# **Deep-inelastic scattering** – *from HERA to LHC* –

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Deep-inelastic scattering – p.1

## **Proton colliders**

- HERA: deep structure of proton at highest  $Q^2$  and smallest x
- LHC: Higgs boson search at highest energies  $\sqrt{S} = 14$ TeV



- Quantum Chromodynamics (QCD) ubiquitous at proton colliders
  - reliable understanding essential for precision and discovery physics

## **Perturbative QCD at colliders**

Hard scattering at hadron colliders

• constituent partons from incoming hadrons interact at short distance (large momentum transfer  $Q^2$ )





- QCD factorization
  - separate sensitivity to dynamics from different scales
- DIS: photon momentum  $Q^2 = -q^2$ , Bjorken's  $x = Q^2/(2p \cdot q)$

• **pp**: energy  $Q^2 = 2p_1 \cdot p_2$ , ratio to W/Z-boson mass  $x = M_{W/Z}^2/Q^2$ 

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#### Hard scattering cross section

QCD factorization

$$\sigma_{had} = \begin{cases} \sum_{i} f_{i} \otimes \hat{\sigma}_{\gamma i \to X} \left( \alpha_{s}(\mu^{2}), Q^{2}, \mu^{2} \right) & \text{DIS} \\ \sum_{ij} f_{i} \otimes f_{j} \otimes \hat{\sigma}_{ij \to X} \left( \alpha_{s}(\mu^{2}), Q^{2}, \mu^{2} \right) & \text{pp} \end{cases}$$

- Parton cross sections  $\hat{\sigma}_{\gamma i \to X}$  and  $\hat{\sigma}_{ij \to X}$  calculable pertubatively in powers of  $\alpha_s$ 
  - short distance interaction of constituent partons
- Parton distributions  $f_i \rightarrow$  parton luminosity
  - proton: very complicated multi-particle bound state
  - colliders: wide-band beams of quarks and gluons
- Standard approach to uncertainties in theoretical predictions

• variation of factorization scale 
$$\mu$$
:  $\frac{d}{d \ln \mu^2} \sigma_{had} = O(\alpha_s^{l+1})$ 



## **Asymptotic freedom of QCD**



• At large scales: application of perturbation theory (but  $\alpha_s \gg \alpha_{QED}$ )

#### Approaches to the calculation of $\sigma_{had}$

- LO (leading order)
  - Automated tree level calculations in Standard Model, MSSM, .... (Madgraph, Alpgen, CompHEP, ...)
  - LO + parton shower
  - String inspired techniques
- NLO (next-to-leading order)
  - Analytical (or numerical) calculations of diagrams yield parton level Monte Carlos (NLOJET++, MCFM, ...)
  - NLO + parton shower (MC@NLO, VINCIA)
- NNLO (next-to-next-to-leading order)
  - selected results known (mostly inclusive kinematics)
- N<sup>3</sup>LO (next-to-next-to-next-to-leading order)
  - very few ...

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## **Parton luminosity**



Feynman diagrams in leading order



• Proton in resolution  $1/Q \rightarrow$ sensitive to lower momentum partons





## **Parton luminosity**



Feynman diagrams in leading order



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- Evolution equations for parton distributions  $f_i$ 
  - predictions from fits to reference processes (universality)

$$\frac{d}{d\ln\mu^2} f_i(x,\mu^2) = \sum_k \left[ P_{ik}(\alpha_s(\mu^2)) \otimes f_k(\mu^2) \right](x)$$

Splitting functions *P* 

$$P = \alpha_{s} P^{(0)} + \alpha_{s}^{2} P^{(1)} + \alpha_{s}^{3} P^{(2)} + \dots$$

NLO: standard approximation (large uncertainties)

#### **Parton distributions in proton**

• Valence  $q - \bar{q}$  (additive quantum numbers) sea (part with  $q + \bar{q}$ )



- Parameterization (bulk of data from deep-inelastic scattering)
  - structure function  $F_2 \longrightarrow$  quark distribution
  - scale evolution (perturbative QCD)  $\longrightarrow$  gluon distribution

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#### PDFs from HERA to LHC

• HERA  $\rightarrow$  LHC: scale evolution in  $Q^2$  over three orders of magnitude

## **Higgs boson production at LHC (I)**

• Dominant channel  $gg \rightarrow H + X$  via top-quark loop



• Estimate of uncertainty: apparent convergence, variation of scale  $\mu$ 

NLO approximation insufficient for reliable predictions

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QQ

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## **Our calculation in deep-inelastic scattering**



## **Our calculation in deep-inelastic scattering**



- more than 10 FTE years and a few CPU years
  - computer algebra updates: Form  $\rightarrow$  Form 3.1  $\rightarrow$  Form 3.2  $\rightarrow$  ...
  - $> 10^5$  tabulated symbolic integrals ( > 3GB)

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## **Splitting functions for a quarter of a century**

$$\begin{split} P_{\rm ns}^{(0)}(x) &= C_F(2p_{\rm qq}(x) + 3\delta(1-x)) \\ P_{\rm ps}^{(0)}(x) &= 0 \\ P_{\rm qg}^{(0)}(x) &= 2n_f p_{\rm qg}(x) \\ P_{\rm gq}^{(0)}(x) &= 2C_F p_{\rm gq}(x) \\ P_{\rm gg}^{(0)}(x) &= C_A \Big( 4p_{\rm gg}(x) + \frac{11}{3}\delta(1-x) \Big) - \frac{2}{3}n_f \delta(1-x) \end{split}$$

1973



"for the discovery of asymptotic freedom in the theory of the strong interaction"





David J. Gross

H. David Politzer Frank

Frank Wilczek

$$\begin{split} P_{\rm ns}^{(1)+}(x) &= 4C_AC_F\left(p_{\rm qq}(x)\Big[\frac{67}{18} - \zeta_2 + \frac{11}{6}H_0 + H_{0,0}\Big] + p_{\rm qq}(-x)\Big[\zeta_2 + 2H_{-1,0} - H_{0,0}\Big] \\ &+ \frac{14}{3}(1-x) + \delta(1-x)\Big[\frac{17}{24} + \frac{11}{3}\zeta_2 - 3\zeta_3\Big]\Big) - 4C_Fn_f\left(p_{\rm qq}(x)\Big[\frac{5}{9} + \frac{1}{3}H_0\Big] + \frac{2}{3}(1-x) \\ &+ \delta(1-x)\Big[\frac{1}{12} + \frac{2}{3}\zeta_2\Big]\Big) + 4C_F^2\Big(2p_{\rm qq}(x)\Big[H_{1,0} - \frac{3}{4}H_0 + H_2\Big] - 2p_{\rm qq}(-x)\Big[\zeta_2 + 2H_{-1,0} \\ &- H_{0,0}\Big] - (1-x)\Big[1 - \frac{3}{2}H_0\Big] - H_0 - (1+x)H_{0,0} + \delta(1-x)\Big[\frac{3}{8} - 3\zeta_2 + 6\zeta_3\Big]\Big) \\ P_{\rm ns}^{(1)-}(x) &= P_{\rm ns}^{(1)+}(x) + 16C_F\Big(C_F - \frac{C_A}{2}\Big)\Big(p_{\rm qq}(-x)\Big[\zeta_2 + 2H_{-1,0} - H_{0,0}\Big] - 2(1-x) \\ &- (1+x)H_0\Big) \\ P_{\rm ps}^{(1)}(x) &= 4C_Fn_f\Big(\frac{20}{9}\frac{1}{x} - 2 + 6x - 4H_0 + x^2\Big[\frac{8}{3}H_0 - \frac{56}{9}\Big] + (1+x)\Big[5H_0 - 2H_{0,0}\Big]\Big) \\ P_{\rm qg}^{(1)}(x) &= 4C_An_f\Big(\frac{20}{9}\frac{1}{x} - 2 + 25x - 2p_{\rm qg}(-x)H_{-1,0} - 2p_{\rm qg}(x)H_{1,1} + x^2\Big[\frac{44}{3}H_0 - \frac{218}{9}\Big] \\ &+ 4(1-x)\Big[H_{0,0} - 2H_0 + xH_1\Big] - 4\zeta_2x - 6H_{0,0} + 9H_0\Big) + 4C_Fn_f\Big(2p_{\rm qg}(x)\Big[H_{1,0} + H_{1,1} + H_2\Big] \end{split}$$

 $P_{gq}^{(1)}(x) = 4C_{A}C_{F}\left(\frac{1}{x} + 2p_{gq}(x)\left[H_{1,0} + H_{1,1} + H_{2} - \frac{11}{6}H_{1}\right] - x^{2}\left[\frac{8}{3}H_{0} - \frac{44}{9}\right] + 4\zeta_{2} - 2$ -7H<sub>0</sub> + 2H<sub>0,0</sub> - 2H<sub>1</sub>x + (1 + x)\left[2H\_{0,0} - 5H\_{0} + \frac{37}{9}\right] - 2p\_{gq}(-x)H\_{-1,0}\right) - 4C\_{F}n\_{f}\left(\frac{2}{3}x - p\_{gq}(x)\left[\frac{2}{3}H\_{1} - \frac{10}{9}\right]\right) + 4C\_{F}^{2}\left(p\_{gq}(x)\left[3H\_{1} - 2H\_{1,1}\right] + (1 + x)\left[H\_{0,0} - \frac{7}{2} + \frac{7}{2}H\_{0}\right] - 3H\_{0,0} + 1 - \frac{3}{2}H\_{0} + 2H\_{1}x\right)

 $-\zeta_{2}$  + 4x<sup>2</sup>  $\left[H_{0} + H_{0,0} + \frac{5}{2}\right]$  + 2(1-x)  $\left[H_{0} + H_{0,0} - 2xH_{1} + \frac{29}{4}\right] - \frac{15}{2} - H_{0,0} - \frac{1}{2}H_{0}$ 

$$\begin{split} P_{gg}^{(1)}(x) &= 4C_A n_f \left( 1 - x - \frac{10}{9} p_{gg}(x) - \frac{13}{9} \left( \frac{1}{x} - x^2 \right) - \frac{2}{3} (1 + x) H_0 - \frac{2}{3} \delta(1 - x) \right) + 4C_A^2 \left( 27 + (1 + x) \left[ \frac{11}{3} H_0 + 8H_{0,0} - \frac{27}{2} \right] + 2p_{gg}(-x) \left[ H_{0,0} - 2H_{-1,0} - \zeta_2 \right] - \frac{67}{9} \left( \frac{1}{x} - x^2 \right) - 12H_0 \\ &- \frac{44}{3} x^2 H_0 + 2p_{gg}(x) \left[ \frac{67}{18} - \zeta_2 + H_{0,0} + 2H_{1,0} + 2H_2 \right] + \delta(1 - x) \left[ \frac{8}{3} + 3\zeta_3 \right] \right) + 4C_F n_f \left( 2H_0 + \frac{2}{3} \frac{1}{x} + \frac{10}{3} x^2 - 12 + (1 + x) \left[ 4 - 5H_0 - 2H_{0,0} \right] - \frac{1}{2} \delta(1 - x) \right) . \end{split}$$

#### NNLO splitting functions S.M. Vermaseren, Vogt

 $P_{pi}^{(2)}(x) = 16C_A C_F n_f \left(\frac{4}{3}(\frac{1}{x} + x^2) \left[\frac{13}{3}H_{-1,0} - \frac{14}{9}H_0 + \frac{1}{2}H_{-1}\zeta_2 - H_{-1,-1,0} - 2H_{-1,0,0}\right]$  $-H_{-1,2}] + \frac{2}{3} \left(\frac{1}{x} - x^2\right) \left[\frac{16}{3} \zeta_2 + H_{2,1} + 9\zeta_3 + \frac{9}{4} H_{1,0} - \frac{676}{216} + \frac{571}{22} H_1 + \frac{10}{4} H_2 + H_1\zeta_2 - \frac{1}{6} H_{1,1}\right]$  $-3H_{1,0,0} + 2H_{1,1,0} + 2H_{1,1,1} + (1-x) \left[ \frac{182}{9} H_1 + \frac{158}{3} + \frac{397}{36} H_{0,0} - \frac{13}{2} H_{-2,0} + 3H_{0,0,0} \right]$  $+\frac{13}{6}H_{1,0}+3xH_{1,0}+H_{-3,0}+H_{-2}\zeta_{2}+2H_{-2,-1,0}+3H_{-2,0,0}+\frac{1}{2}H_{0,0}\zeta_{2}+\frac{1}{2}H_{1}\zeta_{2}-\frac{9}{4}H_{1,0,0}$  $-\frac{3}{4}H_{1,1}+H_{1,1,0}+H_{1,1,1}\right]+(1+x)\left[\frac{7}{12}H_0\zeta_2+\frac{31}{6}\zeta_3+\frac{91}{18}H_2+\frac{71}{12}H_3+\frac{113}{18}\zeta_2-\frac{426}{27}H_0\right]$  $+ \frac{3}{5} H_{2,0} + \frac{1}{3} H_{-1,0} + 6x H_{-1,0} + \frac{31}{6} H_{0,0,0} - \frac{17}{6} H_{2,1} + \frac{117}{20} \zeta_2^2 + 9 H_0 \zeta_3 + \frac{5}{2} H_{-1} \zeta_2 + 2 H_{2,1,0}$  $+\frac{1}{2}H_{-1,0,0}-2H_{-1,2}+H_2\zeta_2-\frac{7}{2}H_{2,0,0}+H_{-1,-1,0}+2H_{2,1,1}+H_{3,1}-\frac{1}{2}H_4\Big]+5H_{-2,0}+H_{2,1}$  $+H_{0,0,0,0} - \frac{1}{5}\zeta_2^2 + 4H_{-3,0} + 4H_0\zeta_3 - \frac{32}{9}H_{0,0} - \frac{29}{17}H_0 - \frac{235}{12}\zeta_2 - \frac{511}{12} - \frac{97}{12}H_1 + \frac{33}{4}H_2 - H_3$  $-\frac{11}{2}H_0\zeta_2 - \frac{11}{2}\zeta_3 - \frac{3}{2}H_{2,0} - 10H_{0,0,0} + \frac{2}{3}x^2 \Big[\frac{83}{4}H_{0,0} - \frac{243}{4}H_0 + 10\zeta_2 + \frac{11}{8H} + \frac{9}{8}H_1 - \frac{4}{3}H_2$  $-4\zeta_3 - H_0\zeta_2 + H_3 + H_{2,0} - 6H_{-2,0}$ ) +  $16C_F n_f^2 (\frac{2}{2\pi}H_0 - 2 - H_2 + \zeta_2 + \frac{2}{3}x^2 |H_2 - \zeta_2 + 3|$  $-\frac{19}{6}H_0$  +  $\frac{2}{9}(\frac{1}{x}-x^2)[H_{1,1}+\frac{5}{3}H_1+\frac{2}{3}] + (1-x)[\frac{1}{6}H_{1,1}-\frac{7}{6}H_1+xH_1+\frac{35}{27}H_0+\frac{185}{54}]$  $+\frac{1}{2}(1+x)\left[\frac{4}{2}H_2-\frac{4}{2}\zeta_2+\zeta_3+H_{2,1}-2H_3+2H_0\zeta_2+\frac{29}{4}H_{0,0}+H_{0,0,0}\right]\right)+16C_F^2n_f\left(\frac{85}{12}H_1-2H_1+2H_0\zeta_2+\frac{29}{4}H_{0,0}+H_{0,0,0}\right]$  $\begin{array}{c} 3^{(1)}_{-} (13^{(1)}_{-}) & 3^{(2)}_{-} (13^{(1)}_{-}) & 3^{(2)}_{-} (13^{(2)}_{-}) & 5^{(2)}_{$  $+\frac{22}{3}H_{0,0,0}\left]+\frac{4}{3}\left(\frac{1}{r}-r^{2}\right)\left[\frac{23}{12}H_{1,0}-\frac{523}{144}H_{1}-3\zeta_{3}+\frac{55}{16}+\frac{1}{2}H_{1,0,0}+H_{1,1}-H_{1,1,0}-H_{1,1,1}\right]$  $+(1-x)\left[\frac{1}{2}H_{1,0,0}+\frac{7}{12}H_{1,1}-\frac{2743}{22}H_0-\frac{53}{12}H_{0,0}-\frac{251}{22}H_1-\frac{5}{4}\zeta_2+\frac{5}{4}H_2-\frac{8}{2}H_{1,0}+3xH_{1,0}-\frac{5}{4}G_2+\frac{5}{4}H_2-\frac{8}{4}H_2+\frac{5}{4}$  $+3H_0\zeta_2 - 3H_3 - H_{1,1,0} - H_{1,1,1} + (1+x) \left[ \frac{169}{124} + \frac{5}{2}H_{0,0,0} + 4H_{2,1} + 7H_{2,0} + 10x\zeta_3 - \frac{37}{10}\zeta_2^{-2} \right]$  $-7H_0\zeta_3 + 6H_{0,0}\zeta_2 - 4H_{0,0,0,0} + H_{2,0,0} - 2H_{2,1,0} - 2H_{2,1,1} - 4H_{3,0} - H_{3,1} - 6H_4$ 

$$\begin{split} P^{(2)}_{41}(x) &= 16 C_{40} C_{\gamma} n_{\ell} \left( \rho_{40}(x) \Big| \frac{39}{21} H_1 \xi_3 - 4 H_{1,1,1} + 3 H_{2,0,0} - \frac{15}{4} H_{1,2} + \frac{9}{4} H_{1,1,0} + 3 H_{2,1,0} \right. \\ + H_0 \xi_3 - 2 H_{2,1,1} + 4 H_1 \xi_2 - \frac{172}{12} H_0 \xi_3 - \frac{551}{72} H_{0,0} + \frac{64}{3} \xi_3 - \xi_3^2 - \frac{49}{9} H_{2,-} - \frac{3}{2} H_{1,0,0,0} - \frac{1}{3} H_{1,0,0} \\ - \frac{382}{72} H_{1,0} - \frac{31}{2} H_{1,1} - \frac{113}{12} H_1 + \frac{49}{4} H_{2,0} + \xi_3 H_{\xi_3} + \frac{9}{6} H_{0,0,0} + \frac{172}{12} H_3 - \frac{1259}{32} + \frac{2833}{216} H_0 \\ + 0 H_{2,1} + 3 H_{1,-2} + 9 H_{1,0} \xi_3 + 4 H_{1,1,0,0} + 3 H_{1,1,0,0} - 4 H_{1,1,1} - 3 H_{1,1,0} - 3 H_{1,1,1} - 0 H_{1,2,1} - 0 H_{1,2,$$

 $-2H_{3,0} - \frac{13}{2}H_0\zeta_2 - 13H_{-3,0} - \frac{13}{2}H_{3,1} + \frac{15}{2}H_3 - \frac{2005}{64} + \frac{157}{4}\zeta_2 + 8\zeta_3 + \frac{1291}{432}H_1 + \frac{55}{12}H_{1,1}$  $+\frac{3}{2}H_{2}+\frac{1}{2}H_{2,1}+\frac{27}{4}H_{-1,0}-\frac{11}{2}H_{1,0,0}-8H_{2,0,0}-4\zeta_{2}^{2}+\frac{3}{2}H_{1,2}-H_{2,2}+\frac{5}{2}H_{1}\zeta_{2}+8H_{-1,-1,0}$  $+4H_{2,0}+\frac{3}{2}H_{2,1,1}-H_{-1}\zeta_{2}+7H_{2}\zeta_{2}+6H_{-2}\zeta_{2}+12H_{-2,-1,0}-6H_{-2,0,0}+x\Big[3H_{1,1,1}-H_{0,0}\zeta_{2}+2H_{-2,0,0}+2H_{-2,$  $+\frac{9}{2}H_{-1,0,0}-\frac{35}{8}H_{1,0}+2H_4+3H_{1,1,0}+H_{-1,2}\right)+16C_A^2C_F\left(x^2\left[\frac{2}{3}H_1\zeta_2-\frac{2105}{81}-\frac{77}{18}H_{0,0}-\frac{1}{3}H_{1,0}+H_{-1,2}\right]\right)$  $-6H_3 + \frac{16}{3}\zeta_3 - 10H_{-1,0} - \frac{14}{3}H_{2,0} - \frac{2}{3}H_{-1}\zeta_2 - \frac{14}{3}H_{0,0,0} + \frac{104}{9}H_2 - \frac{4}{3}H_{1,0,0} + \frac{37}{9}H_{1,1}$  $+\frac{4}{3}H_{-1,-1,0}-\frac{104}{9}\zeta_2-\frac{8}{3}H_{2,1}+\frac{145}{18}H_{1,0}+\frac{4}{3}H_{-1,2}+\frac{2}{3}H_{1,1,1}-\frac{109}{27}H_1+\frac{8}{3}H_{-1,0,0}+6H_0\zeta_2$  $\begin{array}{c} 3 \\ +4H_{-2,0} + \frac{584}{27}H_0 \\ \end{array} + p_{gq}(x) \Big[ \frac{7}{2}H_1\zeta_3 + \frac{138305}{2592} - \frac{1}{3}H_{2,0} + \frac{13}{4}H_{-1}\zeta_2 + 2H_{2,1,1} + \frac{11}{2}H_{1,0,0} \\ \end{array}$  $+4H_{3,1} - \frac{43}{6}H_{1,1,1} - \frac{109}{12}\zeta_2 - \frac{17}{3}H_{2,1} - \frac{71}{24}H_{1,0} - \frac{11}{6}H_{-2,0} - \frac{21}{2}\zeta_3 + \frac{3}{2}H_{1,0,0,0} - H_{1,-2,0}$  $+\frac{395}{54}H_0-2H_{1,0}\zeta_2-H_{1,1}\zeta_2-\frac{55}{12}H_{1,1,0}+2H_{1,1,0,0}+4H_{1,1,1,0}+2H_{1,1,1,1}+4H_{1,1,2}-\frac{55}{12}H_{1,2}$  $+6H_{1,2,0} + 4H_{1,2,1} + 4H_{1,3} + 3H_{2,1,0} + 3H_{2,2} + p_{10}(-x) \left[\frac{23}{2}H_{-1}\zeta_3 + 5H_{-2}\zeta_2 + 2H_{-2,-1,0}\right]$  $+\frac{109}{12}H_{-1,0}+H_0\zeta_3+\frac{17}{5}\zeta_2{}^2+\frac{1}{6}H_1\zeta_2+2H_2\zeta_2-\frac{65}{74}H_{1,1}-\frac{19}{2}H_{-1,-1,0}-4H_{3,0}-3H_{2,0,0}$  $\begin{array}{c} 12 \\ -7H_{-2,0,0} - \frac{3}{2}H_{-1,2} + \frac{3379}{216}H_{1} - 4H_{-2,2} - \frac{49}{6}H_{-1,0,0} - \frac{11}{2}H_{-1,0,0,0} - 13H_{-1,-1}\zeta_{2} - 8H_{-1,3} \end{array}$  $-7H_{1}\zeta_{2} + \frac{97}{12}H_{1,0,0} + \frac{10}{3}H_{-1,0,0} + \frac{245}{12}H_{3} - 8H_{0,0,0,0} + (1+x)\left[4H_{3,1} - H_{2,1,1} + \frac{29}{6}H_{-1,2}\right]$  $+\frac{17}{6}H_{-2,0}-12H_{2,0}-\frac{31}{12}H_{2,1}+\frac{1}{9}H_{2,0,0}-H_2\zeta_2+\frac{61}{26}H_{1,0}-4H_0\zeta_3-\frac{13}{2}H_{-1}\zeta_2-\frac{46}{2}H_{-1,-1,0}$  $+\frac{\frac{25}{4}}{4}H_4+\frac{93}{4}H_0\zeta_2-\frac{55}{9}H_{1,1}-\frac{71}{18}H_2+\frac{49}{18}H_{0,0}-\frac{13}{2}H_{0,0}\zeta_2-\frac{47}{40}\zeta_2^{-2}\Big]+\frac{6131}{2502}-\frac{31}{2}H_{-2}\zeta_2$  $-\frac{67}{40}\zeta_2^2 + \frac{29}{6}H_{-1,2} - H_{-1,0} + 8H_{-2,2} + 25H_0\zeta_2 + \frac{412}{9}H_1 + \frac{928}{9}H_0 + \frac{1}{4}H_4 - 65H_3 - 38H_{0,0}$  $-9H_{-3,0} - \frac{17}{2}H_{0,0,0} + x \Big[\frac{27}{2}H_{-1,0} - \frac{1}{5}H_{0,0,0,0} + \frac{3}{4}H_{0,0}\zeta_2 + \frac{1}{5}H_{-3,0} - 14H_{0,0,0} + \frac{1}{12}H_{1,1,1} + \frac{1}{5}H_{1,0,0,0} + \frac{1}{12}H_{0,0,0} + \frac{1}{12}H_{0,0$  $\begin{array}{c} -\frac{43}{36}\zeta_2-\frac{1}{2}H_1\zeta_2+\frac{72}{72}H_0+\frac{749}{54}H_1+\frac{135}{4}\zeta_3+\frac{97}{24}H_{1,0}+\frac{43}{12}H_1\zeta_2-\frac{85}{12}H_{-1,\zeta_2}-\frac{13}{3}H_{1,0,0}\\ +\frac{43}{12}H_2+\frac{39}{4}H_{1,1}-2H_{3,1}+\frac{13}{6}H_{-1,-1,0}+\frac{7}{4}H_{2,0,0}-4H_{1,1,0}-4H_{1,2} \Big) +16C_Fn_f^2\Big(\frac{1}{9}-\frac{1}{9\frac{1}{2}}\right) \\ \end{array}$  $+\frac{2}{6}x - \frac{1}{6}xH_1 + \frac{1}{6}p_{\mathbb{R}^{\mathbb{Q}}}(x)\left[H_{1,1} - \frac{5}{3}H_1\right] + 16C_F^2n_f\left(\frac{4}{6}x^2\left[H_{0,0} - \frac{11}{6}H_0 - \frac{7}{2} + H_{-1,0}\right]\right]$ 

 $\begin{array}{l} -6H_{1,3}+\frac{49}{4}\zeta_2\Big]+\rho_{4g}(-x)\Big[\frac{17}{2}H_{-1}\zeta_3-\frac{5}{2}H_{-1,-1,0}-\frac{5}{2}H_{-1,2}-\frac{9}{2}H_{-1,0}+\frac{5}{2}H_{-2,0}+\frac{3}{2}H_{-1,0,0}\\ -2H_{3,1}-2H_{4}-6H_{-2,2}+6H_{-2,-1,0}-6H_{-2,0,0}+2H_{0}\zeta_2+9H_{-2}\zeta_2+3H_{-1,-2,0}-2H_{-1,2,1}\\ \end{array}$  $-6H_{-1,-1,-1,0} + 6H_{-1,-1,0,0} + 6H_{-1,-1,2} + 9H_{-1,0}\zeta_2 - 9H_{-1,-1}\zeta_2 - 2H_{-1,2,0} - \frac{11}{2}H_{-1,0,0,0}$  $-6H_{-1,3}$  +  $(\frac{1}{2}-x^2)\left[\frac{55}{12}-4\zeta_3+\frac{23}{9}H_{1,0}-\frac{4}{2}H_{1,1,0}\right]$  +  $(\frac{1}{2}+x^2)\left[\frac{2}{2}H_{1,0,0}-\frac{371}{109}H_1+\frac{23}{9}H_{1,1}\right]$  $-\frac{1189}{100}H_1 - \frac{67}{24}H_{2,1} - 29H_{2,0} - \frac{949}{26}\zeta_2 - \frac{67}{2}H_{0,0,0} - \frac{142}{2}H_3 + \frac{215}{22} - \frac{3989}{48}H_0 + 2H_{-3,0}$ +(1+x) $\begin{bmatrix}H_{-1,0,0} - 10H_{-2}\zeta_2 + 6H_{-2,0,0} + 2H_{0,0}\zeta_2 - 9H_{-1,-1,0} - 7H_{-1,2} - 9H_{-2,0} - 2H_{3,1}\end{bmatrix}$  $-4H_{-2,-1,0} - 4H_4 - 4H_{3,0} - 4H_{0,0,0,0} + \frac{37}{2}H_{-1,0} + \frac{5}{2}(1+x)H_{-1}\zeta_2 - 4H_{-2,0,0} + 2H_{0,0}\zeta_2$  $+H_{2}\zeta_{2}-3H_{1,1,0}+2H_{0,0,0,0}+H_{-3,0}-9H_{2,1,0}-\frac{9}{2}H_{2,1,1}+\frac{11}{2}H_{1,1,1}+\frac{19}{2}H_{2,0,0}+\frac{9}{2}H_{1,1,0}+\frac{9}{2}H_{2,0,0}+\frac{9}{2}H_{1,1,0}+\frac{9}{2}H_{2,0,0}+\frac{9}{2}H_{1,1,0}+\frac{9}{2}H_{2,0,$  $+\frac{1}{2}\frac{1}{2}\frac{1}{2}-\frac{1}{2}\frac{1}{1}\frac{1}{1}\frac{1}{2}+\frac{1}{2}\frac{1}{1}\frac{1}{2}\frac{1}$  $\begin{array}{c} 2 & 0 & 0 \\ -2 & 0 & 0 \\ -175 & 0 & 9 \\ -175 & 0 & 9 \\ -175 & 0 & 9 \\ -175 & 0 & 9 \\ -175 & 0 & 0 \\ -17$  $\frac{4}{12} + \frac{9}{12} + \frac{4}{12} + \frac{192}{12} + \frac{48}{12} + \frac{1}{12} + \frac{1}{1$  $\frac{13}{18}H_0 + \frac{17}{6}H_{0,0} - \zeta_3 + \frac{11}{18}\zeta_2 - \frac{139}{108} + \frac{1}{3}p_{qg}(-x)H_{-1,0,0} - \frac{53}{162}(\frac{1}{x} - x^2) - \frac{2}{9}(1-x)\Big[6H_{0,0,0} - \frac{13}{16}(\frac{1}{x} - x^2) - \frac{2}{9}(1-x)\Big] + \frac{10}{16}H_{0,0,0} - \frac{10}{16}H_{0,0,0} \frac{7}{6}xH_{1}-H_{0,0}+\frac{7}{2}xH_{1,1}]+\frac{7}{6}x(1+x)H_{-1,0}+\frac{7}{4}H_{0}-\frac{19}{54}H_{1}+H_{0,0,0}+\frac{5}{9}H_{1,1}+\frac{5}{9}H_{-1,0}$ 
$$\begin{split} & -\frac{1}{6} \frac{4H_1}{1} + H_{0,0} + \frac{1}{2} \frac{4H_1}{1} + \frac{1}{9} \frac{4(1+2)H_{1,1,0}}{4} + \frac{1}{8} \frac{1}{6} - \frac{1}{84} \frac{4H_1}{1} + \frac{1}{9} \frac{4H_1}{1} - \frac{1}{9} \frac{1}{84} H_{1,0,0} - \frac{1}{12} \frac{1}{84} H_2 - \frac{1}{2} \frac{2H_1}{12} + \frac{1}{3} \frac{2}{2} \frac{2}{15} \frac{5H_{1,1,0}}{1} + \frac{3}{12} \frac{1}{14} H_3 - \frac{3}{2} \frac{1}{4} H_{1,0} - \frac{1}{2} \frac{1}{84} H_2 - \frac{3}{2} \frac{2}{12} \frac{1}{12} \frac{1}{84} \frac{1}{12} \frac{$$
 $\frac{72}{12}$   $\frac{2}{12}$   $\frac{2}{10}$   $\frac{12}{12}$   $\frac{2}{10}$   $\frac{2}{10}$   $\frac{2}{12}$   $\frac{72}{10}$ + $\frac{12}{11}$  + $\frac{$  $-2H_{1,1,2} - 2H_{1,2,0} \Big] + p_{qg}(-x) \Big[ H_{-1,-1}\zeta_2 - 2H_{-1,2} - 6H_{-1,-1,0} + H_{1,1,1} + 2H_{-2}\zeta_2 - H_{-2,0,0} \Big] + 2H_{-1,2,0} \Big$  $+\frac{727}{24}H_{-1,0}-H_{-1}\zeta_{2}-2H_{-2,2}-\frac{5}{2}H_{-1}\zeta_{3}-H_{-1,-2,0}+2H_{-1,-1,0,0}+2H_{-1,-1,2}-\frac{3}{2}H_{-1,0,0,0}$  $+6H_{-1,-1,-1,0}-2H_{-1,3}+2H_{-1,2,1}] + (\frac{1}{v}-x^2) \left[\frac{2}{3}H_{2,1}+\frac{32}{9}\zeta_2-2H_{1,0,0}+\frac{4}{3}H_{1,1,0}-\frac{10}{9}H_{1,1}\right]$  $-\frac{8}{3}H_{-1,0,0}+\frac{3}{2}H_{1,0}+6\zeta_3+\frac{161}{36}H_1-\frac{23511}{108}\Big]+\frac{2}{3}\Big(\frac{1}{\nu}+x^2\Big)\Big[\frac{26}{3}H_{-1,0}-\frac{28}{9}H_0-2H_{-1,-1,0}\Big]$ 

 $+\frac{1}{2}p_{\theta 9}(x)\left[H_{1,2}-H_{1,0}-H_{1}\zeta_{2}+9\zeta_{3}+\frac{83}{12}H_{1,1}+2H_{-2,0}-\frac{7}{26}H_{1}+2H_{0}\zeta_{2}-\frac{1625}{49}+\frac{3}{2}H_{1,0,0}\right]$  $+2H_{1,1,0} - \frac{5}{2}H_{1,1,1} + \frac{31}{18}p_{gg}(-x) \left[\frac{95}{93}H_0 - \zeta_2 - H_{-1,0}\right] + \frac{1}{3}(2-x) \left[6H_{0,0,0,0} - H_3 - \frac{13051}{288}\right]$  $-\frac{13}{2}\zeta_3 - 4H_{-2,0} - H_{2,0} - \frac{1}{2}H_{1,0} - \frac{1}{2}H_{2,1} + 2H_{0,0,0} - \frac{653}{24}H_{0,0} + (1+x)\left[H_0\zeta_2 - \frac{1187}{216}H_0\right]$  $\frac{2}{8}H_{2} - \frac{85}{18}H_{-1,0} - \frac{101}{18}\zeta_{2} - \frac{80}{27}H_{0} + \frac{23}{18}\zeta_{2} - \frac{1}{3}H_{1,1} + \frac{5}{4}xH_{1,1} - \frac{1}{9}H_{1} - \frac{37}{17}xH_{1} + \frac{210}{18}H_{-1,0}$  $+\frac{1501}{54}+H_0\zeta_2-H_{0,0,0}+\frac{101}{2}H_{0,0}-\frac{1}{2}H_{1,0}\Big)+16C_F^{-3}\Big(p_{89}(x)\Big[3H_{1,1}\zeta_2+3H_1\zeta_2+\frac{7}{2}\zeta_2$  $-\frac{23}{9}H_{1,1} - 8H_1\zeta_3 - 6H_{1,-2,0} - 2H_{1,0}\zeta_2 + 3H_{1,1,0} - 3H_{1,1,0,0} - H_{1,1,1,0} + 2H_{1,1,1,1} - 3H_{1,1,2}$  $-2H_{1,2,0} - 2H_{1,2,1} - \frac{9}{2}H_{1,1,1} - \frac{3}{2}H_{1,0,0} - \frac{47}{16} - \frac{47}{16}H_1 - \frac{15}{2}\zeta_3 + p_{gq}(-x) \left[2H_{-1,-2,0}\right]$  $+6H_{-1,-1,0} + 3H_{-1}\zeta_2 + \frac{7}{4}H_{1,0} - \frac{16}{5}\zeta_2^2 - 6H_{-1,0,0} - \frac{7}{2}H_{-1,0} + 4H_{-1,-1,0,0} - 2H_{-1,0}\zeta_2$  $-H_{-1,0,0,0}$  + (1-x) 9 $H_{1,0,0}$  +  $H_{1,1,1}$  -  $10H_1\zeta_2$  +  $3H_0\zeta_3$  +  $H_{2,2}$  -  $H_2\zeta_2$  +  $H_{0,0,0}$  +  $5H_{2,0,0}$  $-4H_3 + H_{2,1,1} + 3H_{0,0}\zeta_2 + 3H_{3,1} - 3H_4 + \frac{211}{16}H_1 + \frac{49}{20}\zeta_2^{-2} + (1+x) \left[11\zeta_3 + \frac{1}{4}H_{1,1} + \frac{1}{4}H_{1,0}\right]$  $+\frac{91}{16}H_0+36H_{-1,0}+8H_{-1,0,0}-14H_{-1,-1,0}-7H_{-1}\zeta_2+2H_{1,2}+4H_0\zeta_2-H_{2,1}+2H_{-2,0,0}$  $+5H_{-2,0} + \frac{11}{2}H_2 - 2H_{0,0,0,0} - 2H_{-1,-1,0} - H_{-1}\zeta_2 - \frac{13}{4}\zeta_2 + \frac{9}{4}H_{1,0} + \frac{9}{20}\zeta_2^2 + \frac{287}{32} + \frac{11}{16}H_1$  $+4H_{-1,0,0}+16H_{-3,0}-4H_{-2}\zeta_2-8H_{-2,-1,0}-5H_2\zeta_2+\frac{19}{4}H_2+H_{2,2}-\frac{35}{8}H_{0,0}+9H_0\zeta_3$  $+25H_{-2,0}+6H_{-2,0,0}+\frac{3}{2}x\Big[\frac{58}{3}\zeta_2-\frac{7}{2}H_1\zeta_2+4H_{1,1}-\frac{3}{2}H_{1,1,1}+\frac{5}{2}H_{1,0,0}-\frac{175}{96}+H_{3,1}+\frac{19}{3}\zeta_3-\frac{175}{96}+H_{3,1}+\frac{19}{3}\zeta_3-\frac{175}{96}+H_{3,1}+\frac{19}{3}\zeta_3-\frac{175}{96}+H_{3,1}+\frac{19}{3}\zeta_3-\frac{175}{96}+H_{3,1}+\frac{19}{3}\zeta_3-\frac{19}{9}+H_{3,1}+\frac{19}{3}\zeta_3-\frac{19}{9}+H_{3,1}+\frac{19}{3}\zeta_3-\frac{19}{9}+H_{3,1}+\frac{19}{3}\zeta_3-\frac{19}{9}+H_{3,1}+\frac{19}{3}$  $+2H_{2,0}-14H_0+H_{0,0}\zeta_2-H_{-1,0}-H_4-\frac{3}{2}H_{2,1}+\frac{1}{3}H_{2,1,1}+3H_{2,0,0}-\frac{5}{\epsilon}H_3-H_{1,2}-\frac{7}{\epsilon}H_0\zeta_2$  $+\frac{2}{2}H_{1,1,0}-\frac{29}{6}H_{0,0,0}-\frac{185}{8}H_{0,0}\right)$ 

$$\begin{split} \mu^{(2)}_{01}(x) &= 16C_{0}C_{0}rr_{0}\left(z^{2}\left[\frac{4}{9}H_{2}+3H_{4,0}-\frac{97}{12}H_{1}+\frac{8}{8}H_{-2,0}-\frac{2}{3}H_{0}C_{2}+\frac{123}{12}H_{1}-\frac{46}{9}L_{2}+2H_{3}\right)\\ &-6H_{-1,0}+2H_{2,0}+\frac{127}{12}H_{0,0}-\frac{511}{2}H_{+}r_{0}(x)\left[2C_{0}-\frac{52}{2}H\right]+\frac{4}{3}\left[\frac{1}{2}-x^{2}\right]\left[\frac{1}{2}H_{1,0}-\frac{43}{8}H_{0}\right]\\ &-\frac{121}{14}H_{-}-\frac{643}{432}-\frac{1}{2}H_{21}+2H_{1}C_{2}+H_{0}C_{2}-2H_{40,0}+\frac{1}{3}H_{1,1}-H_{1,1}-H_{1,1}-\frac{1}{17}H_{2}\\ &+6H_{-1,0}+8H_{0}c_{2}-6H_{-1,0}-\frac{53}{6}H_{0}c_{2}-\frac{49}{2}H_{0}+\frac{188}{4}c_{2}+\frac{511}{12}-\frac{1}{2}H_{2}-3H_{1,0}-4H_{0,0,0,0}\\ &-\frac{71}{12}H_{0,0}+\frac{3}{2}C_{2}-H_{2,1}+\frac{97}{2}H_{1,1}-4C_{2}-\frac{2}{2}H_{0}-8H_{-1,0}-\frac{32}{2}H_{0,0,0}+\frac{4}{3}\left[\frac{1}{4}+x^{2}\right]\left[\frac{1}{2}H_{1}-H_{2}\right]\\ &+\frac{1}{4}H_{-1,0}+H_{-1,2}+H_{2}-H_{2}C_{2}+2C_{2}-\frac{2}{2}H_{0}-8H_{-1,0}-\frac{3}{2}H_{0,0,0}+\frac{4}{3}\left[\frac{1}{4}+x^{2}\right]\left[\frac{1}{2}H_{1}-H_{2}\right]\\ &+\frac{1}{4}H_{-1,0}+H_{-1,2}+H_{-2}-\frac{4}{9}C_{2}+2C_{2}-H_{1,1}-\frac{1}{2}H_{1,1}-H_{1,1}-H_{1,1}-H_{1,1}\right] \\ &+\frac{1}{4}H_{-1,0}+H_{-2,2}+H_{2}-H_{2}-4H_{-1,1}C_{2}-4H_{-1,1}-\frac{1}{2}H_{-1,0,0}-H_{-1,1}\right] \\ &+\frac{1}{4}H_{-1,0}+H_{-2,2}+H_{2}-H_{2}-2H_{$$

 $-2H_{-1,2}+H_{1}\zeta_{2}+H_{-1}\zeta_{2}+\frac{10}{2}H_{2}+H_{1,1,1}\Big]+(1-x)\Big[15H_{0,0,0,0}-5H_{2}\zeta_{2}-\frac{65}{4}\zeta_{3}+\frac{11}{4}H_{1,1,1}\Big]$  $-\frac{3}{2}H_4 + \frac{5}{2}H_{0,0}\zeta_2 + H_{1,1,0} - \frac{31}{6}H_{2,0} + \frac{17}{12}H_{1,0} - \frac{551}{20}\zeta_2^2 - \frac{29}{4}H_{1,0,0} - \frac{113}{4}H_2 + \frac{18691}{72}H_0$  $+\frac{2243}{109}+\frac{265}{6}H_{-1,0,0}+\frac{33}{2}H_{2,0,0}+19H_{2,1}+\frac{31}{12}H_{1,1}+\frac{23}{2}H_{-2,0}-\frac{497}{36}\zeta_2+\frac{29}{6}H_1\zeta_2-\frac{143}{12}H_{-2,0}+\frac{143}{36}H_1\zeta_2+\frac{143}{12}H_1$  $-\frac{11}{6}H_{1,1,1} - \frac{19}{13}H_0\zeta_2 + \frac{1223}{22}H_1 - \frac{43}{6}H_{0,0,0} - \frac{3011}{26}H_{0,0} + (1+x) \left[8H_{2,1,0} - 4H_{-1,2}\right]$  $6 \frac{12}{12} \frac{12}{12} \frac{5}{12} \frac{35}{12} \frac{11}{12} \frac{12}{11} \frac{15}{12} \frac{15}{12} \frac{15}{12} \frac{15}{12} \frac{11}{12} \frac{15}{12} \frac{1$  $+5H_2\zeta_2 + 4H_{2,1,1} - H_{-3,0} + 36H_0\zeta_3 - 5H_2\zeta_2 + 2H_{-1,2} + 6H_{-1,-1,0} - 6H_{2,1,0} - 3H_{2,1,1}$  $-11 H_{0,0,0,0} - 5 H_{3,1} + \frac{25}{4} H_{1,1,1} + \frac{13}{2} H_{-2} \zeta_2 + \frac{27}{2} H_{-2,0,0} + \frac{11}{2} H_{-3,0} + \frac{13}{2} H_2 \zeta_2 - \frac{17}{4} H_{1,0,0}$  $+13H_{-2,-1,0} - \frac{17}{12}H_{1,1,1} - \frac{3}{4}H_4 - \frac{1}{4}H_{0,0}\zeta_2 + H_{1,2} + \frac{11}{2}H_{1,1,0} + \frac{79}{12}H_{2,0} + \frac{67}{8}H_{1,0} + \frac{263}{8}\zeta_2^2$  $\begin{array}{l} 1.000-10^{-1}\overline{12}^{011,11}-\frac{7}{12}^{011,11}-\frac{7}{4}u_{1-2}-\frac{7}{4}^{100}0_{2}^{1-1}n^{11,2}+\frac{7}{2}n^{11,0}+\frac{7}{12}H_{1,0}+\frac{8}{8}H_{1,0}+\frac{8}{8}V_{2}^{1-1}\\ +\frac{119}{5}V_{2}+\frac{97}{24}H_{2}-\frac{305}{10}H_{-1,0}-24H_{0}5_{1}+H_{-1}\zeta_{2}-\frac{1335}{275}H_{0}-\frac{188}{18}-38H_{-1,0,0}-\frac{21}{2}H_{2,1}\\ -\frac{7}{2}H_{0,0,0}-\frac{24}{24}H_{1,1}-\frac{7}{2}H_{-2,0}+\frac{79}{72}\zeta_{3}+\frac{4}{3}H_{1}\zeta_{2}+\frac{17}{12}H_{1,1}+\frac{17}{12}H_{0}\zeta_{3}+\frac{31}{18}H_{1}+3H_{0,0,0}\\ \end{array}$  $\begin{array}{c} 4\frac{4}{16} & -\frac{23}{16} & -\frac{1}{12} & -\frac{2}{16} & -\frac{72}{16} & -\frac{3}{16} & -\frac{11}{12} & -\frac{11}{16} & -\frac{18}{16} & -\frac{36}{16} \\ +\frac{11}{12} & +\frac{1}{12} & +\frac{1}{24} & +\frac{1}{16} & +\frac{1$  $+\frac{7}{9}xH_2 + \frac{8}{9}xH_{1,0} - \frac{7}{7}x\zeta_2 - (1+x)\left[\frac{3475}{216}H_0 + \frac{103}{12}H_{0,0}\right] + 16C_F^{-2}n_f \left(p_{qg}(x)\left[7H_{1,3} + 7H_4\right]\right) + \frac{1}{12}H_{0,0} + \frac{1}{12}H_{$  $-2H_{-3,0}-7H_1\zeta_3+5H_{2,2}+6H_{3,0}+6H_{3,1}+H_{2,1,0}+4H_{2,0,0}+3H_{2,1}+2H_{2,1,1}+\frac{5}{2}H_{2,0}$  $+\frac{61}{8}H_2-\frac{61}{8}\zeta_2+\frac{87}{8}H_1+\frac{11}{2}H_{1,2}+\frac{61}{8}H_{1,1}+\frac{17}{2}H_{1,0}-7H_{0,0}\zeta_2+\frac{5}{2}H_{1,0,0}+\frac{5}{2}H_{1,1,0}-\frac{19}{2}\zeta_3$  $\begin{array}{c} 8 \\ + \frac{81}{32} + \frac{11}{2} H_3 - \frac{11}{2} H_0 \zeta_2 - \frac{7}{2} H_1 \zeta_2 + \frac{15}{2} H_{0,0,0} - \frac{87}{8} H_0 + \frac{11}{5} \zeta_2^2 + 3H_{1,1,1} - 5H_2 \zeta_2 - 7H_0 \zeta_3 \\ + 11 H_{0,0} - 2H_{1,-2,0} - 7H_{1,0} \zeta_2 + 3H_{1,0,0,0} - 5H_{1,1} \zeta_2 + 4H_{1,1,0,0} + H_{1,1,1,0} + 2H_{1,1,1,1} + 5H_{1,1,2} \end{array}$  $+6H_{1,2,0}+6H_{1,2,1}\Big]+4p_{qg}(-x)\Big[H_{0,0,0,0}-H_{-2,0}+H_{-1,-1,0}-H_{-2,0,0}+\frac{1}{2}H_{-1,-2,0}-\frac{5}{2}H_{-1,0}$  $-\frac{5}{4}H_{-1,0,0} - \frac{1}{2}H_{-3,0} + \frac{1}{2}H_{-1}\zeta_2 + H_{-1,-1,0,0} - \frac{1}{4}H_{-1,0,0,0} \Big] + 2(1-x)\Big[H_{2,1,0} - H_{2,0,0} - H_{2,2}]$  $-H_{3,1} - 2H_{3,0} - 2H_{-1}\zeta_2 + H_{1,2} - H_{1,0,0} - H_{1,1,0} + H_2\zeta_2 - \zeta_2^2 + \frac{43}{9}H_2 + \frac{49}{9}\zeta_2 + \frac{13}{9}H_{1,1}$  $-\frac{33}{16}H_1+\frac{5}{2}H_{1,0}+\frac{7}{2}H_{0,0}\zeta_2+\frac{21}{4}\zeta_3+\frac{479}{64}-\frac{1}{2}H_{1,1,1}-\frac{1}{2}H_3+\frac{1}{4}H_{2,1}+\frac{1}{2}H_{2,1,1}+\frac{3}{2}H_0\zeta_2$  $\frac{1}{2}H_{0}\zeta_{3} - \frac{7}{2}H_{4} + H_{1}\zeta_{2} - \frac{19}{2}H_{0,0,0} - \frac{29}{16}H_{0,0} - \frac{405}{32}H_{0} + 8(1+x)\left[H_{-1,-1,0} - H_{-1,0,0}\right]$ 

 $+12H_{0,0,0,0}-\frac{293}{108}+\frac{61}{6}H_0\zeta_2-\frac{7}{2}H_{1,0}-\frac{857}{26}H_1-9H_0\zeta_3+16H_{-2,-1,0}-4H_{-2,0,0}+8H_{-2}\zeta_2$  $-\frac{13}{2}H_{1,0,0}+\frac{3}{4}H_{1,1}-H_{1,1,0}-H_{1,1,1}\right]+(1+x)\left[\frac{1}{6}H_{2,0}-\frac{95}{3}H_{-1,0}-\frac{149}{36}H_{2}+\frac{3451}{108}H_{0}-\frac{149}{108}H_{1,0}-\frac{149}{108}H_{1$  $\begin{array}{c} 2 & 1000 & 4 & 110 & 1000 \\ -7H_{-2,0} + \frac{302}{9}H_{0,0} + \frac{19}{6}H_3 - \frac{991}{36}\zeta_2 - \frac{163}{6}\zeta_3 - \frac{35}{6}J_{0,0} + \frac{17}{6}H_{2,1} - \frac{43}{10}\zeta_2^{-2} + 13H_{-1}\zeta_2 \end{array}$  $+18H_{-1,-1,0} - H_{3,1} - 6H_4 - 4H_{-1,2} + 6H_{0,0}\zeta_2 + 8H_2\zeta_2 - 7H_{2,0,0} - 2H_{2,1,0} - 2H_{2,1,1} - 4H_{3,0}$  $-9H_{-1,0,0}\Big] - \frac{241}{288}\delta(1-x)\Big) + 16C_{A}n_{f}^{-2}\Big(\frac{19}{54}H_{0} - \frac{1}{24}xH_{0} - \frac{1}{27}p_{gg}(x) + \frac{13}{54}(\frac{1}{x}-x^{2})\Big[\frac{5}{3}-H_{1}\Big]$  $+ (1-x) \Big[ \frac{11}{12} \frac{7}{11} \frac{11}{12} - \frac{7}{216} \Big] + \frac{2}{9} (1+x) \Big[ \zeta_2 + \frac{13}{12} x H_0 - \frac{1}{2} H_{0,0} - H_2 \Big] + \frac{29}{288} \delta(1-x) \Big)$  $+16C_{A}^{2}n_{f}\left(x^{2}\left[\zeta_{3}+\frac{11}{9}\zeta_{2}+\frac{11}{9}H_{0,0}-\frac{2}{3}H_{3}+\frac{2}{3}H_{0}\zeta_{2}+\frac{1639}{108}H_{0}-2H_{-2,0}\right]+\frac{1}{3}p_{gg}(x)\left[\frac{10}{3}\zeta_{2}+\frac{10}{108}H_{0}-\frac{2}{3}H_{0}+\frac$  $-\frac{209}{26}-8\zeta_3-2H_{-2,0}-\frac{1}{2}H_0-\frac{10}{2}H_{0,0}-\frac{20}{2}H_{1,0}-H_{1,0,0}-\frac{20}{2}H_2-H_3\Big]+\frac{10}{9}\rho_{gg}(-x)\Big[\zeta_2$  $-\frac{37}{9}H_{-1,0} + \frac{2}{3}H_{-1,2} + (1-x) \left[\frac{5}{6}H_{-2,0} + H_{-3,0} + 2H_{0,0,0} - \frac{269}{36}\zeta_2 - \frac{4097}{216} - 3H_{-2}\zeta_2\right]$  $-6H_{-2,-1,0} + 3H_{-2,0,0} - \frac{7}{2}H_1\zeta_2 + \frac{677}{72}H_1 + H_{1,0} + \frac{7}{4}H_{1,0,0}\Big] + (1+x)\Big[\frac{193}{126}H_2 - \frac{11}{2}H_{-1}\zeta_2$  $+\frac{39}{20}\zeta_{2}^{2}-\frac{7}{12}H_{3}-\frac{53}{9}H_{0,0}+\frac{7}{12}H_{0}\zeta_{2}-\frac{5}{2}H_{0,0}\zeta_{2}+5\zeta_{3}-7H_{-1,-1,0}+\frac{77}{6}H_{-1,0}+\frac{9}{2}H_{-1,0,0}$  $+2H_{-1,2} - 3H_2\zeta_2 - \frac{2}{2}H_{2,0} + \frac{3}{2}H_{2,0,0} + \frac{3}{2}H_4\Big] + \frac{1}{6}\zeta_2 + 7H_{-2,0} + 2H_2 + \frac{458}{27}H_0 + H_{0,0}\zeta_2$  $+\frac{3}{2}\zeta_{2}^{2}+4H_{-3,0}-x\left[\frac{131}{12}H_{0,0}-\frac{8}{3}H_{0}\zeta_{2}+\frac{7}{2}H_{3}-H_{0,0,0,0}+\frac{7}{6}H_{0,0,0}+\frac{1943}{216}H_{0}+6H_{0}\zeta_{3}\right]$  $-\delta(1-x)\left[\frac{233}{288}+\frac{1}{6}\zeta_2+\frac{1}{12}\zeta_2^2+\frac{5}{3}\zeta_3\right]\right)+16C_A{}^3\left(x^2\left[33H_{-2,0}+33H_0\zeta_2-\frac{1249}{18}H_{0,0}-\frac{1249}{1$  $-44H_{0,0,0} - \frac{110}{3}H_3 - \frac{44}{3}H_{2,0} + \frac{85}{6}\zeta_2 + \frac{6409}{108}H_0 \Big] + \rho_{gg}(x) \Big[\frac{245}{24} - \frac{67}{9}\zeta_2 - \frac{310}{30}\zeta_2^2 + \frac{11}{3}\zeta_3 + \frac{110}{3}\zeta_3 +$  $+\frac{134}{9}H_{1,0}+\frac{11}{6}H_{1,0,0}+8H_{1,2,0}+8H_{1,3}+\frac{134}{9}H_2-4H_2\zeta_2+8H_{3,1}+8H_{2,2}+\frac{11}{6}H_3+10H_{3,0}$ 
$$\begin{split} & + \frac{1}{9} \cdot \Pi h - \frac{1}{6} \cdot \Pi \eta h - \frac{1}{6} \cdot \Pi \eta h - \frac{1}{6} \cdot \Pi h - \frac{1}{2} \cdot \Pi h - \frac{1}{9} \cdot \Pi h - \frac{$$
  $\begin{array}{l} -H_{0,0,0,0}+\frac{3}{4}H_{-2,0,0}-\frac{9}{4}H_{-1,0}\Big]-4H_{-1,-1,0}+5H_{0,0,0,0}+5H_{-1,0,0}-13H_{-2,0}+\frac{1}{2}H_{-1,0}\\ +4_{4}H_{-2,0,0}-\frac{113}{6}H_{2}-\frac{7}{16}\zeta_{2}-\frac{3}{2}H_{1}-\frac{11}{2}H_{1,2}-\frac{3}{3}H_{1,1}-\frac{7}{2}H_{0,0}-\frac{7}{2}H_{0,0}\zeta_{2}-\frac{5}{2}H_{1,0}\\ -\frac{5}{2}H_{1,1,0}-\frac{7}{2}G_{-}-\frac{157}{6}-\frac{9}{4}H_{1,1,1}-\frac{9}{4}H_{-}-\frac{5}{4}H_{2,1}-\frac{1}{4}H_{2,1,1}+\frac{1}{4}H_{0}\zeta_{2}+H_{2}\zeta_{2}+\frac{5}{2}H_{0}\zeta_{3}\\ +\frac{9}{5}\zeta_{5}^{2}+\frac{7}{2}H_{+}\frac{7}{2}H_{1}\zeta_{5}+\frac{49}{4}H_{0,0,0}+\frac{30}{16}H_{0}-H_{2,0,0}-H_{2,1,0}+H_{2,2}+H_{3,1}\\ +H_{3,1}+6H_{-1}\zeta_{1}\zeta_{1}+\frac{1}{4}H_{3}+2H_{-2}\zeta_{2}+4H_{-2,-1,0}\end{array}$ 

 $P_{gq}^{(2)}(x) = 16C_A C_F n_f \left(\frac{2}{9}x^2 \left[\frac{25}{6}H_1 - \frac{131}{4} + 3\zeta_2 - H_{-1,0} - 3H_2 + H_{1,1} + \frac{125}{6}H_0 - H_{0,0}\right]\right)$  $+\frac{5}{6}p_{gg}(x)\left[H_{1,2}+H_{2,1}+\frac{967}{120}+\frac{251}{90}H_1-\frac{39}{10}H_{1,1}-3\zeta_3-\frac{2}{5}H_0\zeta_2-\frac{1}{5}H_1\zeta_2-\frac{4}{3}H_{1,0}+H_{1,1,0}\right]$  $-\frac{2}{5}H_{1,0,0} + H_{1,1,1} + \frac{2}{5}H_{2,0} + \frac{2}{2}p_{89}(-x) \left[2H_{-1}\zeta_2 + \frac{7}{4}\zeta_2 + \frac{41}{12}H_{-1,0} - \frac{151}{22}H_0 + \frac{1}{2}H_{-2,0}\right]$  $+\frac{5}{2}H_2 + 2H_{-1,-1,0} - H_{-1,0,0} - H_{-1,2} + \frac{2}{2}(1-x)[H_{-2,0} + 2\zeta_3 - H_3] + (1+x)[\frac{179}{109}H_1]$  $+\frac{5}{6}\zeta_2 + \frac{25}{9}H_{-1,0} - \frac{5}{26}H_{1,1} - \frac{167}{36}H_{0,0} - \frac{1}{2}H_{2,1} - \frac{4}{3}H_0\zeta_2 - \frac{193}{72} + \frac{1}{4}H_1 + \frac{1}{9}H_{-1,0} + 4H_2$  $-\frac{1}{4}H_{1,1} + \frac{227}{18}H_0 - \frac{35}{12}H_{0,0} - H_{2,1} - \frac{2}{2}H_0\zeta_2 + \frac{10}{2}H_{-2,0} + 3\zeta_3 + 2H_3 + \frac{2}{2}H_{0,0,0} + x \left[\frac{11}{4}\zeta_2 + \frac{10}{2}H_{-2,0} + \frac{10}{2}H_{-2,$  $\frac{4}{523} + \frac{18}{19} + \frac{12}{108} + \frac{271}{108} + \frac{271}{108} + \frac{2}{5} + \frac{1}{108} + \frac{1}{5} + \frac{1}{108} + \frac{1}{5} + \frac{1}{108} + \frac{1}{$  $-6H_2 + 2H_{2,1} + 6\zeta_2 + \frac{335}{54}H_0 - \frac{28}{9}H_{0,0} - \frac{8}{3}H_{0,0,0} + p_{gq}(x) \left[\frac{3}{2}H_1\zeta_3 + \frac{163}{32} - 5\zeta_2 + \frac{27}{4}\zeta_3\right]$  $\begin{array}{c} \frac{6503}{442} H_1 + \frac{35}{2} H_{1,1} + \frac{35}{3} H_{1,1} + 4 H_2 + \frac{9}{2} H_{2,1} + 4 H_{1,0,0} + 2 H_{2,0,0} - H_2 \zeta_2 + \frac{41}{12} H_{1,2} + H_{2,2} \\ + \frac{191}{24} H_{1,0} + 3 H_{2,0} - 2 H_{2,1,1} - \frac{3}{2} H_{-1} \zeta_2 - \frac{99}{12} H_1 \zeta_2 + 5 H_{1,-2,0} + H_{1,0} \zeta_2 + \frac{5}{2} H_{1,0,00} - 2 H_{1,1} \zeta_2 \\ \end{array}$  $+\frac{1}{12}H_{1,1,0} + 5H_{1,1,0,0} - 3H_{1,1,1,0} - 4H_{1,1,1,1} - H_{1,1,2} - 2H_{1,2,1} + H_{2,1,0}] + p_{89}(-x)[H_{-1,0}]$  $H_{-1,0}\zeta_2 + \frac{3}{2}H_{-1,0,0} + \frac{27}{10}\zeta_2^2 - 3H_{-1,-1,0} - \frac{11}{2}H_{-1}\zeta_3 - 3H_{-1,-2,0} - \frac{3}{2}H_{-1,0,0,0} - 3H_{-1,2,0}$  $+5H_{-1,-1}\zeta_2 - 4H_{-1,-1,0,0} - 2H_{-1,-1,2} + 6H_{-1,-1,-1,0} + 2H_{-1,2,1}\Big] + (1-x)\Big[H_2\zeta_2 - H_{2,2}\Big]$  $+\frac{23}{12}H_{1,0}-\frac{7061}{432}H_0-\frac{4631}{144}H_{0,0}-\frac{38}{3}H_{0,0,0}-H_{-3,0}-2H_{3,0}-\frac{4433}{432}H_1-2H_{2,0,0}-\frac{21}{2}H_{1,0,0}-\frac{4433}{2}H_1-2H_{2,0,0}-\frac{21}{2}H_{1,0,0}-\frac{4433}{2}H_1-2H_{2,0,0}-\frac{443}{2}H_1-2H_{2,0}-\frac{44}{2}H_1-2H_{2,0}-\frac{44}{2}H_1-2H_{2,0}-\frac{44}{2}H_1-2H_{2,0}-\frac{44}{2}H_1-2H_{2,0}-\frac{44}{2}H_1-2H_{2,0}-\frac{44}{2}H_1-2H_{2,0}-\frac{44}{2}H_1-2H_{2,0}-\frac{44}{2}H_1-2H_2-\frac{44}{2}H_1-2H_2-\frac{44}{2}H_1-2H_2-\frac{44}{2$  $\frac{12}{-\frac{2}{5}}\zeta_{2}^{2} - \frac{2}{7}\frac{1}{H_{1,2}} + \frac{23}{2}H_{1}\zeta_{2} - 4H_{0}\zeta_{3} + (1+x)\left[\frac{49}{6}H_{3} - H_{-2,0} - \frac{432}{5}H_{0}\zeta_{2} - \frac{1}{2}H_{3,1} - \frac{1159}{36}\zeta_{2}\right]$  $+\frac{655}{576}-\frac{151}{6}\zeta_3-\frac{185}{18}H_{1,1}+\frac{1}{6}H_{1,1,1}-\frac{95}{9}H_2+\frac{29}{6}H_{2,1}-\frac{171}{4}H_{-1,0}-12H_{-1,0,0}+7H_{-1}\zeta_2$  $+16H_{-1,-1,0} + \frac{5}{2}H_{2,0} + \frac{3}{2}H_{2,1,1} + 4H_{0,0,0,0} - 35H_{-2,0} - \frac{179}{27}H_0 + \frac{2041}{144}H_{0,0} - \frac{19}{6}H_{0,0,0}$ 

$$\begin{split} & -\frac{413}{10}H_1 - \frac{11}{2}H_1\zeta_2 + \frac{33}{2}H_{1,0,0} + 11(\frac{1}{4}+z^2)[\frac{7}{24}H_0 - \frac{1}{6}H_1 - \frac{389}{105}\zeta_2 - \frac{2}{2}H_{-2,0} - \frac{1}{2}H_{-1}\zeta_2 \\ & +H_{-1,-1,0} - \frac{523}{24}H_{1,0,0} - \frac{18}{3}H_{-1,0} + H_{-1,2}] + (1-z)[\frac{33}{24}H_1 + \frac{27}{2}H_{1,0} - \frac{25}{2}H_{1,0,0} - 4H_{-3,0} \\ & -\frac{26}{3}H_{0,0} - \frac{23}{2}H_{0,0} - \frac{19}{3}H_{-2,0} - \frac{1117}{10} - 4H_{-5}\zeta_5 - 8H_{-2,-1,0} - 12H_{-2,0} - \frac{21}{2}H_{5,0} \\ & +(1+z)[\frac{7}{2}H_0\zeta_2 - \frac{33}{2}H_2 - \frac{39}{2}H_{2,0} + \frac{10}{22}H_{2,0} + \frac{10}{2}H_{2,0} + \frac{11}{2}H_{2,1} + 12(1+z)(1-\frac{4}{3}+\zeta_2^2 - \frac{15}{16}H_{-1,1} \\ & -22H_0\zeta_5 - 8H_0\zeta_5 - 8H_{1,-1,0} - 8H_{1,0,0} - 2H_{1,1,0} - 4H_{1,0,0} - 4H_{5,1} + 18H_{1,0,1} + 28H_{1,0,0,0} + \frac{4}{5}\zeta_5 \\ & +\frac{60}{9}H_2 - \frac{53}{2}H_{-1}\zeta_5 - \frac{3}{2}H_{0,0} + \frac{49}{6}H_{0,0}\zeta_7 - 16H_{0,0} - 4H_{5,1} + 18H_{1,0,0,0} + \frac{4}{5}\zeta_5 \\ & +\frac{60}{9}H_2 - 4\zeta_5 + 2\zeta_5 + 2Z_7 + 27H_2 - 4H_{0,0}\zeta_7 - 16H_0\zeta_7 - 16H_{1,0,0} - 4H_5\zeta_7 + 18H_{1,0,0,0} + 6(1-z)][\frac{73}{22}\\ & -\zeta_5\zeta_1 + \frac{1}{6}\zeta_1 + \frac{11}{2}\zeta_5 + \frac{6}{6}\zeta_1 - \zeta_5\zeta_1 \right] + 16C_7n^2(\frac{1}{3}\zeta_7)^2 [\frac{1}{16}H_2 + H_1 - \zeta_5 + 2H_{0,0,0} - \frac{1}{3}] + \frac{1}{14}H_1 + \frac{1}{3}H_{0,0} + \frac{1}{3}(1+z)[\frac{6}{9}H_0 - \frac{4}{3}H_0 + \frac{4}{3}\zeta_7 + \frac{3}{6}C_0 - \zeta_5 + 2H_0\zeta_7 - 10H_0\zeta_7 - 10H_0\zeta_7 + 10H_0\zeta_$$

2004

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• Scale derivatives of quark and gluon distributions at  $Q^2 \approx 30 \text{ GeV}^2$ 



Expansion very stable except for very small momenta  $x \lesssim 10^{-4}$ 

## **Impact on precision of LHC predictions**

•  $W^{\pm}, Z$ -boson rapidity distribution (scale variation  $\frac{m_{W,Z}}{2} \le \mu \le 2m_{W,Z}$ )

Anastasiou, Petriello, Melnikov '05



- NNLO QCD theoretical uncertainties (renormalization / factorization scale) at level of 1% Dissertori et al. '05
  - one of the few cross sections known to NNLO in pQCD
- Standard candle process for parton luminosity

#### **Updates of PDFs (exp)**

- New experimental data
  - results from neutrino-nucleon DIS for strange quark PDFs ( $s \neq \bar{s}$ )
- Uncertainty on  $\bar{u}$ ,  $\bar{d}$  doubles from 1.5% to 3% at  $Q^2 \simeq M_W^2$  MSTW '07

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#### **Updates of PDFs (th)**

- Improved heavy quark (charm) threshold
  - matching consistent with QCD factorization CTEQ '08
- Significant changes due to larger light flavor PDFs



#### **Cross sections at LHC**



- Predictions for  $W^{\pm}/Z$  cross sections at LHC shift by 8% between PDF sets CTEQ6.6 and CTEQ6.1 (improved theory!)
  - sensitivity to PDFs in the  $x \sim 10^{-3}$  range
- $W^{\pm}/Z$ -ratio golden calibration measurement

#### Large extra dimensions

- Sensitivity of LHC dijet cross section to large extra dimensions Ferrag '04
  - large extra dimensions accelerate running of  $\alpha_s$  as compactification scale  $M_c$  is approached
- PDF uncertainties
  - potential sensitivity to  $M_c$  reduced from 6 TeV to 2 TeV

 $M_c = 2$  TeV no PDF error

 $M_c = 2$  TeV with PDF error



## **Drell-Yan process and Higgs production**

- Mapping of DIS to Drell-Yan lepton-pair production (or to Higgs production in gluon fusion)
  - re-engineering the soft and collinear limit
  - threshold enhanced terms at N<sup>3</sup>LO (numerically most important)
    S.M., Vogt '05; Laenen, Magnea '05; Idilbi et al. '05



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#### Anatomy of DIS result (1 loop)



 $\mathcal{T}_1^{\mathrm{b}} = 2 \operatorname{\mathsf{Re}} \mathcal{F}_1 \,\delta(1-x) + \mathcal{S}_1$ 

- Forward Compton amplitude  $T_n$  in  $D = 4 2\epsilon$ -dimensions combines
  - virtual corrections  $\mathcal{F}_n$  (dependent on  $\delta(1-x)$ )
  - pure real-emission contributions  $S_n$ (dependent on *D*-dimensional +-distributions)
- Infrared finiteness implies cancellation of poles between  $\mathcal{F}_n$  and  $\mathcal{S}_n$ Kinoshita '62; Lee, Nauenberg '64
- Constructive approach to form factor  $\mathcal{F}_n$  and  $\mathcal{S}_n$

**Sven-Olaf Moch** 

#### Anatomy of DIS result (2 loops)









 $T_2^{\rm b} = (2 \operatorname{Re} \mathcal{F}_2 + |\mathcal{F}_1|^2) \delta(1-x) + 2 \operatorname{Re} \mathcal{F}_1 \mathcal{S}_1 + \mathcal{S}_2$ 

#### Anatomy of DIS result (3 loops)



 $\mathcal{T}_{3}^{b} = (2 \operatorname{\mathsf{Re}} \mathcal{F}_{3} + 2 |\mathcal{F}_{1}\mathcal{F}_{2}|) \,\delta(1-x) + (2 \operatorname{\mathsf{Re}} \mathcal{F}_{2} + |\mathcal{F}_{1}|^{2}) \mathcal{S}_{1}^{\mathcal{S}^{(3)}} + 2 \operatorname{\mathsf{Re}} \mathcal{F}_{1}\mathcal{S}_{2} + \mathcal{S}_{3}$ 

#### **Drell-Yan (1 loop)**



- Construction of cross sections for hadron-hadron scattering
  - form factor with time-like kinematics  $Q^2 > 0$
  - soft emission with *D*-dimensional +-distributions
- **Drell-Yan lepton-pair production in**  $q\bar{q}$ -annihilation
- Higgs production from gluon fusion

#### **Drell-Yan (2 loops)**











- Checks at two loops
  - Drell-Yan

Hamberg, van Neerven, Matsuura '91; Harlander, Kilgore '02

Higgs production

Harlander, Kilgore '02; Anastasiou, Melnikov '02; Ravindran, Smith, van Neerven '03



## **Higgs boson production at LHC (II)**



N<sup>3</sup>LO<sub>approx.</sub> increase at  $\mu_r = M_H$  5% (NNLO PDF's) S.M., Vogt '05
  $\mu_r$  variation 4%

• Overall accuracy of 5% reached with approx. N<sup>3</sup>LO prediction

## **Instanton induced processes at LHC**

- Multi-gluon production in instanton background
  - Itransfer of DIS phenomenology to Drell-Yan lepton-pair production Brandenburg, Ringwald, Utermann '06; Petermann, Schrempp '08
  - amplitudes related by crossing



- Characteristic final state signature
  - isotropic multi-particle production, very high multiplicity

## **Summary**

#### **HERA**

- Deep-inelastic scattering (electron-proton collision)
  - wealth of experimental information on proton structure
- QCD precision predictions
  - radiative corrections for parton evolution
- HERA PDFs have strong impact on measurements at LHC

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#### Outlook

If past performance is an indicator, we are well prepared for future challenges ...