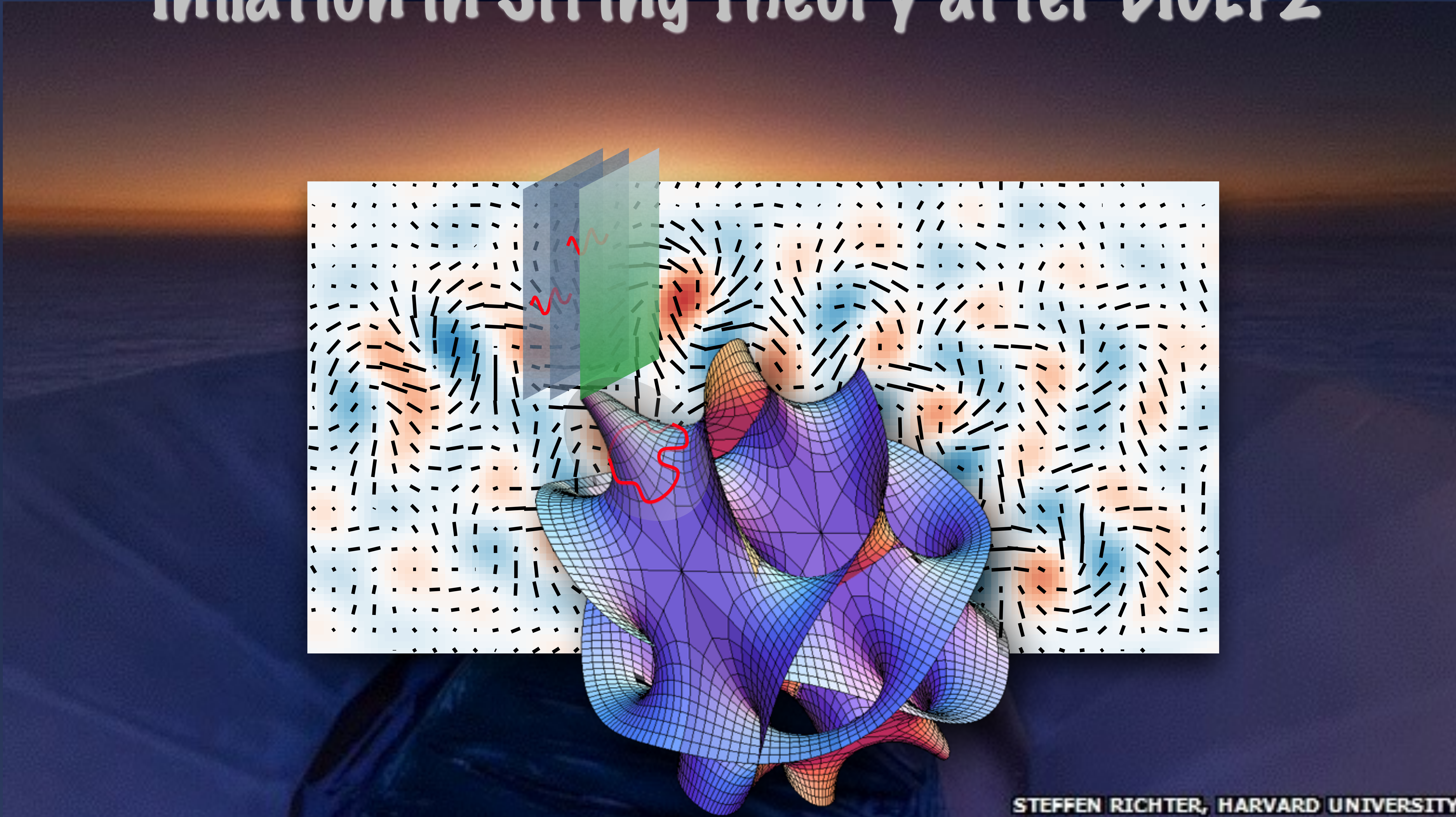
Gravitational Waves from the Big Bang
Inflation in String Theory after BICEP2



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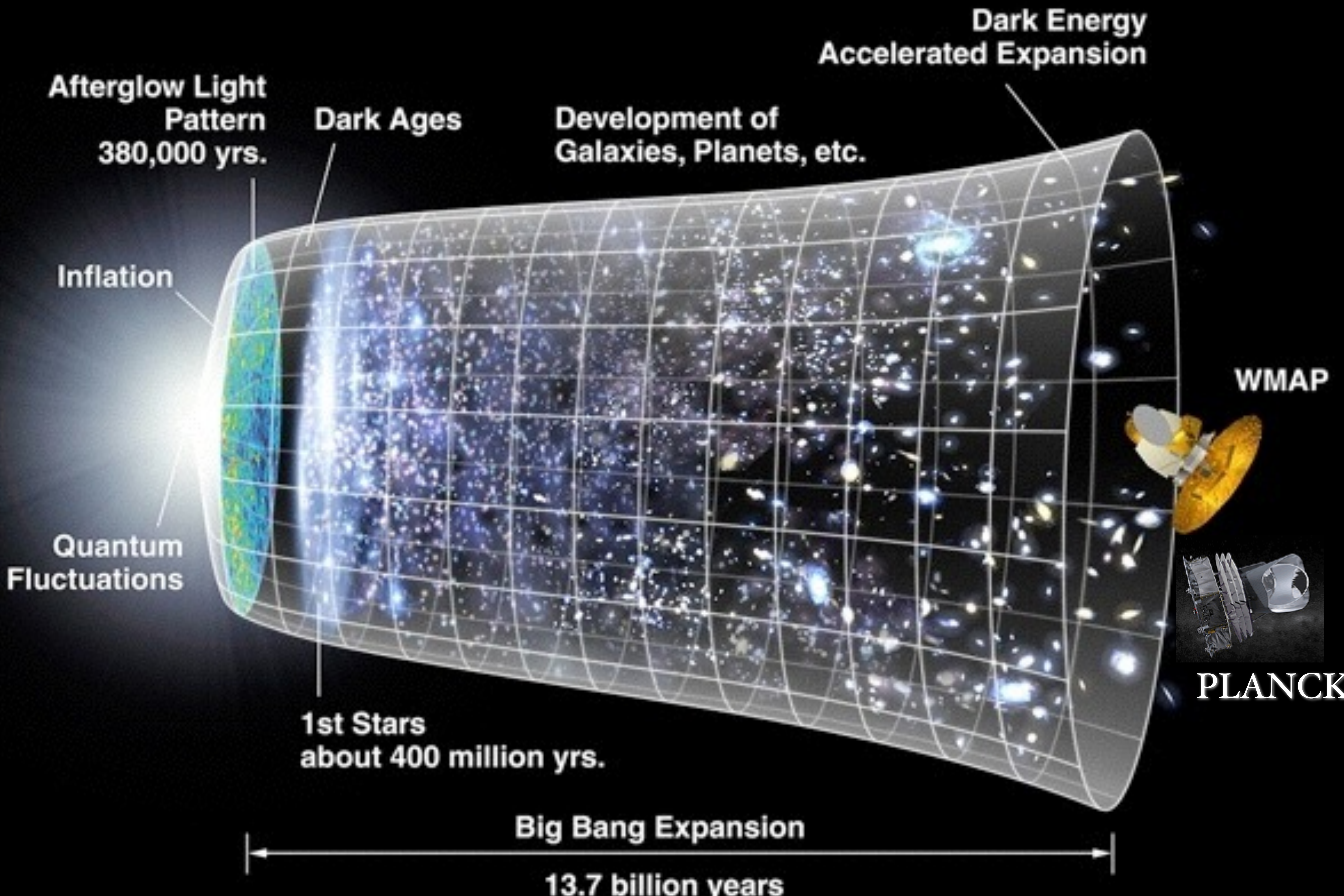
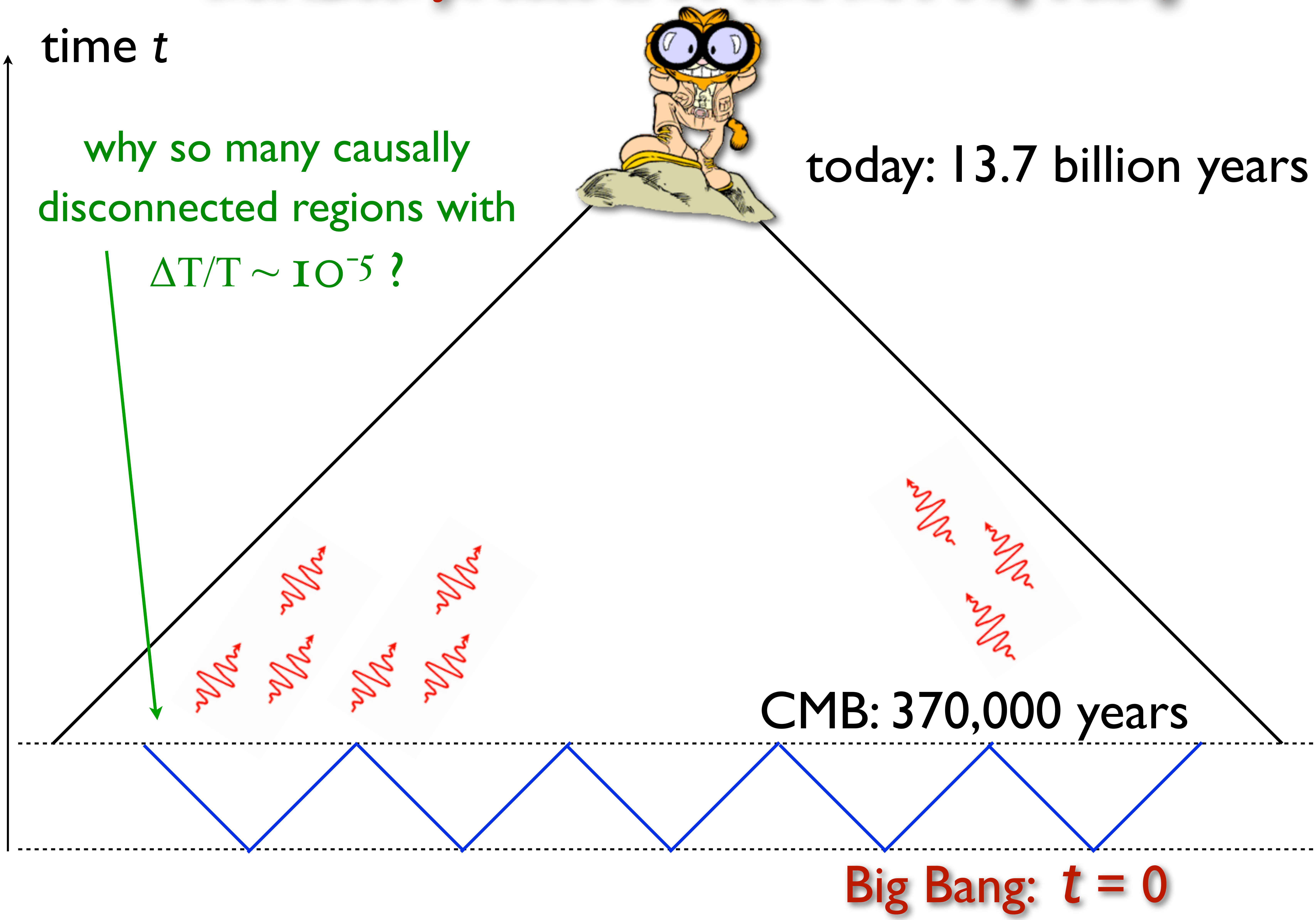
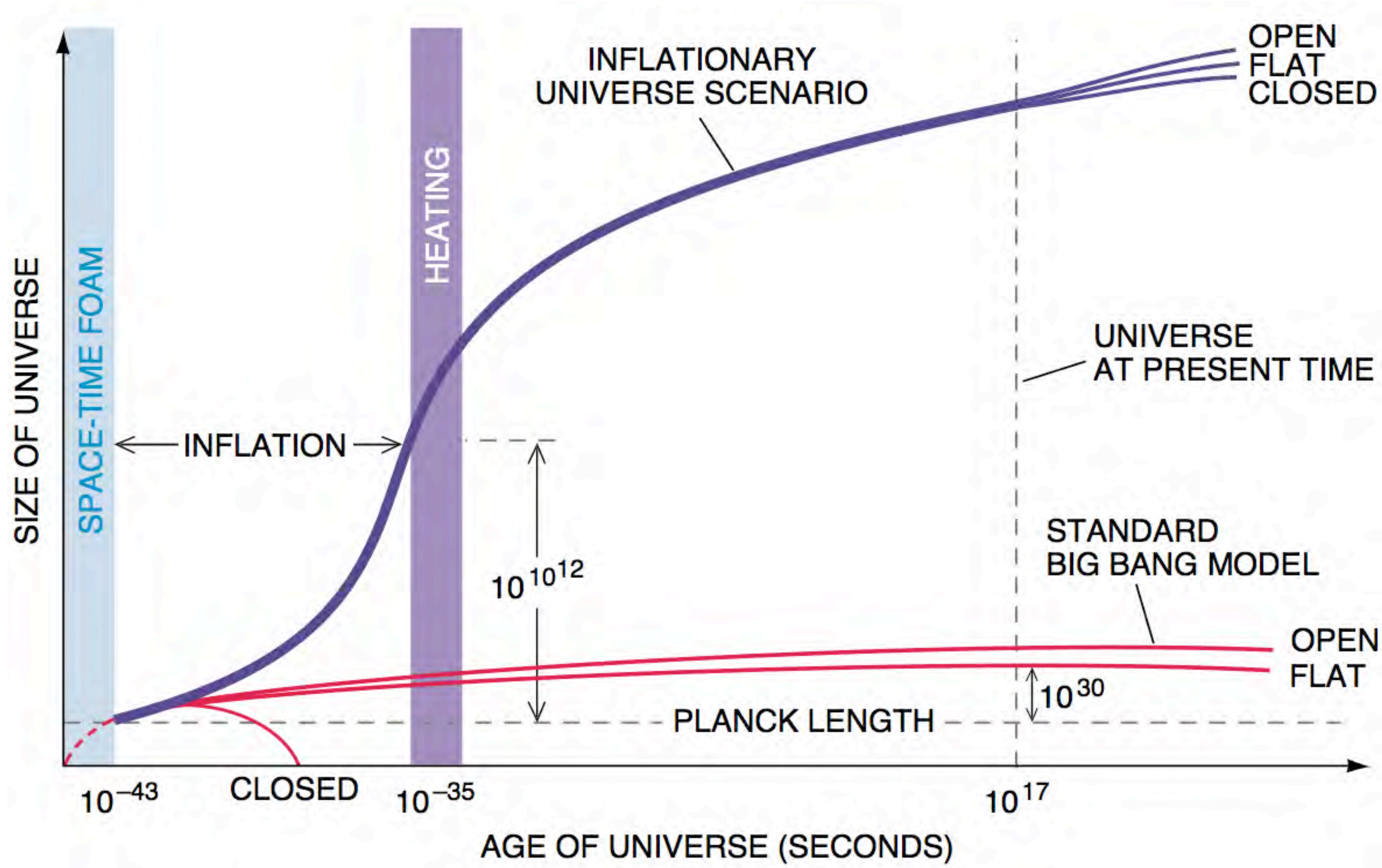
and many friends: Eva Silverstein, Renata Kallosh, Andrei Linde, Shamit Kachru, Jan, Louis, Shanta de Alwis, Liam McAllister, Raphael Flauger, Enrico Pajer, Gang Xu, Xi Dong, Bart Horn, Francisco Pedro, Pascal Vaudrevange, Koushik Dutta, Ido-Ben-Dayan, Markus Rummel ...


Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
380,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
WMAP
Quantum
Fluctuations
PLANCK
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years

horizon problem of the hot Big Bang



Inflationary Universe



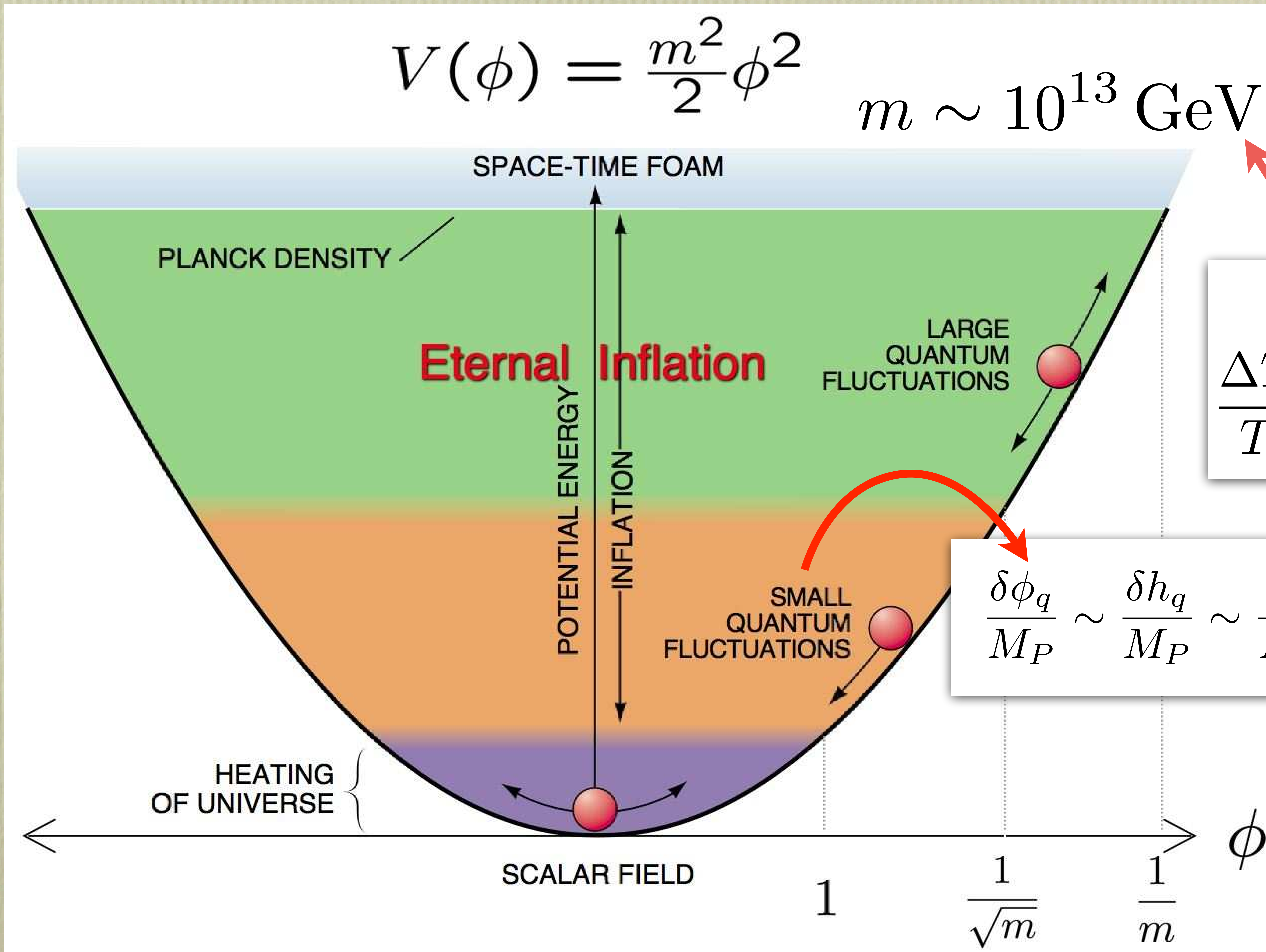
Inflation is an extremely rapid acceleration in the universe soon after its creation.

[picture from Munich lectures: Linde '07]

slow-roll inflation ...

[Linde '82]

$$V(\phi) = \frac{m^2}{2}\phi^2 \qquad m \sim 10^{13}\,\mathrm{GeV}$$

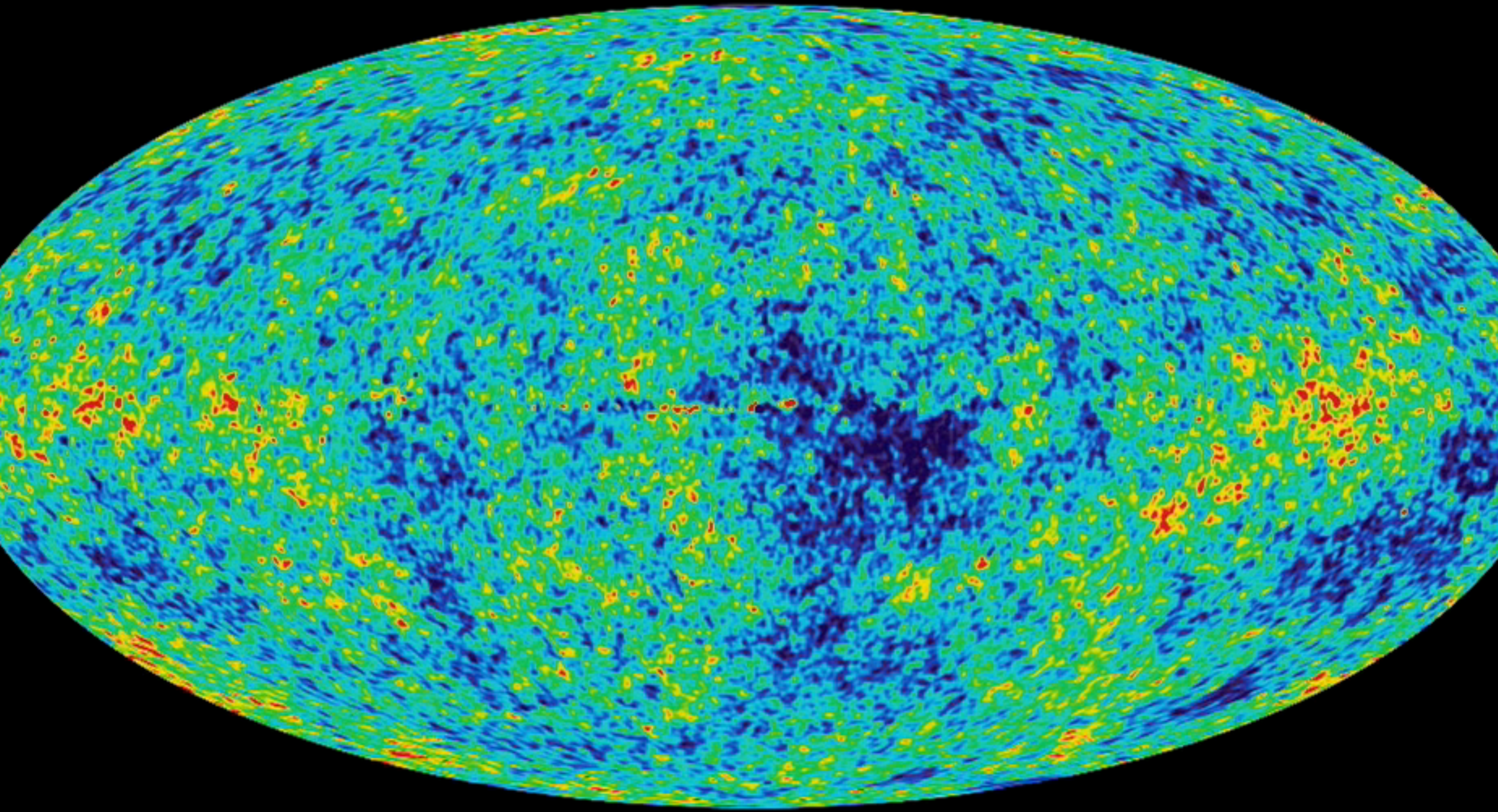


CMB:

$$\frac{\Delta T}{T} \sim 10^{-5}$$

$$\frac{\delta\phi_q}{M_P} \sim \frac{\delta h_q}{M_P} \sim \frac{H}{M_P} \sim 10^{-5}$$

[picture from lecture notes: Linde '07]



Inflation ...

- inflation: period quasi-exponential expansion of the very early universe

  (solves horizon, flatness problems of hot big bang ...)

[Guth '80]

- driven by the vacuum energy of a slowly rolling light scalar field:

e.o.m.: $\cancel{\ddot{\phi}} + 3H\dot{\phi} + V' = 0$

[Linde; Albrecht & Steinhardt '82]

Inflation ...

- slow-roll inflation: [Linde; Albrecht & Steinhardt '82]

$$\text{scale factor grows exponentially} : a \sim e^{Ht} \quad \text{if} : \begin{cases} \dot\phi^2 \ll V \\ |\ddot\phi| \ll |3H\dot\phi| \end{cases}$$

$$\Rightarrow \quad \epsilon \equiv -\frac{\dot H}{H^2} \simeq \frac{1}{2}\left(\frac{V'}{V}\right)^2 \cancel{\ll} 1 \quad , \quad \eta \equiv \frac{\dot\epsilon}{\epsilon H} \simeq \frac{V''}{V} \ll 1$$

with the Hubble parameter: $H^2 = \frac{\dot a^2}{a^2} \simeq const. \sim V$

e-folds N_e in $a \sim e^{N_e}$: $N_e = \int H \mathrm{d}t = \int_{\phi_E}^{\phi_E+\Delta\phi} \frac{\mathrm{d}\phi}{\sqrt{2\epsilon}}$

Inflation ...

- inflation generates metric perturbations: scalar (us) & tensor

$$\mathcal{P}_S \sim \frac{H^2}{\epsilon} \sim \left(\frac{\delta\rho}{\rho}\right)^2 \quad \text{and} \quad \mathcal{P}_T \sim H^2 \sim V$$

$$\sim k^{n_S - 1}$$

window to GUT scale & direct measurement of inflation scale

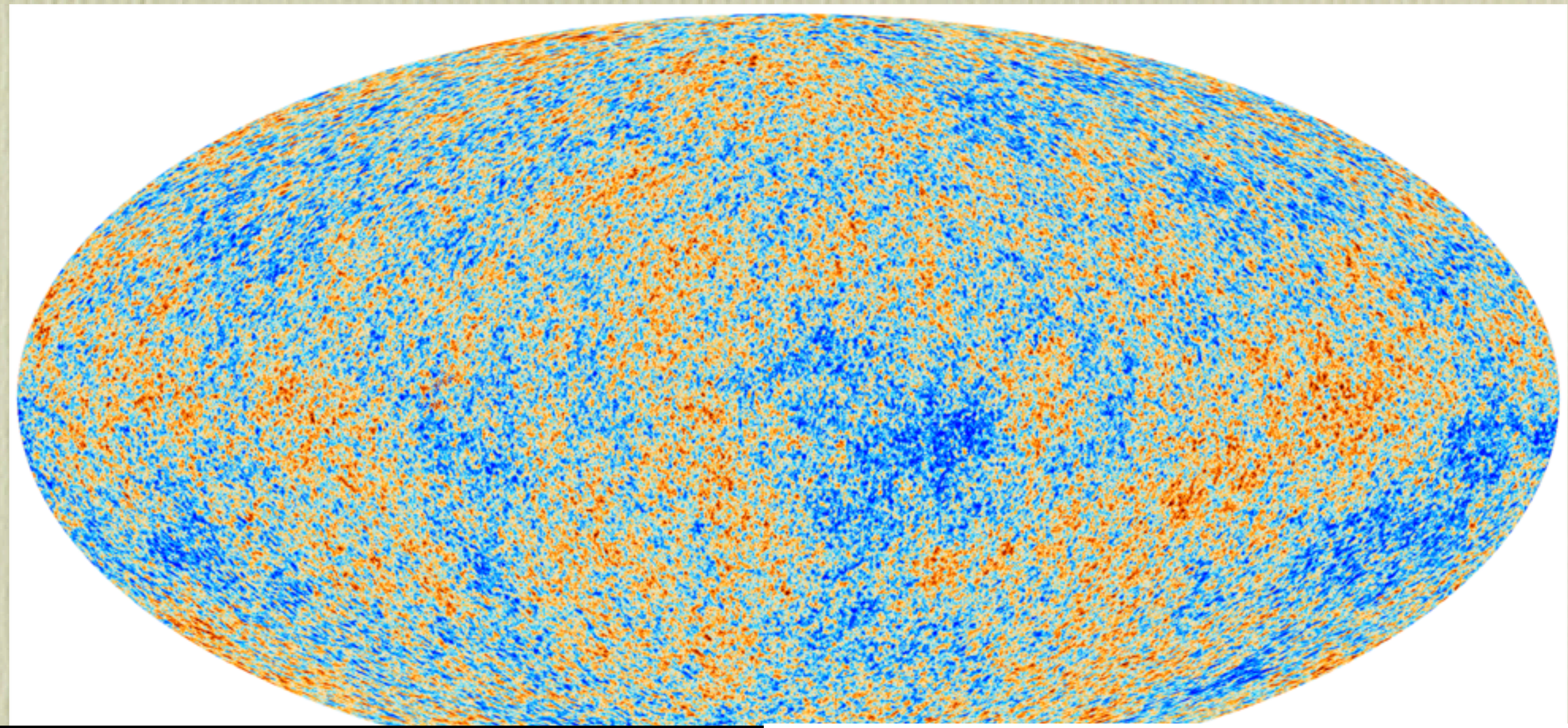
- scalar spectral index:

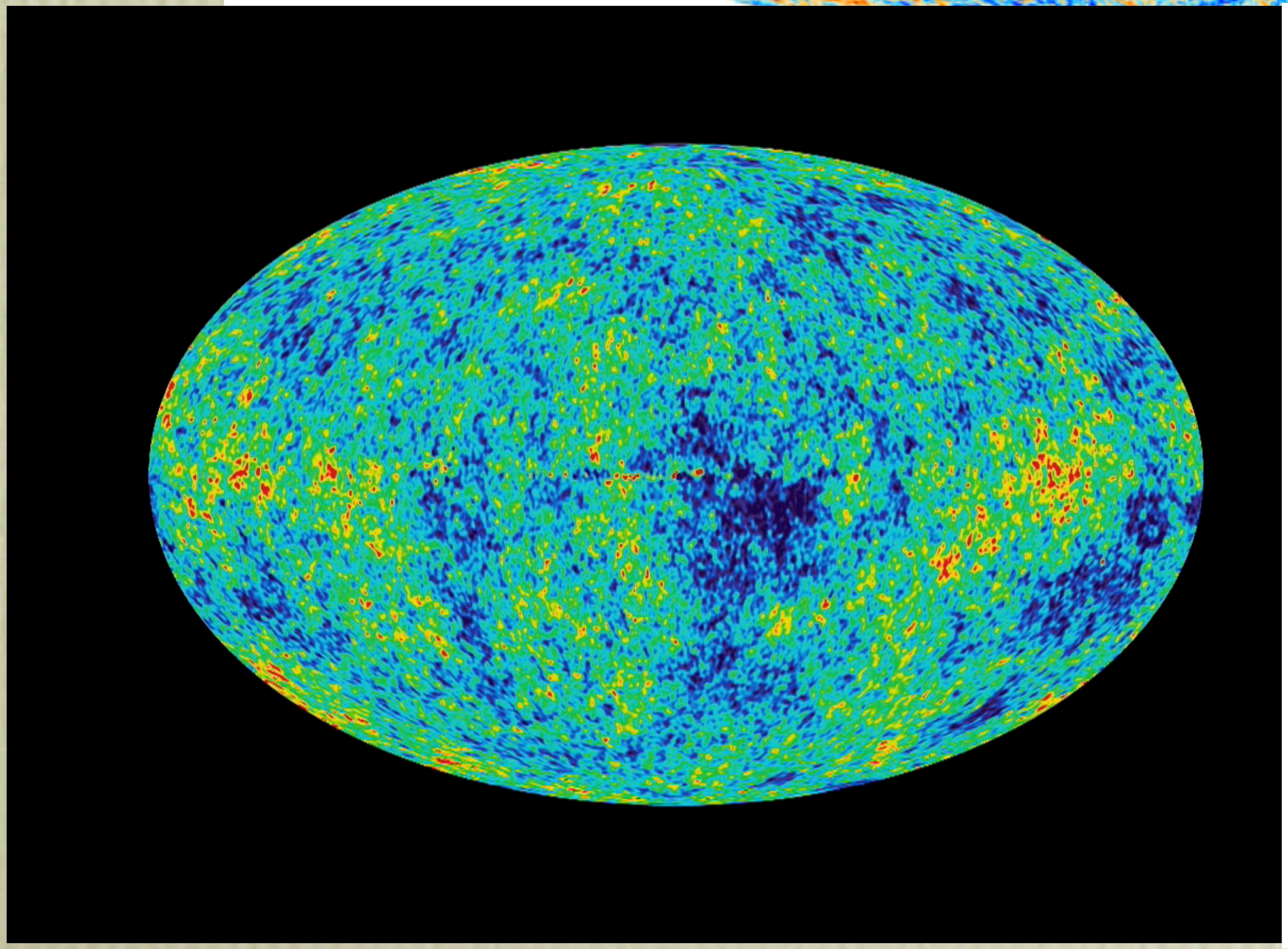
$$n_S = 1 - 6\epsilon + 2\eta$$

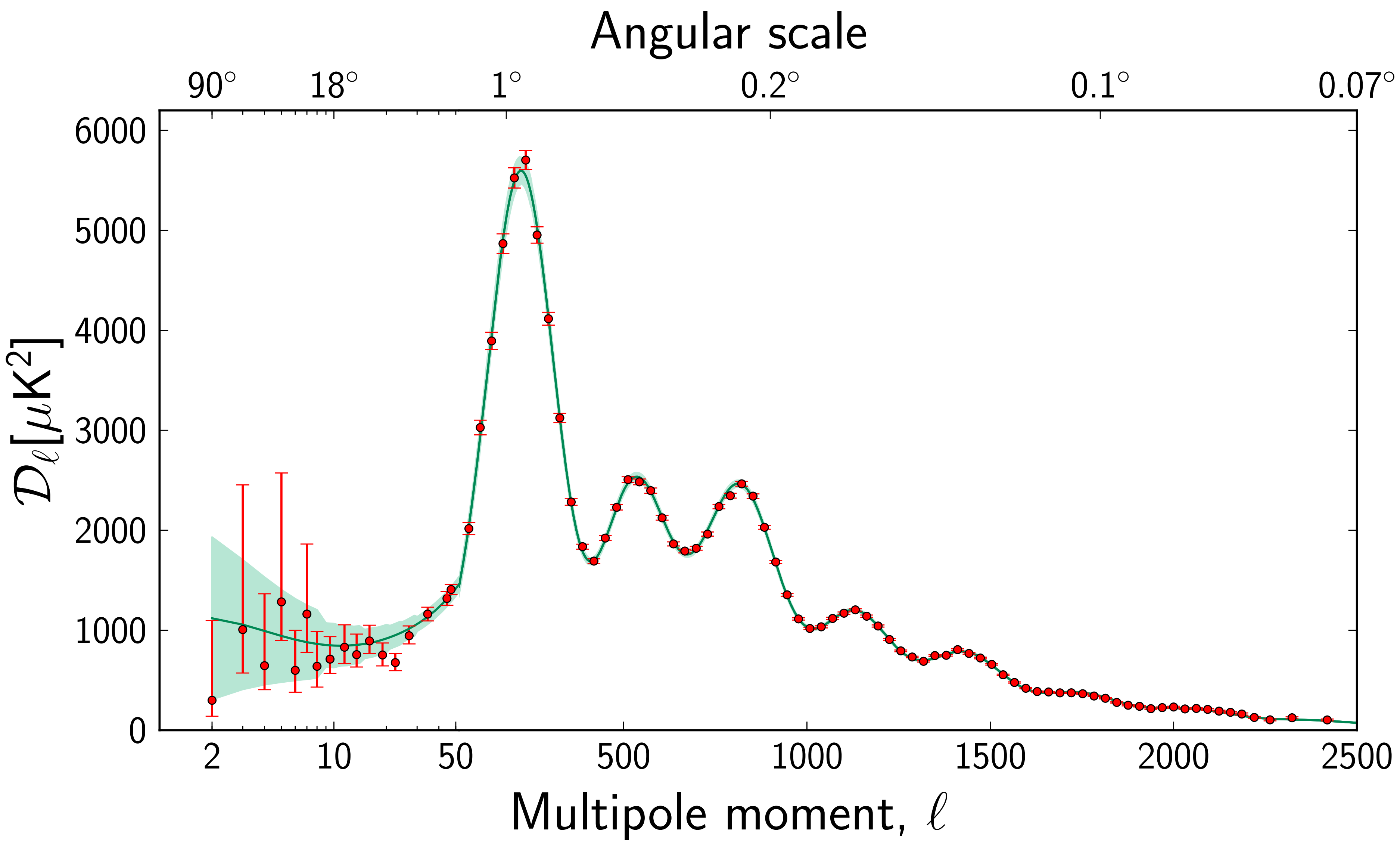
- tensor-to-scalar ratio:

$$r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_S} = 16\epsilon$$

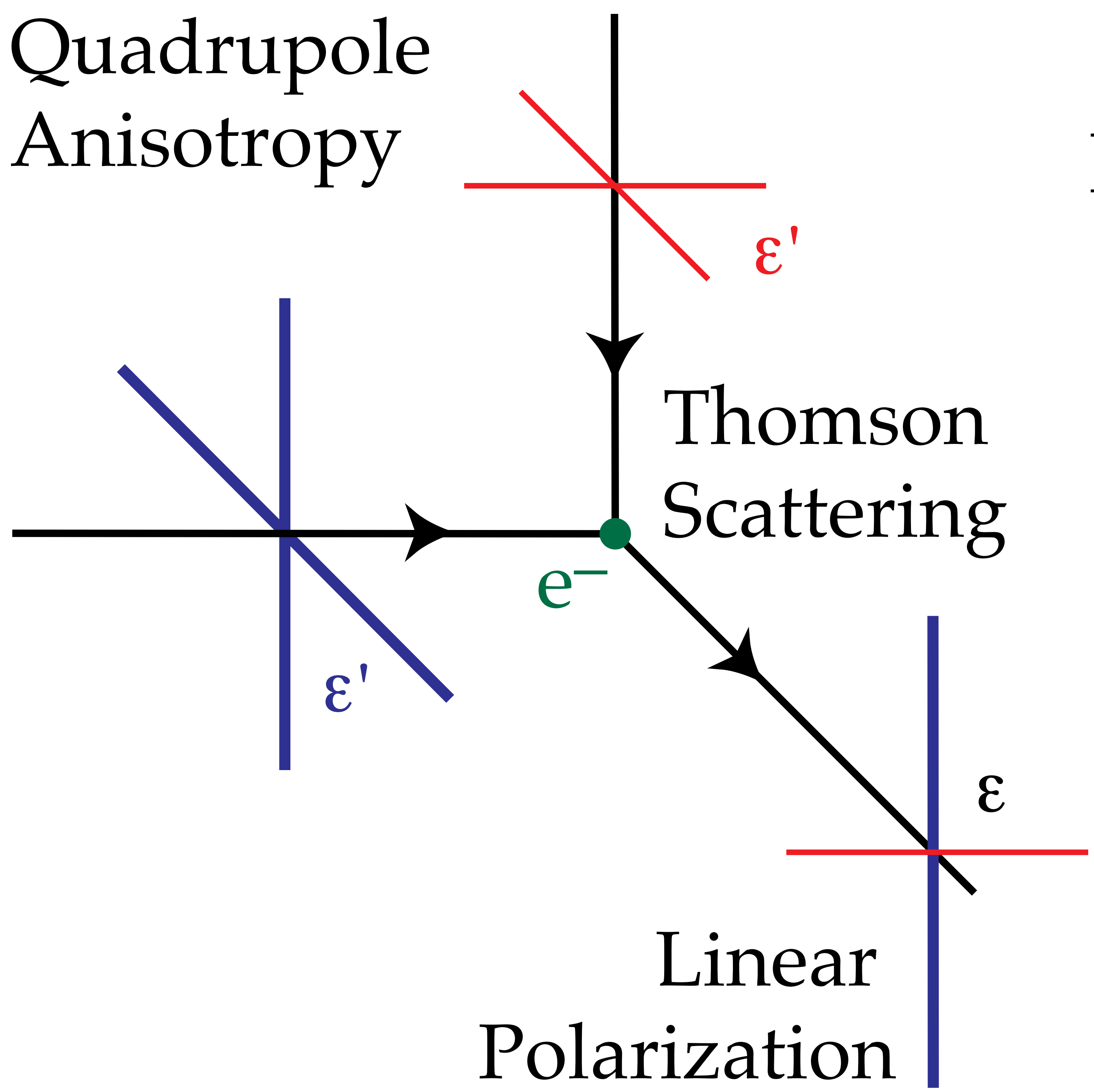
Cosmic Microwave Background: PLANCK cosmology results 2013!







temperature anisotropy produces non-isotropic linear polarization field



fluctuation density field κ

E-mode/B-mode components κ_E , κ_B

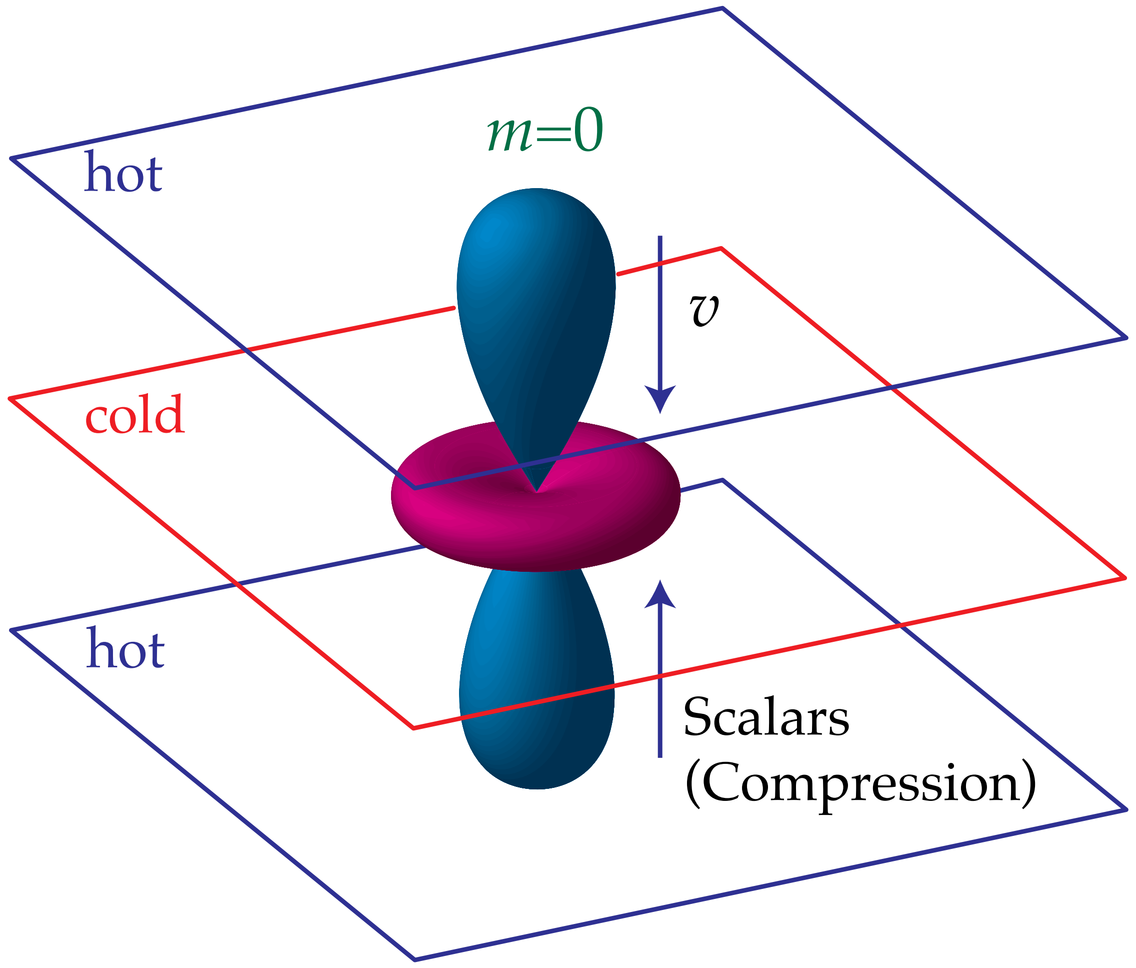
polarization vector field $\vec{\nabla}\kappa = \vec{u}$

$$\nabla^2 \kappa^{\mathrm{E}} = \nabla \cdot \mathbf{u} \quad ; \quad \nabla^2 \kappa^{\mathrm{B}} = \nabla \times \mathbf{u}$$

div u: polarization vector field has **non-vanishing divergence** & zero curl — **E-mode** polarization

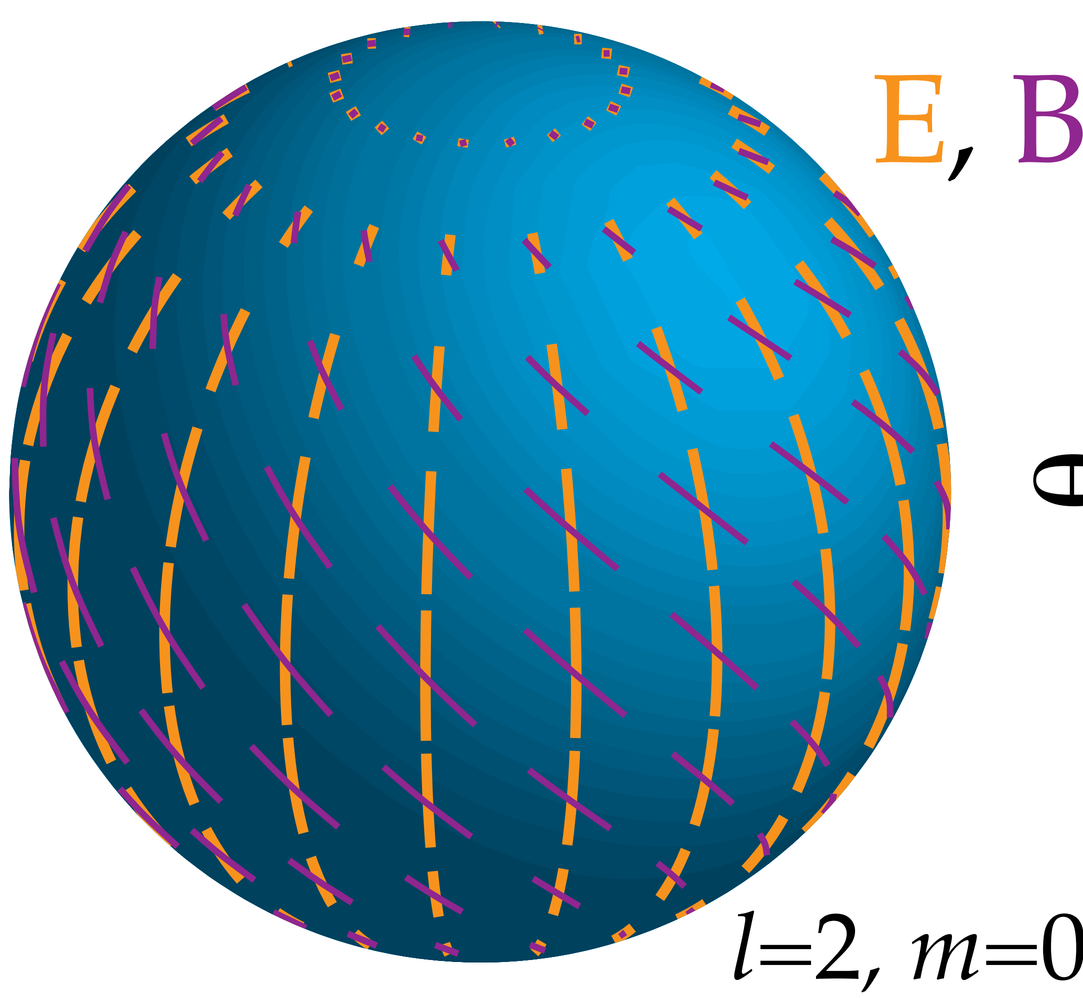
curl u: polarization vector field has **non-vanishing curl** & zero divergence — **B-mode** polarization

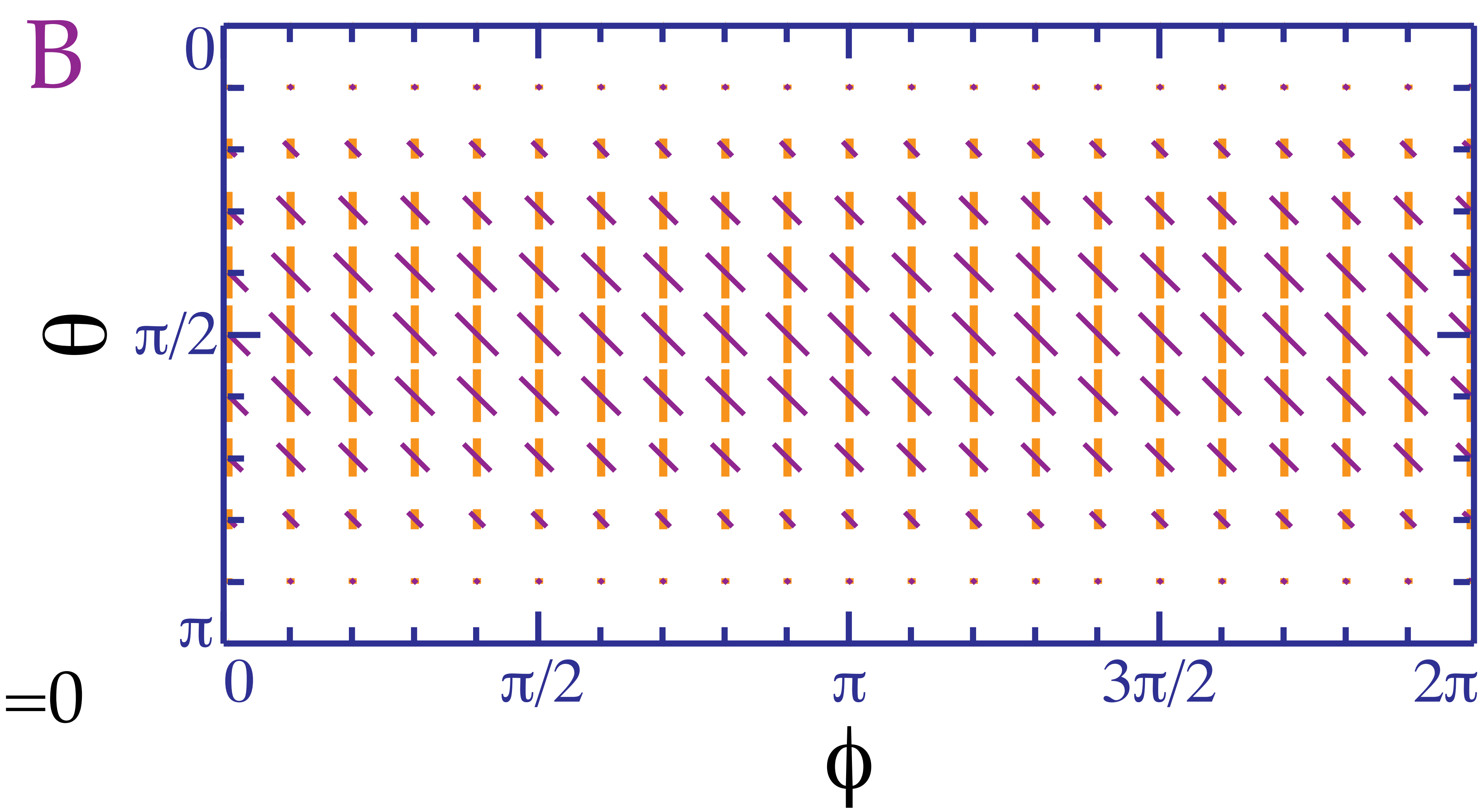
temperature scalar anisotropy



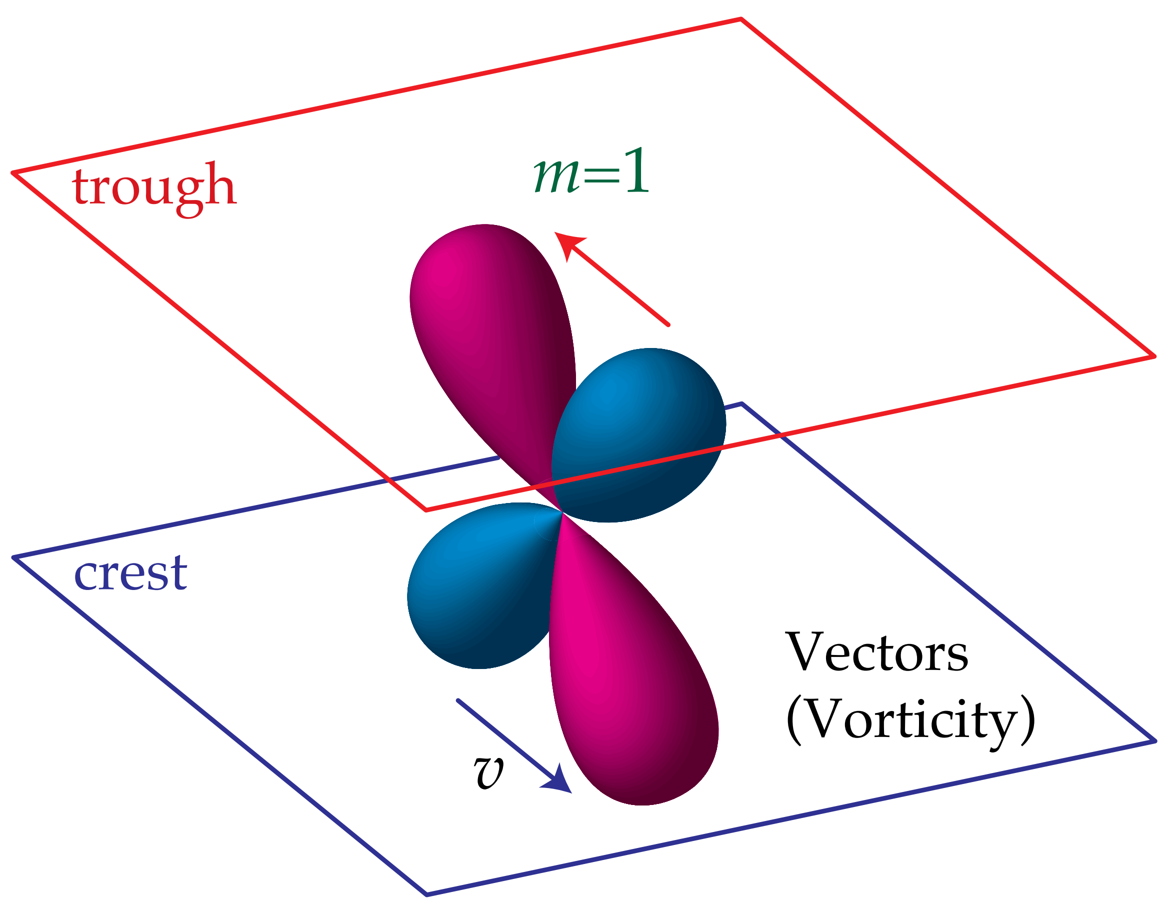
symmetric metric tensor decomposes into 3 parts:

- **scalar (S)**
- **vector (V)**
- **tensor (T)**

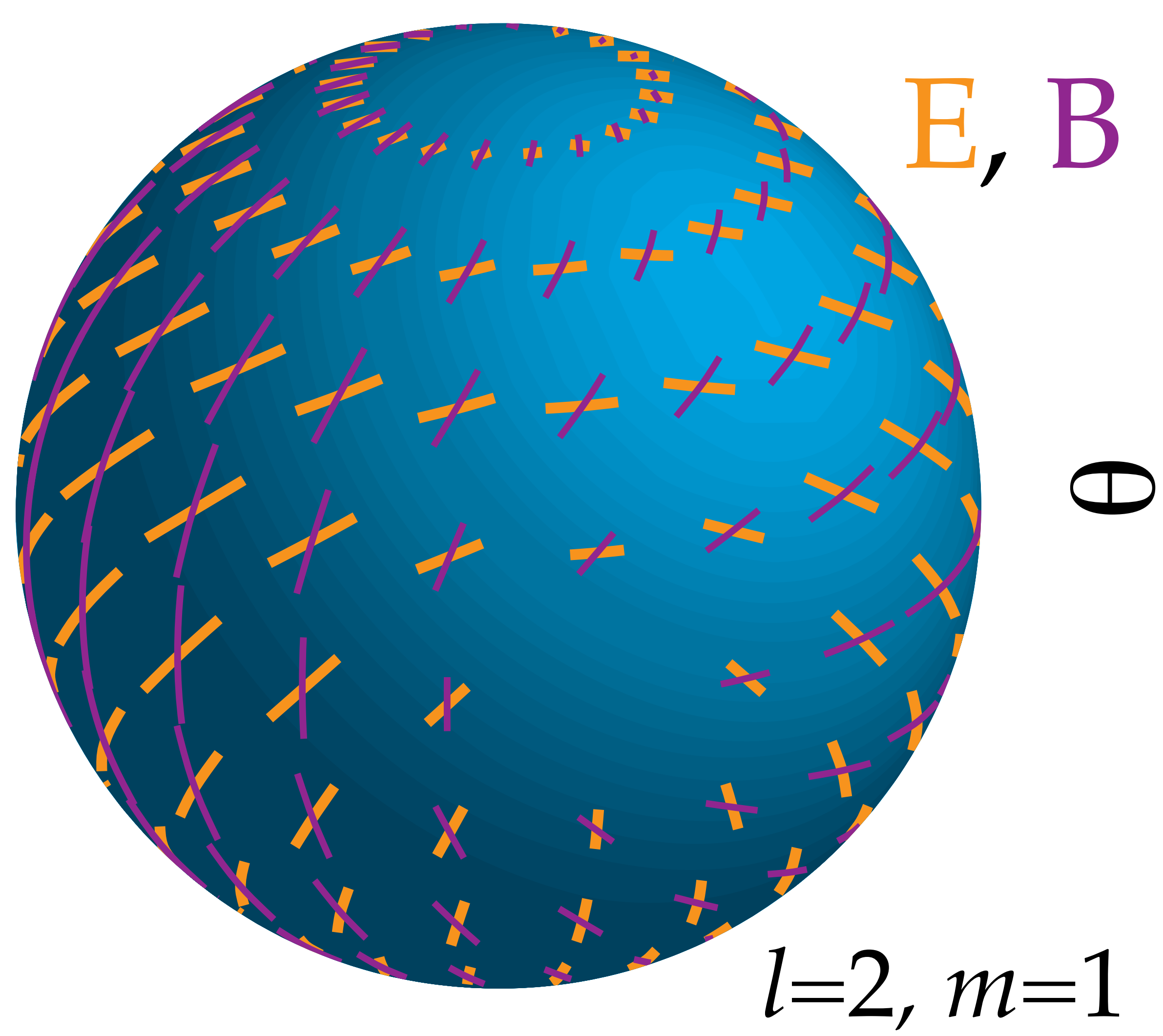


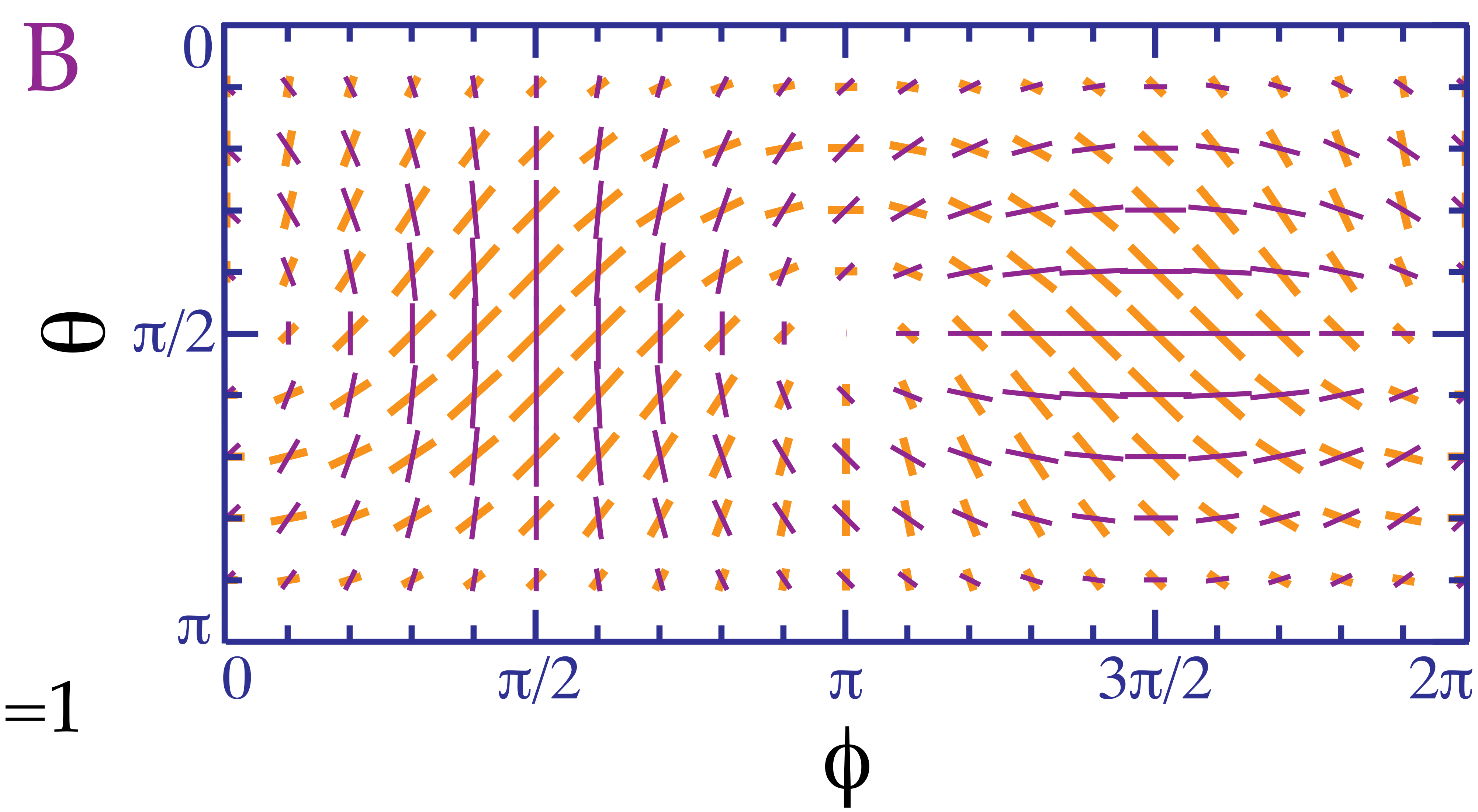


temperature vector anisotropy

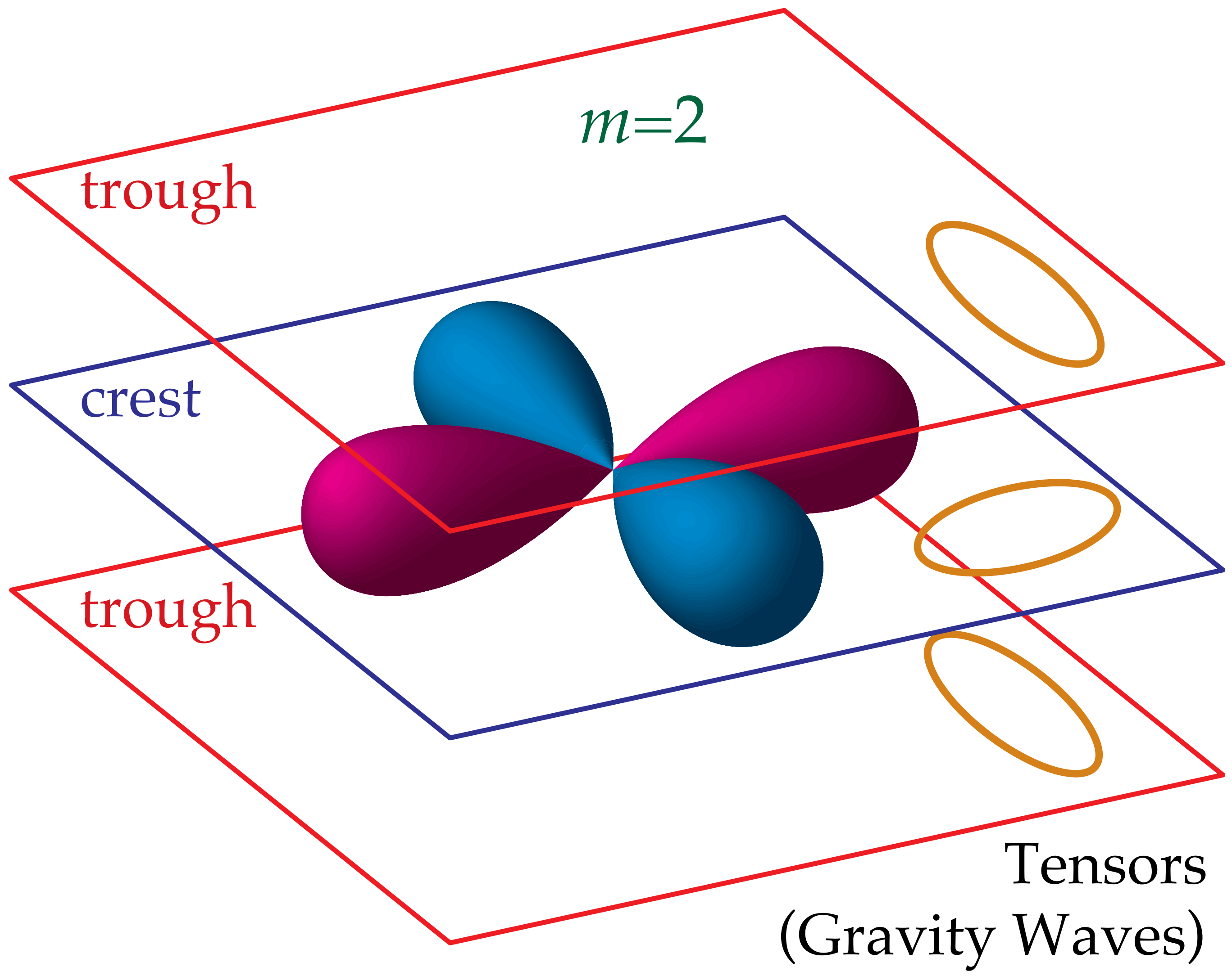


E-mode polarization pattern from quadrupole temperature vector anisotropy



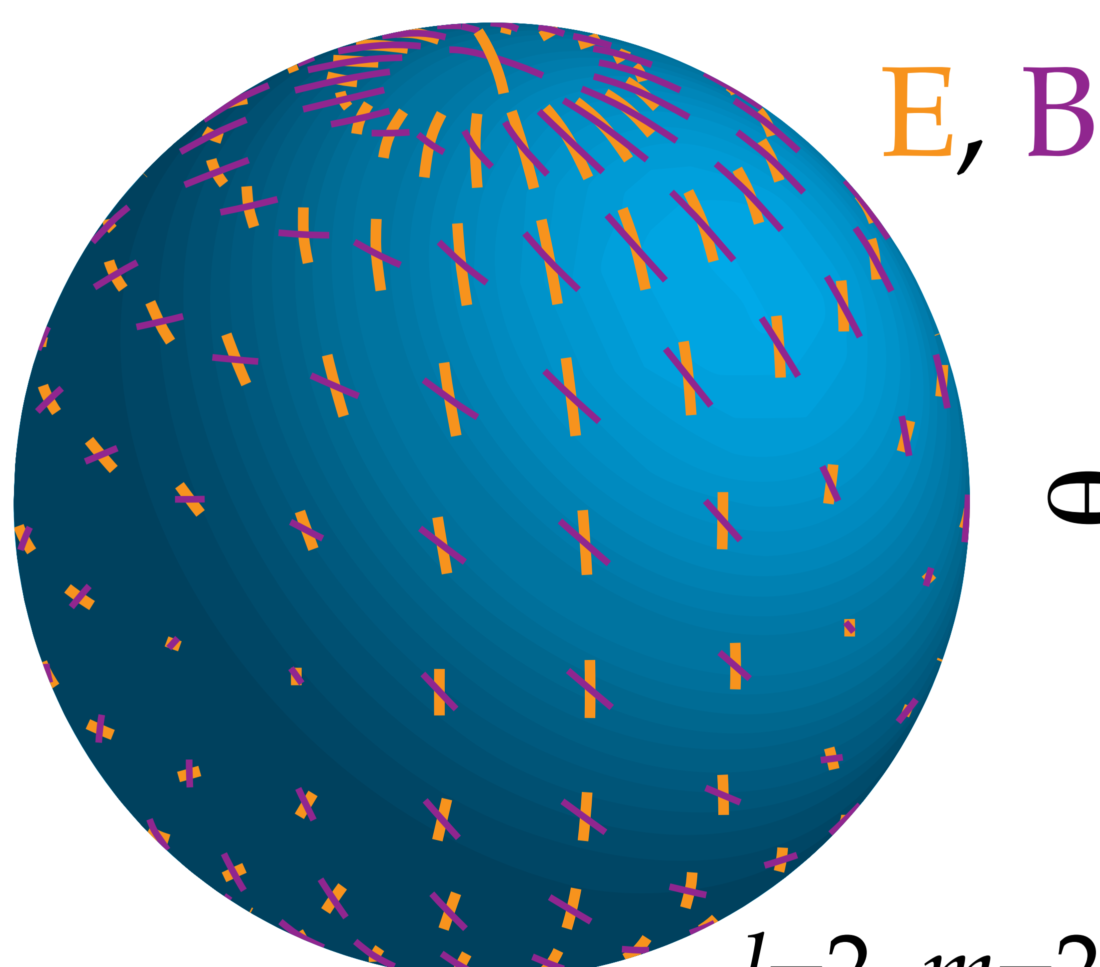


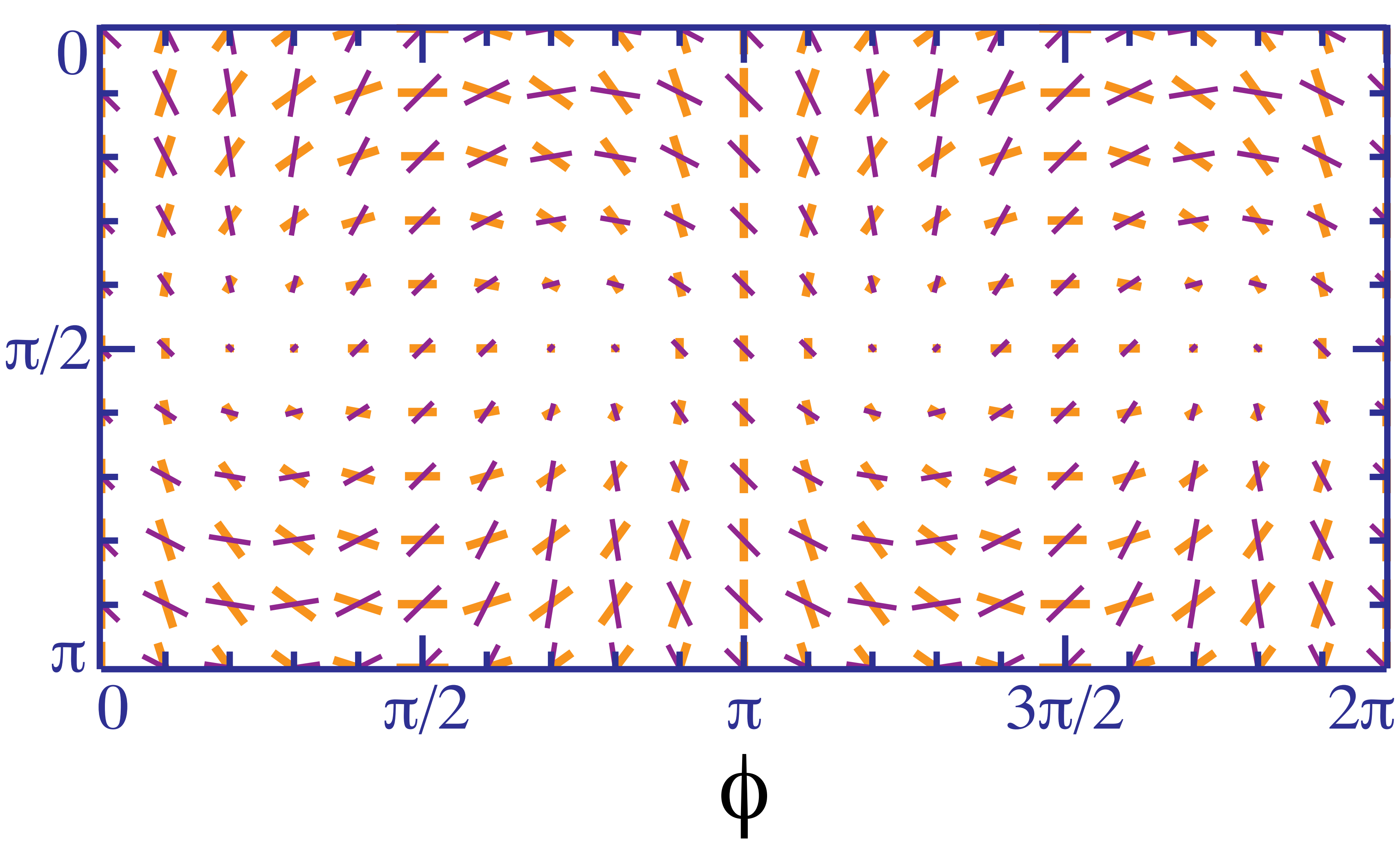
temperature tensor anisotropy

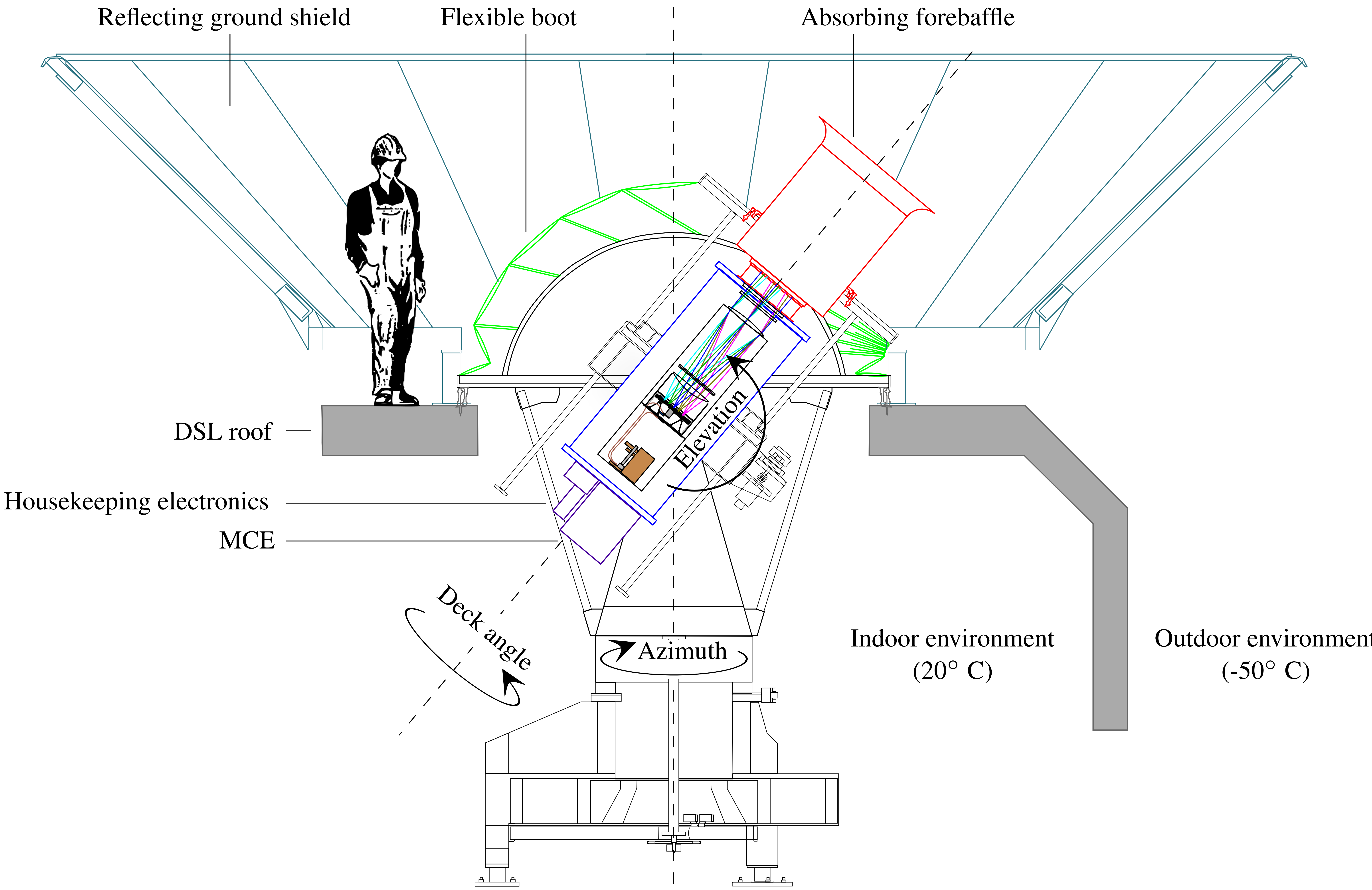


before CMB-formation — only gravitational waves can generate tensor anisotropy

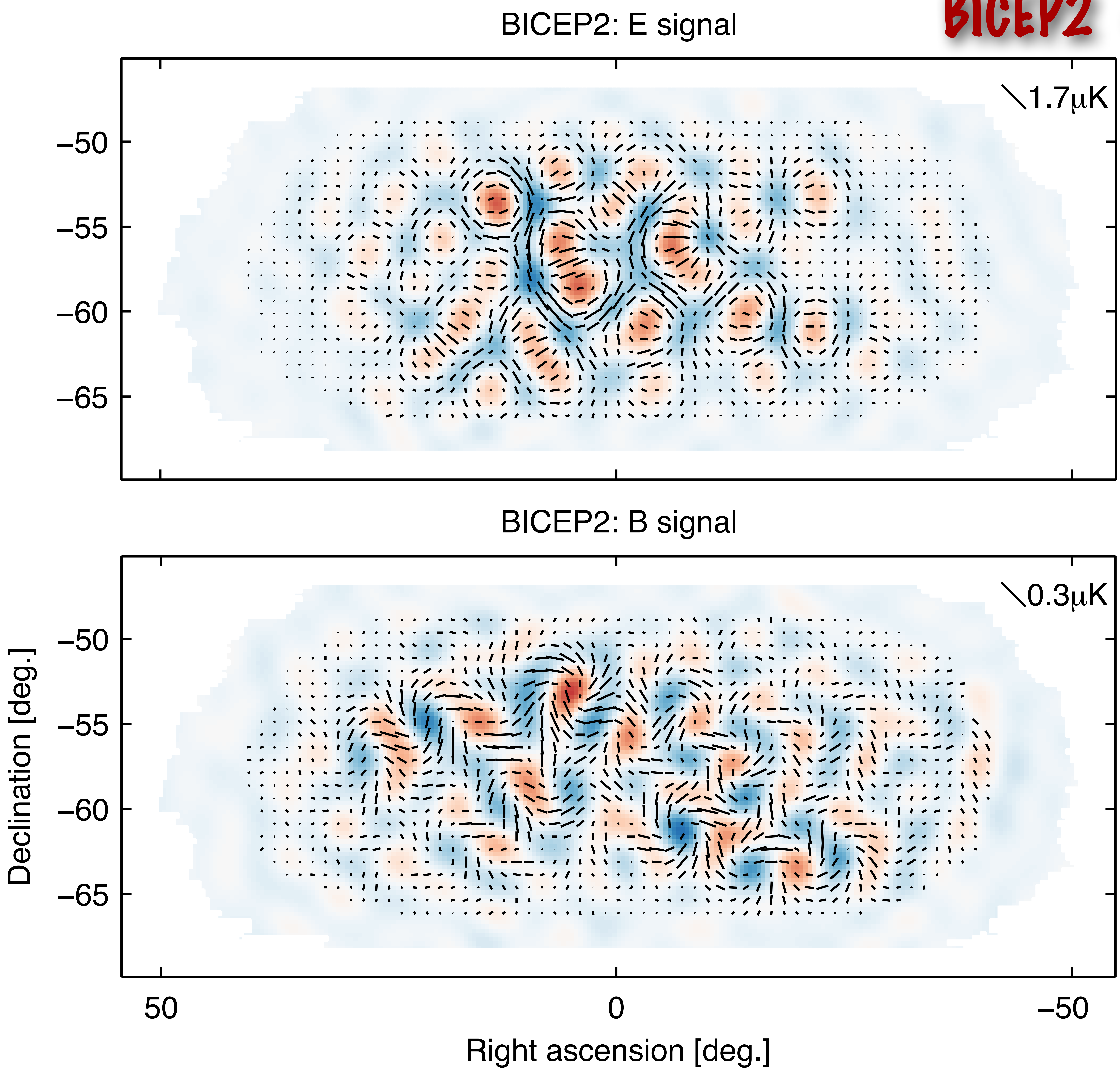
B-mode polarization pattern from quadrupole temperature tensor anisotropy

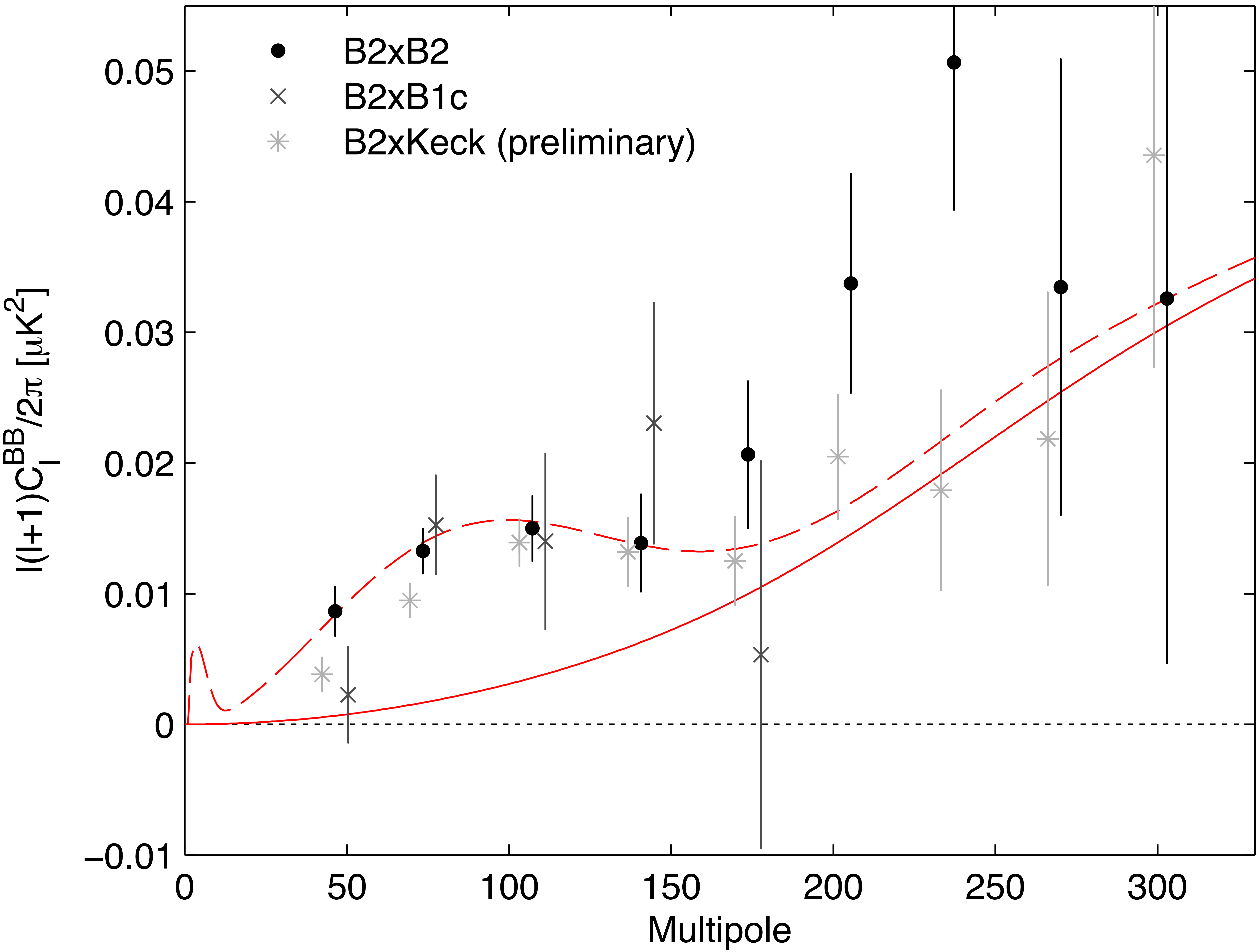


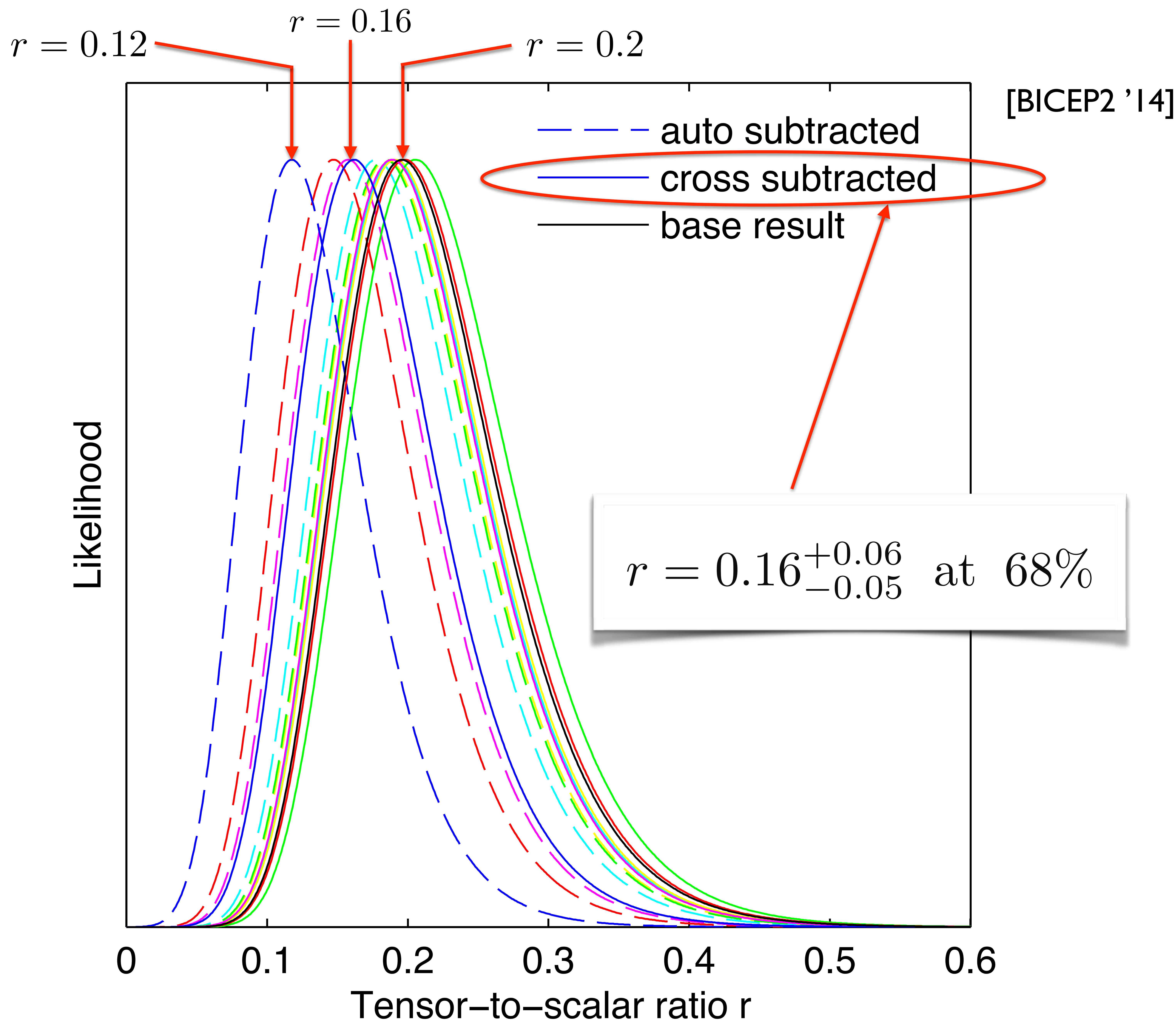



Reflecting ground shield
Flexible boot
Absorbing forebaffle
DSL roof
Housekeeping electronics
MCE
Elevation
Deck angle
Azimuth
Indoor environment
(20° C)
Outdoor environment
(-50° C)

BICEP2 — 2014


BICEP2: E signal
1.7μK
BICEP2: B signal
0.3μK
−50
−55
−60
−65
Declination [deg.]
50
0
−50
Right ascension [deg.]


B2xB2
B2xB1c
B2xKeck (preliminary)
0.05
0.04
0.03
0.02
0.01
0
−0.01
0
50
100
150
200
250
300
Multipole
$l(l+1)C_l^{BB}/2\pi\ [\mu K^2]$



$r = 0.12$
$r = 0.16$
$r = 0.2$
[BICEP2 '14]
auto subtracted
cross subtracted
base result
$r = 0.16^{+0.06}_{-0.05}$ at 68%
Likelihood
0
0.1
0.2
0.3
0.4
0.5
0.6
Tensor–to–scalar ratio r

single field models ...

- monomial — **large-field, r ~ 0.1** [Linde '83]
  (n = 2/3 , 4/5, 1 , 2 , 3 , 4):

$$V(\phi) = \lambda M_{\rm pl}^4 \left(\frac{\phi}{M_{\rm pl}}\right)^n$$

$$n_s = 1 - \frac{n+2}{2N_e} \quad , \quad r = \frac{4n}{N_e}$$

$$\Delta\phi(N_e) = \sqrt{2nN_e}\, M_{\rm P}$$

- natural (axion) inflation — **large & small-field:**

$$V(\phi) = \Lambda^4 \left[1 + \cos\left(\frac{\phi}{f}\right)\right]$$

$$f \gtrsim 1.5\, M_{\rm P} \quad : \quad \text{large-field } (m^2\phi^2): \quad n_s = 1 - \frac{2}{N_e} \quad , \quad r = \frac{8}{N_e}$$

$$f \lesssim 1.5\, M_{\rm P} \quad : \quad \text{small-field}: \quad n_s \approx 1 - \frac{M_{\rm P}^2}{f^2} \quad , \quad r \to 0$$

single field models ...

- **hill-top — small-field, r < 0.01** (except when quadratic):

$$V(\phi) \approx \Lambda^4 \left(1 - \frac{\phi^p}{\mu^p} + \ldots\right)$$

$$p = 2 \;:\; \text{ large-field, fits Planck for } \mu \gtrsim 9\, M_{\mathrm{P}}$$

$$p \geq 3 \;:\; \text{ small-field : } \; n_s = 1 - \frac{2}{N_e}\frac{p-1}{p-2} \;,\; r \to 0 \,, \; \text{ fits Planck for } p \geq 4$$

- (D-term) hybrid inflation — **small-field, r < 0.01** $\alpha = \alpha_{1\text{-loop}} << 1$:

$$V(\phi,\chi) = \Lambda^4 \left(1 - \frac{\chi^2}{\mu^2}\right)^2 + U(\phi) + \frac{g^2}{2}\phi^2\chi^2$$

$$U(\phi) = \alpha_{\mathrm{h}} \Lambda^4 \ln\left(\frac{\phi}{\mu}\right)$$

$$n_s = 1 - \frac{1 + 3\alpha_h/2}{N_e}$$

$$r = \frac{8\alpha_h}{N_e}$$

single field models ...

- $R+R^2$ / Higgs inflation / fibre inflation in LVS string scenarios — $O(M_P)$ field range, **$r < 0.01$**:

$$S = \int d^4x \sqrt{-g}\frac{M_{\rm pl}^2}{2}\left(R + \frac{R^2}{6M^2}\right) \quad \Rightarrow \quad V(\phi) = \frac{3M^2}{4}\left(1 - e^{-\sqrt{\frac{2}{3}}\phi}\right)^2$$

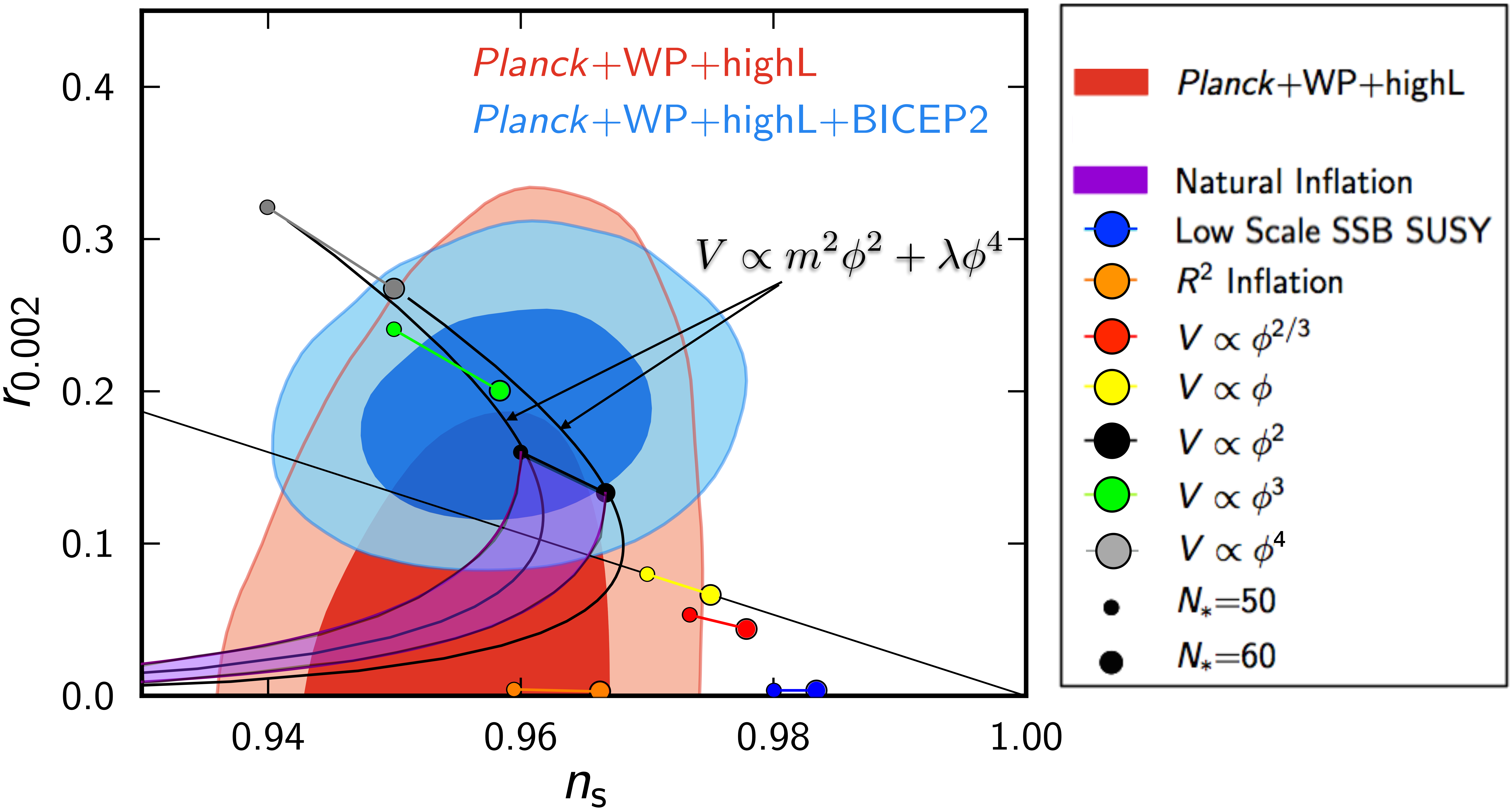
$$\text{or fibre inflation}: \quad V(\phi) \sim \left(1 - \frac{4}{3}e^{-\sqrt{\frac{1}{3}}\phi}\right)$$

[Cicoli, Burgess & Quevedo '08]

$$V \sim 1 - e^{-\sqrt{\frac{2}{3\alpha}}\phi} \quad \Rightarrow \quad n_s = 1 - \frac{2}{N_e} \quad , \quad r = \alpha\frac{12}{N_e^2} < 0.01$$

[Kallosh, Linde & Roest '13]

BICEP2 + PLANCK + WP + highL (with running) ...



shades of difficulty ...

- observable tensors link levels of difficulty:

$$r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_S} = 16\epsilon \leq 0.003 \left(\frac{50}{N_e}\right)^2 \left(\frac{\Delta\phi}{M_{\rm P}}\right)^2$$ [Lyth '97]

- $r << O(1/N_e^2)$ models:

$\Delta\phi \ll \mathcal{O}(M_P) \quad \Rightarrow$

Small-Field inflation ... needs control of leading dim-6 operators

➥ enumeration & fine-tuning reasonable

- $r = O(1/N_e^2)$ models:

$\Delta\phi \sim \mathcal{O}(M_P) \quad \Rightarrow$

needs *severe* fine-tuning of all dim-6 operators, or accidental cancellations

- $r = O(1/N_e)$ models:

$\Delta\phi \sim \sqrt{N_e} M_P \gg M_P \quad \Rightarrow$

Large-Field inflation ... needs suppression of all-order corrections

➥ symmetry is essential!

why strings?

- We need to understand generic dim ≥ 6 operators

$$\mathcal{O}_{p\geq 6} \quad \sim \quad V(\phi)\left(\frac{\phi}{M_{\rm P}}\right)^{p-4}$$

$$\Rightarrow \quad \Delta\eta \quad \sim \quad \left(\frac{\phi}{M_{\rm P}}\right)^{p-6} \gtrsim 1 \quad \forall p \geq 6 \quad \text{if} \quad \phi > M_{\rm P}$$

- requires <u>UV-completion</u>, e.g. string theory: need to know string and α'-corrections, backreaction effects, ...
- typical approximations (tree-level, large-volume/non-compact, probe ...) often insufficient
- detailed information about moduli stabilization necessary!
- string theory manifestation of the supergravity eta problem

shades of difficulty ...

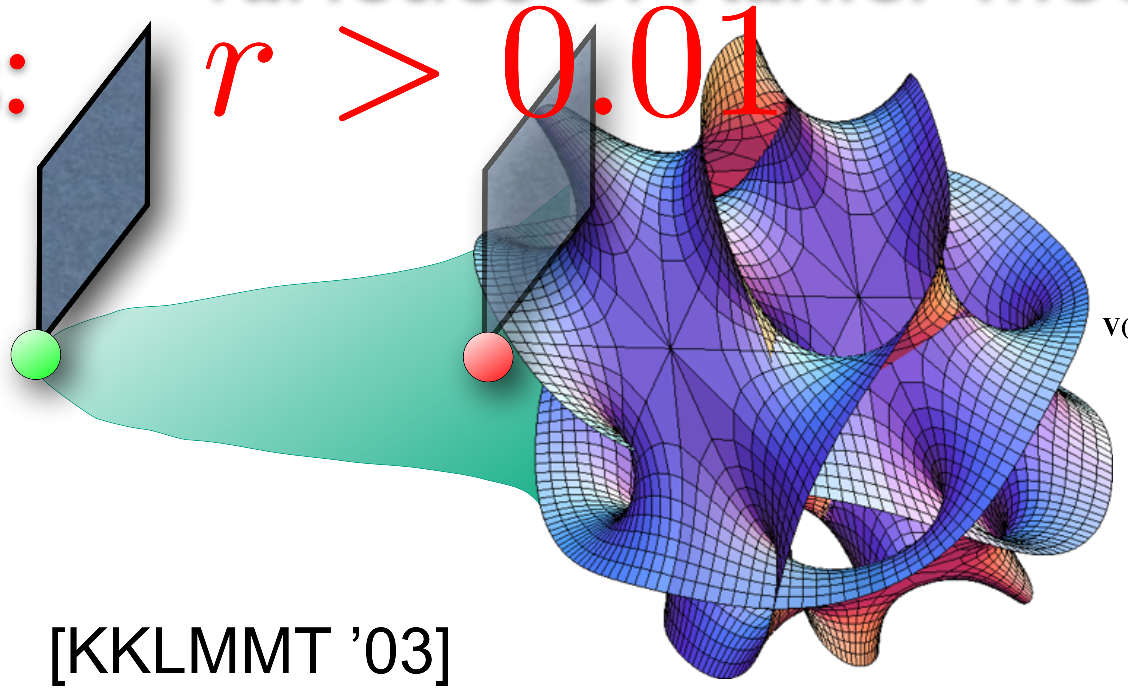
- observable tensors link levels of difficulty:

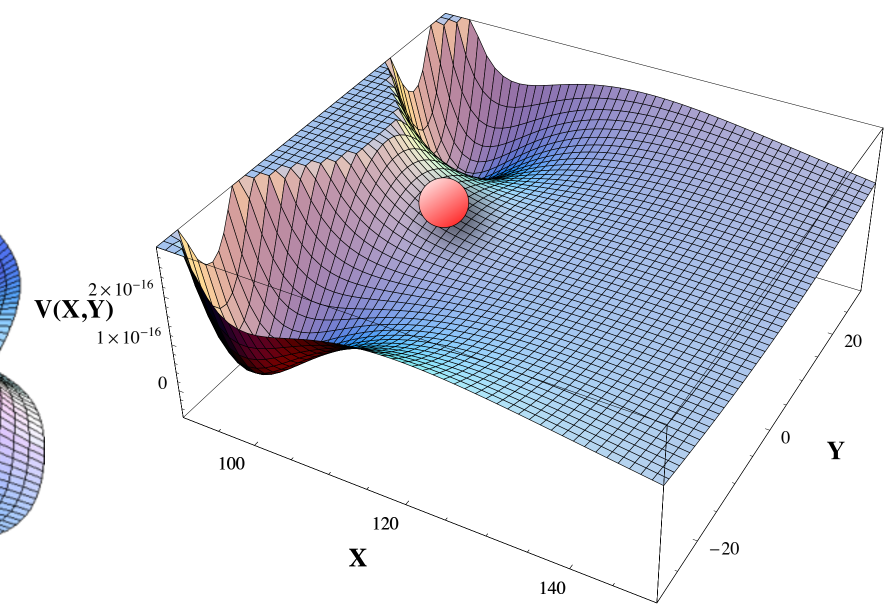
$$r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_R} = 16\epsilon \leq 0.003 \left(\frac{50}{N_e}\right)^2 \left(\frac{\Delta\phi}{M_{\rm P}}\right)^2$$ [Lyth '97]

- r << $O(1/N_e^2)$ models:

$\Delta\phi \ll \mathcal{O}(M_P) \quad \Rightarrow$ warped D-brane inflation & DBI; varieties of Kahler moduli inflation

observable tensors: $r > 0.01$





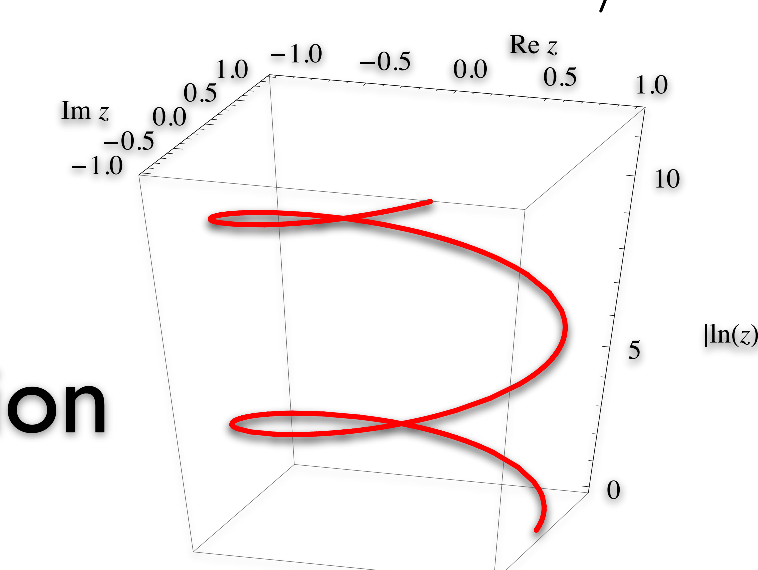
[KKLMMT '03]
[Baumann, Dymarsky, Klebanov, McAllister & Steinhardt '07]

- $r = O(1/N_e^2)$ models:

$\Delta\phi \sim \mathcal{O}(M_P) \quad \Rightarrow$ fibre inflation in LARGE volume scenarios (LVS)

[Cicoli, Burgess & Quevedo '08]

$T_{D5/NS5}$



- $r = O(1/N_e)$ models:

$\Delta\phi \sim \sqrt{N_e} M_P \gg M_P \quad \Rightarrow$ axion monodromy inflation

large field inflation in string theory ...

large field inflation ...

- either need many fields in lockstep:

$$\Delta\phi_{diag.} \sim \sqrt{\sum_{i=1\ldots N} \Delta\phi_i^2} \gg 1 \qquad \text{with} \qquad |\Delta\phi_i| < 1$$

called " *N-flation* "

[Dimopoulos, Kachru, McGreevy & Wacker '05]

- string theory embedding is challenging, due to need for large number of fields w/ instanton potentials ...

see: [Grimm '07] for coming very close

and: [Cicoli, Dutta & Maharana '14]

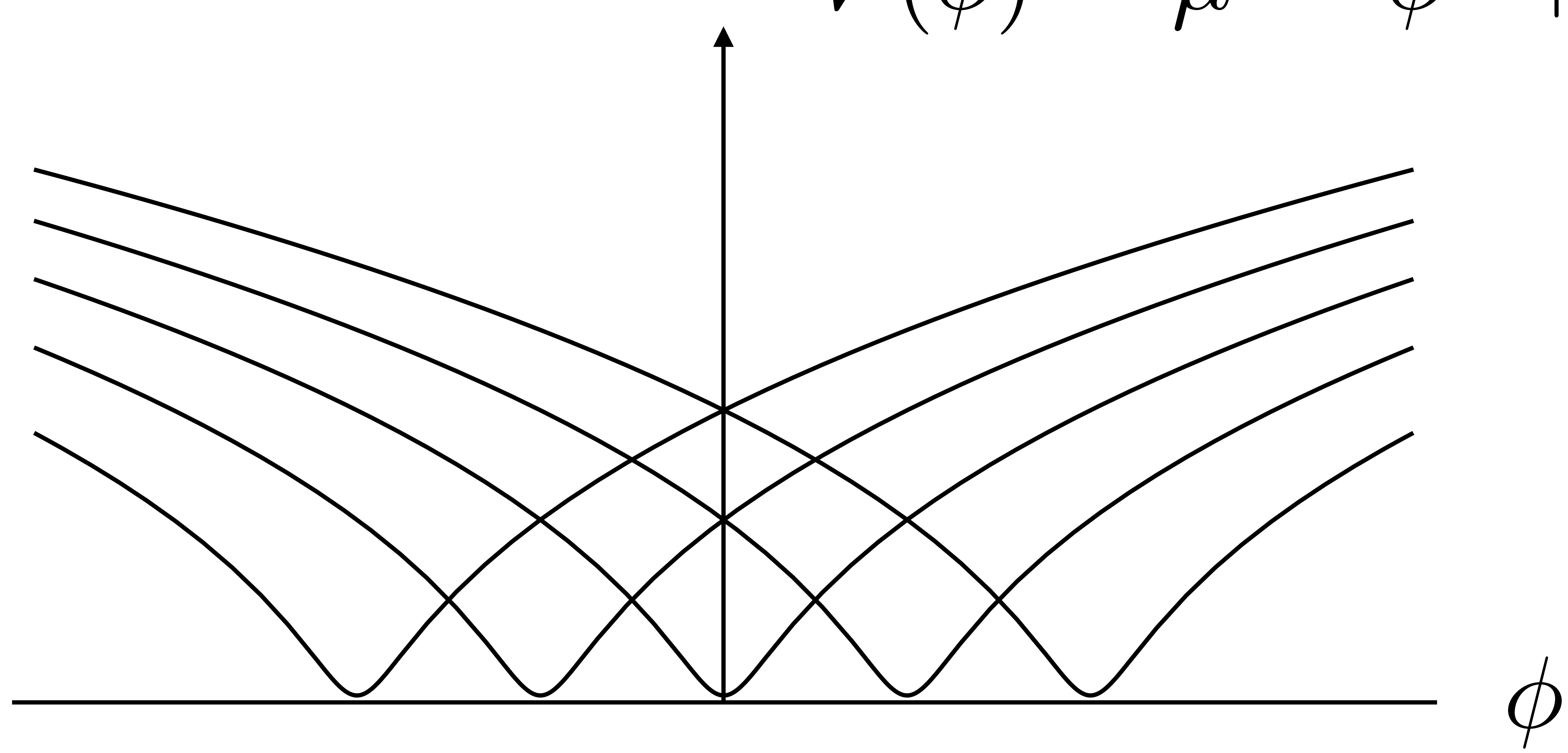
but constraints from G_N renormalization & dS entropy: [Conlon '12]

axion monodromy inflation - general story

[McAllister, Silverstein & AW '08]
[Kaloper & Sorbo '08]
[Flauger, McAllister, Pajer, Xu & AW '09]
[Berg, Pajer & Sjörs '09]
[Dong, Horn, Silverstein & AW '10]
[Lawrence, Kaloper & Sorbo '11]
[Dubovsky, Lawrence & Roberts '11]

- p-form axions get **non-periodic** potentials from coupling to **branes** or **fluxes/field-strengths**
- produces periodically spaces set of multiple branches of large-field potentials:

$$V(\phi) \sim \mu^{4-p}\phi^p + \Lambda^4 \cos(\phi/f)$$



axion monodromy inflation - an example

[McAllister, Silverstein & AW '08]

- now let's take a 5-brane:

$$\text{wraps:} \quad (3+1)_{\text{large}} + 2_{\text{small}}$$

space time e.g. a 2-sphere

- put a B_2 (or C_2) field on small 2-sphere with volume v :

$$S_{5-\text{brane}} \sim \frac{1}{g_s} \int_{\mathcal{M}_4 \times \text{2-sphere}} d^6\xi \sqrt{\det(G+B)}$$

$$= \frac{1}{g_s} \int_{\mathcal{M}_4} d^4x \sqrt{-g} \sqrt{v^2 + b^2}$$

monodromy; breaks perturbative shift symmetry in B_2

$$\Rightarrow V(b) \sim b \;, \quad b \text{ large, } \textit{non}\text{-periodic}$$

$$n_S \simeq 0.975$$
$$r \simeq 0.08$$

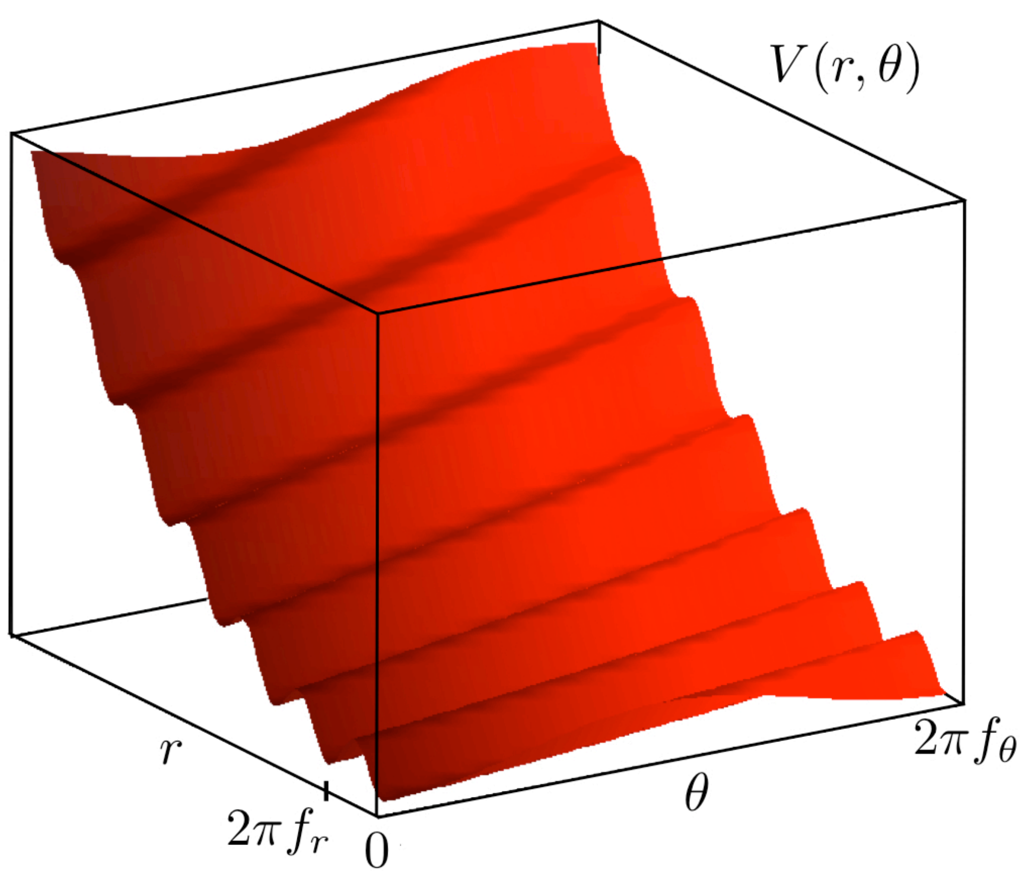
axion monodromy inflation - a 2nd example

"Dante's Inferno": [Berg, Pajer & Sjörs '09]

- 2 axions, a 5-brane, and an ED1-instanton:

$$V \sim \mu^4 \sqrt{1+r^2} + \Lambda^4 \left[1 - \cos\left(\frac{r}{f_r} + \frac{\theta}{f_\theta} \right) \right] \quad , \quad f_r \ll f_\theta < M_{\rm P}$$

$$V_{eff.}(\tilde{\phi}) \simeq m^2 \tilde{\phi}^2 \quad \text{for} \quad \tilde{\phi} \gg M_{\rm P} \quad \text{while} \quad r, \theta < M_{\rm P}$$



brane-free variant:
[Ben-Dayan, Pedro & AW: work in progress]

axion monodromy inflation from fluxes

[Dong, Horn, Silverstein & AW '10]

- we can compute the inflaton potential using the N=2 gauge theory data, or using the gravity dual:

$$V_{\mathcal{N}=2}(\phi) \sim V_{grav.\,side}(\phi) \sim \int F_5 \wedge \star F_5 \sim \phi$$

  the same as for the 5-brane monodromy, where lots of extra flux kept the throats open

- if only the inflaton B_2 (or C_2) keeps the throats open, they shrink during inflation, and:

$$V_{grav.\,side}(\phi) \sim \int F_5 \wedge \star F_5 \sim \phi^{4/5}$$

axion monodromy inflation from fluxes

[Dong, Horn, Silverstein & AW '10]

- a question arises:
  the flux terms are quadratic - so why not just quadratic potentials?

- this is a general tendency:
  heavy moduli fields & fluxes backreact, and *flatten* the potential - a simple energetic argument:

$$V(\phi_L, \phi_H) = g^2\phi_L^2\phi_H^2 + m^2(\phi_H - \phi_0)^2$$

light field

heavy field

$$\Rightarrow \quad V(\phi_L, \phi_{H,min}(\phi_L)) = \frac{g^2\phi_L^2}{g^2\phi_L^2 + m^2} m^2\phi_0^2 \quad \rightarrow \quad const.$$

axion monodromy inflation from fluxes

[McAllister, Senatore, Silverstein, Wrase & AW: work in progress]

see also: [Palti & Weigand; Marchesano, Shiu & Uranga;
Hebecker, Kraus & Witkowski '14]

- type IIB string theory:

$$\int \mathrm{d}^{10}x \left(\frac{|dB|^2}{g_s^2} + |F_1|^2 + |F_3|^2 + \left|\tilde{F}_5\right|^2 \right)$$

$$\text{with:} \quad \tilde{F}_5 = dC_4 - B_2 \wedge F_3 + C_2 \wedge H_3 + F_1 \wedge B_2 \wedge B_2$$

- ϕ^2 , ϕ^3, ϕ^4 terms …

  - generically flattening of the potential from adjusting moduli and/or flux rearranging its distribution on its cycle - 'sloshing', while preserving flux quantization [Dong, Horn, Silverstein & AW '10]

axion monodromy inflation from fluxes

[McAllister, Senatore, Silverstein, Wrase & AW: work in progress]

- simple torus example: $$ds^2 = \sum_{i=1}^{3} L_1^2 (dy_1^{(i)})^2 + L_2^2 (dy_2^{(i)})^2$$

axion $$B = \sum_{i=1}^{3} \frac{b}{L^2} dy_1^{(i)} \wedge dy_2^{(i)}$$

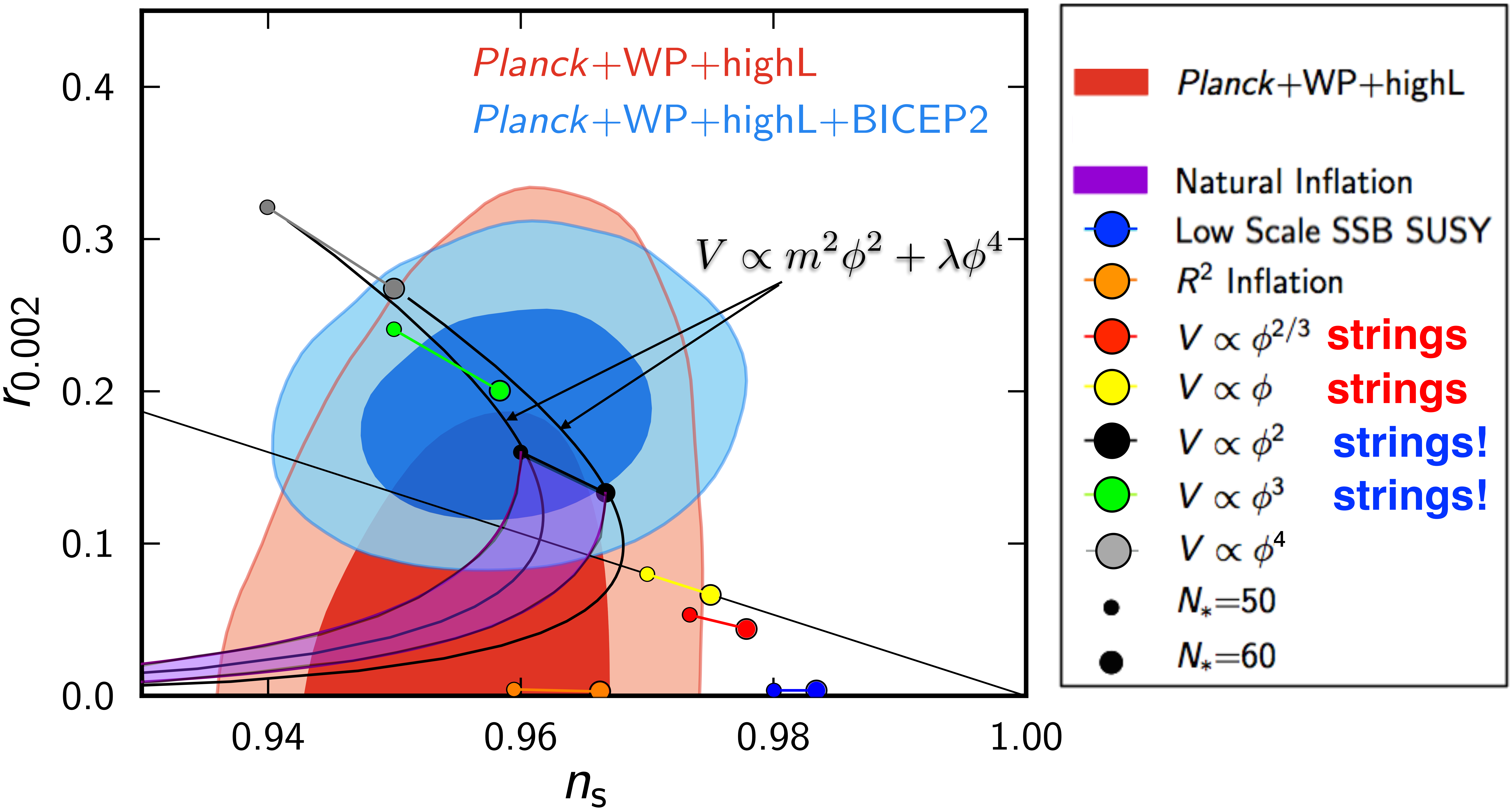
fluxes
$$F_3 = Q_{31} dy_1^{(1)} \wedge dy_1^{(2)} \wedge dy_1^{(3)} + Q_{32} dy_2^{(1)} \wedge dy_2^{(2)} \wedge dy_2^{(3)}$$
$$F_1 = \frac{Q_1}{L_1} \sum_{i=1}^{3} dy_1^{(i)}$$

- effective 4d action gives ϕ^3-potential: $$u = \frac{L_2}{L_1} \quad , \quad \frac{\phi}{M_{\mathrm{P}}} = \frac{b}{L^2}$$

$$\mathcal{L} \sim M_{\mathrm{P}}^2 \frac{\dot{b}^2}{L^4} + M_{\mathrm{P}}^4 \frac{g_s^4}{L^12} \left[Q_1^2 L^4 \left(\frac{b}{L^2} \right)^4 u + Q_{31}^2 u^3 + \frac{Q_{32}^2}{u^3} \right] \sim \dot{\phi}^2 + \mu\, \phi^3$$

- use Riemann surfaces: can fix *Vol = L*^{6} as well & get *m*^{2}ϕ^2

BICEP2 + PLANCK + WP + highL (with running) ...



open questions ...

- BICEP2 provides strong evidence for primordial tensor modes with $r = 0.1 \ \ldots\ 0.2$ — only large-field inflation survives …

- axion monodromy provides one avenue for large field inflation in string theory - technically natural & distinctive predictions ...

- many powers $\phi^{2/3} \ \ldots\ \phi^4$ possible ; we need generalizations ... harder look at universality, generic distinctiveness from field theory models