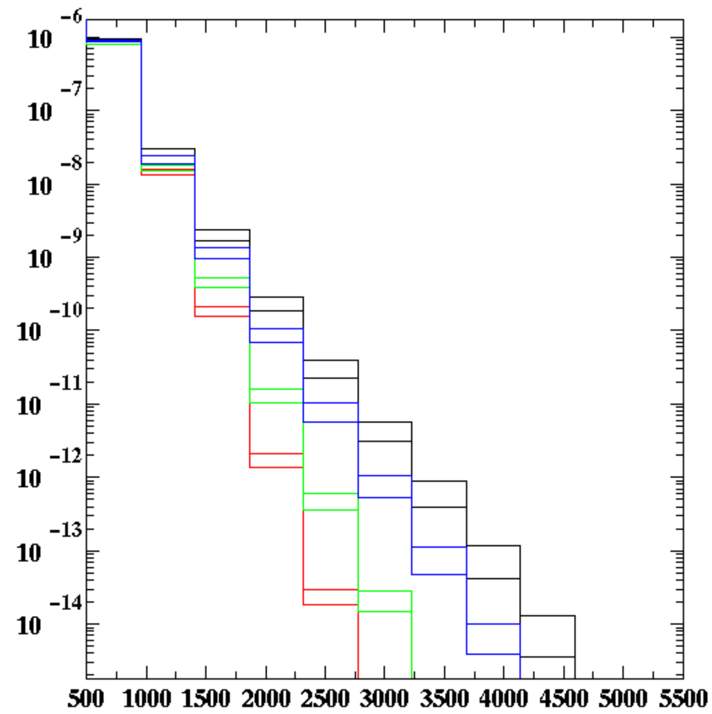
Large extra dimensions

- Sensitivity of LHC dijet cross section to large extra dimensions Ferrag ‘04
  - large extra dimensions accelerate running of α_s as compactification scale M_c is approached
- PDF uncertainties
  - potential sensitivity to M_c reduced from 6 TeV to 2 TeV

$M_c = 2$ TeV no PDF error

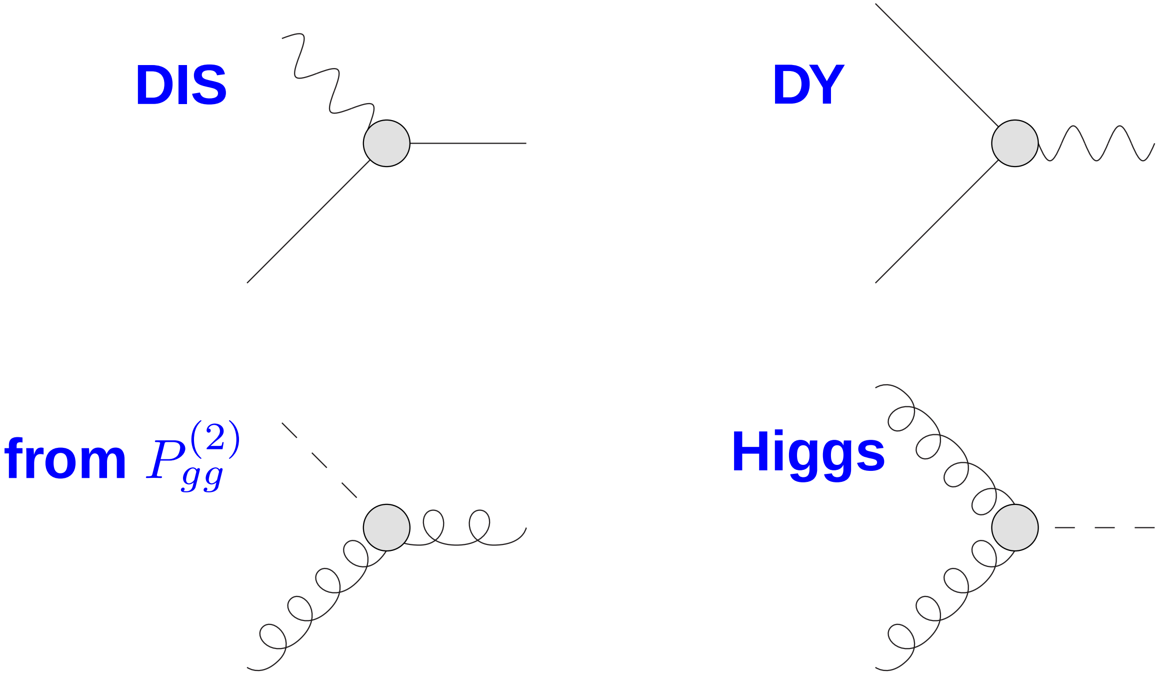


$M_c = 2$ TeV with PDF error



Drell-Yan process and Higgs production

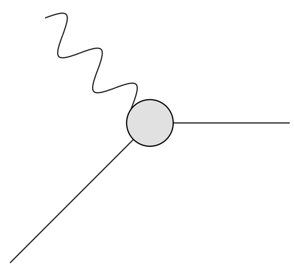
- Mapping of DIS to Drell-Yan lepton-pair production (or to Higgs production in gluon fusion)
  - re-engineering the soft and collinear limit
  - threshold enhanced terms at N^{3}LO (numerically most important)
    S.M., Vogt '05; Laenen, Magnea '05; Idilbi et al. '05



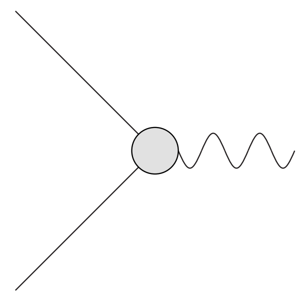
Drell-Yan process and Higgs production

- Mapping of DIS to Drell-Yan lepton-pair production (or to Higgs production in gluon fusion)
  - re-engineering the soft and collinear limit
  - threshold enhanced terms at N^{3}LO (numerically most important)
    S.M., Vogt ‘05; Laenen, Magnea ‘05; Idilbi et al. ‘05

DIS

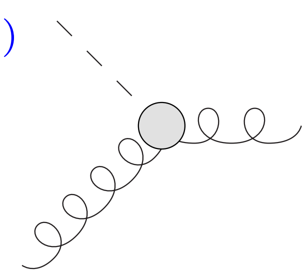


DY

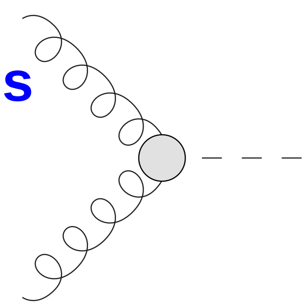


Recycling of theory

from $P_{gg}^{(2)}$



Higgs

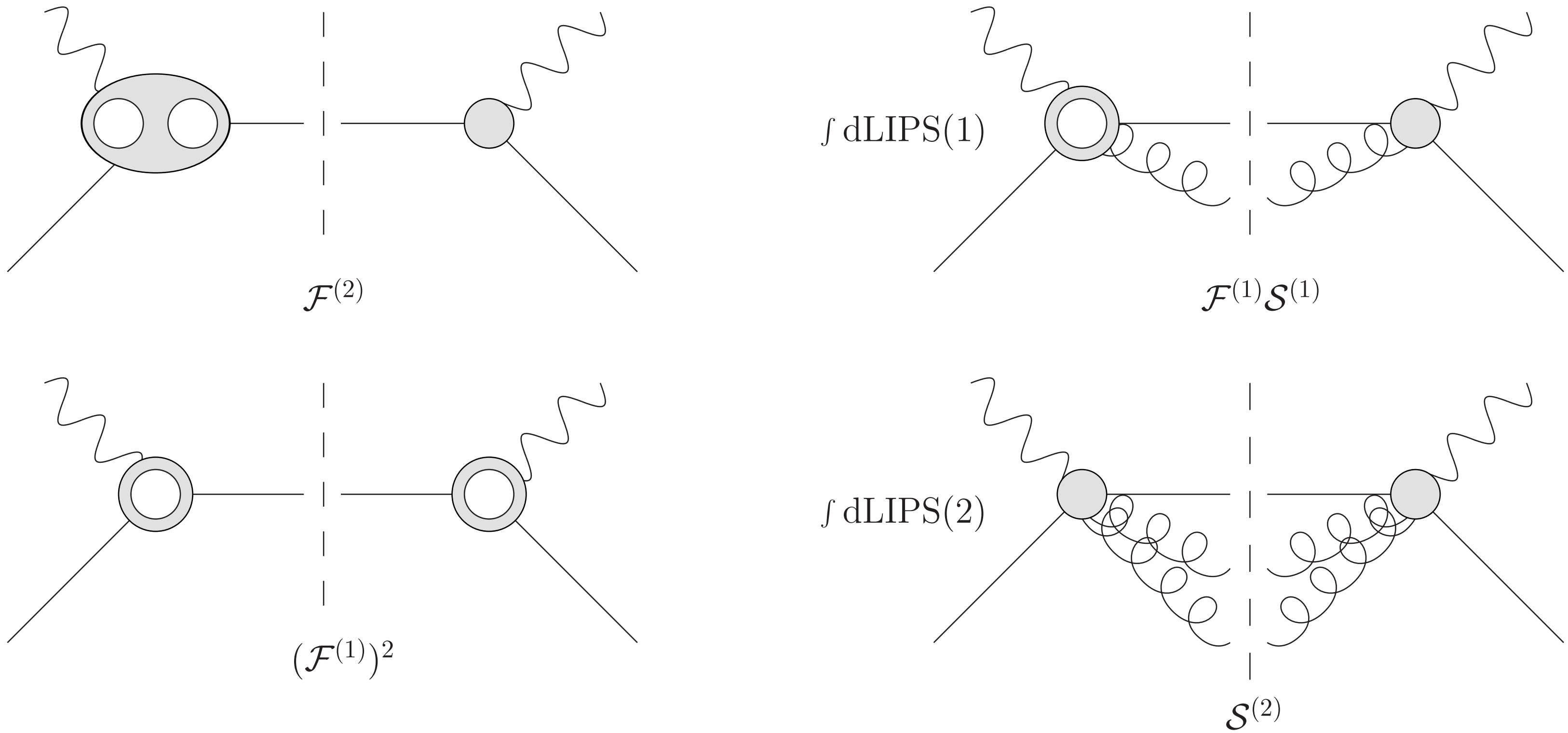


Anatomy of DIS result (1 loop)

$$\mathcal{T}_1^{\mathrm{b}} \quad = \quad 2\,\mathrm{Re}\,\mathcal{F}_1\,\delta(1-x) + \mathcal{S}_1$$

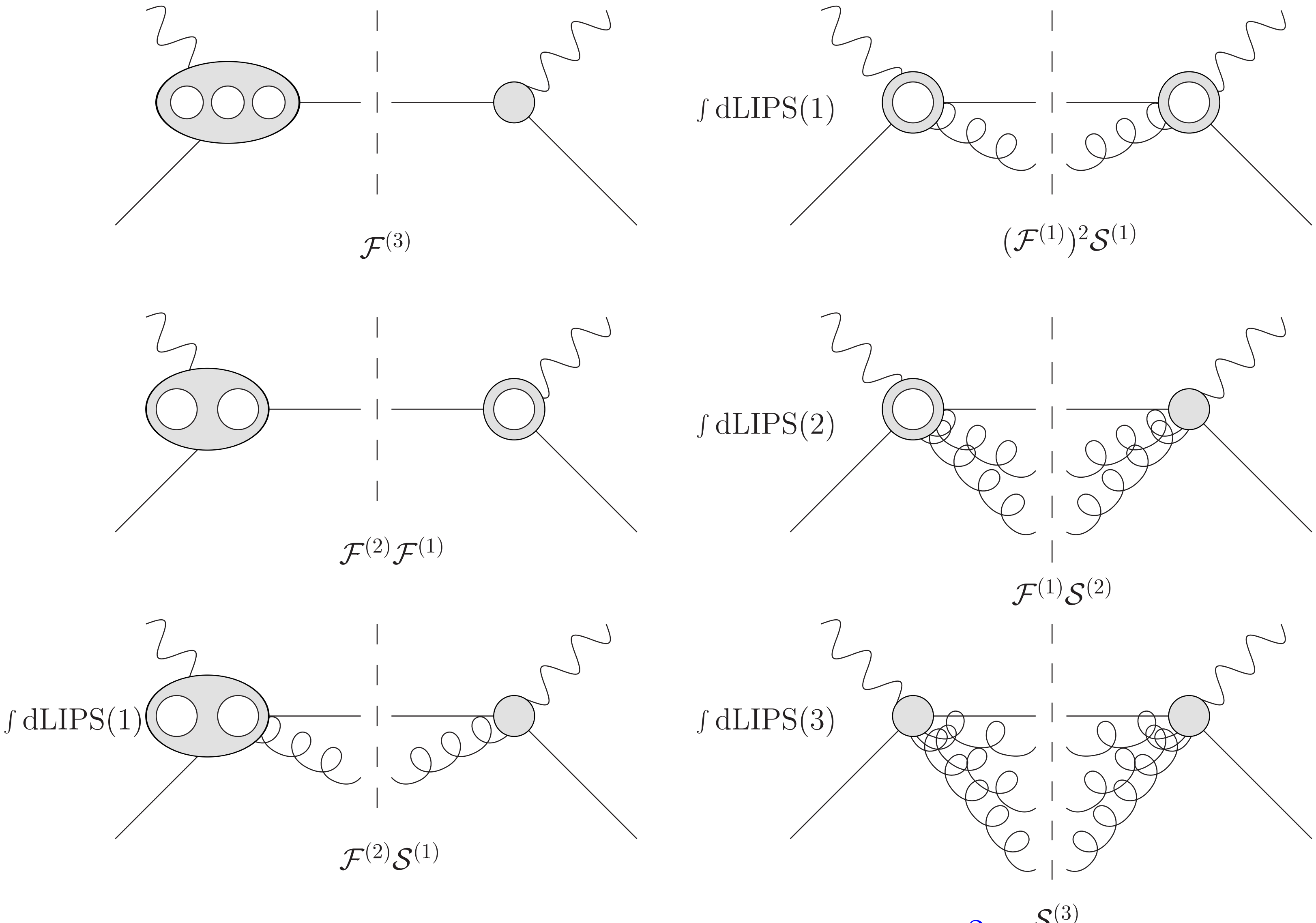
- Forward Compton amplitude $\mathcal{T}_n$ in $D = 4 - 2\epsilon$-dimensions combines
  - virtual corrections $\mathcal{F}_n$ (dependent on $\delta(1-x)$)
  - pure real-emission contributions $\mathcal{S}_n$ (dependent on D-dimensional +-distributions)
- Infrared finiteness implies cancellation of poles between $\mathcal{F}_n$ and $\mathcal{S}_n$
  Kinoshita '62; Lee, Nauenberg '64
- Constructive approach to form factor $\mathcal{F}_n$ and $\mathcal{S}_n$

Anatomy of DIS result (2 loops)



$$\mathcal{T}_2^{\mathrm{b}} \quad = \quad (2\,\mathrm{Re}\,\mathcal{F}_2 + |\mathcal{F}_1|^2)\delta(1-x) + 2\,\mathrm{Re}\,\mathcal{F}_1\mathcal{S}_1 + \mathcal{S}_2$$

Anatomy of DIS result (3 loops)

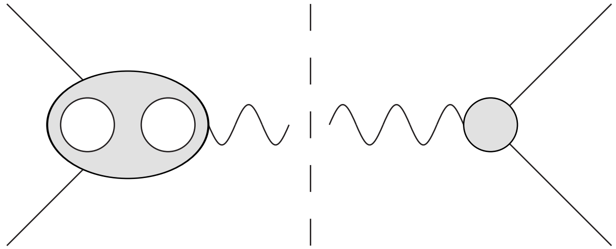


$$\mathcal{T}_3^{\rm b} \;=\; (2\,\text{Re}\,\mathcal{F}_3 + 2\,|\mathcal{F}_1\mathcal{F}_2|)\,\delta(1-x) + (2\,\text{Re}\,\mathcal{F}_2 + |\mathcal{F}_1|^2)\mathcal{S}_1 + 2\,\text{Re}\,\mathcal{F}_1\mathcal{S}_2 + \mathcal{S}_3$$

Drell-Yan (1 loop)

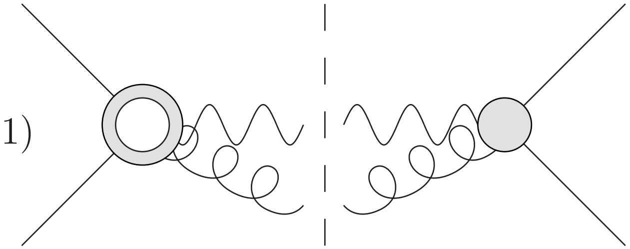
- Construction of cross sections for hadron-hadron scattering
  - form factor with time-like kinematics $Q^2 > 0$
  - soft emission with D-dimensional +-distributions
- Drell-Yan lepton-pair production in $q\bar{q}$-annihilation
- Higgs production from gluon fusion

Drell-Yan (2 loops)

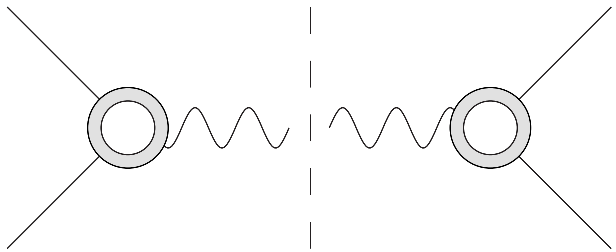


$\mathcal{F}^{(2)}$

$\int \mathrm{dLIPS}(1)$

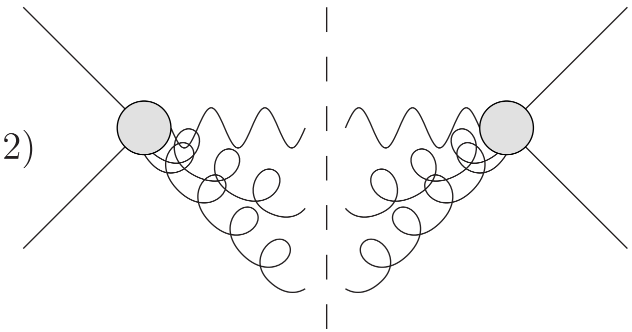


$\mathcal{F}^{(1)}\mathcal{S}^{(1)}$



$(\mathcal{F}^{(1)})^2$

$\int \mathrm{dLIPS}(2)$



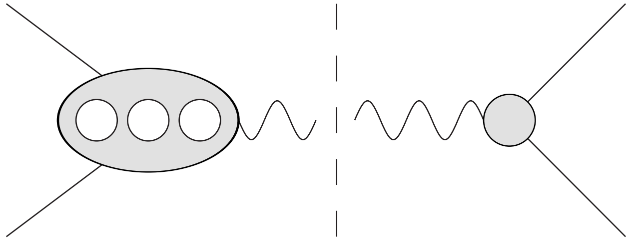
$\mathcal{S}^{(2)}$

- Checks at two loops
  - Drell-Yan

    Hamberg, van Neerven, Matsuura '91; Harlander, Kilgore '02
  - Higgs production

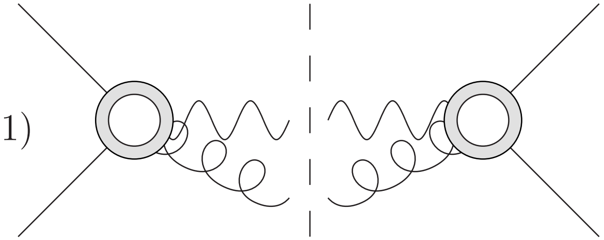
    Harlander, Kilgore '02; Anastasiou, Melnikov '02; Ravindran, Smith, van Neerven '03

Drell-Yan (3 loops)

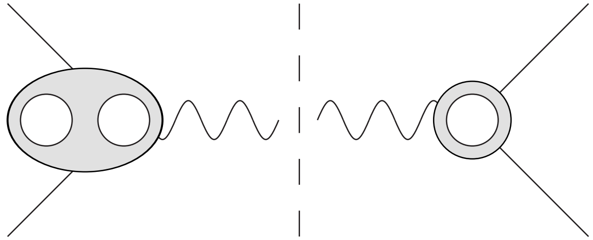


$\mathcal{F}^{(3)}$

$\int \mathrm{dLIPS}(1)$

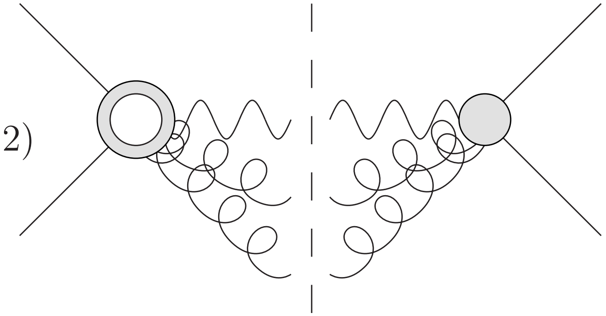


$(\mathcal{F}^{(1)})^2 \mathcal{S}^{(1)}$



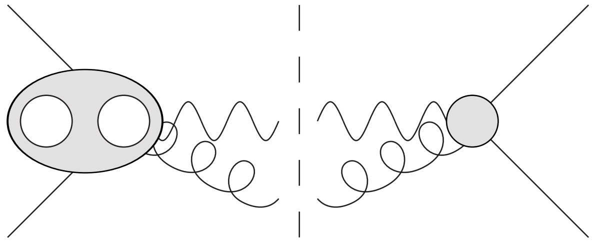
$\mathcal{F}^{(2)} \mathcal{F}^{(1)}$

$\int \mathrm{dLIPS}(2)$



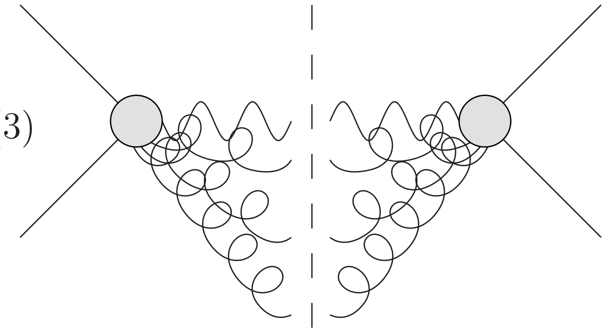
$\mathcal{F}^{(1)} \mathcal{S}^{(2)}$

$\int \mathrm{dLIPS}(1)$



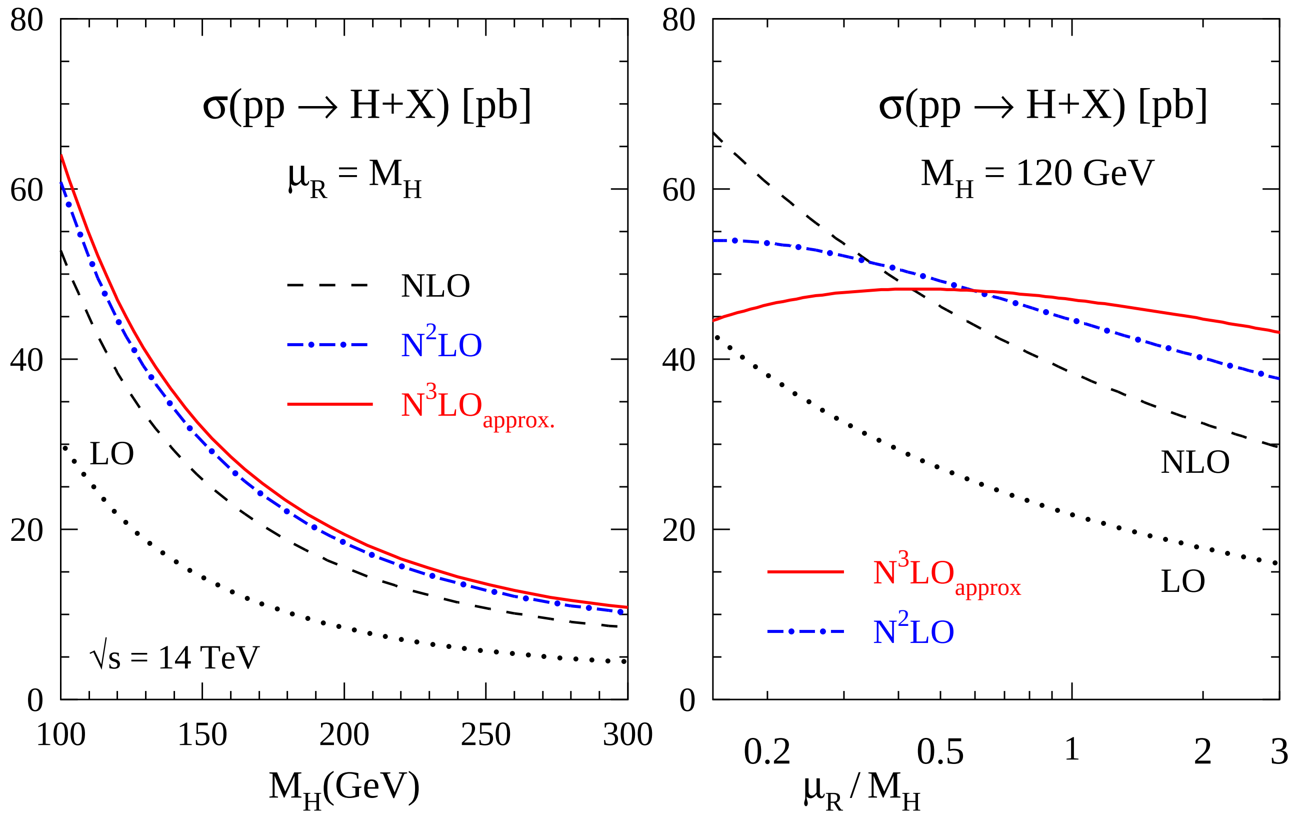
$\mathcal{F}^{(2)} \mathcal{S}^{(1)}$

$\int \mathrm{dLIPS}(3)$



$\mathcal{S}^{(3)}$

Higgs boson production at LHC (II)

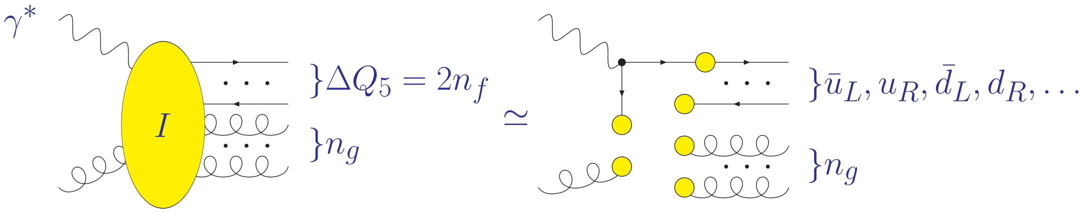


- **N^{3}LO$_{\rm approx.}$** increase at $\mu_r = M_H$ 5% (NNLO PDF's) S.M., Vogt '05
  - μ_r variation 4%
- Overall accuracy of 5% reached with approx. N^{3}LO prediction

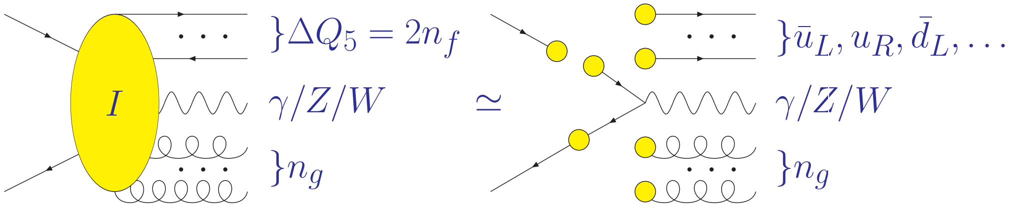
Instanton induced processes at LHC

- Multi-gluon production in instanton background
  - transfer of DIS phenomenology to Drell-Yan lepton-pair production
    Brandenburg, Ringwald, Utermann '06; Petermann, Schrempp '08
  - amplitudes related by crossing

- **DIS**: I-induced photon-gluon scattering



- **pp**: I-induced $q\bar{q}$-annihilation



- Characteristic final state signature
  - isotropic multi-particle production, very high multiplicity

Summary

HERA

- Deep-inelastic scattering (electron-proton collision)
  - wealth of experimental information on proton structure
- QCD precision predictions
  - radiative corrections for parton evolution
- HERA PDFs have strong impact on measurements at LHC

Summary

HERA

- Deep-inelastic scattering (electron-proton collision)
  - wealth of experimental information on proton structure
- QCD precision predictions
  - radiative corrections for parton evolution
- HERA PDFs have strong impact on measurements at LHC

LHC

- Theory results transferred to proton-proton collision
  - Higgs boson production
- Instanton induced multi-particle production

Summary

HERA

- Deep-inelastic scattering (electron-proton collision)
  - wealth of experimental information on proton structure
- QCD precision predictions
  - radiative corrections for parton evolution
- HERA PDFs have strong impact on measurements at LHC

LHC

- Theory results transferred to proton-proton collision
  - Higgs boson production
- Instanton induced multi-particle production

Outlook

- If past performance is an indicator, we are well prepared for future challenges . . .