News from the proton

Lamb shift and hyperfine splitting in muonic hydrogen

Randolf Pohl

for the *CREMA* collaboration

Outline

- The problem:

  Proton rms charge radius r_{p} from muonic hydrogen $\mu\mathrm{p}$ is 4 % smaller than the values from elastic electron-proton scattering and hydrogen spectroscopy.

  That's 5σ ... 9.4σ.

  But the $\boldsymbol{\mu}$**p** result is 10 times more accurate than any other measurement.

- Introduction:

  Hydrogen, fundamental constants, QED tests and all that.

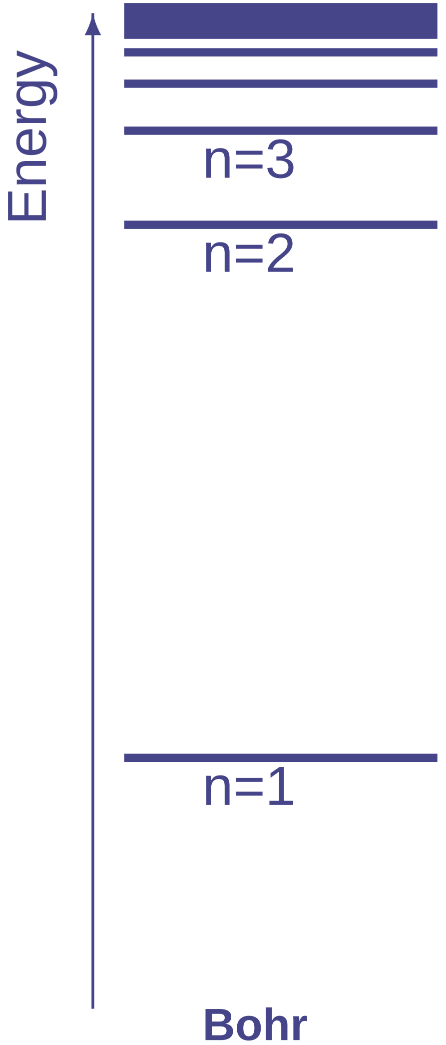
  How large is the proton?

- Muonic hydrogen:

  Size does matter!

- Experiment:
  - Principle
  - Muon beam
  - Laser system
  - NEW DATA
- A solution of the "proton size puzzle"

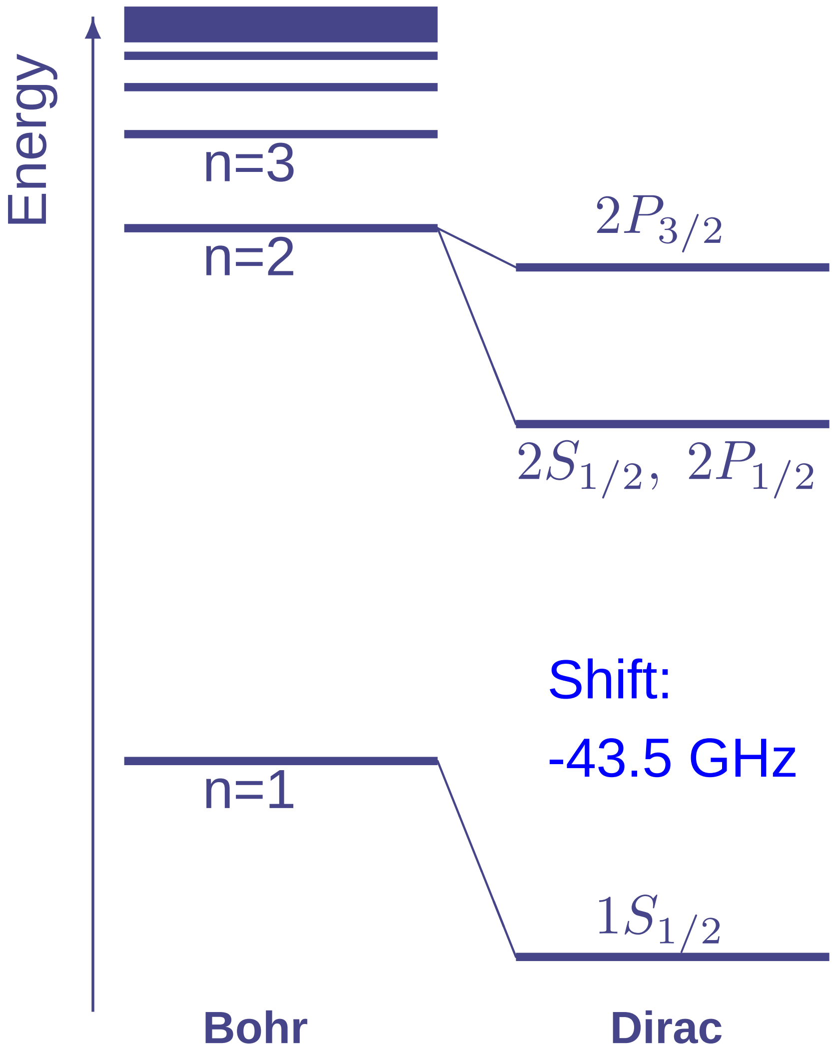
Hydrogen energy levels



$$E = R_\infty / n^2$$

$$V \sim 1/r$$

Hydrogen energy levels



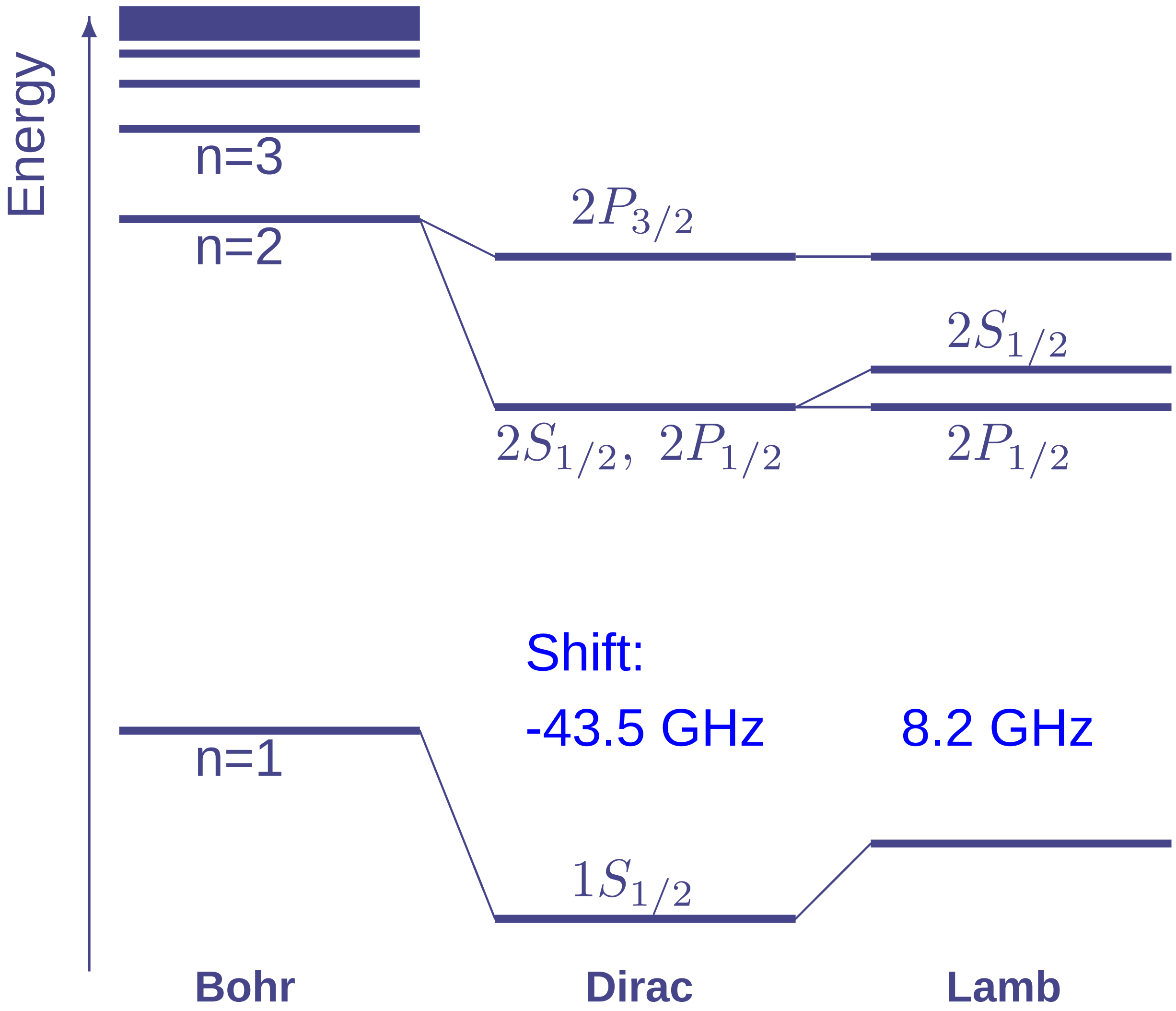
$E = R_\infty / n^2$

$V \sim 1/r$

e^- spin

relativity

Hydrogen energy levels



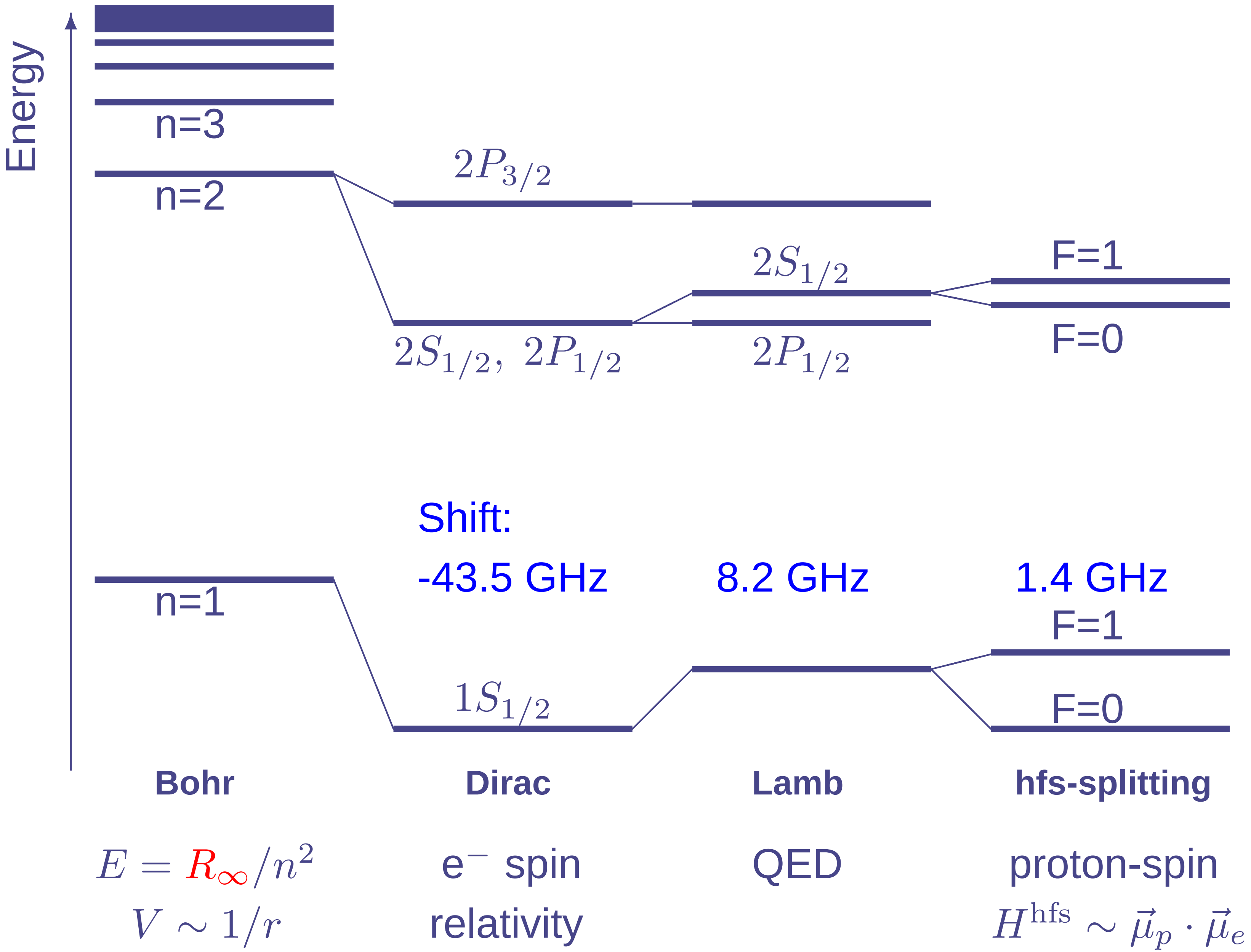
$E = R_\infty / n^2$

$V \sim 1/r$

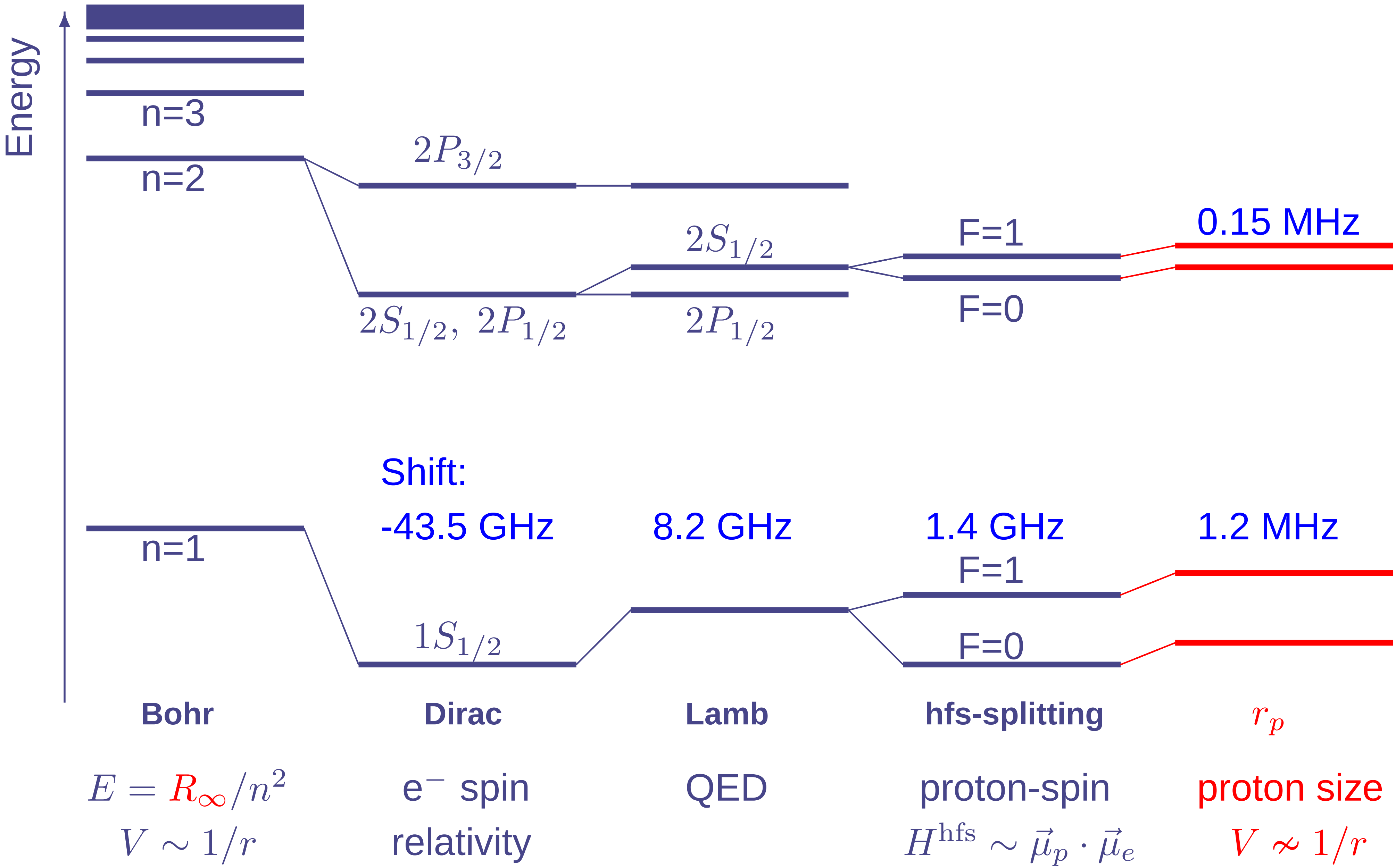
e^- spin relativity

QED

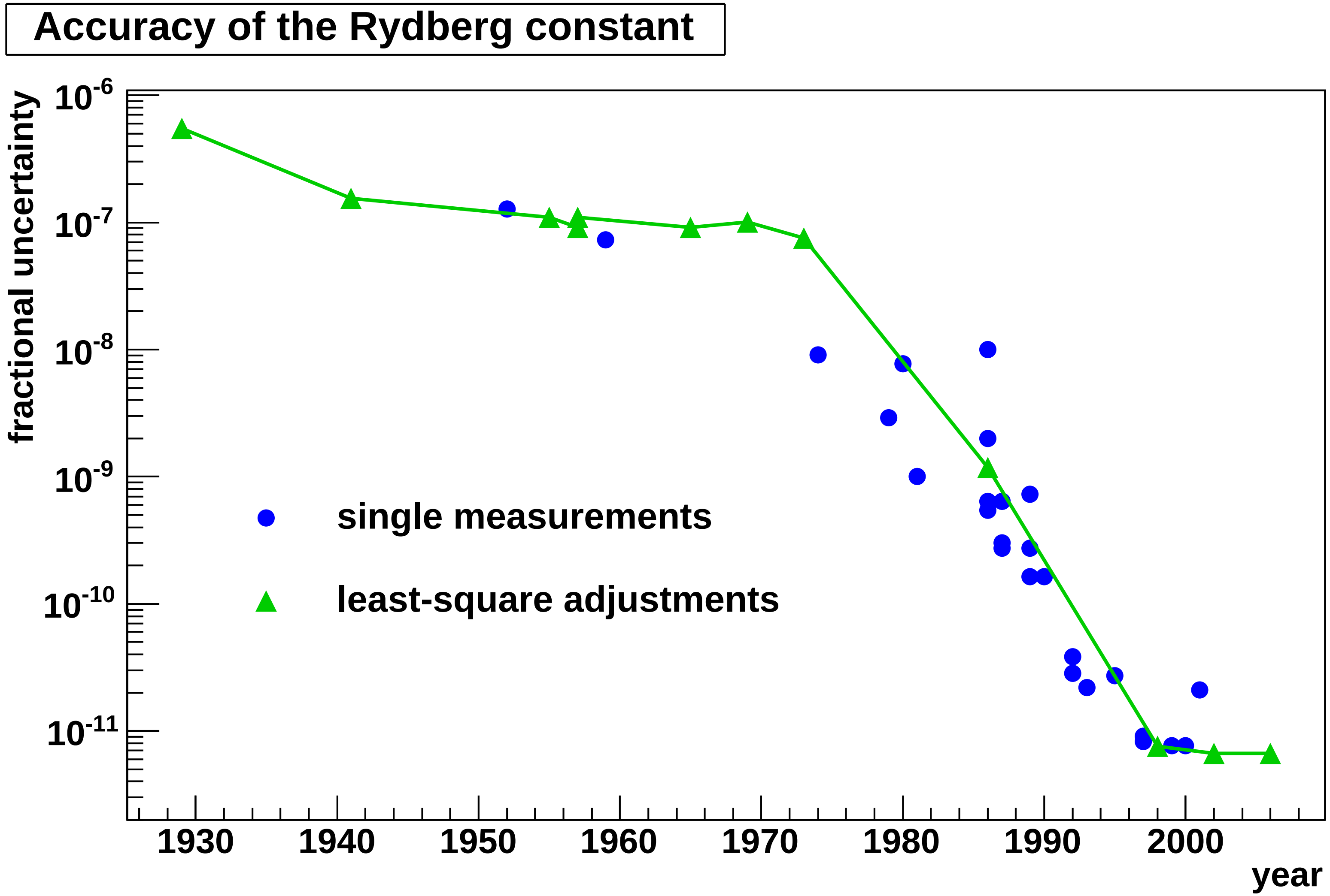
Hydrogen energy levels



Hydrogen energy levels



The Rydberg constant

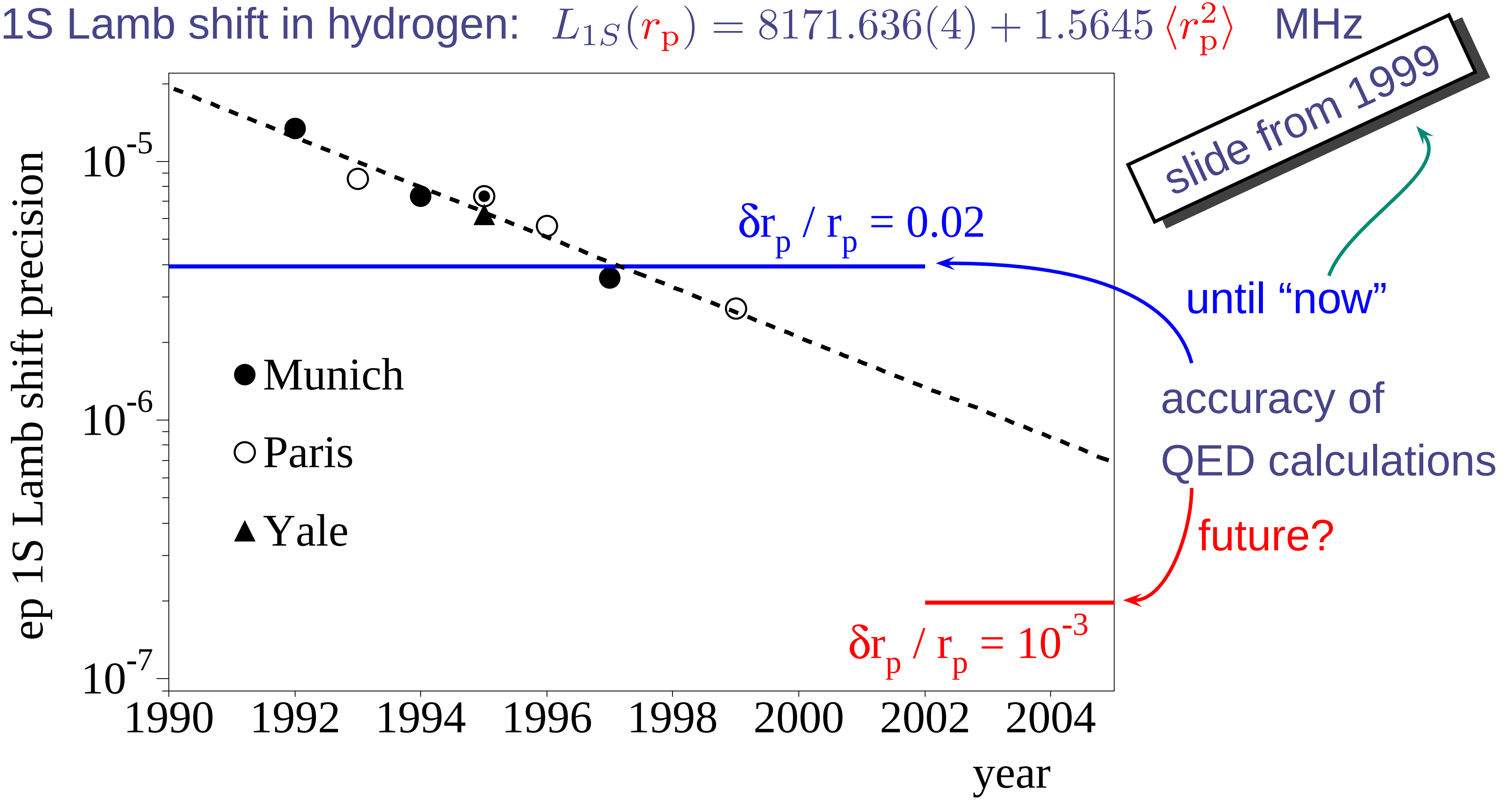


2006: R_∞ = $10\,973\,731.568\,525 \pm 0.000\,073\,\mathrm{m}^{-1}$ $(u_r = 6.6 \cdot 10^{-12})$

is the most accurately determined fundamental constant.

Test of bound-state QED

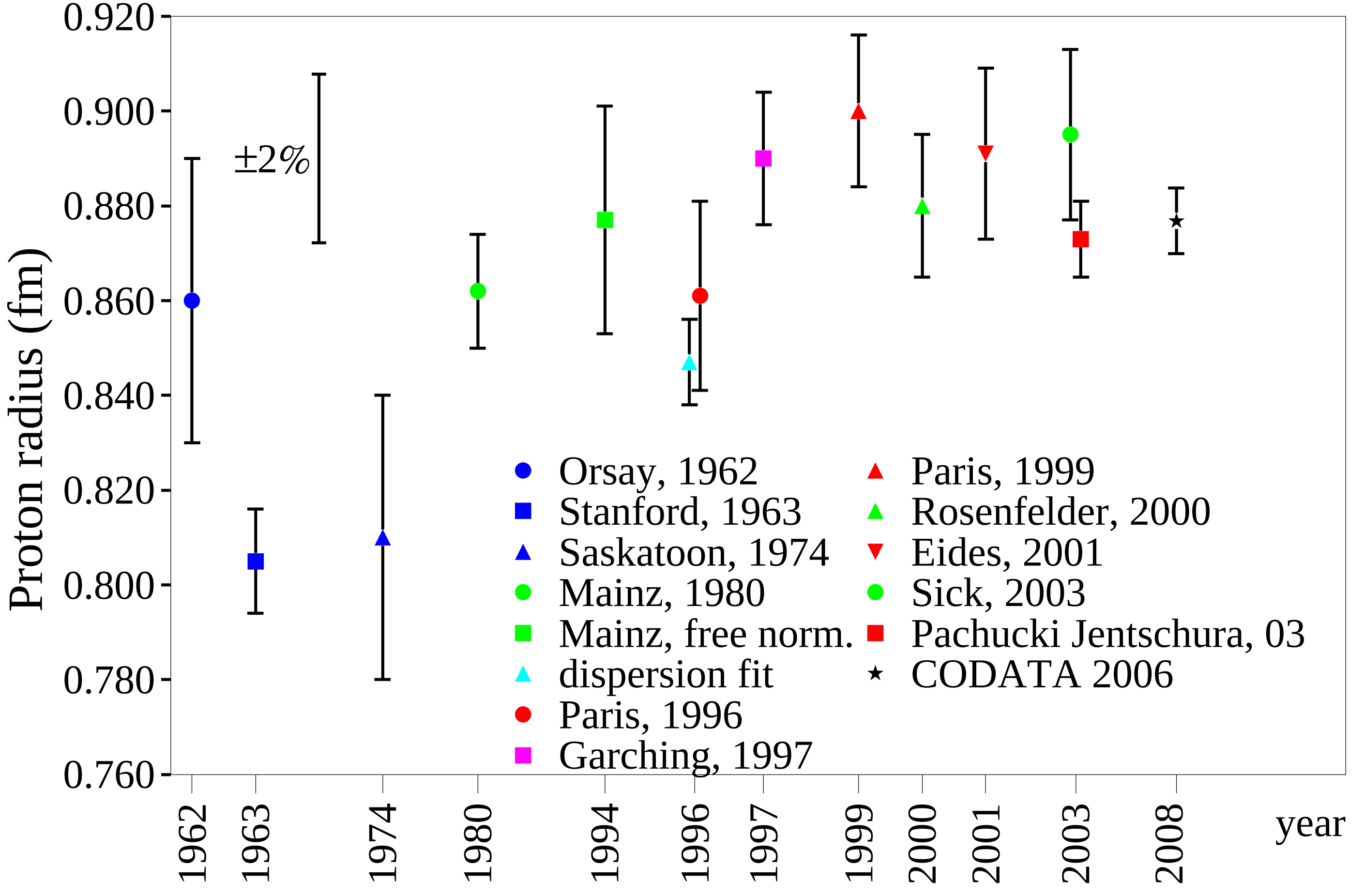
1S Lamb shift in hydrogen: $L_{1S}(r_p) = 8171.636(4) + 1.5645 \langle r_p^2 \rangle$ MHz



QED-test is limited by the uncertainty of the proton rms charge radius.

Proton radius vs. time

The proton rms charge radius is not the most accurate quantity in the universe.



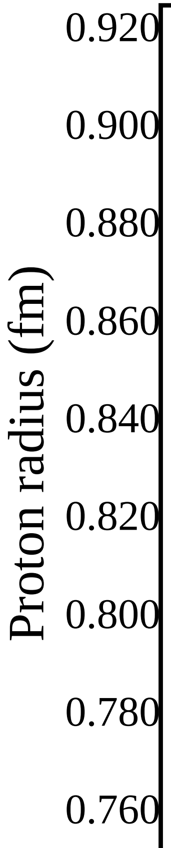
- electron scattering
  slope of G_E at $Q^2 = 0$
- hydrogen spectr.
  Lamb shift (S-states)

e-p scattering: r_p = 0.895(18) fm $(u_r = 2\,\%)$

CODATA: r_p = 0.8768(69) fm $(u_r = 0.8\,\%)$

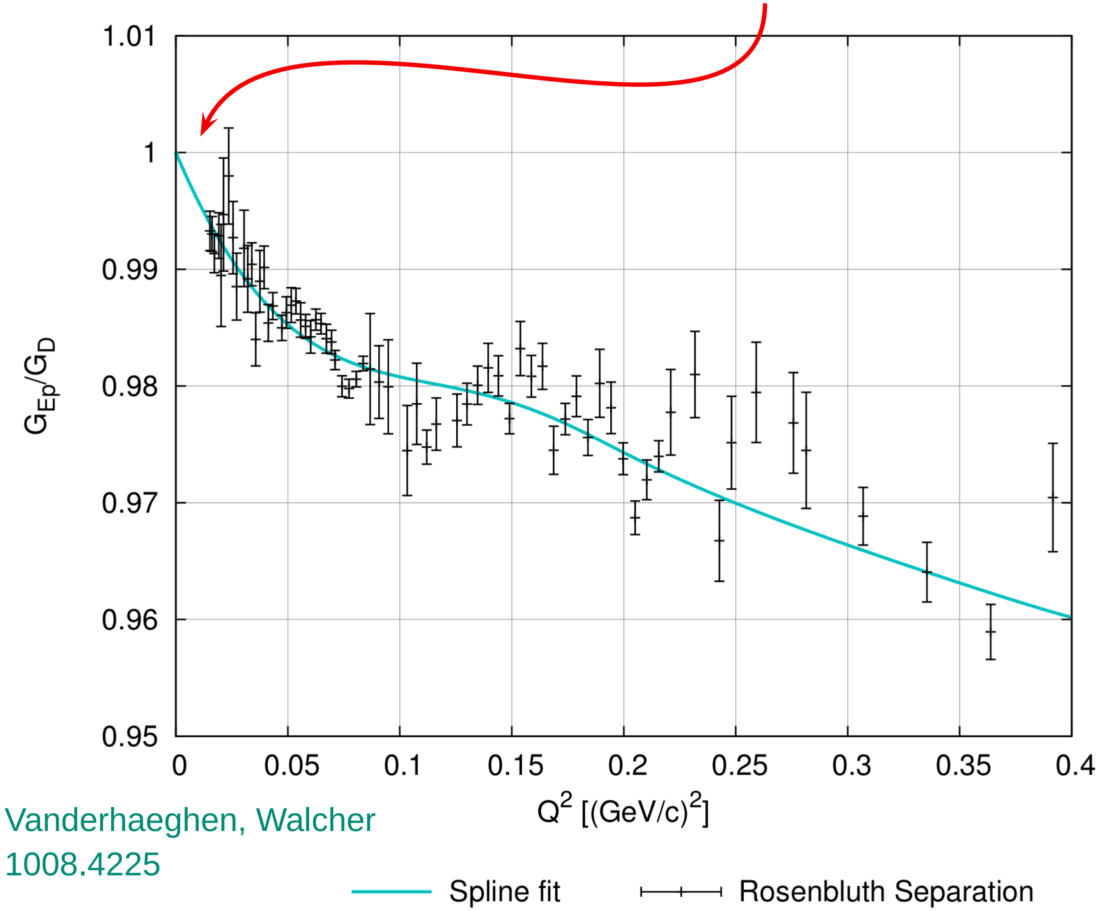
Proton radius vs. time

The proton rms charge radius is not the most accurate quantity in the universe.



Electron scattering:

$$\langle r_p^2 \rangle = -6\hbar^2 \frac{dG_E(Q^2)}{dQ^2}\Big|_{Q^2=0} \Rightarrow \text{slope of } G_E \text{ at } Q^2 = 0$$



Vanderhaeghen, Walcher
1008.4225

e-p
COD

- electron scattering

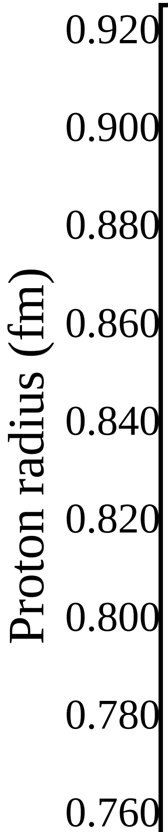
  slope of G_E at $Q^2 = 0$

- hydrogen spectr.

  Lamb shift (S-states)

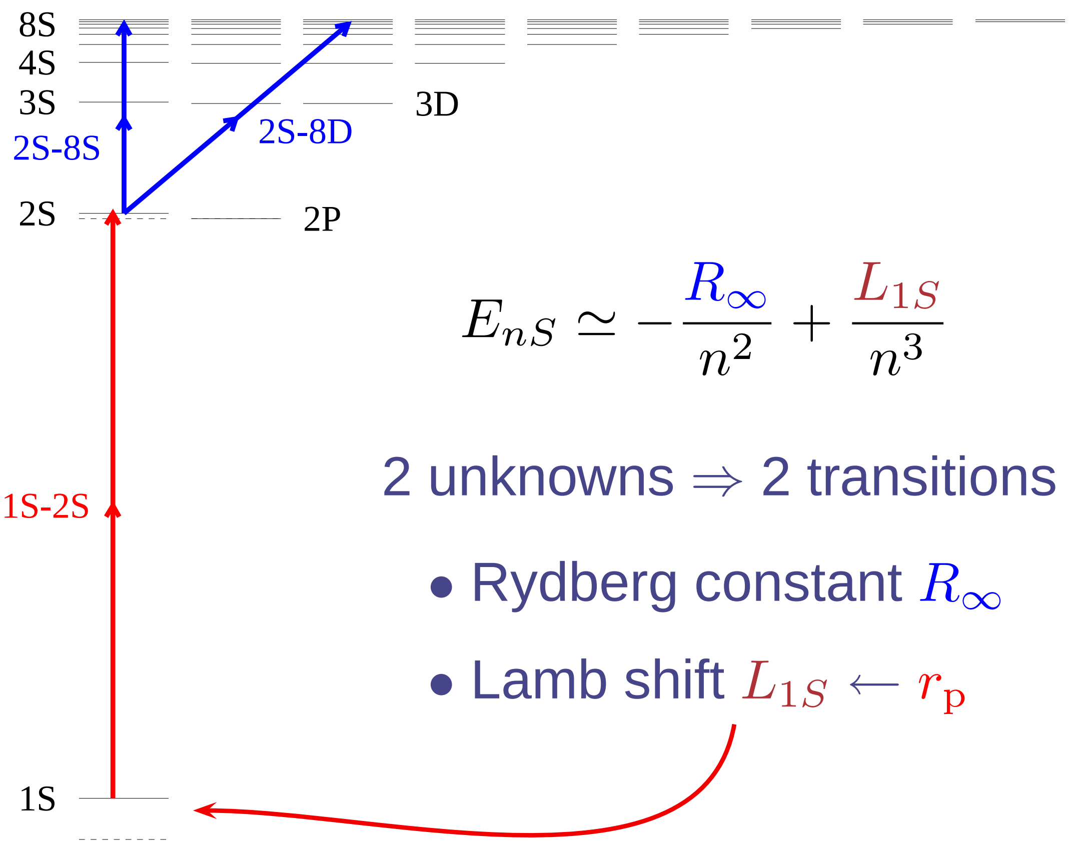
Proton radius vs. time

The proton rms charge radius is not the most accurate quantity in the universe.



Hydrogen spectroscopy (Lamb shift):

$$L_{1S}(r_p) = 8171.636(4) + 1.5645 \langle r_p^2 \rangle \quad \text{MHz}$$



$$E_{nS} \simeq -\frac{R_\infty}{n^2} + \frac{L_{1S}}{n^3}$$

2 unknowns ⇒ 2 transitions

- Rydberg constant R_∞
- Lamb shift $L_{1S} \leftarrow r_p$

e-p

COD

- electron scattering

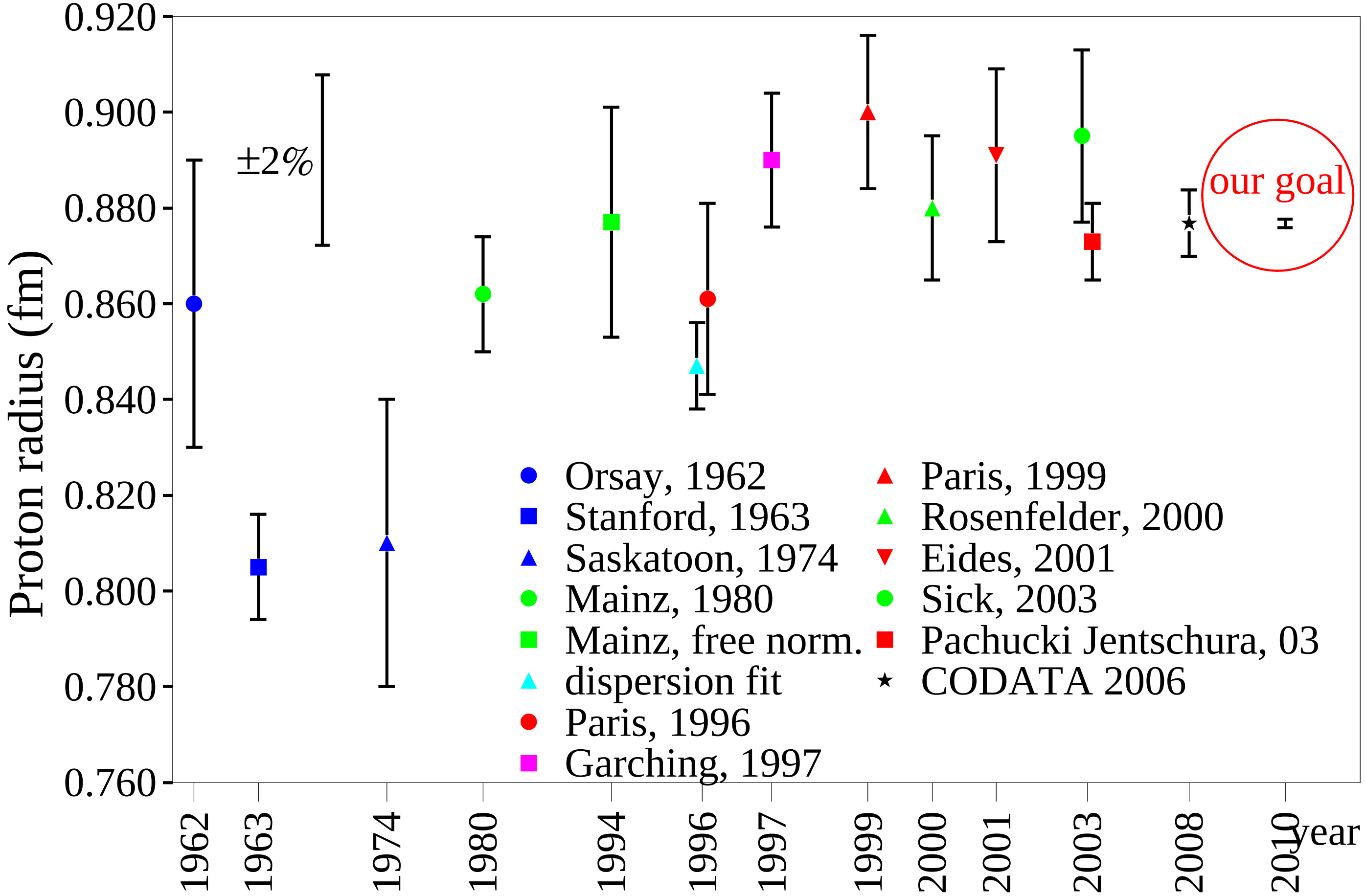
slope of G_E at $Q^2 = 0$

- hydrogen spectr.

Lamb shift (S-states)

Proton radius vs. time

The proton rms charge radius is not the most accurate quantity in the universe.



- electron scattering
  slope of G_E at $Q^2 = 0$
- hydrogen spectr.
  Lamb shift (S-states)

e-p scattering: r_p = 0.895(18) fm ($u_r = 2\,\%$)

CODATA: r_p = 0.8768(69) fm ($u_r = 0.8\,\%$)

muonic hydrogen goal (1998): $u_r = 0.1\,\%$

20x improvement

(aim: 10x better QED test in H)

Proton charge radius and muonic hydrogen

muonic hydrogen = μ^- p   mass m_μ = 207 m_e

$\Rightarrow$ Bohr: $\langle r^{\text{orbit}} \rangle \sim \frac{\hbar}{Z\alpha\, m_r\, c} n^2$

$\Delta E_{\text{finite size}}(nl) \sim r_p^2 \, |\Psi(r=0)|^2$

$$\Rightarrow \boxed{\Delta E_{\text{finite size}}(nl) = \frac{2(Z\alpha)^4 c^4}{3\hbar^2 n^3} m_r^3\, r_p^2\, \delta_{l0}}$$

Lamb shift in μp: $\Delta E(2P_{3/2}^{F=2} - 2S_{1/2}^{F=1})$ =

$$\boxed{209.9779(49) - 5.2262\, r_p^2 + 0.0347\, r_p^3 \ \text{[meV]}}$$

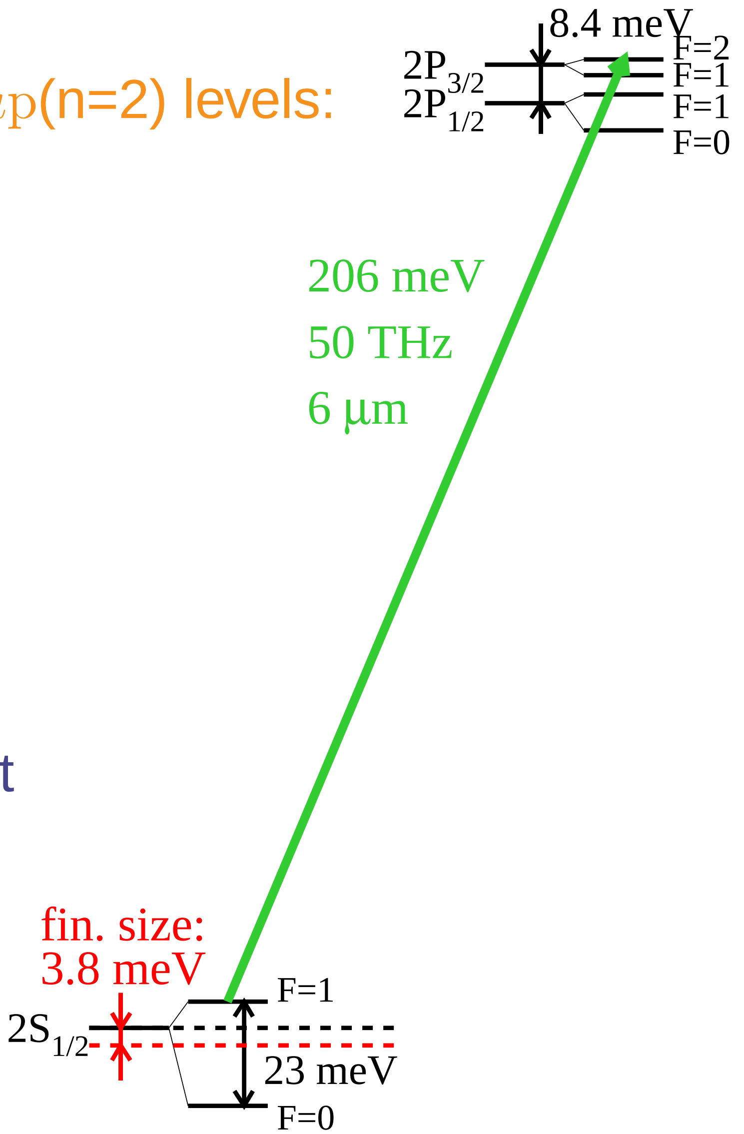
finite size contribution is 2% of the μp Lamb shift

measure ΔE(2S-2P) to 30 ppm = 1.5 GHz

$$\boxed{\Rightarrow r_p \text{ to } 10^{-3}}$$

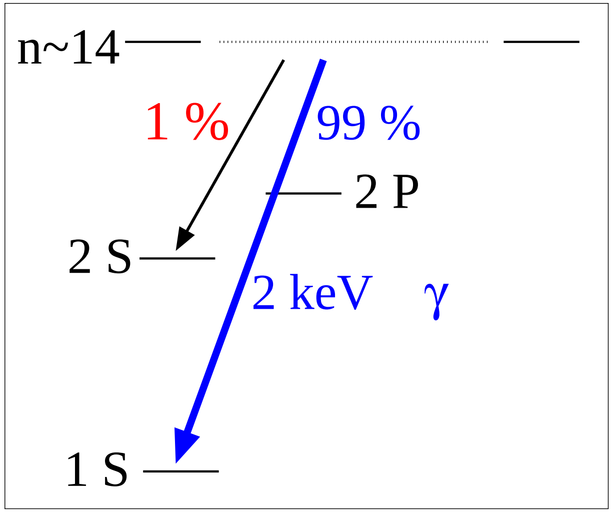
$$\Gamma_{2P} = 18.6 \text{ GHz} \qquad (\Gamma_{\text{rad.}})$$

μp(n=2) levels:

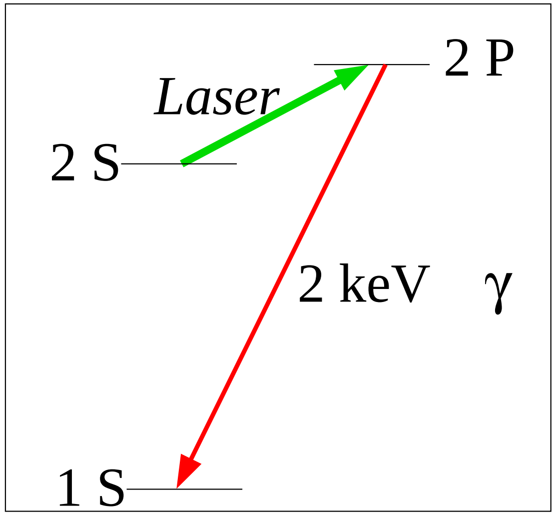


μp Lamb shift experiment: Principle

"prompt" ($t \sim 0$)



"delayed" ($t \sim 1\ \mu$s)



μ^- stop in H_2 gas
$\Rightarrow$ μp* atoms formed ($n \sim 14$)

99%: cascade to μp(1S),
emitting prompt K_α, K_β ...

1%: long-lived μp(2S) atoms
lifetime $\boxed{\tau_{2S} \approx 1\ \mu\text{s}}$ at 1 mbar H_2

R. Pohl *et. al.*, Phys. Rev. Lett. 97, 193402 (2006).

fire laser ($\lambda \approx 6\ \mu$m, $\Delta E \approx 0.2$ eV)

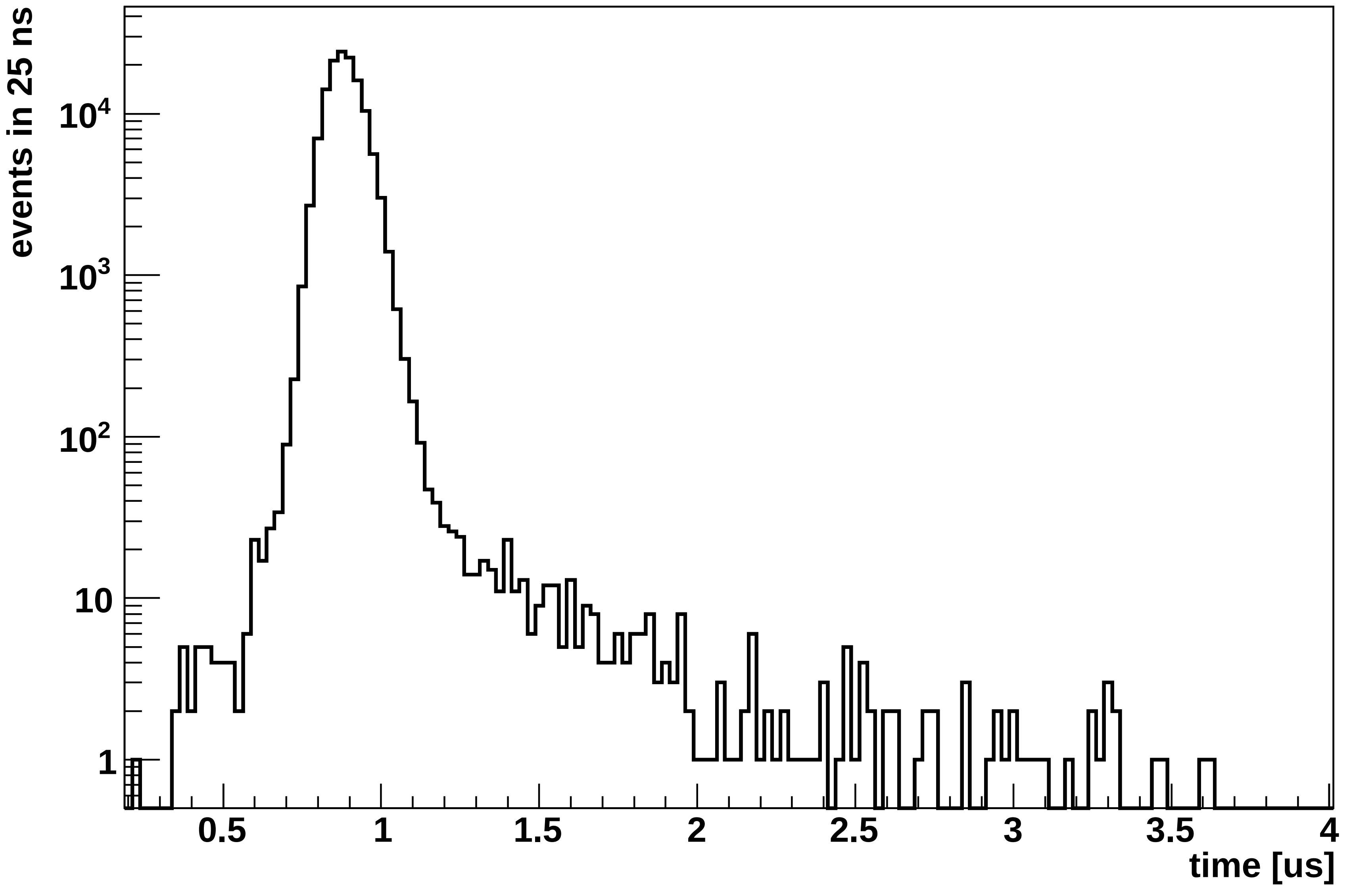
$\Rightarrow$ induce μp(2S) $\rightarrow$ μp(2P)

$\Rightarrow$ observe delayed K_α x-rays

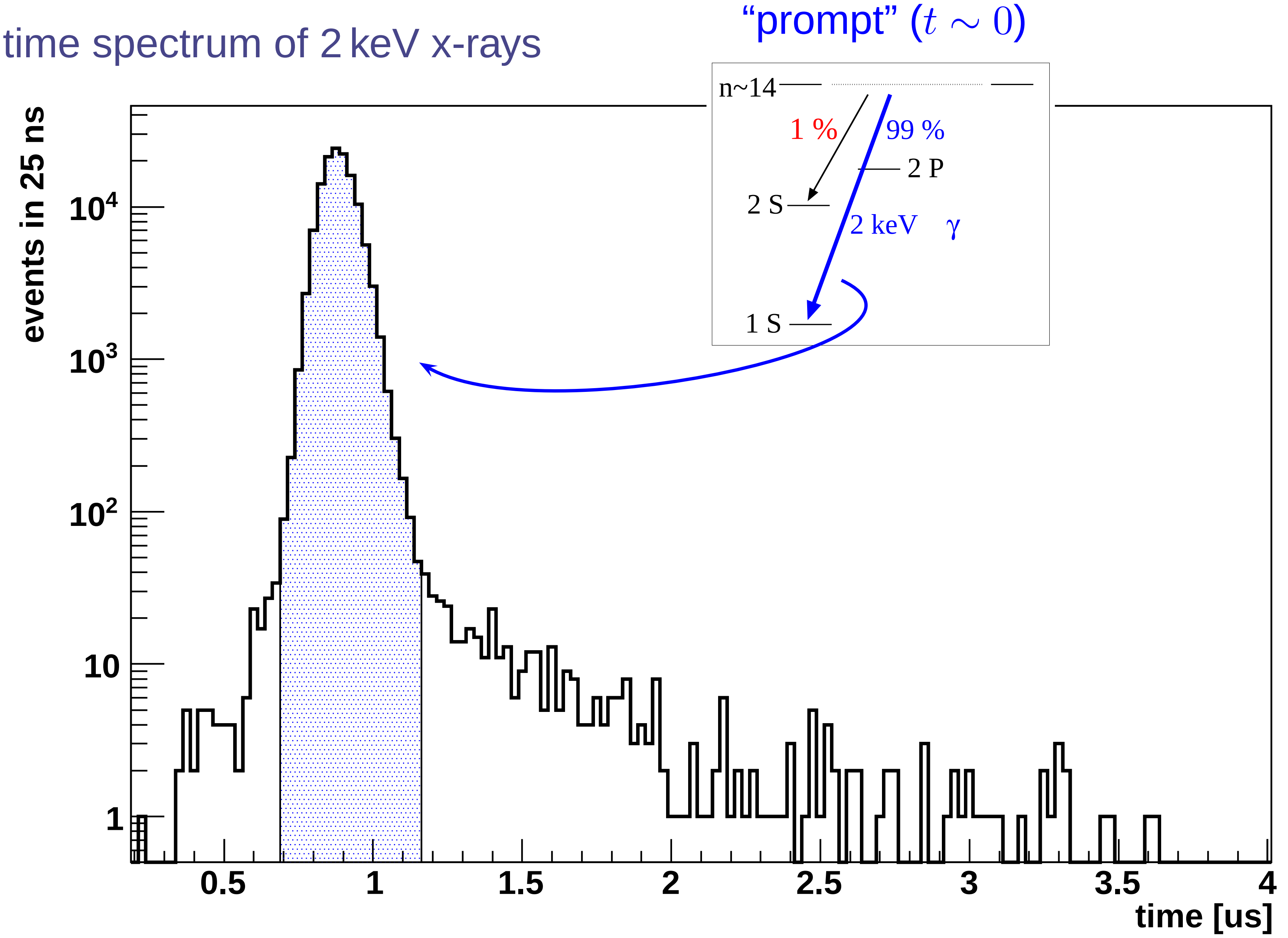
$\Rightarrow$ normalize $\frac{\text{delayed } K_\alpha}{\text{prompt } K_\alpha}$ x-rays

μp Lamb shift experiment: Principle

time spectrum of 2 keV x-rays (∼ 13 hours of data)

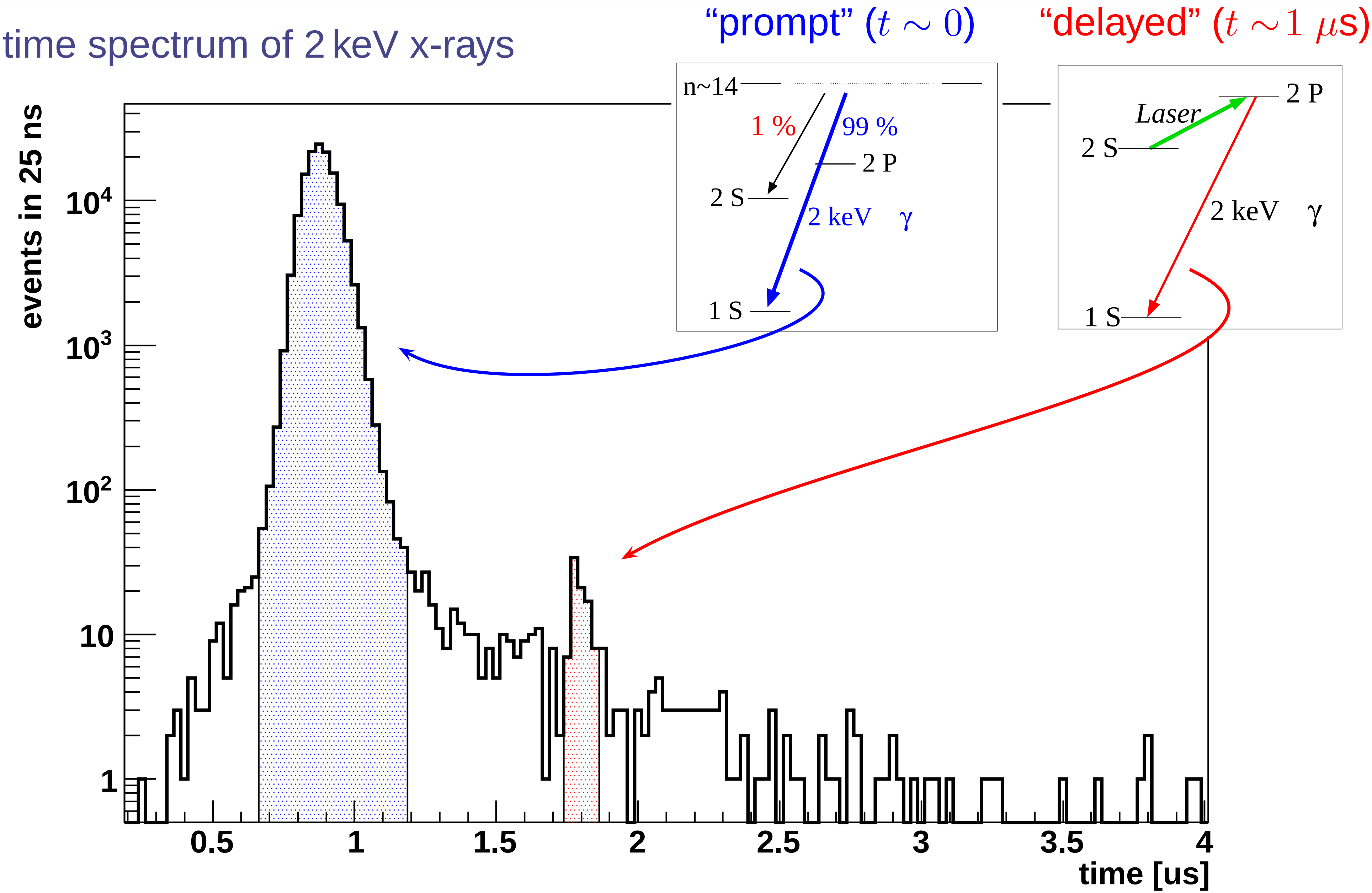


time spectrum of 2 keV x-rays



μp Lamb shift experiment: Principle

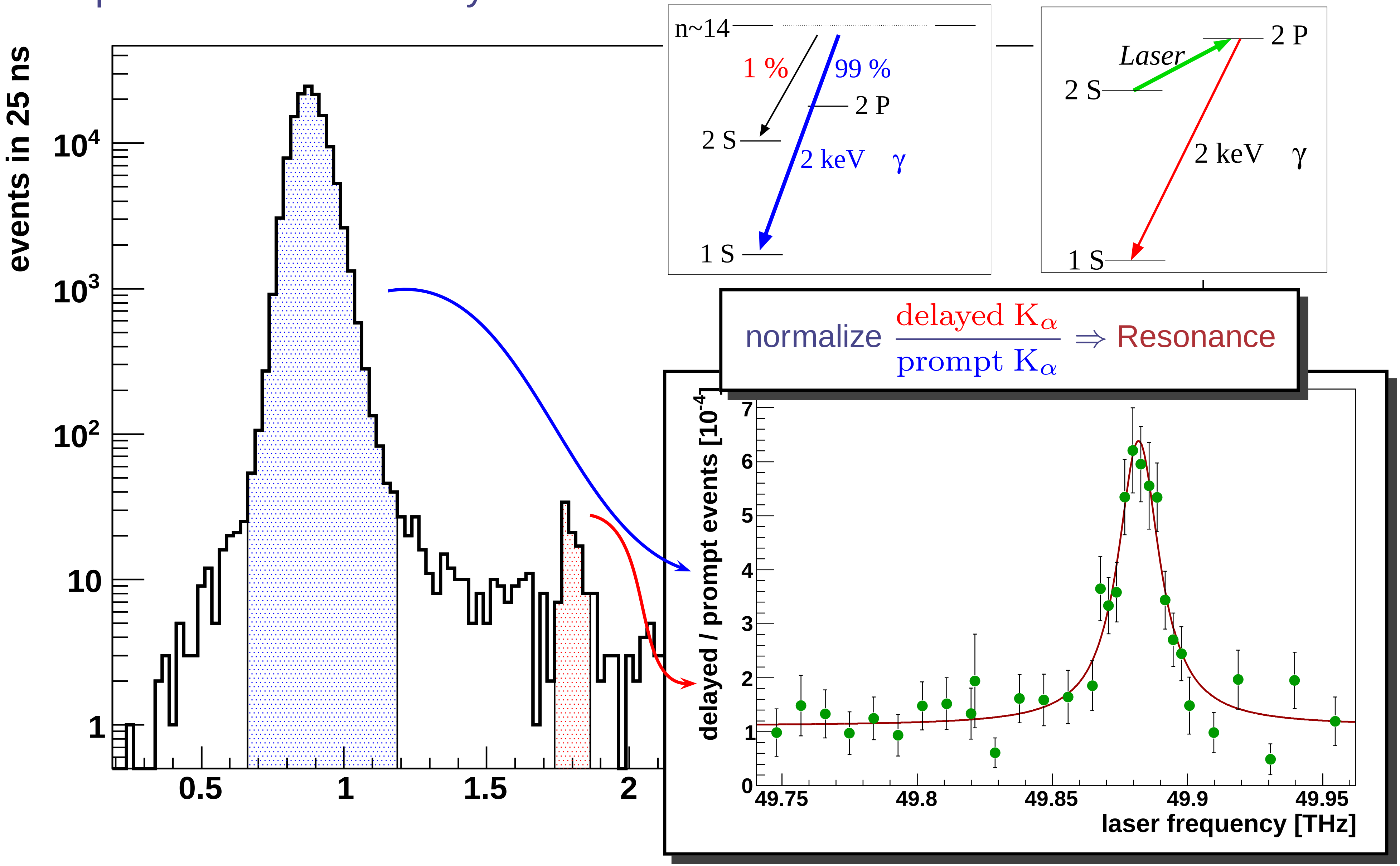
time spectrum of 2 keV x-rays



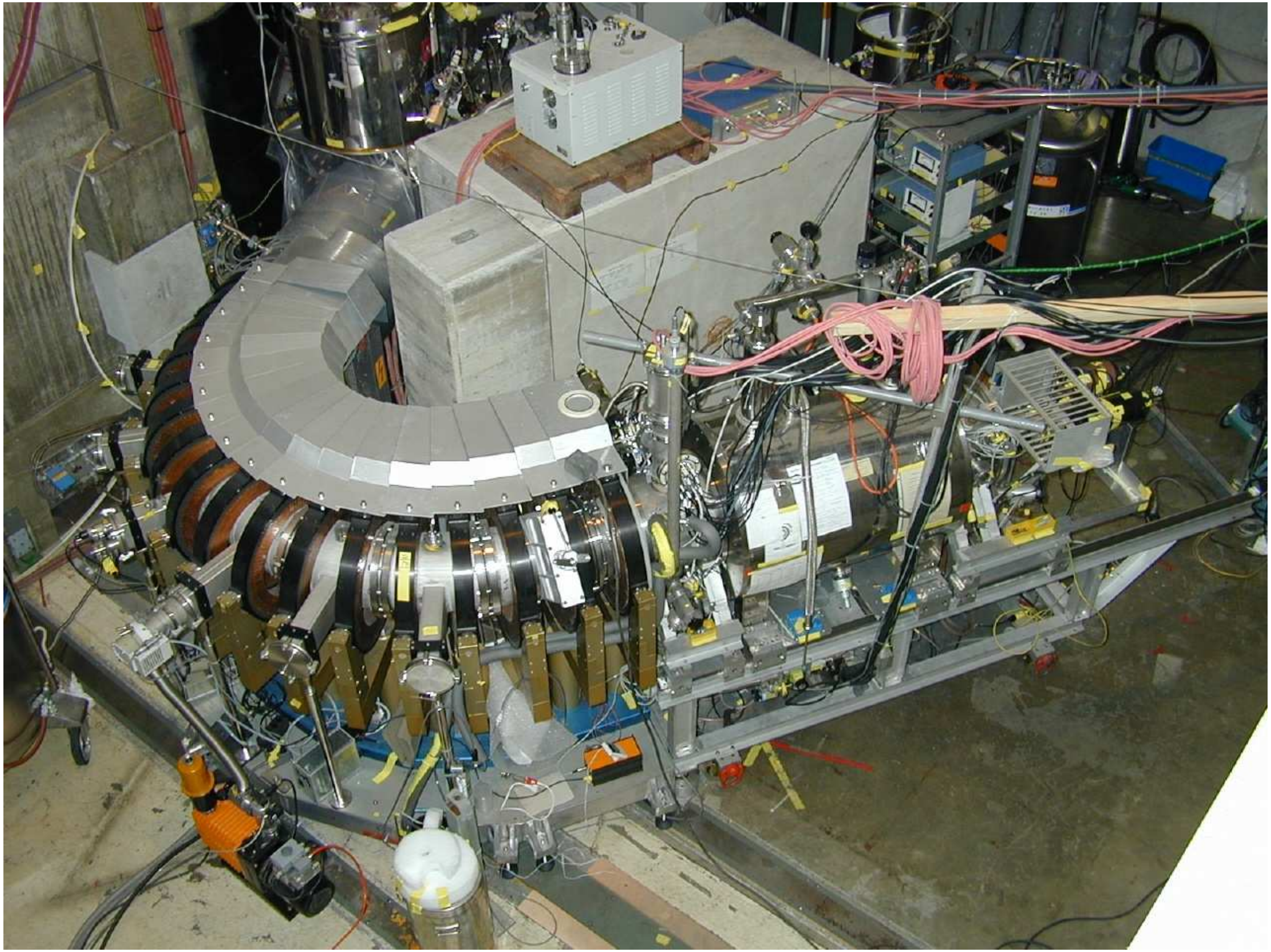
μp Lamb shift experiment: Principle

time spectrum of 2 keV x-rays

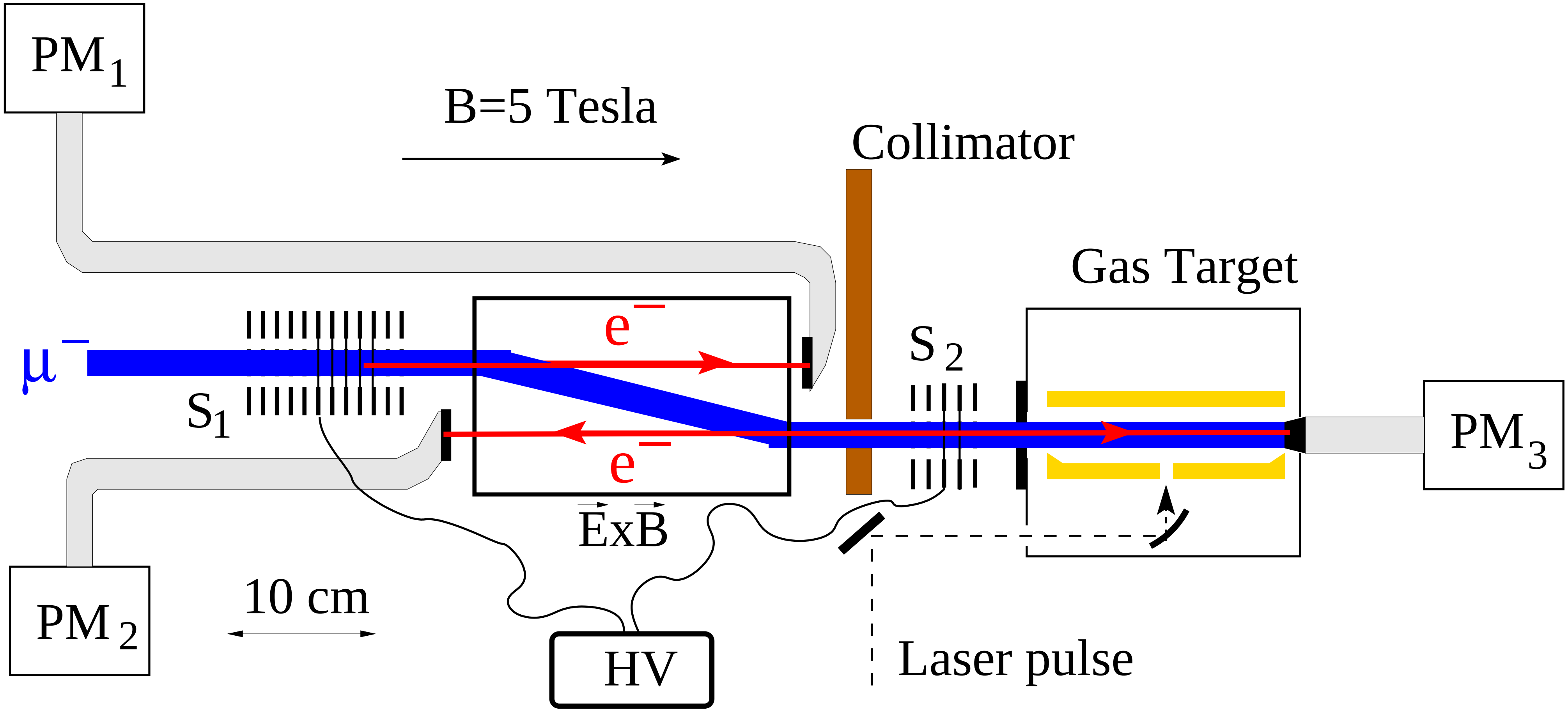
“prompt” ($t \sim 0$) “delayed” ($t \sim 1\ \mu$s)



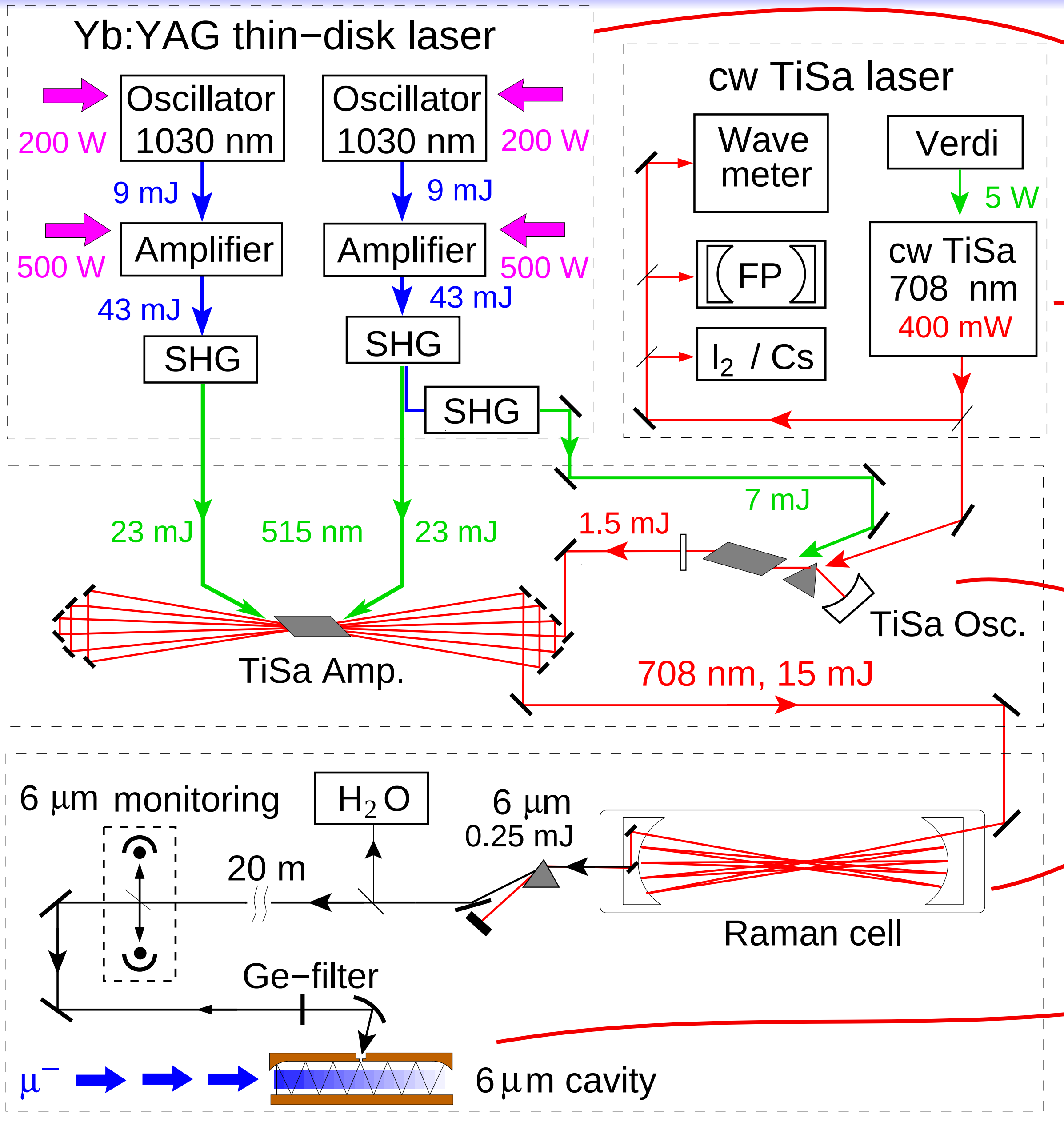
Muon beam line



Muon beam: inside 5 T solenoid



The laser system

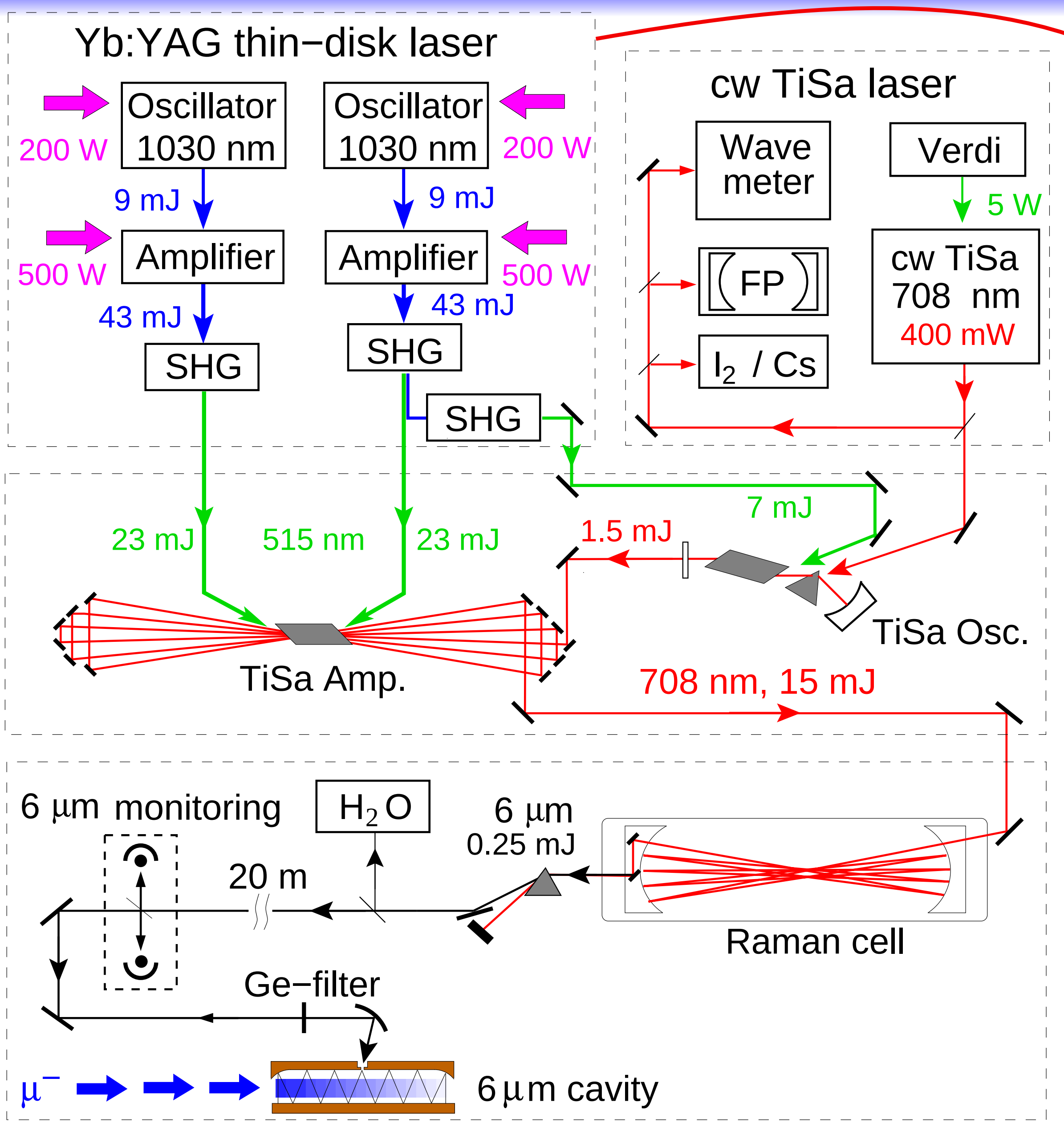


Main components:

- Thin-disk laser
  - fast response to detected μ^-
- Frequency doubling
- TiSa laser:
  - frequency stabilized cw laser
  - injection seeded oscillator
  - multipass amplifier
- Raman cell
  - 3 Stokes: 708 nm $\rightarrow$ 6 μm
  - λ calibration @ 6 μm
- Target cavity

A. Antognini *et. al.*, Opt. Comm. 253, 362 (2005).

The laser system



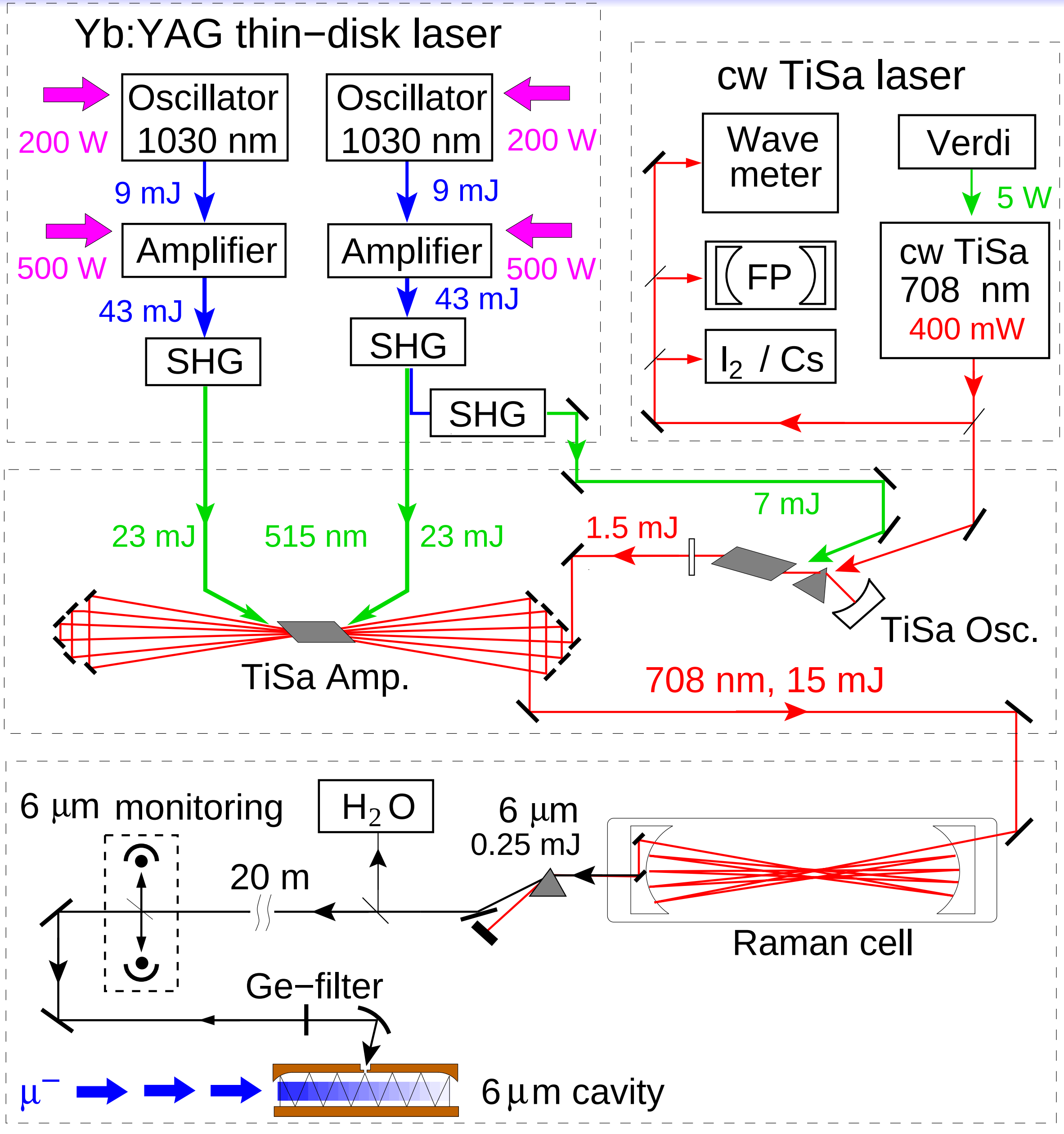
Thin-disk laser

- Large pulse energy: 85 (160) mJ
- Short trigger-to-pulse delay: $\lesssim 400$ ns
- Random trigger
- Pulse-to-pulse delays down to 2 ms (rep. rate $\gtrsim 500$ Hz)

- Each single μ^- triggers the laser system
- $2S$ lifetime $\approx 1\ \mu$s $\rightarrow$ short laser delay

A. Antognini *et. al.*,
IEEE J. Quant. Electr. 45, 993 (2009).

The laser system



MOPA TiSa laser:

Cw frequency stabilized laser

- referenced to a stable FP cavity
- FP cavity calibrated with I_2, Rb, Cs lines

$\nu_{\mathrm{FP}} = N \cdot FSR$

$FSR = 1497.344(6)$ MHz

$\nu^{\mathrm{cw}}_{\mathrm{TiSa}}$ absolutely known to 30 MHz

$\Gamma_{2\mathrm{P}-2\mathrm{S}} = 18.6$ GHz

Seeded oscillator

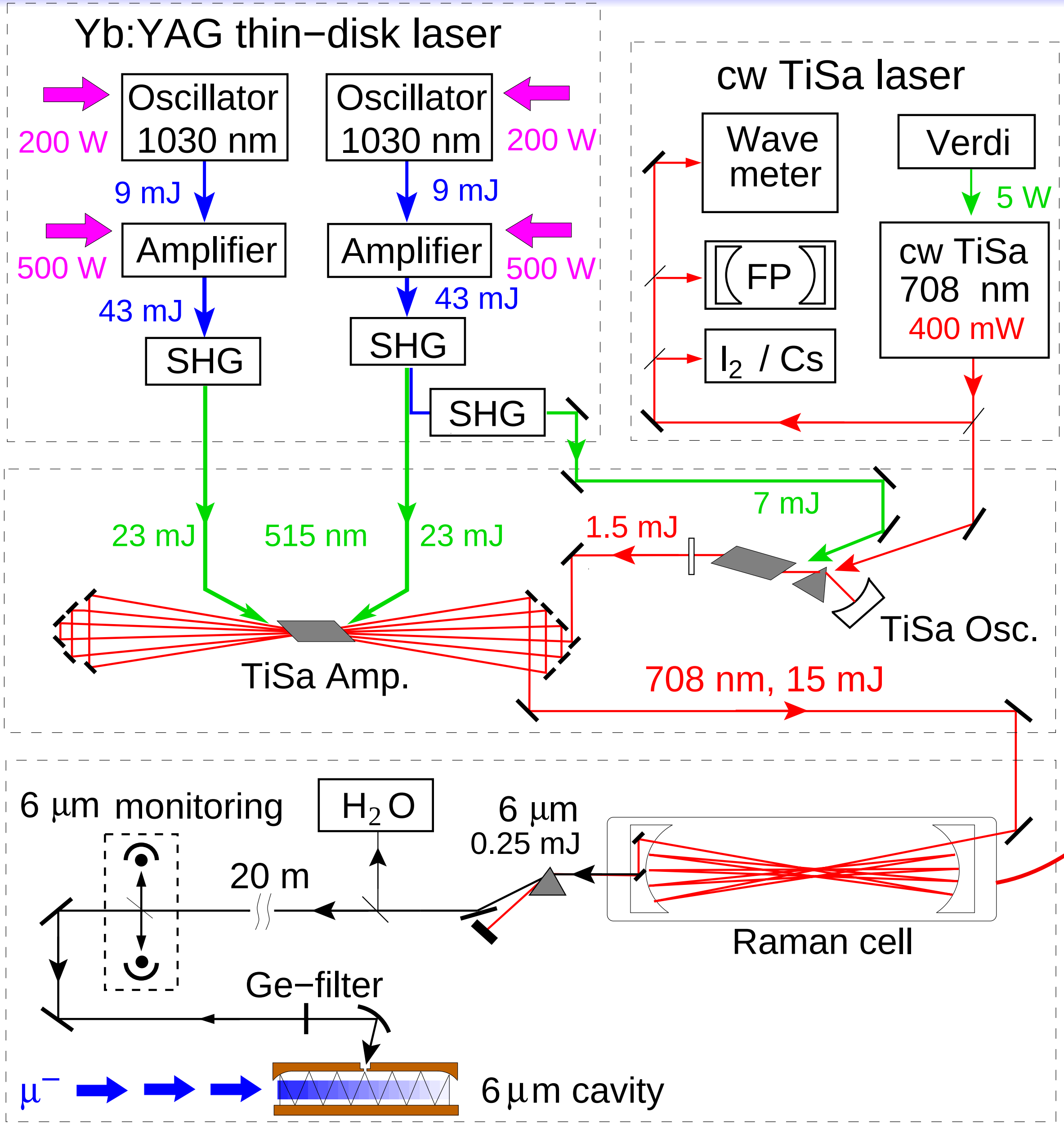
$\rightarrow \nu^{\mathrm{pulsed}}_{\mathrm{TiSa}} = \nu^{\mathrm{cw}}_{\mathrm{TiSa}}$

(frequency chirp ≤ 100 MHz)

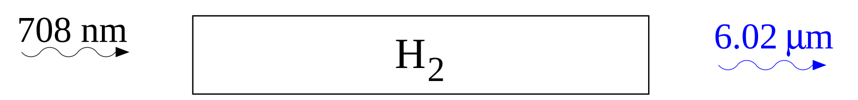
Multipass amplifier (2f- configuration)

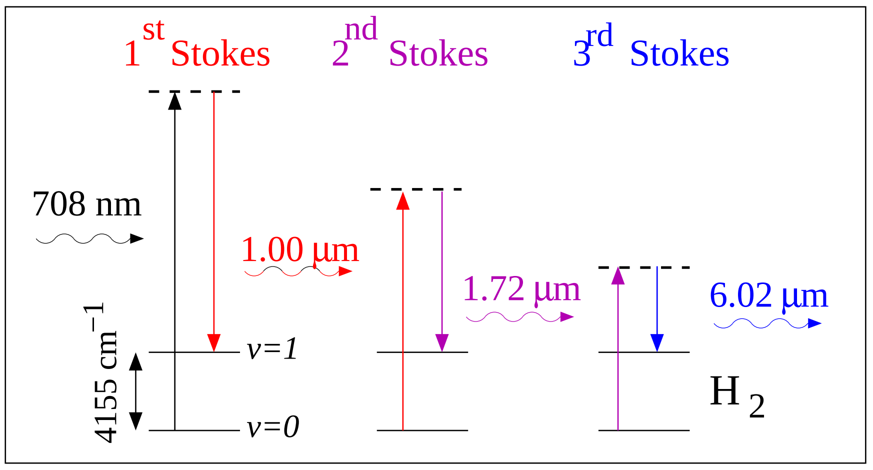
gain=10

The laser system



Raman cell:



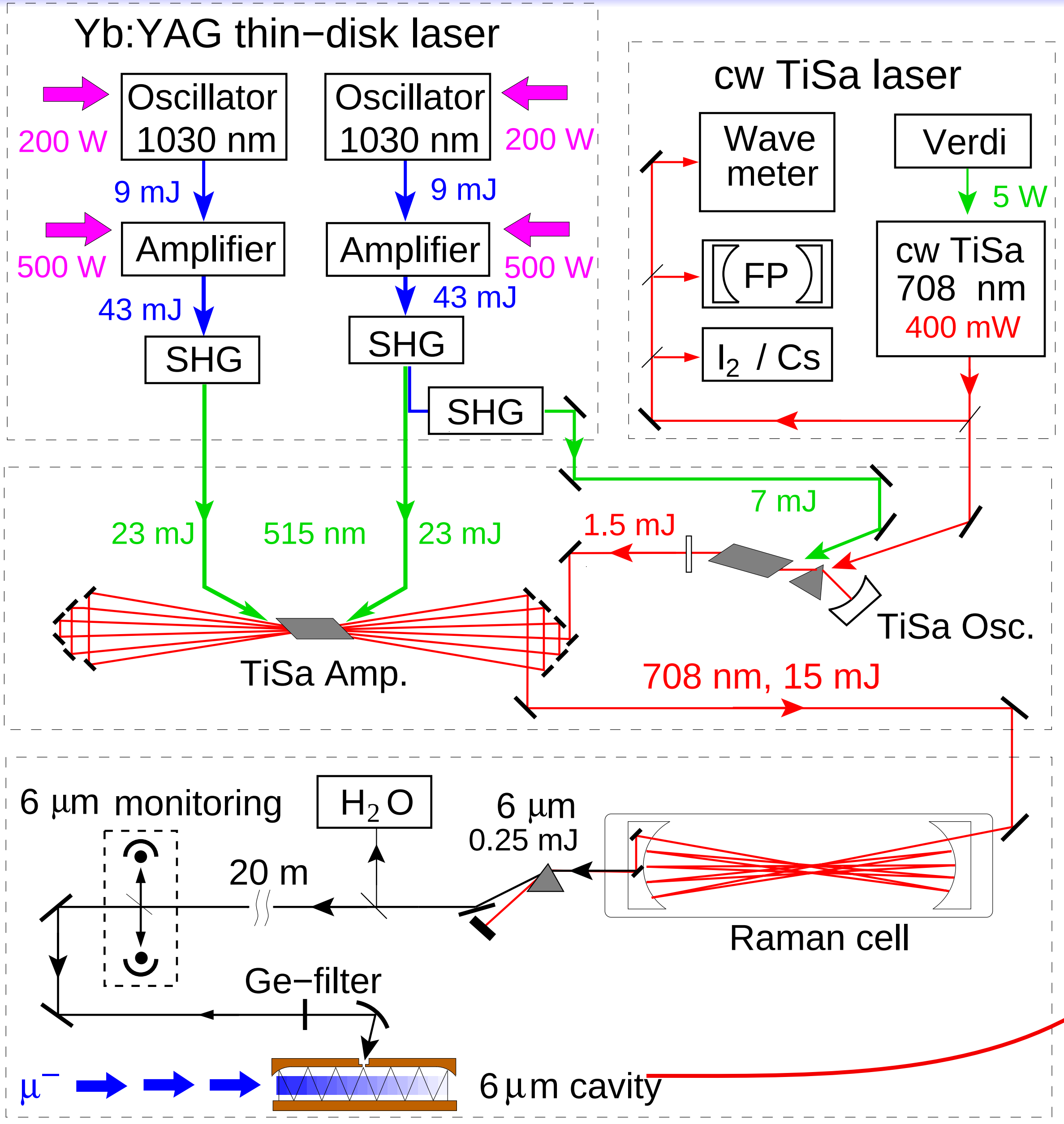


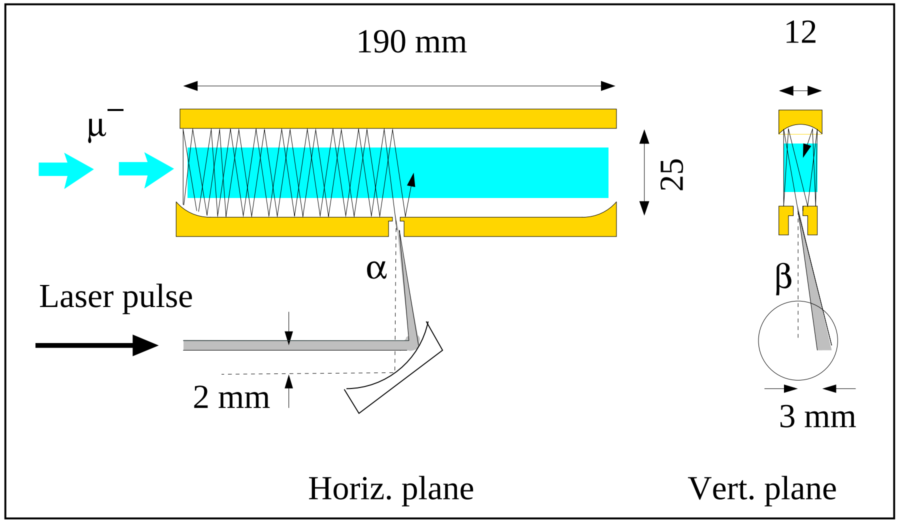
$$\nu^{6\mu m} = \nu^{708\text{nm}} - 3 \cdot \hbar\omega_{\text{vib}}$$

tunable $\qquad \omega_{\text{vib}}(p, T) = \text{const}$

P. Rabinowitz *et. al.*, IEEE J. QE **22**, 797 (1986)

The laser system





Design: insensitive to misalignment

Transverse illumination

Large volume

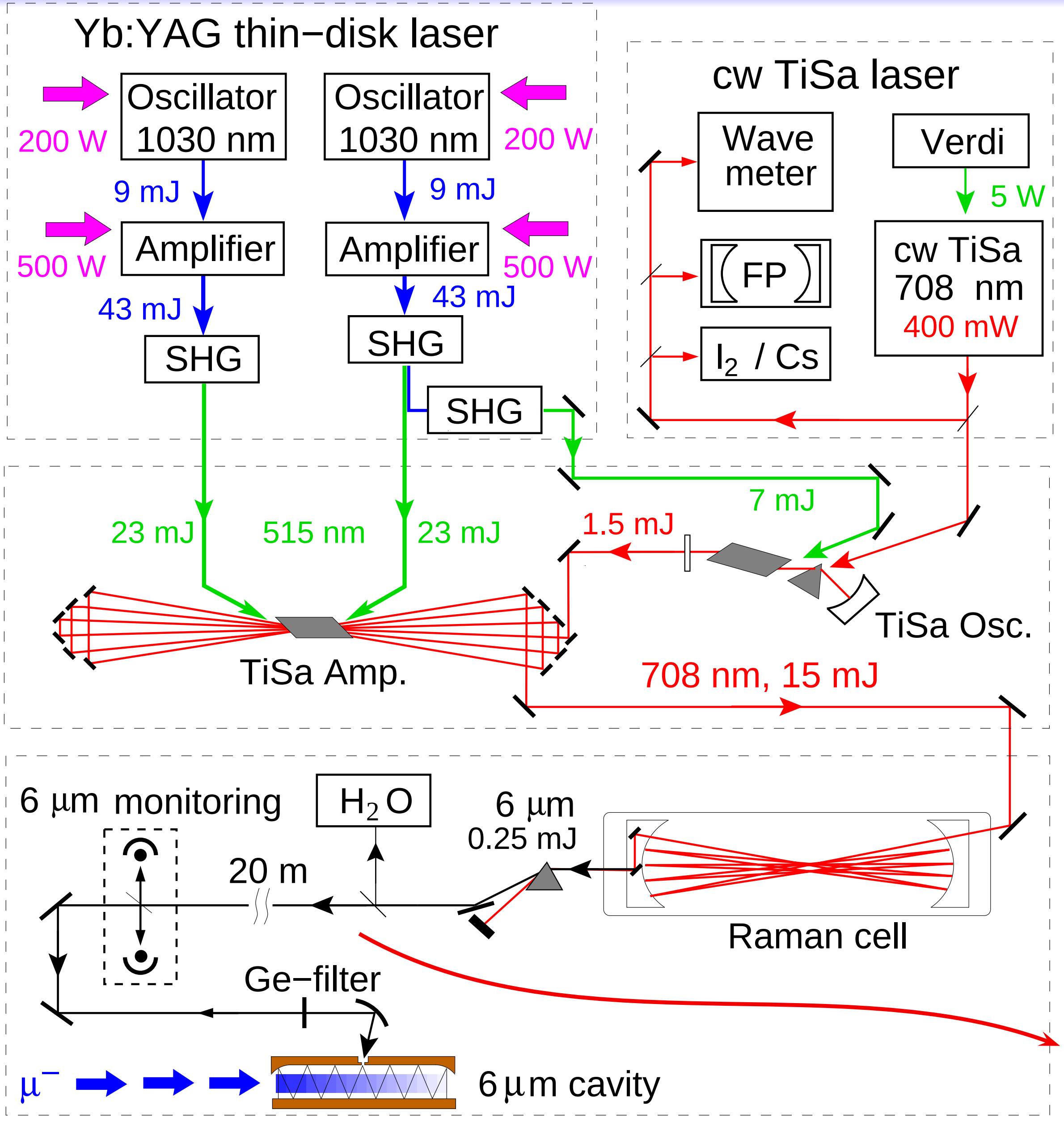
Dielectric coating with $R \geq 99.9\%$ (at 6 μm)

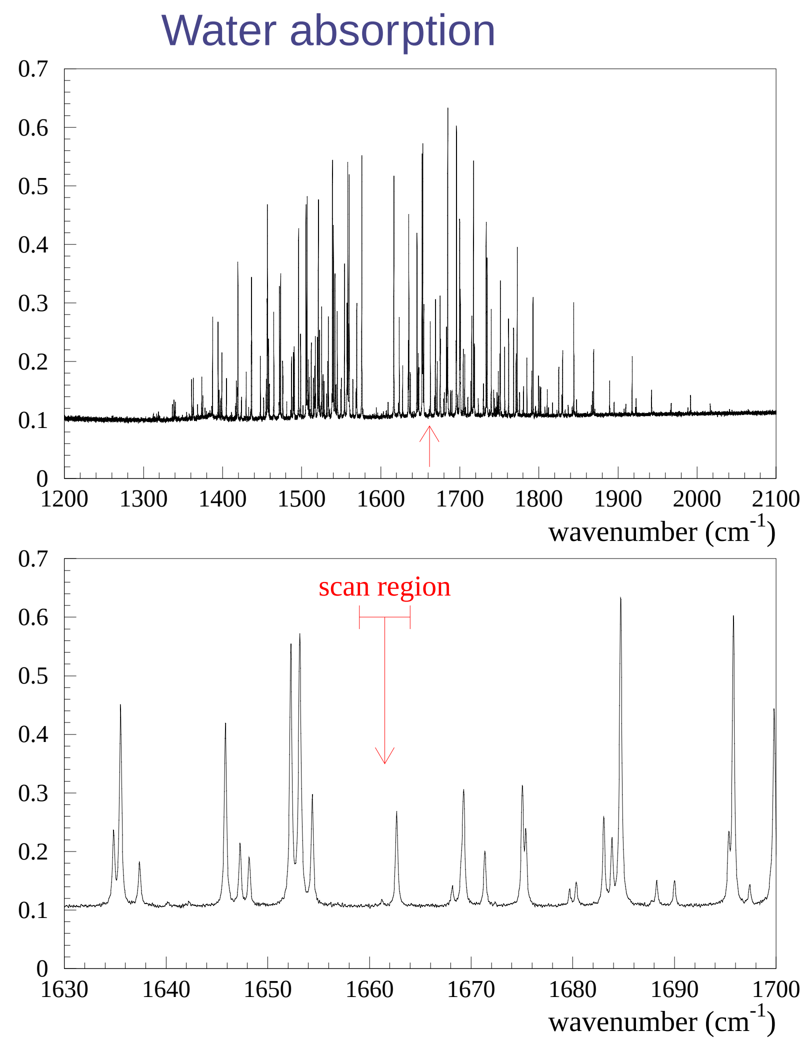
$\rightarrow$ Light makes 1000 reflections

$\rightarrow$ Light is confined for τ=50 ns

$\rightarrow$ 0.15 mJ saturates the $2S - 2P$ transition

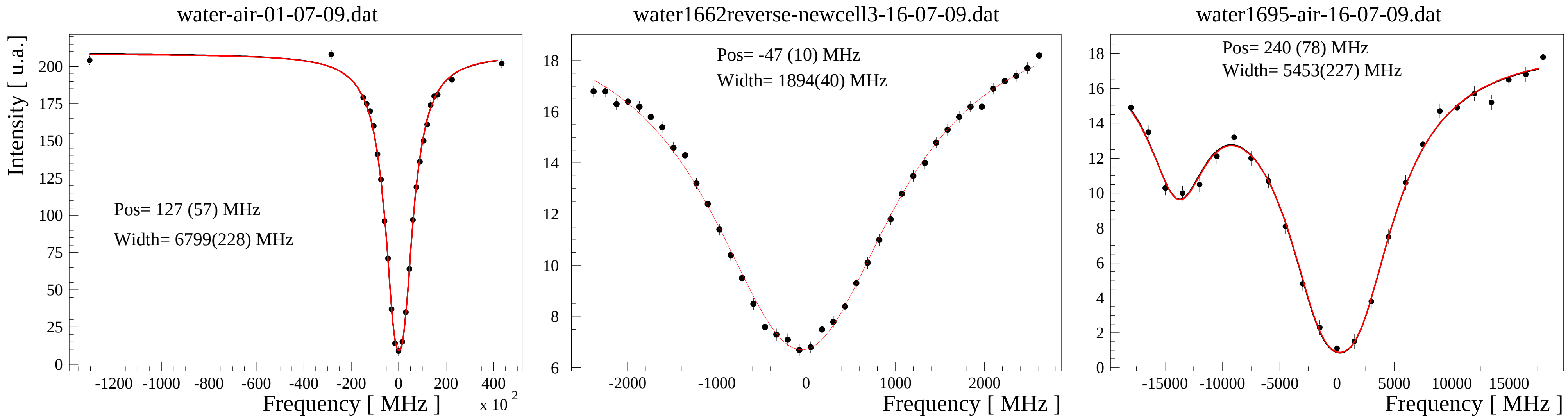
The laser system





- Vacuum tube for 6 μm beam transport.
- Direct frequency calibration at 6 μm.

6 μm wavelength calibration



- 6 μm light calibration: H_2O vapor absoprtion measurement in air / cell

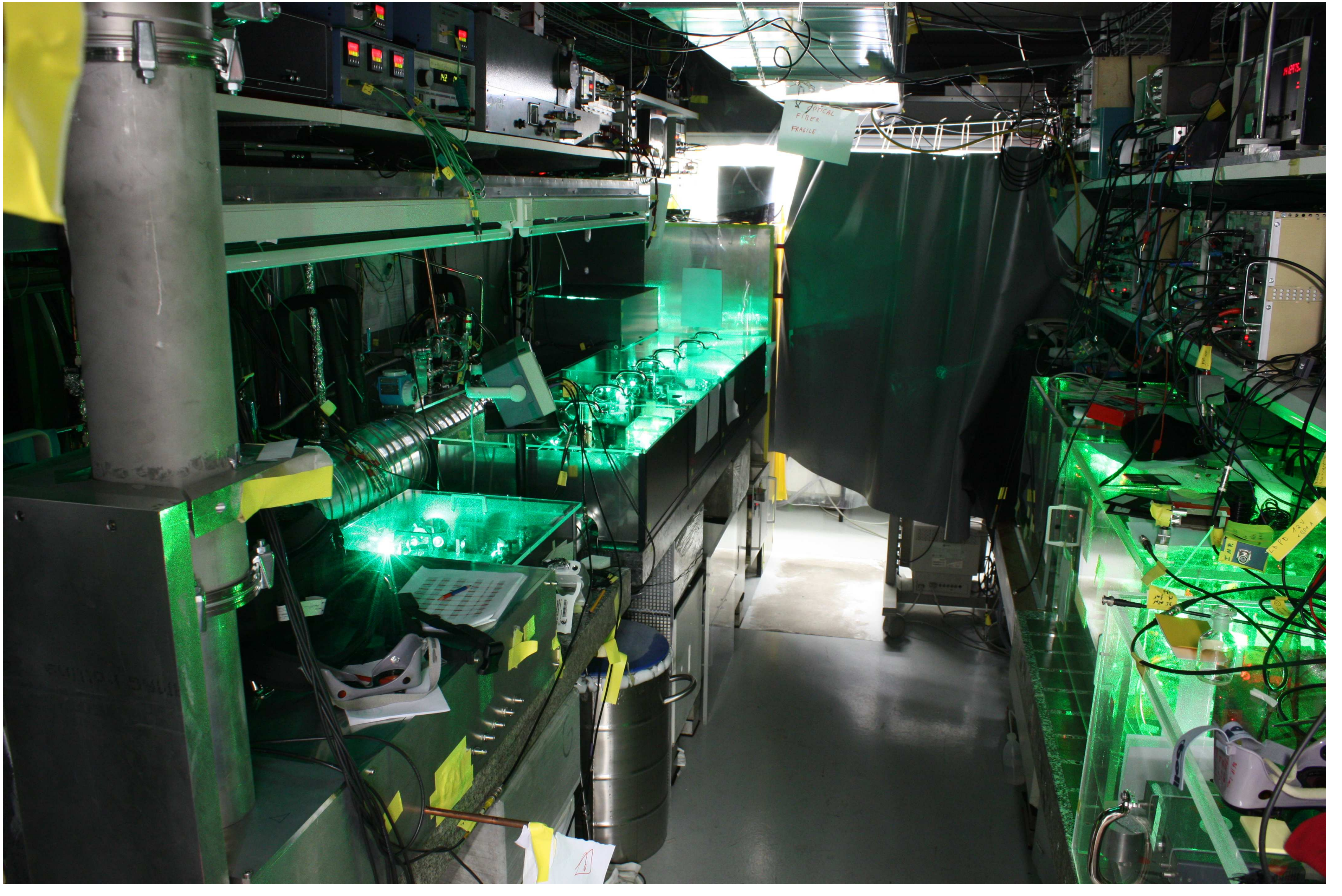
  H_2O absorption lines known to a few MHz (HITRAN)

  $\Rightarrow$ $\delta\nu \approx 300$ MHz uncertainty (6 ppm of ΔE_{2S-2P}) due to our calibration accuracy

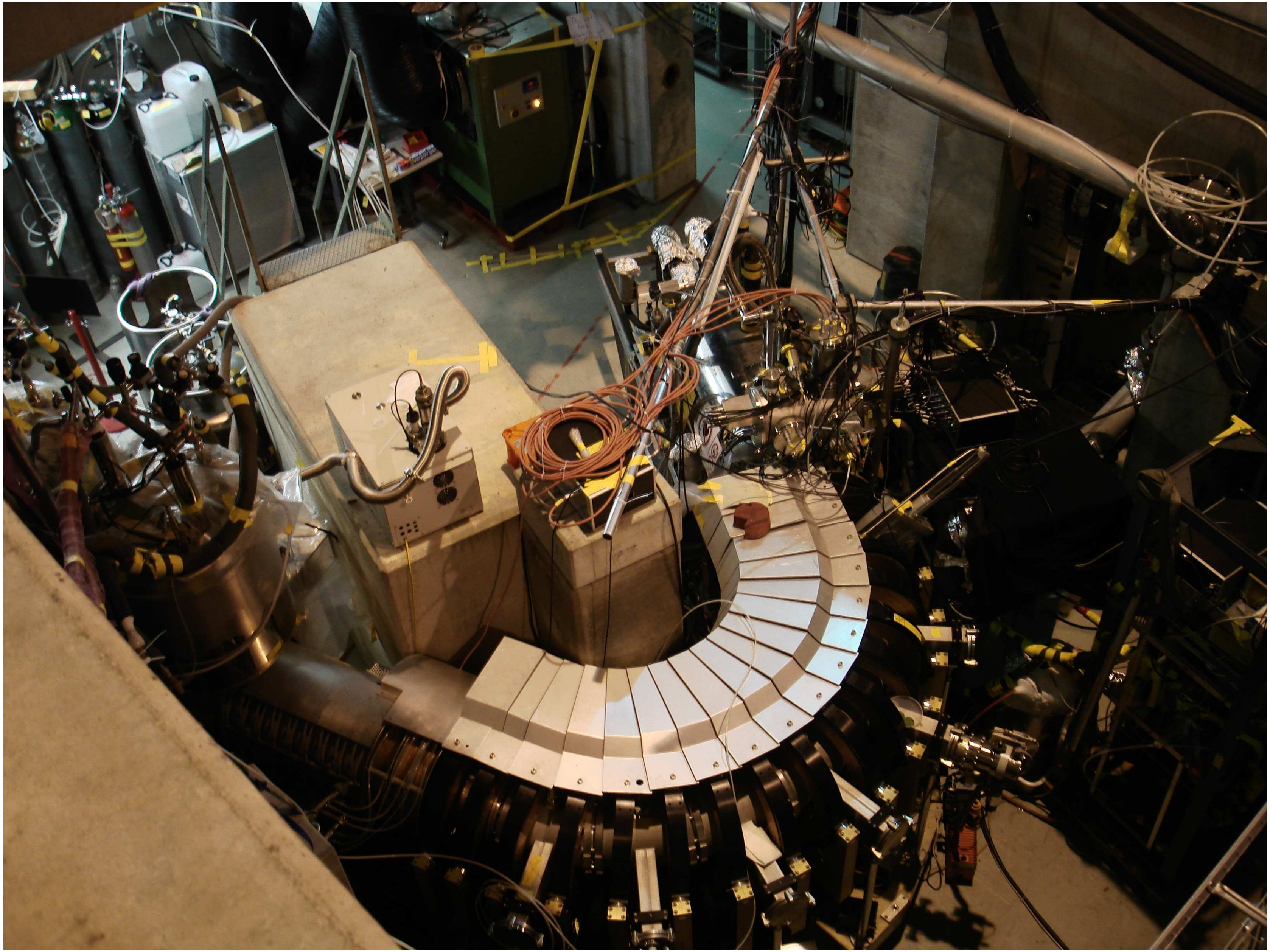
  over the whole wavelength range λ = 5.5 ... 6.1 μm

- Laser frequency detuning is measured in number of Fabry-Perot cavity fringes
  - grid spacing of our measurement: FSR(FP) = 1497.344(6) MHz
  - all measured resonances are within ± 70 FP fringes of a H_2O line

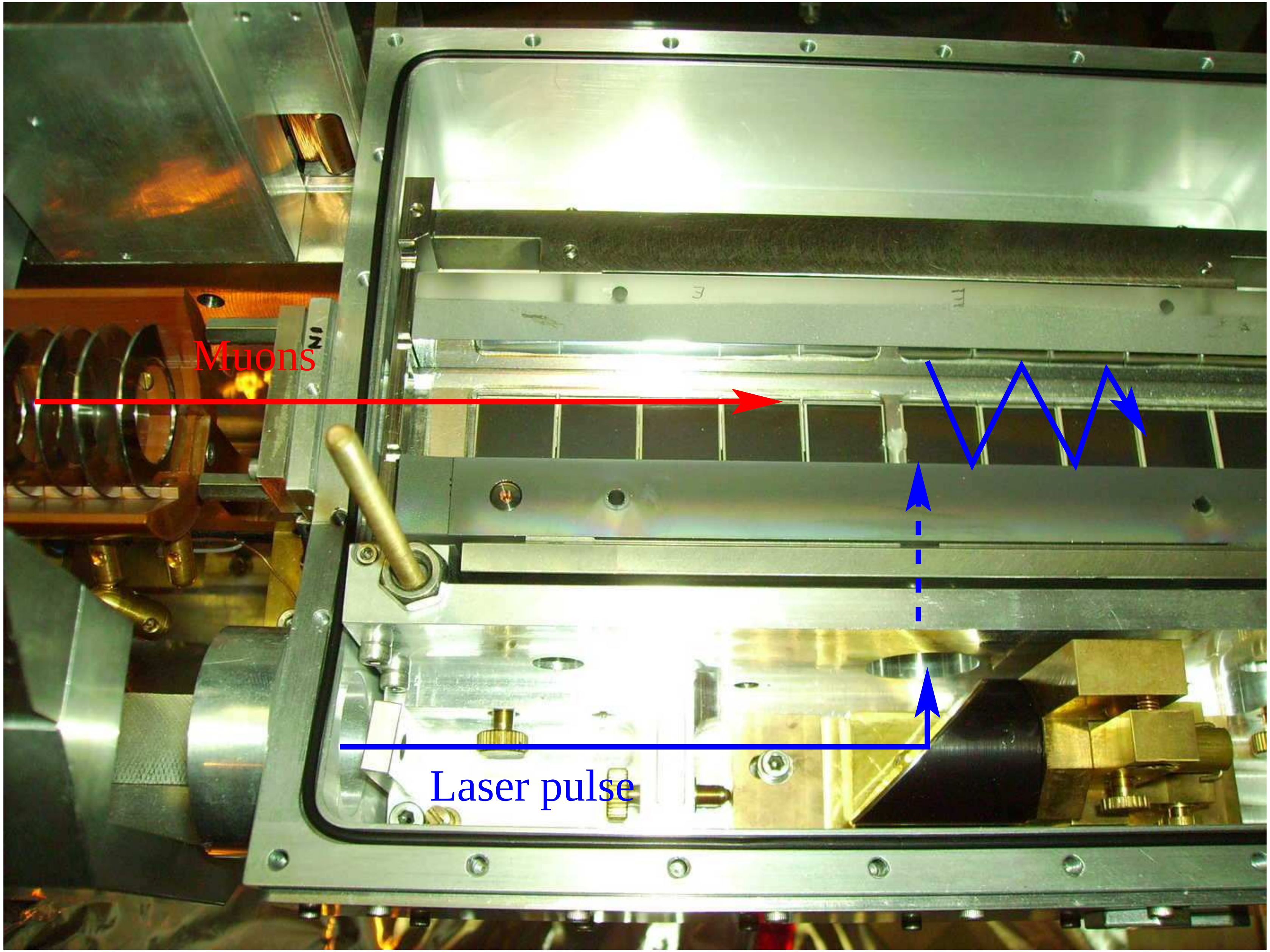
Laser hut



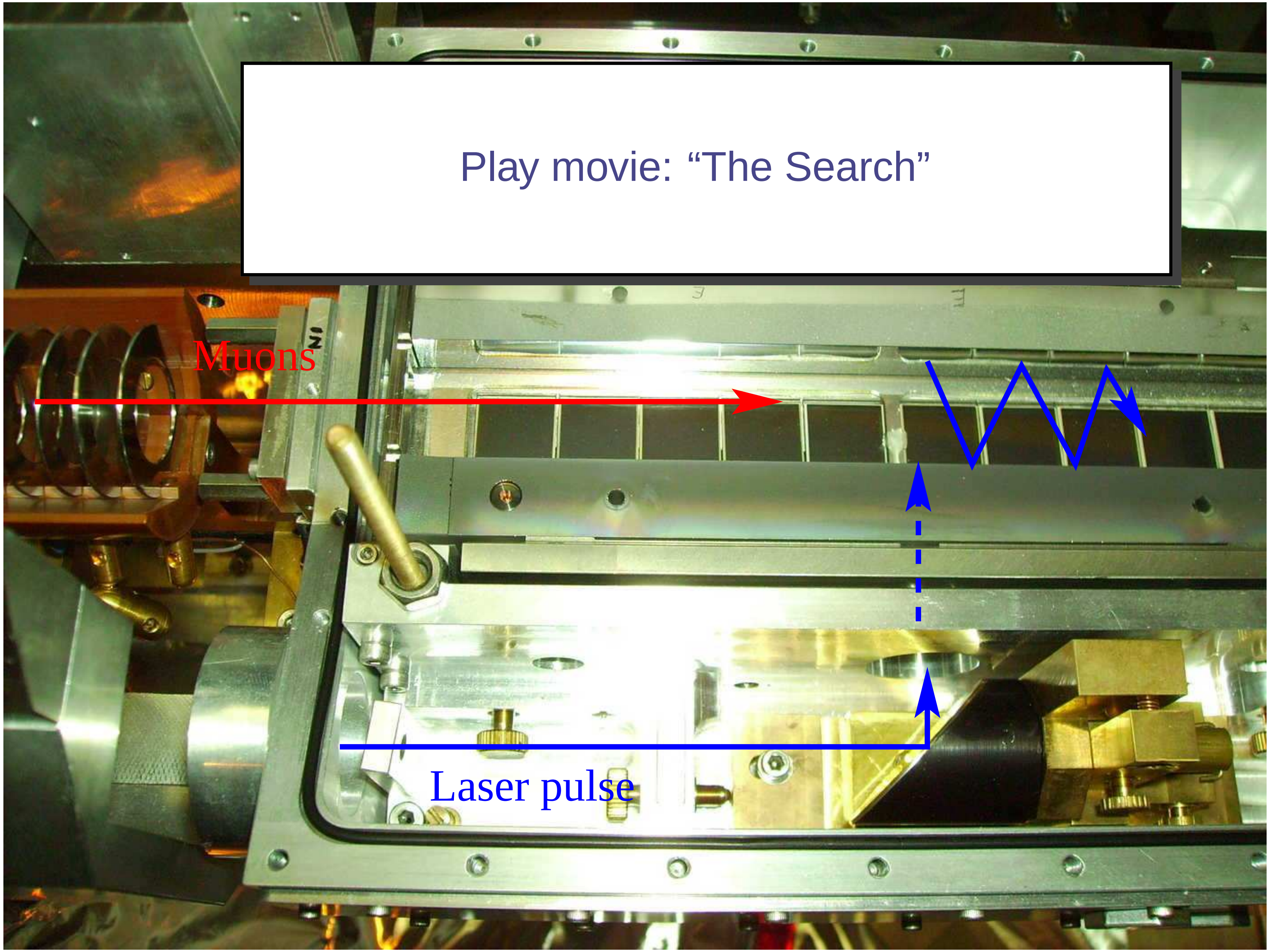
Muon beam line



Target, cavity and detectors



Target, cavity and detectors

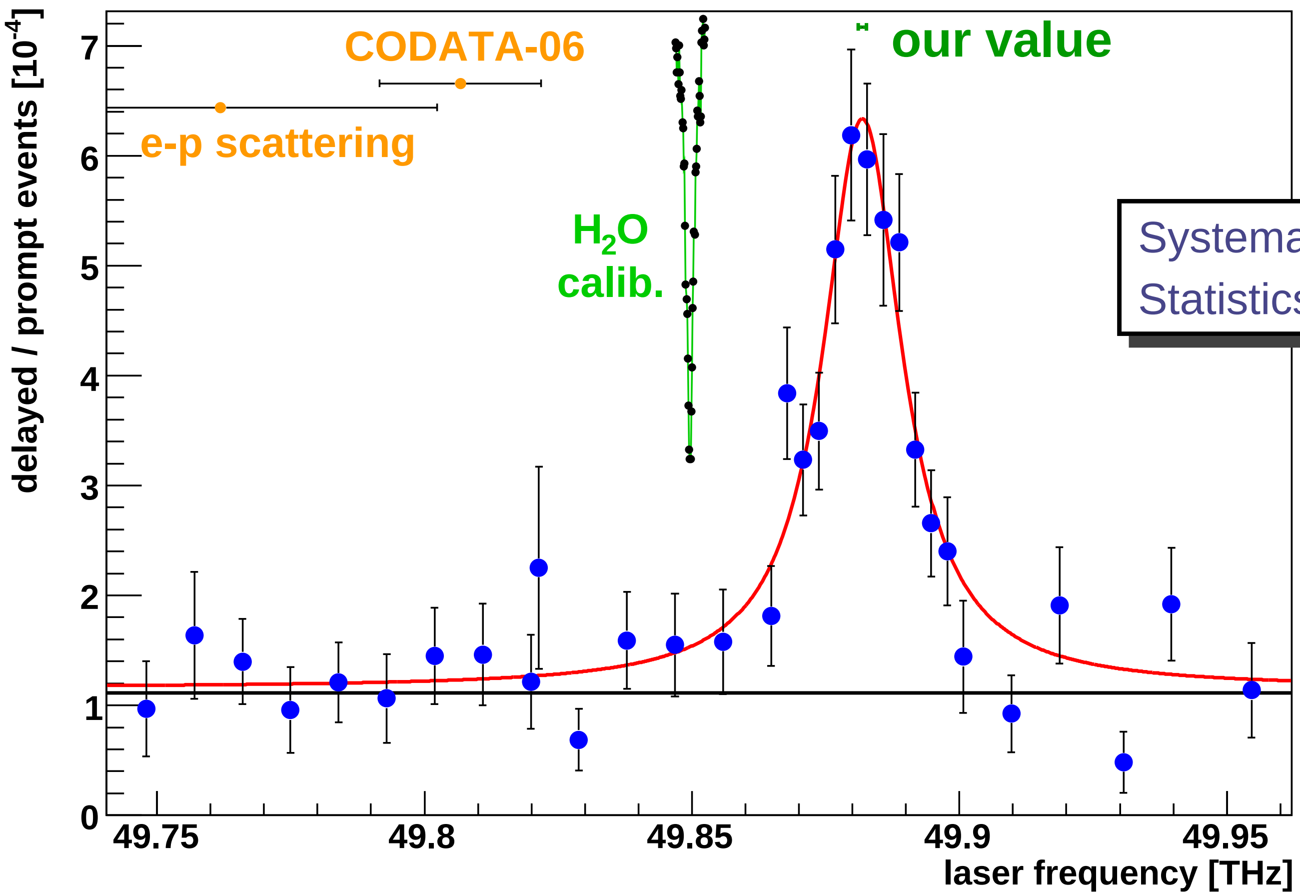


There are surprises in physics.

The resonance: discrepancy, sys., stat.

Water-line/laser wavelength:
300 MHz uncertainty

$\Delta\nu$ water-line to resonance:
200 kHz uncertainty



Systematics: 300 MHz
Statistics: 700 MHz

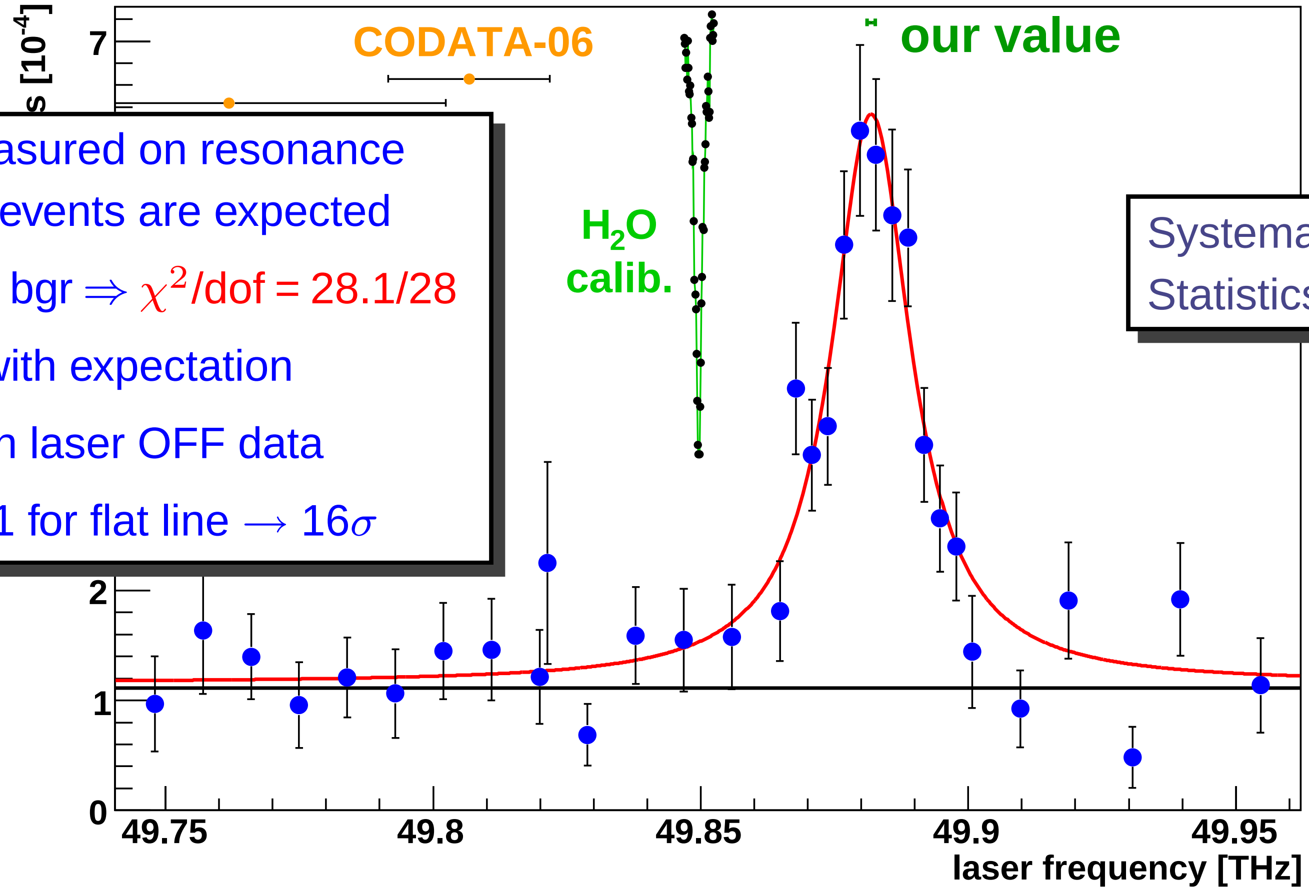
Discrepancy:
$5.0\,\sigma \;\leftrightarrow\; 75\text{ GHz} \;\leftrightarrow\; \delta\nu/\nu = 1.5 \times 10^{-3}$

R. Pohl *et al.*, Nature 466, 213 (2010).

The resonance: discrepancy, sys., stat.

Water-line/laser wavelength:
300 MHz uncertainty

$\Delta\nu$ water-line to resonance:
200 kHz uncertainty



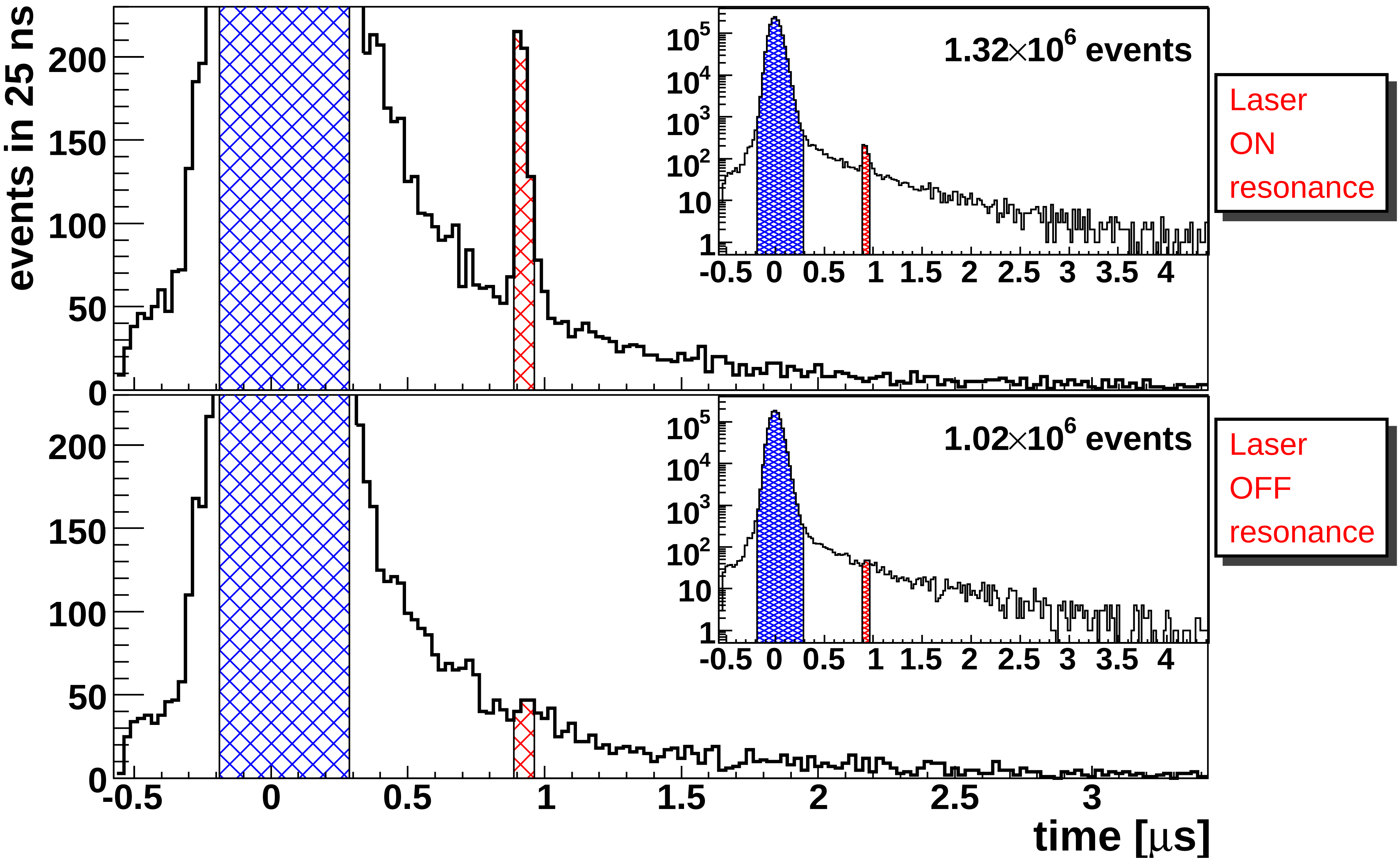
550 events measured on resonance
where 155 bgr events are expected
fit Lorentz + flat bgr $\Rightarrow$ χ^2/dof = 28.1/28
width agrees with expectation
bgr agrees with laser OFF data
χ^2/dof = 283/31 for flat line $\rightarrow$ 16σ

Systematics: 300 MHz
Statistics: 700 MHz

Discrepancy:
$5.0\,\sigma \;\leftrightarrow\; 75$ GHz $\;\leftrightarrow\; \delta\nu/\nu = 1.5 \times 10^{-3}$

R. Pohl *et al.*, Nature 466, 213 (2010).

The time spectra



Uncertainty budget and sensitivity

- Statistics

Center position uncertainty ($\sim 4\%$ of Γ)	700 MHz

- Systematics

Laser frequency ($H_2$0 calibration)	300 MHz
AC and DC stark shift	< 1 MHz
Zeeman shift (5 Tesla)	< 30 MHz
Doppler shift	< 1 MHz
Collisional shift	2 MHz

- Total uncertainty of the line determination — 760 MHz

- Theory: proton polarizability — 1200 MHz

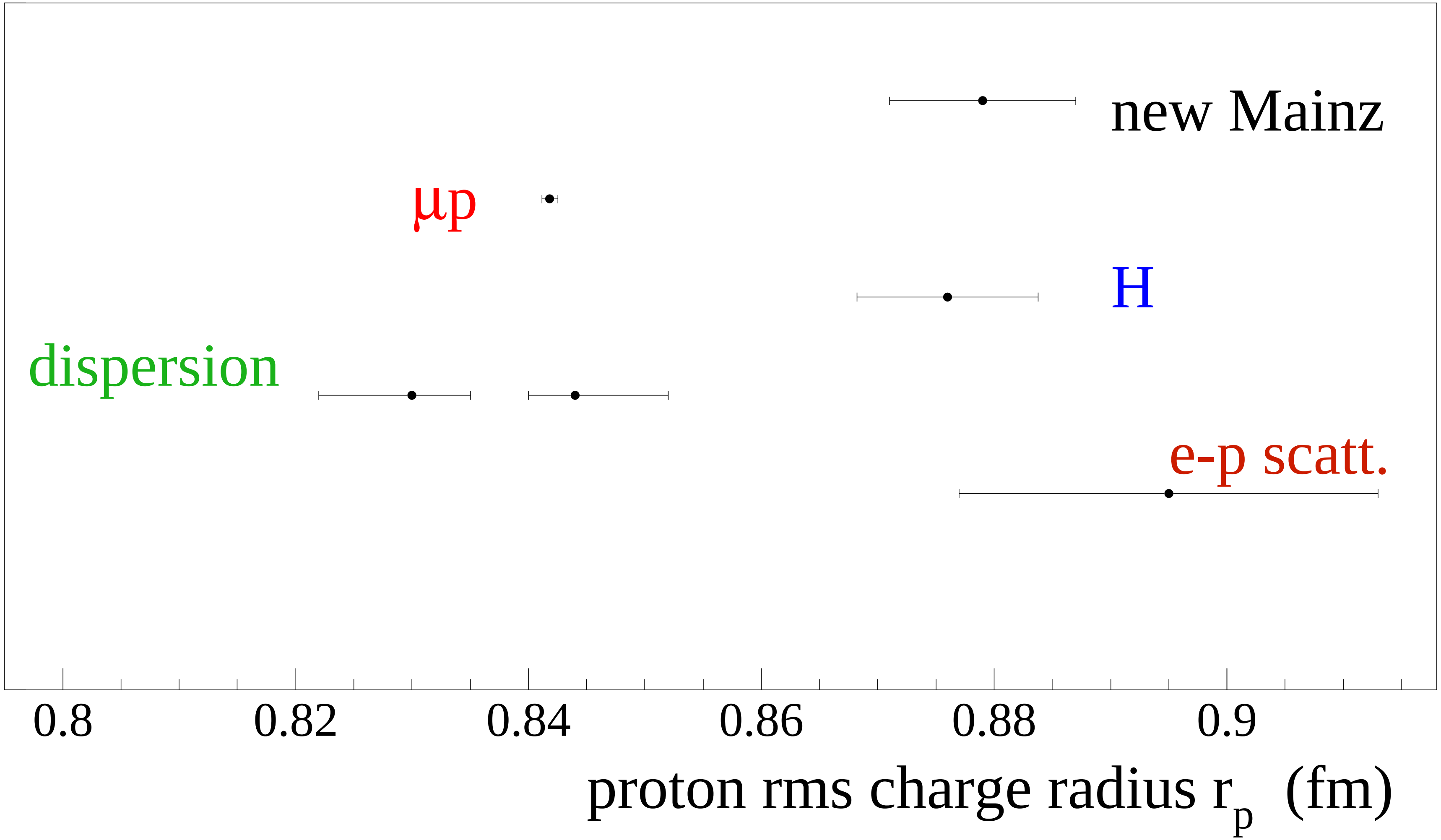
- Discrepancy with CODATA prediction — 75 300 MHz

Systematic effects are small since they scale like $\mathbf{1/m}$

Finite size effect scales like $\mathbf{m^3}$

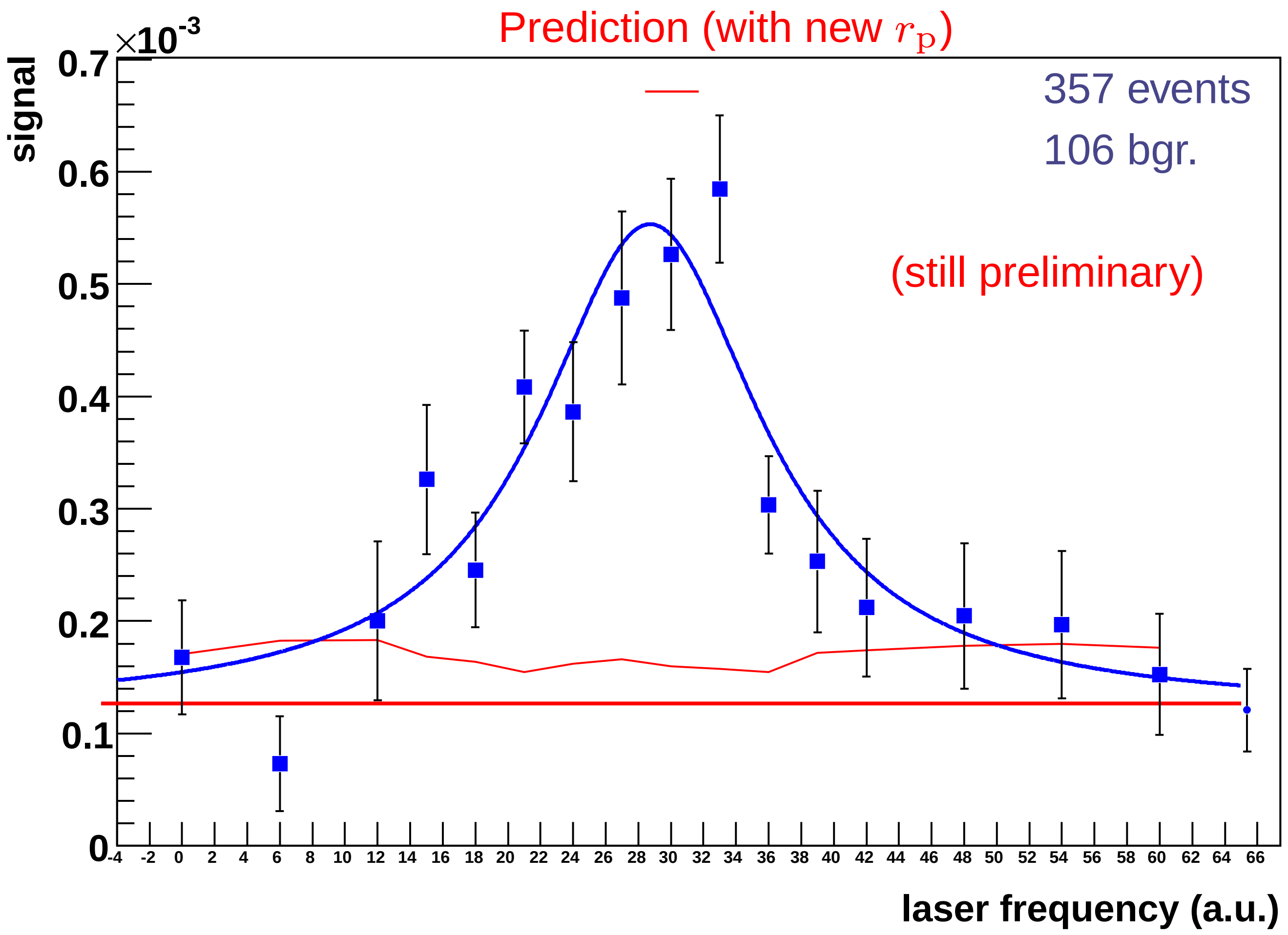
Proton radius 2010

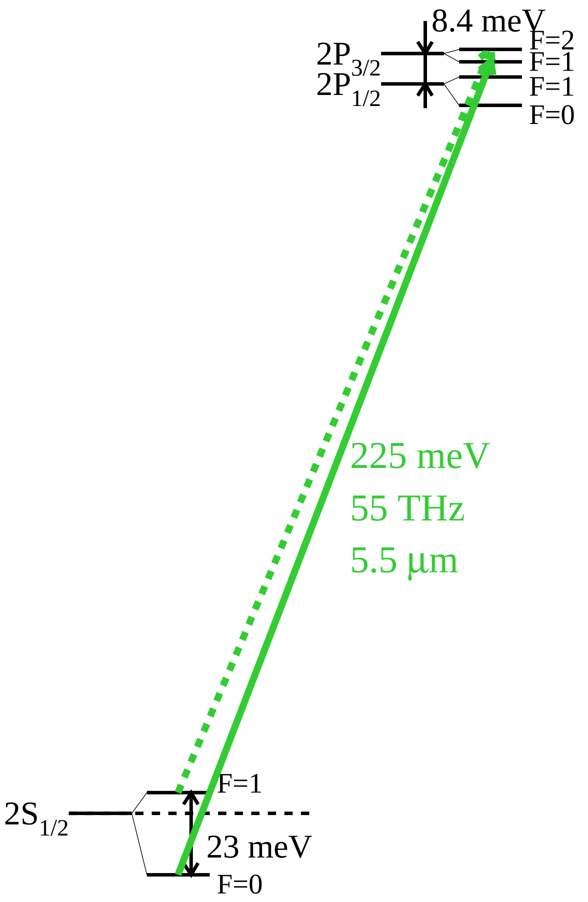
$$r_p = 0.84184\,(36)_{exp}(56)_{theo}\ \text{fm}$$



R. Pohl *et al.*, Nature 466, 213 (2010).

2nd line in muonic hydrogen





- $\sigma_{\text{position}} = 1.1$ GHz $\Longleftrightarrow$ 25 ppm ($\Gamma = 18.6$ GHz)
- Position fits perfectly with theory using new r_p

Extract HFS and r_{Zemach}

Results on muonic hydrogen

Transition frequencies:

$\nu(2S_{1/2}^{F=1} \rightarrow 2P_{3/2}^{F=2})$	=	49881.88(76) GHz	R. Pohl *et al.*, Nature 466, 213 (2010)
		49881.16(62) GHz	PRELIMINARY 2011
$\nu(2S_{1/2}^{F=0} \rightarrow 2P_{3/2}^{F=1})$	=	54611.87(1.01) GHz	PRELIMINARY 2011

Proton radius using 2010 theory

$\nu(2S_{1/2}^{F=1} \rightarrow 2P_{3/2}^{F=2})$	:	0.84184(36)(56) fm	R. Pohl *et al.*, Nature 466, 213 (2010)
		0.84218(29)(56) fm	PRELIMINARY 2011
$\nu(2S_{1/2}^{F=0} \rightarrow 2P_{3/2}^{F=1})$	:	0.84252(48)(85) fm	PRELIMINARY 2011

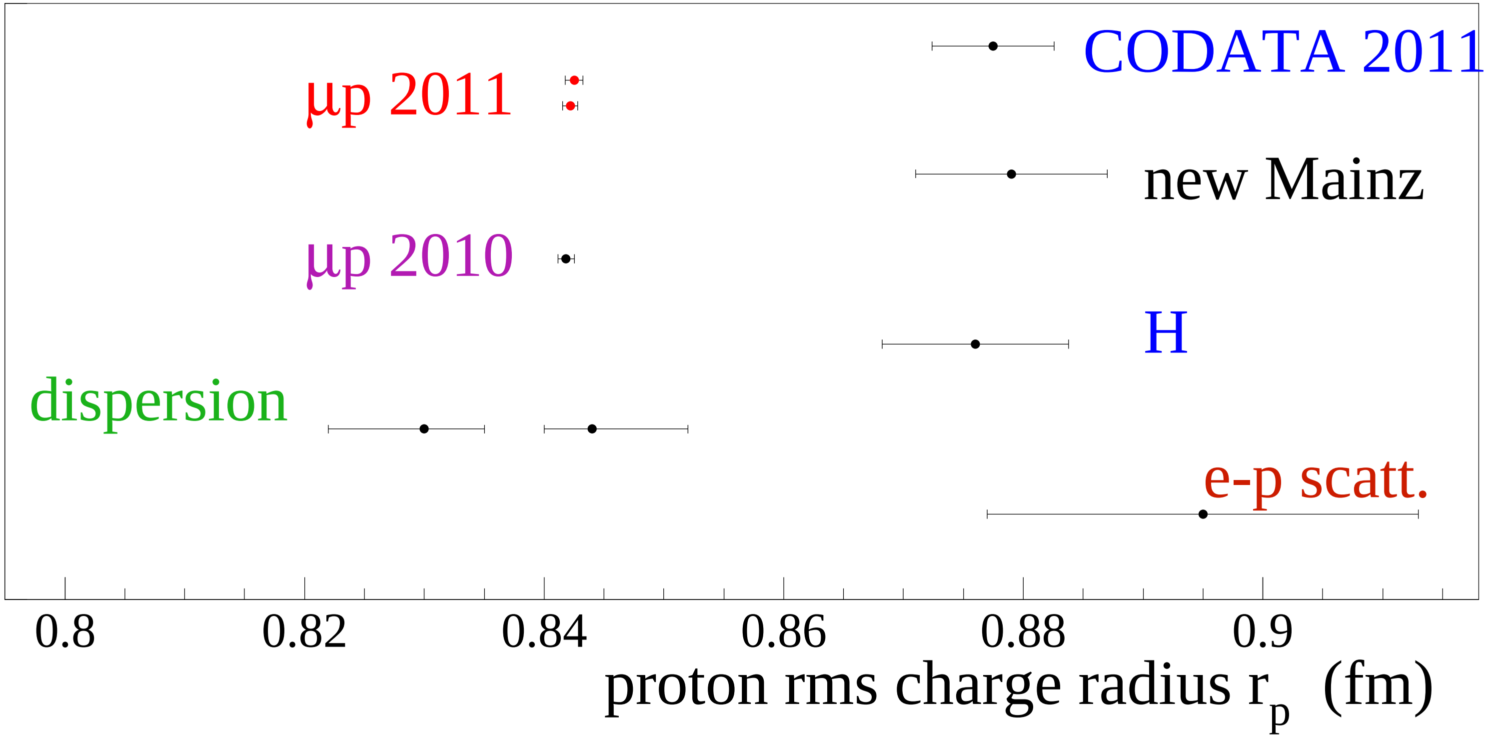
Results on muonic hydrogen

Proton radius using 2010 theory

$\nu(2S_{1/2}^{F=1} \rightarrow 2P_{3/2}^{F=2})$: 0.84184(36)(56) fm R. Pohl *et al.*, Nature 466, 213 (2010)

0.84218(29)(56) fm PRELIMINARY 2011

$\nu(2S_{1/2}^{F=0} \rightarrow 2P_{3/2}^{F=1})$: 0.84252(48)(85) fm PRELIMINARY 2011



Proton radius puzzle

From the two transitions in muonic hydrogen we extract a PRELIMINARY value of the proton rms charge radius of

r_p=0.84284(58) fm $\quad (u_r = 6.8 \times 10^{-4})$

$$u_\mathrm{exp} = 3.0 \times 10^{-4}$$
$$u_\mathrm{theo} = 6.2 \times 10^{-4}$$

BUT:

CODATA 2006: $r_\mathrm{p} = (0.8768 \pm 0.0069)$ fm
Hydrogen: $r_\mathrm{p} = (0.876 \pm 0.008)$ fm
e-p scattering: $r_\mathrm{p} = (0.895 \pm 0.018)$ fm (Sick 2005)

r_p is 4% smaller

5.0σ from CODATA-2006
4.3σ from H
3.1σ from e-p scatt.

Proton radius puzzle

From the two transitions in muonic hydrogen we extract a PRELIMINARY value of the proton rms charge radius of

r_p=0.84284(58) fm $(u_r = 6.8 \times 10^{-4})$

$$u_\mathrm{exp} = 3.0 \times 10^{-4}$$
$$u_\mathrm{theo} = 6.2 \times 10^{-4}$$

BUT:

CODATA 2006: $r_\mathrm{p} = (0.8768 \pm 0.0069)$ fm
Hydrogen: $r_\mathrm{p} = (0.876 \pm 0.008)$ fm
e-p scattering: $r_\mathrm{p} = (0.894 \pm 0.008)$ fm (Sick 2011)
$r_\mathrm{p} = (0.879 \pm 0.008)$ fm (Mainz 2010)
$r_\mathrm{p} = (0.875 \pm 0.010)$ fm (JLab Hall A 2011)

r_p is 4% smaller

5.0σ from CODATA-2006
4.3σ from H
8.4σ from scatt.

Proton radius puzzle

From the two transitions in muonic hydrogen we extract a PRELIMINARY value of the proton rms charge radius of

r_p=0.84284(58) fm $\quad (u_r = 6.8 \times 10^{-4})$

$$u_\mathrm{exp} = 3.0 \times 10^{-4}$$
$$u_\mathrm{theo} = 6.2 \times 10^{-4}$$

BUT:

CODATA 2010: $r_\mathrm{p} = (0.8775 \pm 0.0051)$ fm

CODATA 2006: $r_\mathrm{p} = (0.8768 \pm 0.0069)$ fm

Hydrogen: $r_\mathrm{p} = (0.876 \pm 0.008)$ fm

e-p scattering: $r_\mathrm{p} = (0.894 \pm 0.008)$ fm (Sick 2011)

$r_\mathrm{p} = (0.879 \pm 0.008)$ fm (Mainz 2010)

$r_\mathrm{p} = (0.875 \pm 0.010)$ fm (JLab Hall A 2011)

r_p is 4% smaller

6.8σ from CODATA-2010

5.0σ from CODATA-2006

4.3σ from H

8.4σ from scatt.

What’s wrong ??

What's wrong ??

PRA **81**, 060501	Karshenboim *et al.*: *"NR contrib. in order $\alpha^5 m_\mu c^2$ to the LS"* *(1005.4879)*
JETP Lett. **92**, 8	Karshenboim *et al.*: *"Light-by-light scatt. contrib to energy levels"* *(1005.4880)*
PRL **105**, 242001	Bernauer *et al*: *"High-precision el. and magn. FF of the proton"* *(1007.5076)*
PRD **82**, 125020	Jaeckel, Roy: *"Spectroscopy as a test of Coulomb's law"* *(1008.3536)*
PLB **693**, 555	De Rujula: *"QED is not endangered by the proton's size"* *(1008.3861)*
1008.4225	Vanderhaeghen, Walcher: *"Long range structure of the nucleon"*
PRC **83**, 012201	Cloet, Miller: *"Third Zemach moment of the proton"* *(1008.4345)*
1008.4546	De Rujula: *"Comment on Third Zemach moment of the proton"*
PRD **82**, 113005	Hill, Paz: *"Model independent extraction of r_p from e-scattering"* *(1008.4619)*
PLB **697**, 26	De Rujula: *"QED confronts the proton's radius"* *(1010.3421)*
1011.0692	Jaeckel, Roy: *"Spectroscopic bounds on New Physics"*
PLB **696**, 343	Distler *et al*: *"The RMS radius of the proton and Zemach moments"* *(1011.1861)*
EPJD **61**, 7	Jentschura: *"Proton radius, Darwin-Foldy term and radiative corrections"*
PRL **106**, 153001	Barger *et al.*: *"Proton size anomaly"* *(1011.3519)*
PRA **83**, 012507	Yerokhin: *"Nuclear size corrections to the Lamb shift in 1-e atoms"* *(1011.4272)*
PRD **83**, 101702	Tucker-Smith, Yavin: *"Muonic hydrogen and MeV forces"* *(1011.4922)*
Ann. Phys. **326**, 500	Jentschura: *"LS in μp 1: Verification and update of theory"* *(1011.5275)*
Ann. Phys. **326**, 516	Jentschura: *"LS in μp 2: Analysis of discrepancy"* *(1011.5453)*

Discussions...

PRA **81**, 060501 Karshenboim *et al.*: *"NR contrib. in order $\alpha^5 m_\mu c^2$ to the LS" (1005.4879)*

JETP ... 0)

PRL

PRD

PLB

10

PRC

10

PRD

PLI

10

PLB ... 61)

EP

PRL

PRA ... 2)

PRD

Ann. Ph

$3^{\rm rd}$ Zemach moment of the proton

PLB **693**, 555 De Rujula: *"QED is not endangered by the proton's size" (1008.3861)*

A large third Zemach moment $\langle r_p^3 \rangle_{(2)} = \int d^3r_1\, d^3r_2\, \rho(r1)\, \rho(r_2)\, |\boldsymbol{r}_1 - \boldsymbol{r}_2|^3$ of the proton can explain all three measurements: μp, H, e-p

$\rho(r)$ is not a simple Dipole, but has "core" and "tail"

PRC **83**, 012201 Cloet, Miller: *"Third Zemach moment of the proton" (1008.4345)*

Such a large third Zemach moment is impossible.

$\langle r_p^3 \rangle_{(2)}$ (De Rujula) = 36.6 ± 6.9 fm^3

$\langle r_p^3 \rangle_{(2)}$ (Sick) = 2.71 ± 0.13 fm^3

PLB **696**, 343 Distler *et al*: *"The RMS radius of the proton and Zemach moments"* (1011.1861)

$\langle r_p^3 \rangle_{(2)}$ (Mainz 2010) = 2.85 ± 0.08 fm^3

Ann. Phys. **326**, 516 Jentschura: *"LS in μp 2: Analysis of discrepancy" (1011.5453)*

Discussions

PRA
JETP
PRL
PRD
PLB
10
PRC
10
PRD
PLI
10
PLB
EP
PRL
PRA
PRD
Ann. Ph
Ann. Ph

New Physics

PRD **82**, 125020 Jaeckel, Roy: *"Spectroscopy as a test of Coulomb's law" (1008.3536)*

hidden photons, minicharged particles $\longrightarrow$ deviations from Coulomb's law.

μp transition can NOT be explained this. (contradicts Lamb shift in H)

PRL **106**, 153001 Barger *et al.*: *"Proton size anomaly" (1011.3519)*

decay of Υ, J/ψ, π^0, η, neutron scattering, muon g-2, μ^{24}Mg, μ^{28}Si

$\Rightarrow$ It's NOT a new flavor-conserving spin-0, 1 or 2 particle

PRD **83**, 101702 Tucker-Smith, Yavin: *"Muonic hydrogen and MeV forces" (1011.4922)*

MeV force carrier can explain discrepancies for r_p and (g-2)$_\mu$
and is in agreement with observations for (g-2)$_e$, α, μSi, μMg
IF coupling to e, n is suppressed relative to coupling to μ, p
prediction for μHe$^+$, $\mu^+\mu^-$

PRL **107**, 011803 Batell, McKeen, Pospelov: *"New Parity-violating muonic forces" (1103.0721)*

10...100 MeV heavy photon ("light Higgs") can explain r_p and (g-2)$_\mu$
prediction for μHe$^+$, enhanced PNC in muonic systems

Discussions...

Theory updates

EPJD **61**, 7 Jentschura: *"Proton radius, Darwin-Foldy term and radiative corrections"*

- Darwin-Foldy term: Zitterbewegung of spin-1/2 nucleus
- Atomic physics: Nuclear size is *without* DF term (point-like → r=0)
- Nuclear physics: DF term somethimes pushed into nuclear size
- CODATA: → consistent treatment

⇒ proton charge radius = slope of electric Sachs G_E FF, w/o DF term

- Radiative corrections are treated consistently.

PRA **83**, 012507 Yerokhin: *"Nucl. size corr. to the Lamb shift of 1-e atoms"* (1011.4272)

- Higher-order NS corr. to SE, VP increase LO result by 4.4%
- shifts H(1S) state by ~ 800 Hz ⇔ 110'000 Hz discrepancy

Ann. Phys. **326**, 500 Jentschura: *"LS in μp 1: Theory verification + update"* (1011.5275)

- μp theory ok, minor corrections → r_p = 0.84169 (66) fm

Ann. Phys. **326**, 516 Jentschura: *"LS in μp 2: Analysis of discrepancy"* (1011.5453)

- no millicharged particles, no unstable neutral vector boson
- e^- bound to μp(2S) ?!?!?!

NO!!!

Discussions...

Theory updates

EPJD **61**, 7 Jentschura: *"Proton radius, Darwin-Foldy term and radiative corrections"*

e^- bound to μp(2S) not likely:

- How should this bound state form?
  - "No" free electrons.
- Why should this bound state be stable?
  - Stark mixing !?!
  - Auger effect !!
  - Collisions: every 100 ns.
  - Only 1 line observed: >80% formation rate in 1 state.
    - no peak at expected position
    - observed peak has expected width

- e^- bound to μp(2S) ?!?!?!

NO!!!

Discussions...

More Theory updates

1104.2971 J.D. Carroll, A.W. Thomas, J. Rafelski, and G.A. Miller:
"Nonperturbative relativistic calculation of the muonic hydrogen spectrum", (PRA, accepted)

- Theory in muonic hydrogen confirmed by fully numerical calculations.
- Creative treatment of the 2S HFS
  won't fit with our measured 2nd transition in $\mu\mathrm{p}$
- Other than that: Our r_p confirmed. $\surd$

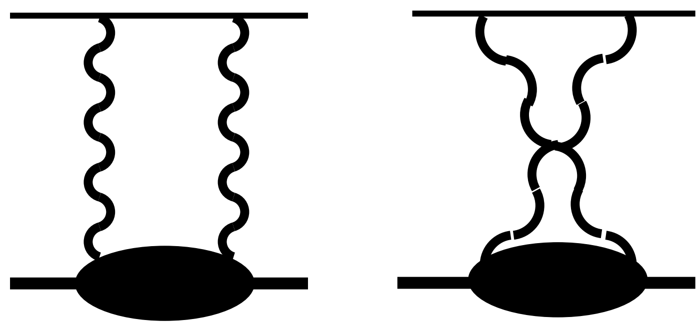
1103.1772 Borie: *"Lamb shift in light muonic atoms - revisited"*

- Recalculation of all 2S–2P energy differences for
  - muonic **hydrogen**
  - muonic **deuterium** (measured 2009)
  - muonic **helium-3** (CREMA 2014)
  - muonic **helium-4** (CREMA 2013)
- Our r_p confirmed. $\surd$

Discussions...

Proton polarizability

proton polarizability aka. two-photon exchange



Seems to be the only contribution which *might* be able to solve the proton size puzzle by changing theory in $\mu\mathrm{p}$.

Keep in mind:

Discrepancy:	0.31 meV
Polarizability:	0.015(4) meV

Discussions...

Proton polarizability

PRA **83**, 042509 (2011)   C.E. Carlson, V. Nazaryan, K. Griffioen:
“Proton-structure corrections to hyperfine splitting in muonic hydrogen” (1101.3239)

- The 2S HFS is confirmed with smaller uncertainty:
  22.8146(49) meV instead of the
  22.8148(78) meV we used.

1101.4073   G.A. Miller, A.W. Thomas, J.D. Carroll, and J. Rafelski:
“Natural resolution of the proton size puzzle”

- New off-mass-shell effect $\sim \alpha \frac{m^4}{M^3}$ solves puzzle.
- Others say: This is already included in standard treatment.
- C.E. Carlson: calculation gives 50 times smaller value.

1103.4617   R.J. Hill, G. Paz:
“Model independent analysis of proton structure for hydrogenic bound states”

- Again an off-shell-effect.
- “Crazy” functional behaviour can give any correction.
- No numbers given.
- Again: The “main stream gurus” (e.g. Pachucki) are not impressed.

Discussions...

Proton polarizability

proton polarizability aka. two-photon exchange

- Our value is based on 3 calculations
  - 0.017(4) meV: Rosenfelder, Phys. Lett. B 463, 317 (1999)
  - 0.012(2) meV: Pachucki, PRA 60, 3593 (1999)
  - 0.018 meV: Martynenko, Faustov, Phys. At. Nucl. 63, 845 (2000)
  - 0.015(4) meV: Borie, PRA 71, 032508 (2005)
- (0.31 meV is our discrepancy)
- No consensus about **validity** of the new treatments
- Independent calculation of new effects give **50 times smaller** value.
- Lack of **numbers**

Until somebody comes up with detailed calculations and **numbers** I don't consider this a solution of the proton size puzzle.

What is wrong?

μp experiment:	discrepancy 75 GHz
	100σ
	$\sim 4\,\Gamma_{\rm nat}$
	two independent wavelength calibrations
	one very significant line, no satellite
	fitted width = natural width
	another transition in μp confirms our $r_{\rm p}$
μp theory:	discrepancy 0.31 meV
	60σ
	0.15% of the total Lamb shift
	4th largest term
	several independent calculations
H theory:	L_{1S} off by 100 kHz
	25σ
	most terms only calculated by 2 groups + methods
	convergence?

What is wrong?

μp experiment:	discrepancy 75 GHz
	100σ
↓	$\sim 4\,\Gamma_{\mathrm{nat}}$
ok $\surd$	two independent wavelength calibrations
	one very significant line, no satellite
	fitted width = natural width
	another transition in μp confirms our r_{p}
μp theory:	discrepancy 0.31 meV
	60σ
	0.15% of the total Lamb shift
	4th largest term
	several independent calculations
H theory:	L_{1S} off by 100 kHz
	25σ
	most terms only calculated by 2 groups + methods
	convergence?

What is wrong?

μp experiment: discrepancy 75 GHz

100σ

$\sim 4\,\Gamma_{\rm nat}$

ok $\surd$ two independent wavelength calibrations

one very significant line, no satellite

fitted width = natural width

another transition in μp confirms our $r_{\rm p}$

μp theory: discrepancy 0.31 meV

60σ

0.15% of the total Lamb shift

ok $\surd$ 4th largest term

several independent calculations

H theory: L_{1S} off by 100 kHz

25σ

most terms only calculated by 2 groups + methods

convergence?

What is wrong?

μp experiment:	discrepancy 75 GHz
↓	100σ
	$\sim 4\,\Gamma_{\rm nat}$
ok $\surd$	two independent wavelength calibrations
	one very significant line, no satellite
	fitted width = natural width
	another transition in μp confirms our $r_{\rm p}$
μp theory:	discrepancy 0.31 meV
↓	60σ
	0.15% of the total Lamb shift
ok $\surd$	4th largest term
	several independent calculations
H theory:	L_{1S} off by 100 kHz
↓	25σ
	most terms only calculated by 2 groups + methods
likely ok	convergence?

What is wrong?

μp experiment: discrepancy 75 GHz

100σ

$\sim 4\,\Gamma_{\mathrm{nat}}$

ok $\surd$ two independent wavelength calibrations

one very significant line, no satellite

μp theory

ok $\surd$

H theory:

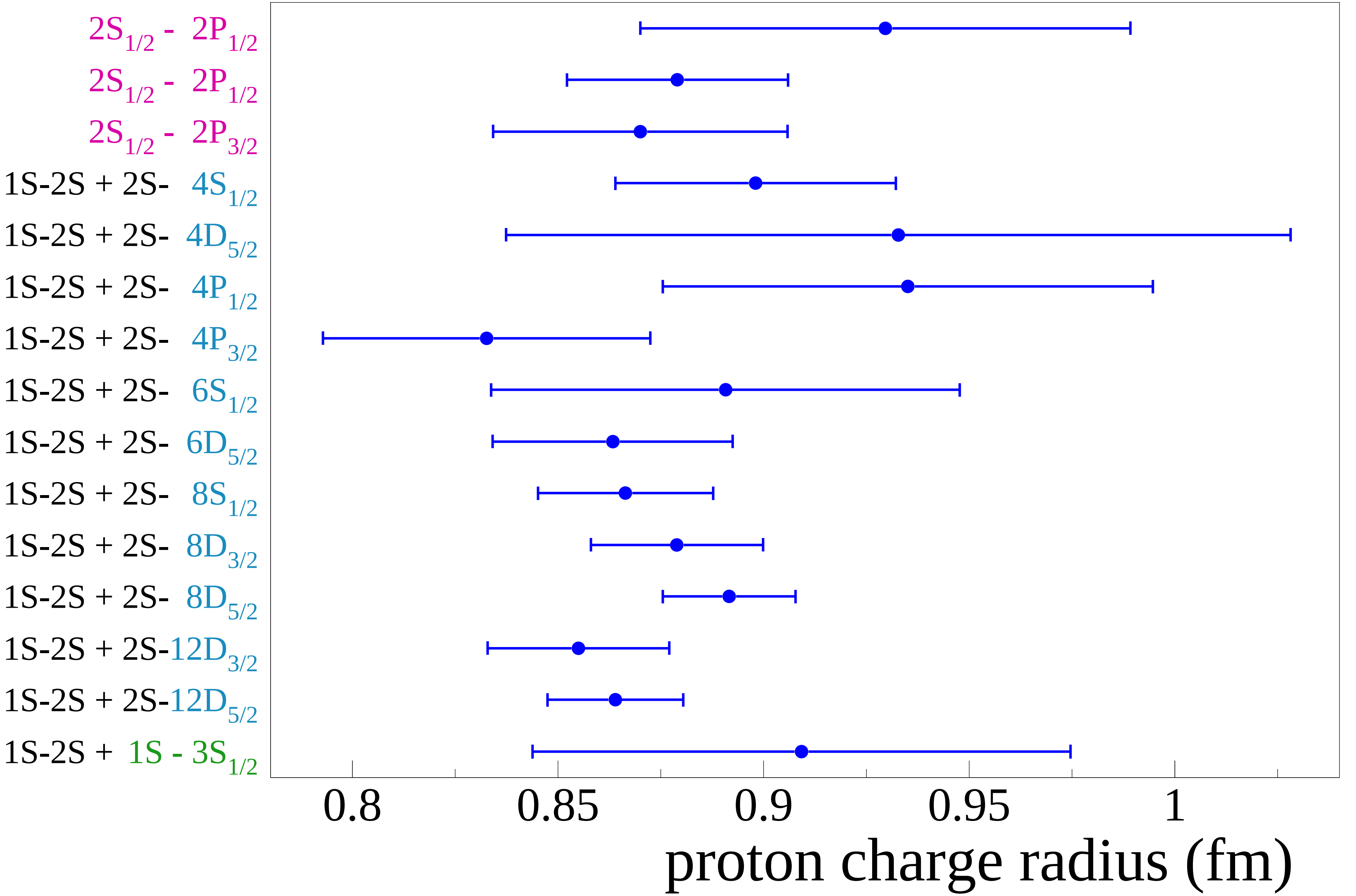
likely ok most terms only calculated by 2 groups + methods

convergence?

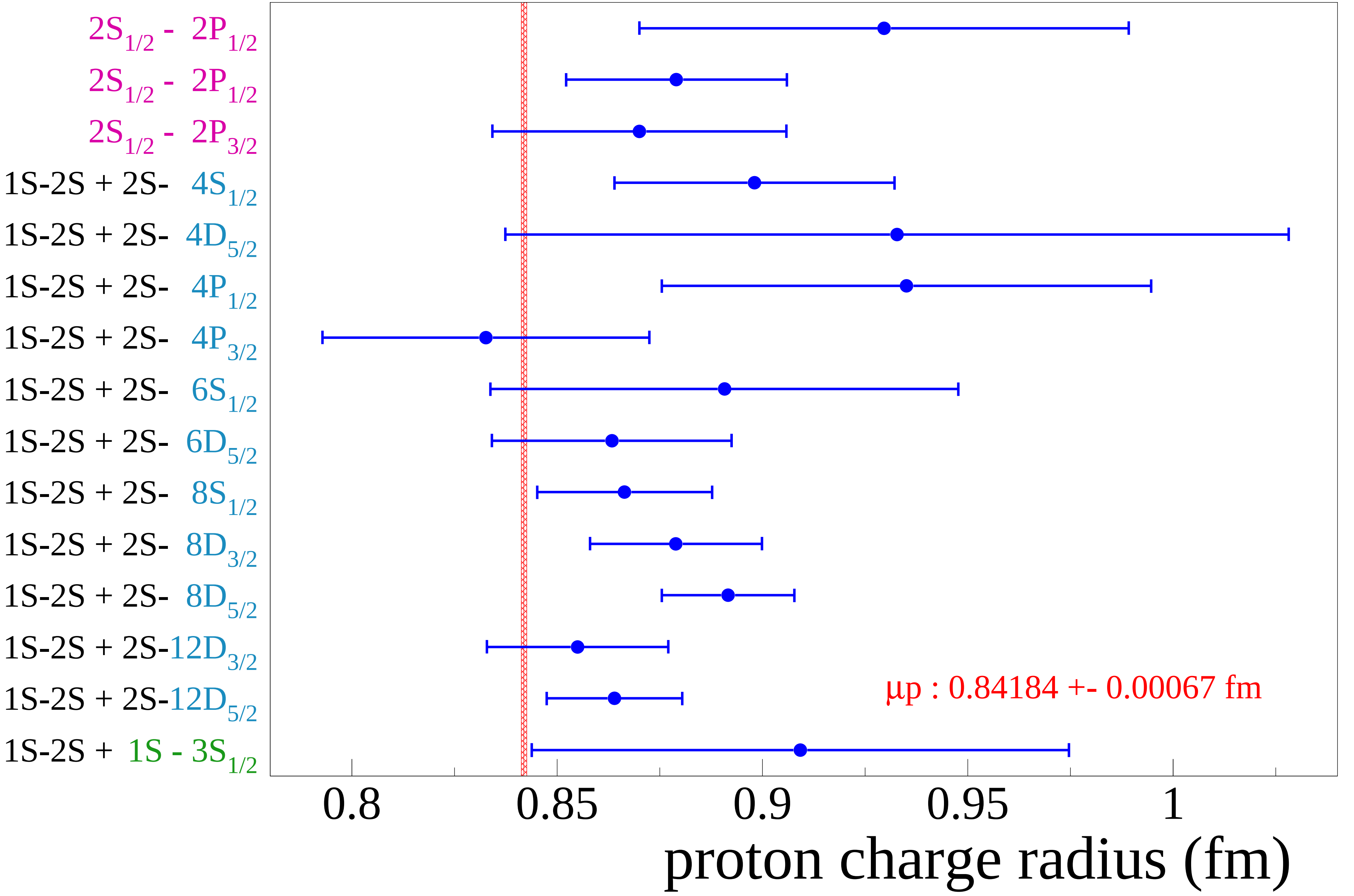
Are H spectroscopy and e-p scattering really

“obviously correct”?

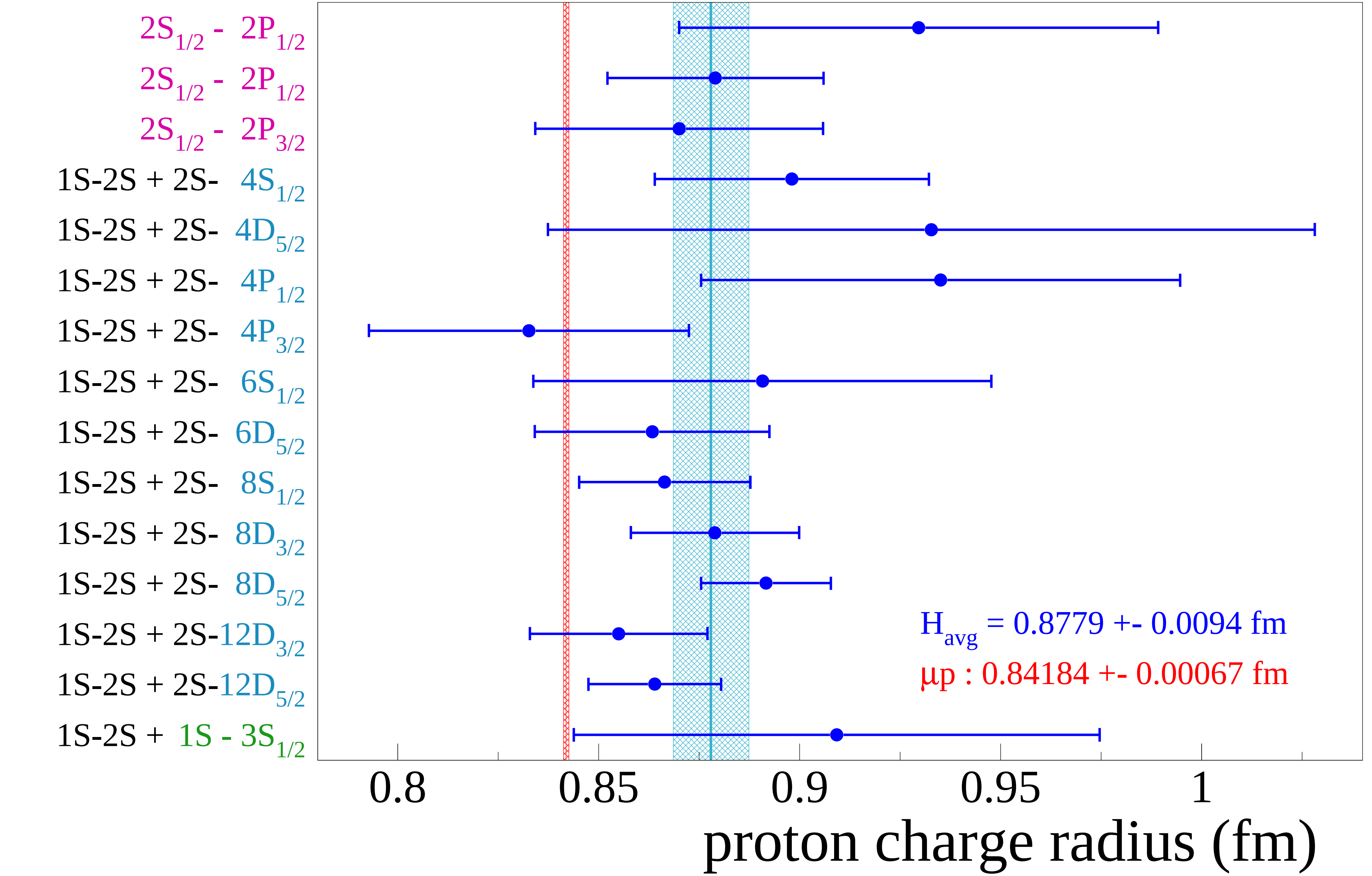
Hydrogen spectroscopy



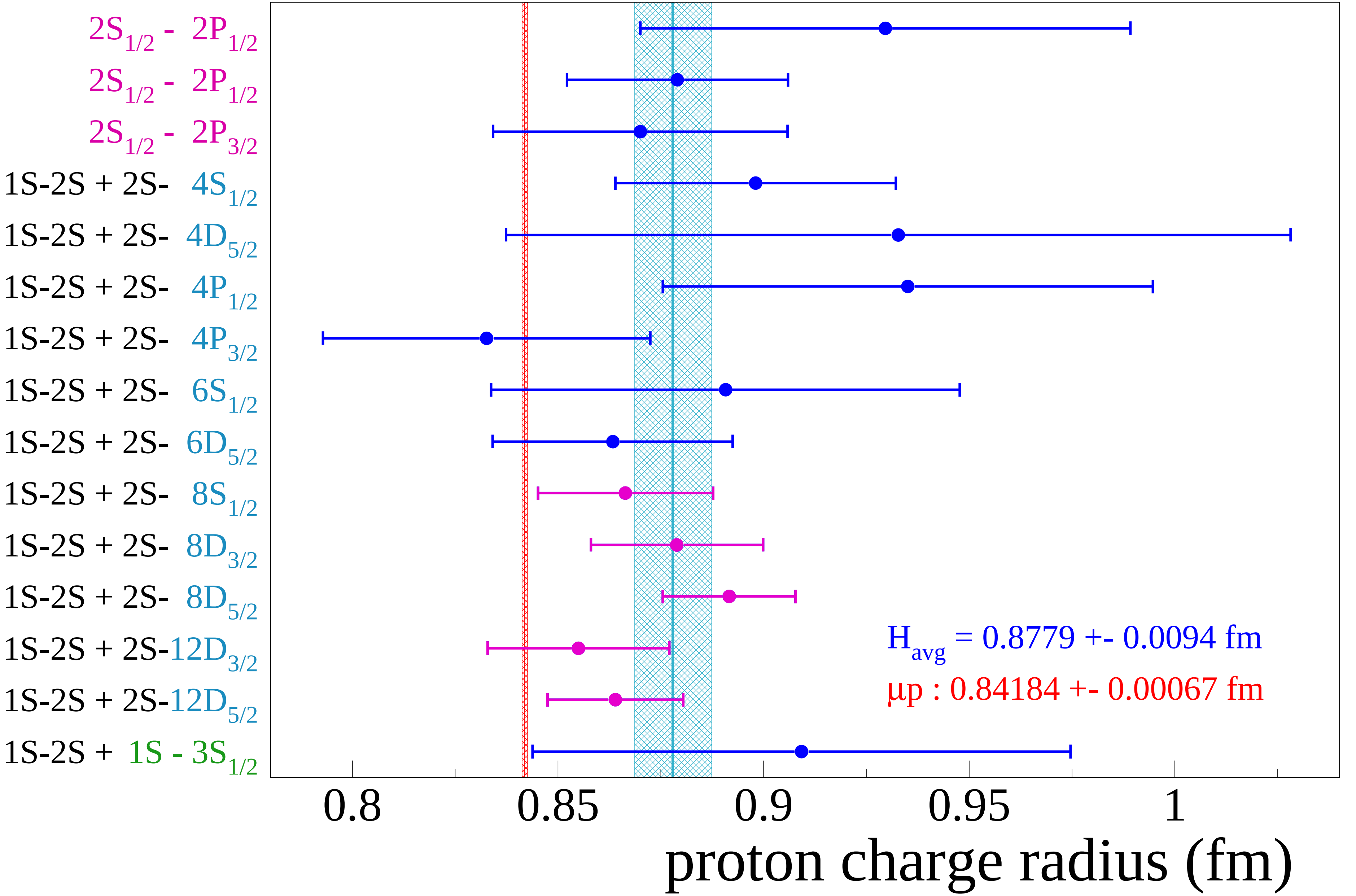
Hydrogen spectroscopy



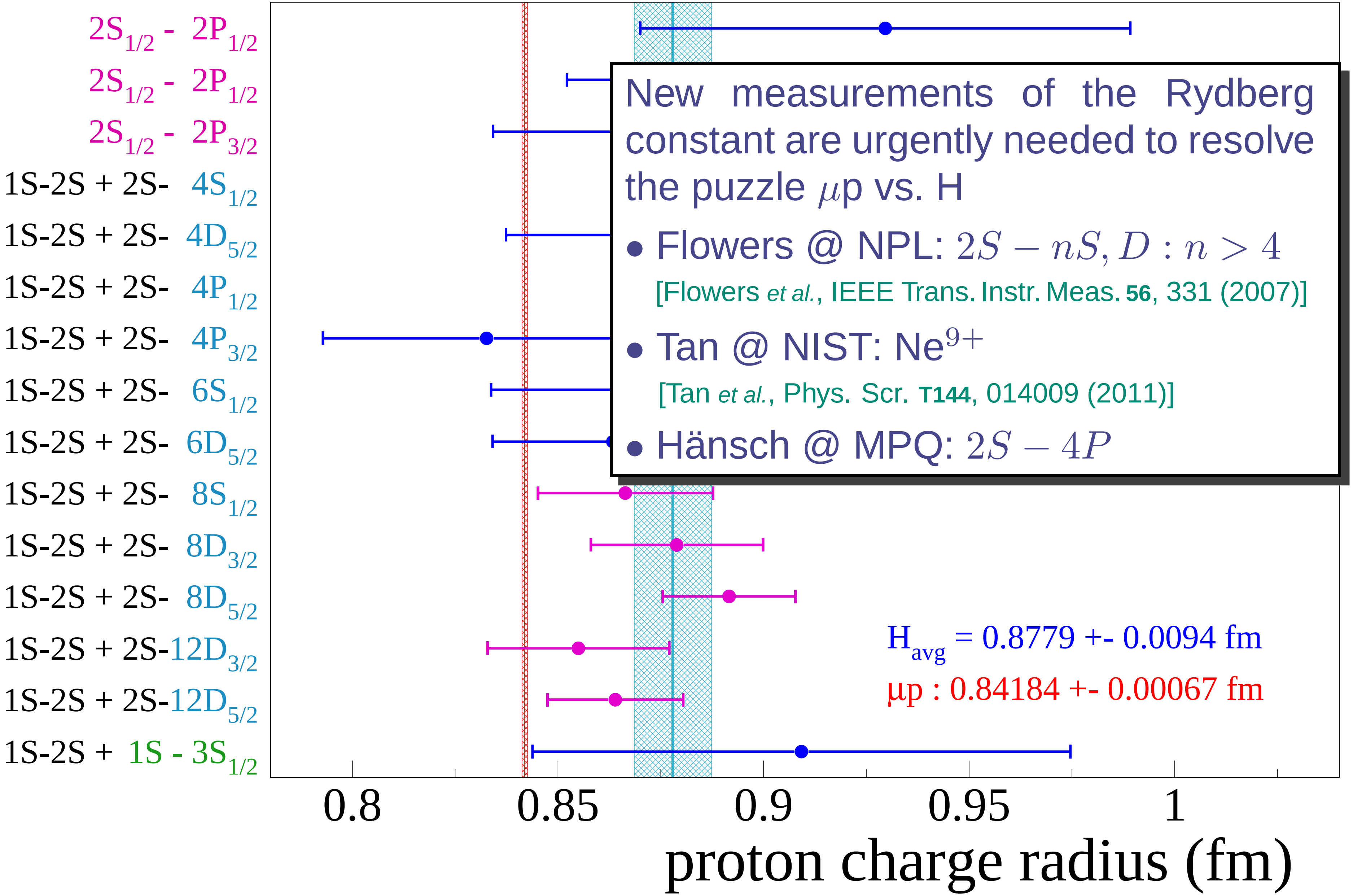
Hydrogen spectroscopy



Hydrogen spectroscopy



Hydrogen spectroscopy



New measurements of the Rydberg constant are urgently needed to resolve the puzzle μp vs. H

- Flowers @ NPL: $2S - nS, D : n > 4$
  [Flowers *et al.*, IEEE Trans. Instr. Meas. **56**, 331 (2007)]
- Tan @ NIST: Ne^{9+}
  [Tan *et al.*, Phys. Scr. **T144**, 014009 (2011)]
- Hänsch @ MPQ: $2S - 4P$

r_p from electron scattering

PhD thesis J.C. Bernauer

- Rosenbluth cross section $\rightarrow$ Sachs form factor $\rightarrow r_\mathrm{p}$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Ros.}} = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} \frac{\varepsilon {G_E}^2 + \tau {G_M}^2}{\varepsilon(1+\tau)} \qquad \varepsilon = \left(1 + 2(1+\tau)\tan^2\frac{\theta}{2}\right)^{-1} \quad ; \quad \tau = \frac{Q^2}{4m_p^2}$$

G_E and G_M are the Fourier transforms of the charge and magnetization distributions

$G_E(0) = 1$ (charge), and $G_M(0) = \mu_p$ (magnetic moment)

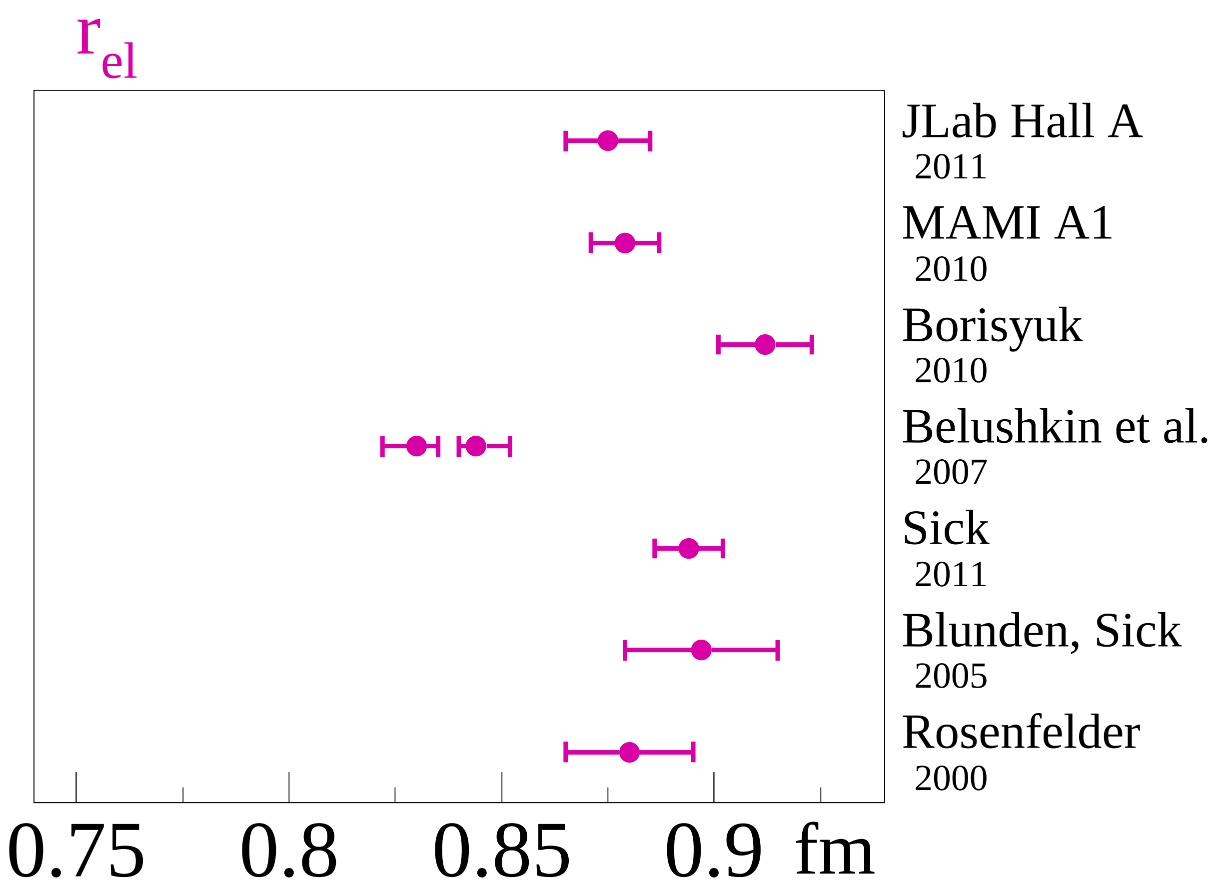
$$\langle r_\mathrm{p}^2 \rangle = -6\hbar^2 \frac{dG_E(Q^2)}{dQ^2}\Big|_{Q^2=0}$$ $\Rightarrow$ rms charge radius = slope of G_E at $Q^2 = 0$

extrapolation to $Q^2 \rightarrow 0$ required

$$Q^2[\,(\mathrm{GeV/c})^2] = \begin{cases} 6 \cdot 10^{-7} & (\mu\mathrm{p}) \\ > 2 \cdot 10^{-2} & (\text{e-p scatt.}) \end{cases}$$

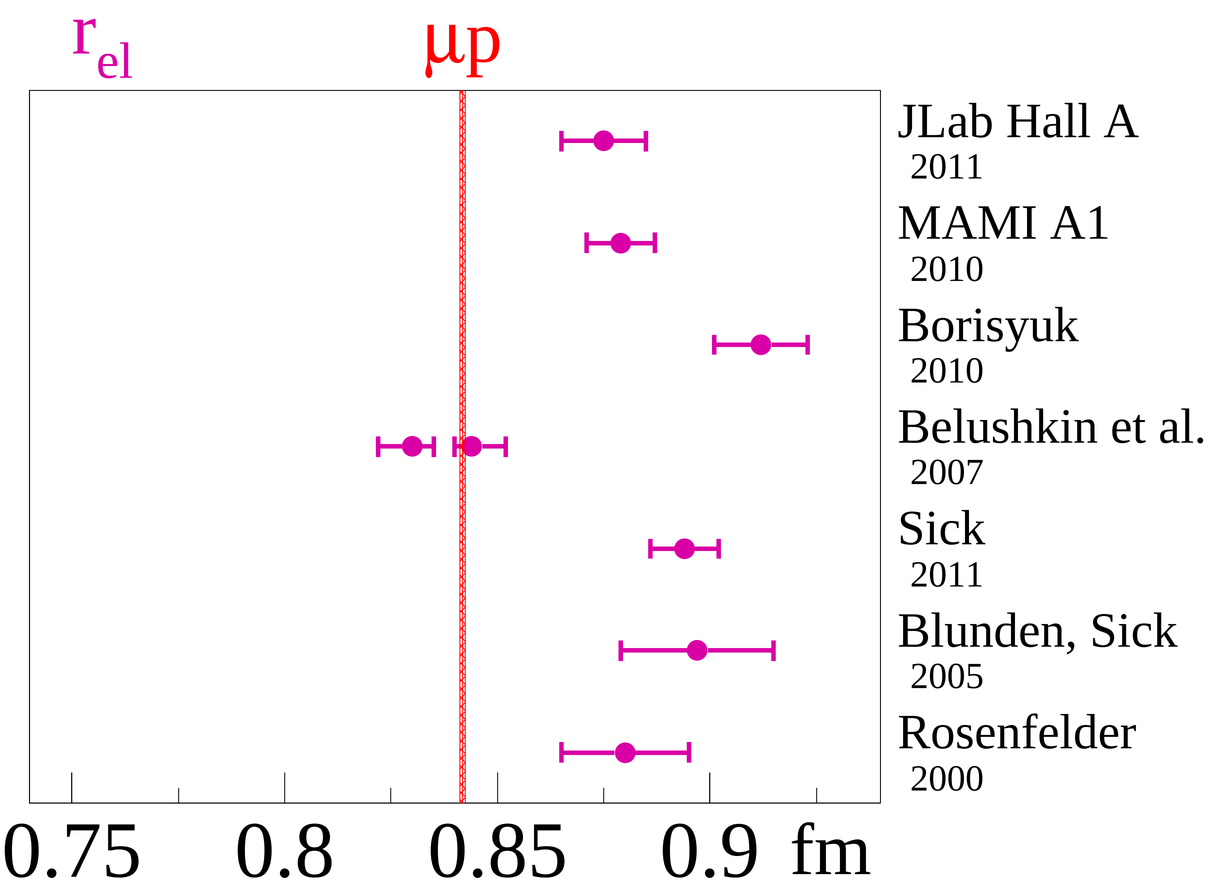
Note: You get **charge** and **magnetic** radius *simultaneously*.

Electron scattering



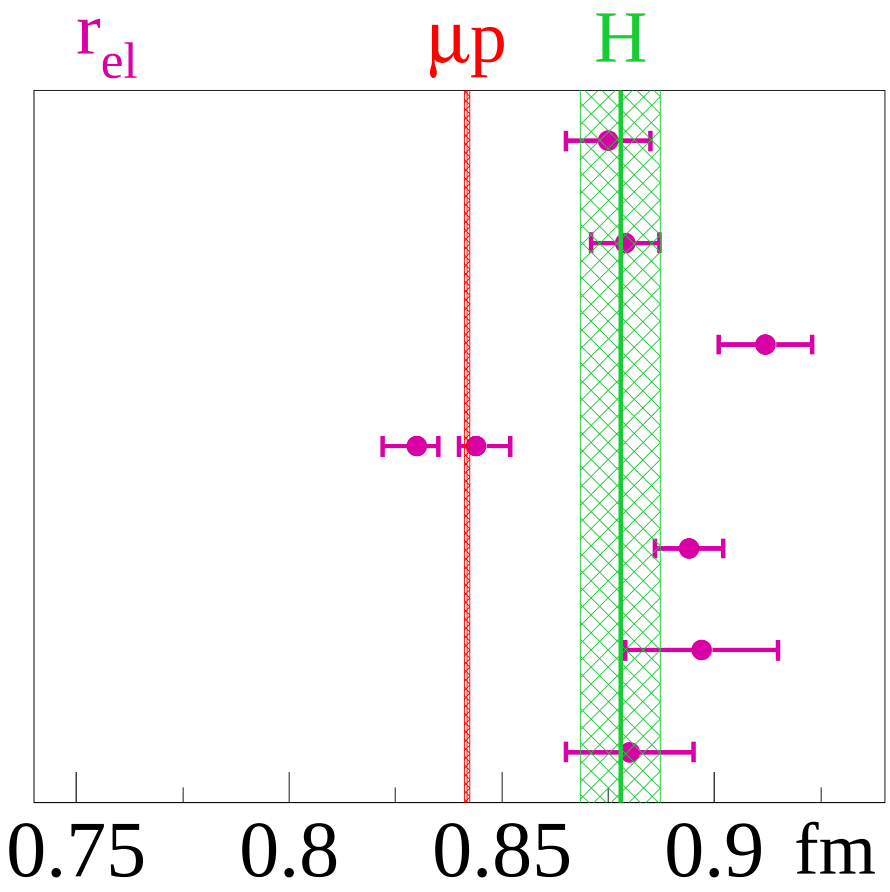
Rosenfelder , Phys Lett B 479, 381 (2000)
Blunden, Sick , PRC 72, 057601 (2005)
Sick , Few Body Syst. (2011)
Belushkin et al. , PRC 75, 035202 (2007)
Borisyuk , Nucl Phys A 843, 59 (2010)
MAMI A1 Bernauer et al., PRL 105, 242001 (2010)
JLab Hall A Zhan et al., 1102.0318 (nucl-ex) (2011)

Electron scattering



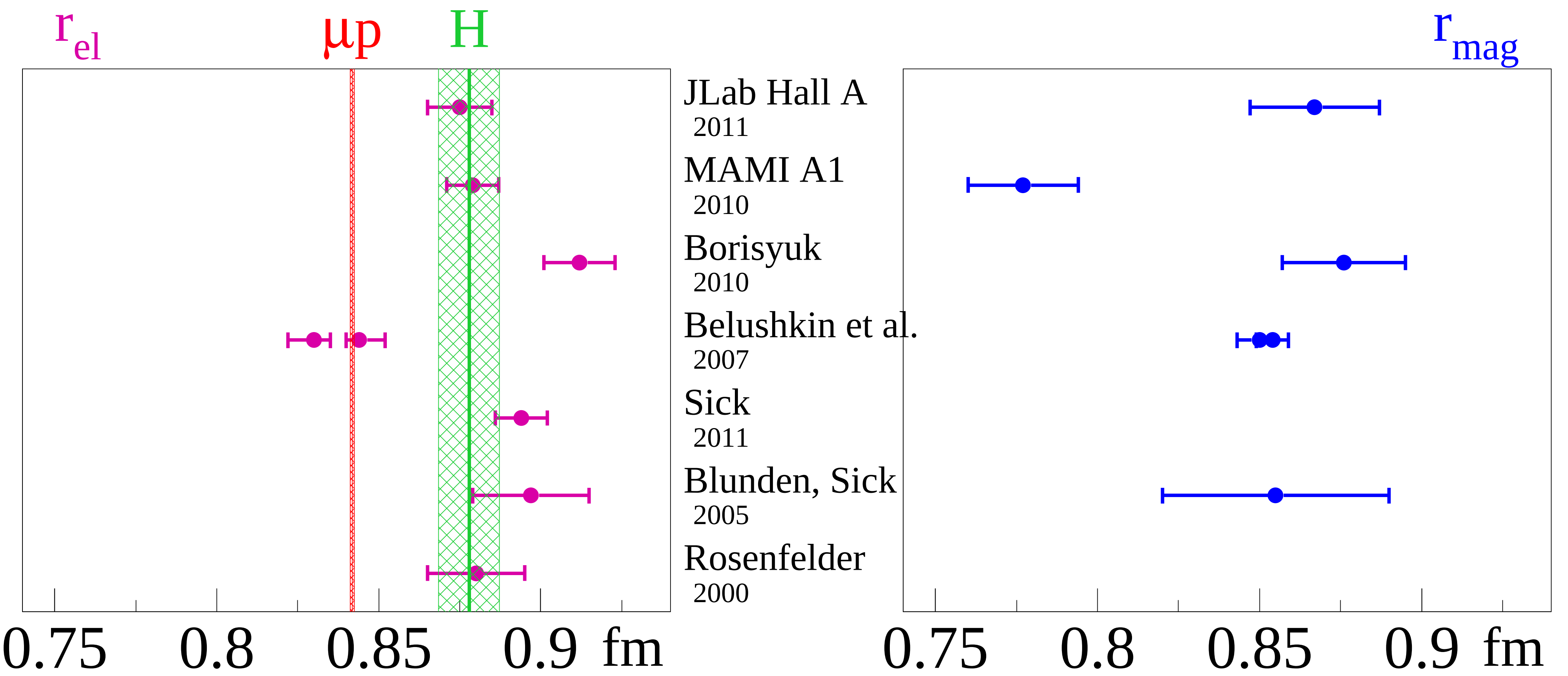
Rosenfelder , Phys Lett B 479, 381 (2000)
Blunden, Sick , PRC 72, 057601 (2005)
Sick , Few Body Syst. (2011)
Belushkin et al. , PRC 75, 035202 (2007)
Borisyuk , Nucl Phys A 843, 59 (2010)
MAMI A1 Bernauer et al., PRL 105, 242001 (2010)
JLab Hall A Zhan et al., 1102.0318 (nucl-ex) (2011)

Electron scattering



Rosenfelder , Phys Lett B 479, 381 (2000)
Blunden, Sick , PRC 72, 057601 (2005)
Sick , Few Body Syst. (2011)
Belushkin et al. , PRC 75, 035202 (2007)
Borisyuk , Nucl Phys A 843, 59 (2010)
MAMI A1 Bernauer et al., PRL 105, 242001 (2010)
JLab Hall A Zhan et al., 1102.0318 (nucl-ex) (2011)

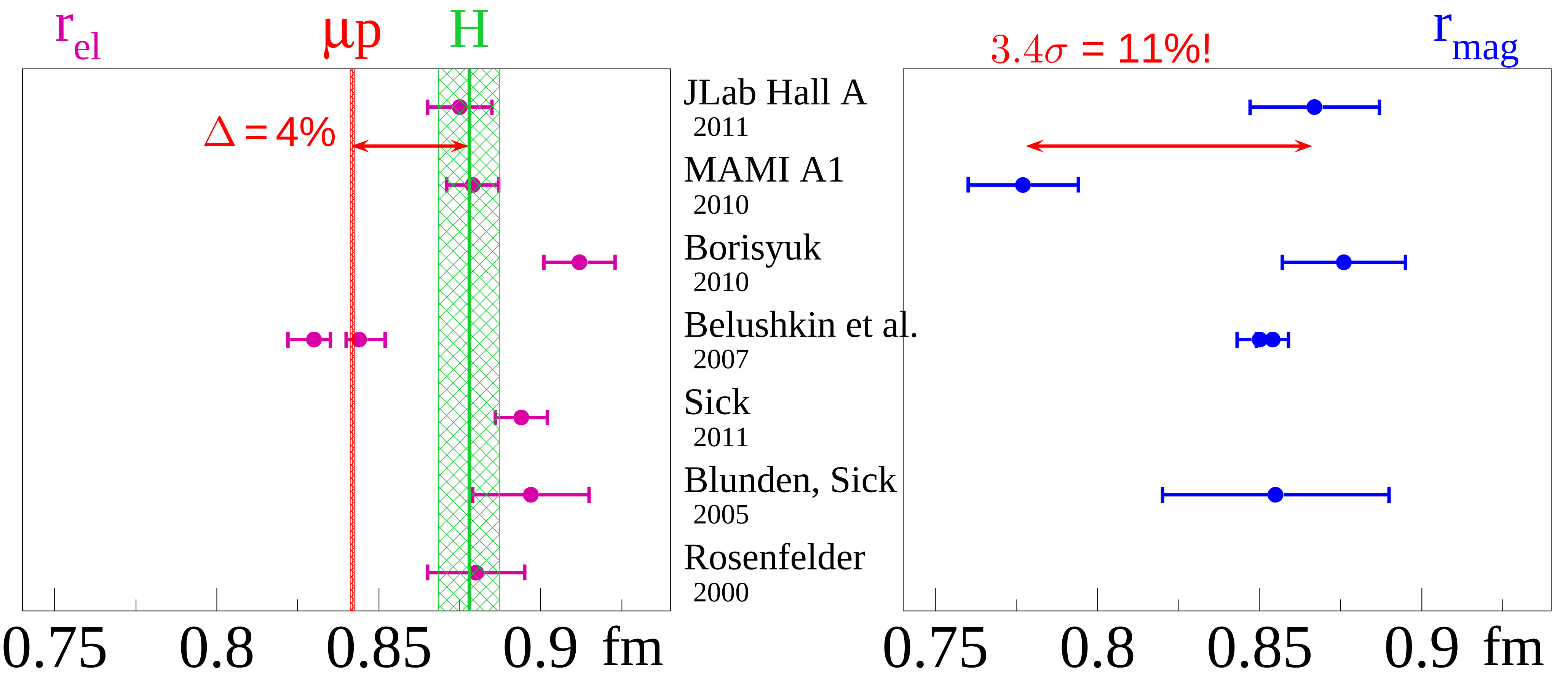
Electron scattering



Rosenfelder , Phys Lett B 479, 381 (2000)
Blunden, Sick , PRC 72, 057601 (2005)
Sick , Few Body Syst. (2011)
Belushkin et al. , PRC 75, 035202 (2007)

Borisyuk , Nucl Phys A 843, 59 (2010)
MAMI A1 Bernauer et al., PRL 105, 242001 (2010)
JLab Hall A Zhan et al., 1102.0318 (nucl-ex) (2011)

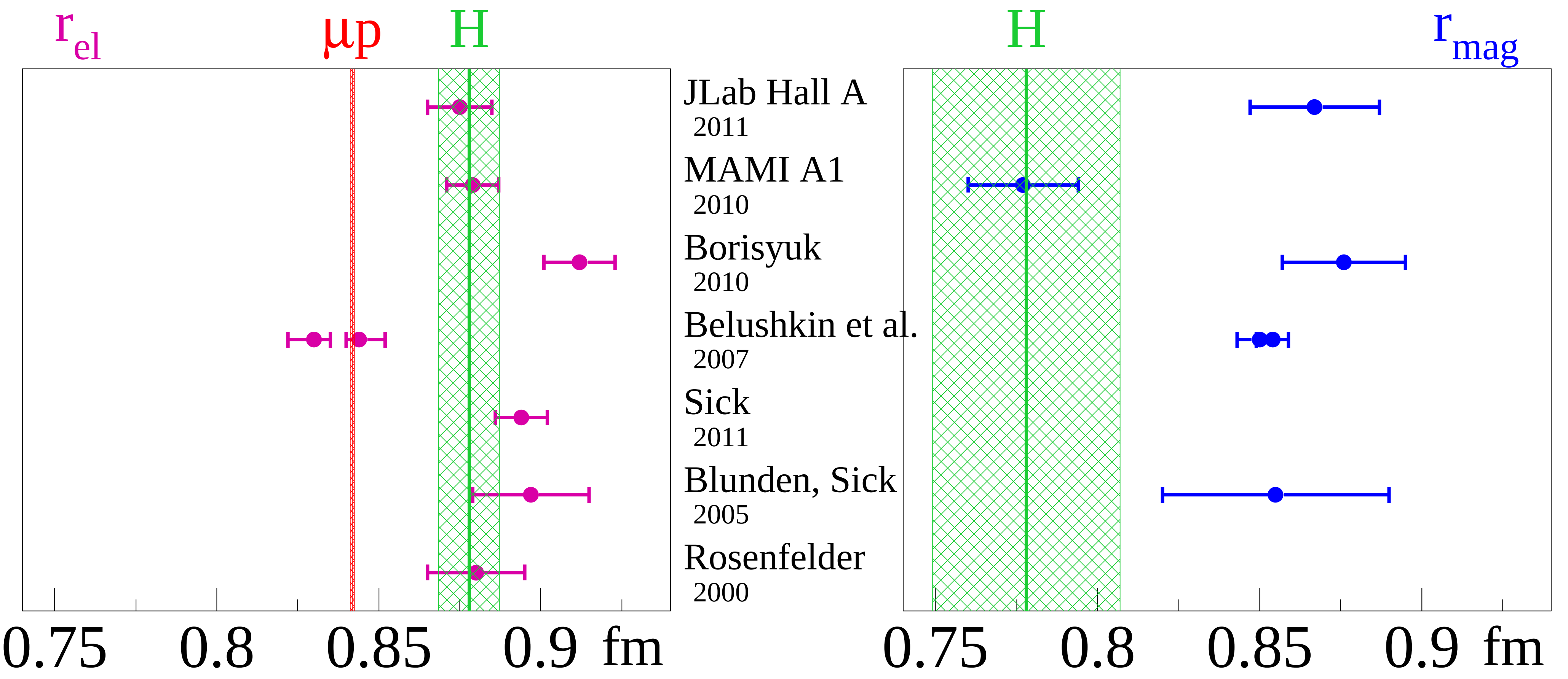
Electron scattering



Rosenfelder , Phys Lett B 479, 381 (2000)
Blunden, Sick , PRC 72, 057601 (2005)
Sick , Few Body Syst. (2011)
Belushkin et al. , PRC 75, 035202 (2007)

Borisyuk , Nucl Phys A 843, 59 (2010)
MAMI A1 Bernauer et al., PRL 105, 242001 (2010)
JLab Hall A Zhan et al., 1102.0318 (nucl-ex) (2011)

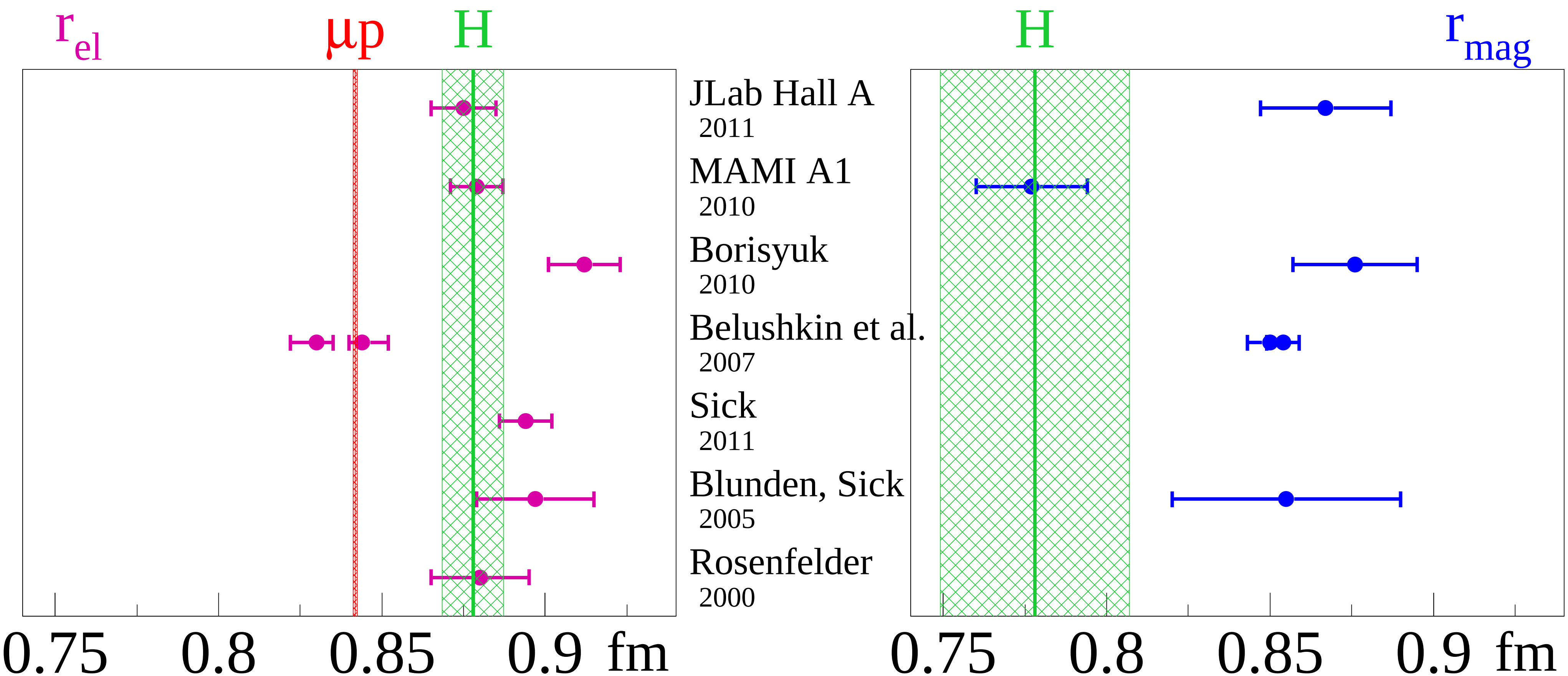
Electron scattering



Rosenfelder , Phys Lett B 479, 381 (2000)
Blunden, Sick , PRC 72, 057601 (2005)
Sick , Few Body Syst. (2011)
Belushkin et al. , PRC 75, 035202 (2007)

Borisyuk , Nucl Phys A 843, 59 (2010)
MAMI A1 Bernauer et al., PRL 105, 242001 (2010)
JLab Hall A Zhan et al., 1102.0318 (nucl-ex) (2011)
r_{mag}(H) Volotka et al., Eur Phys J D33, 23 (2005)

Electron scattering



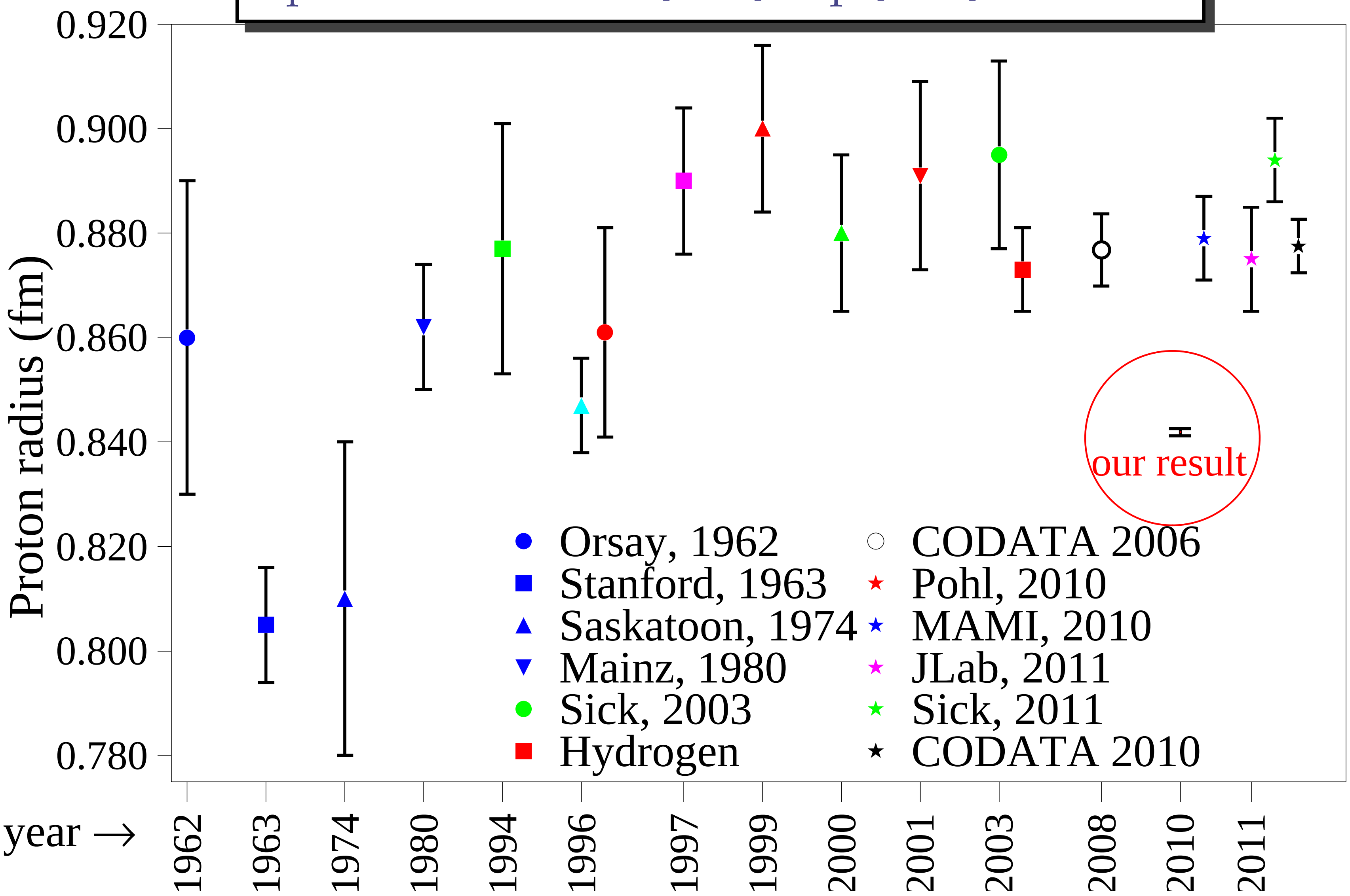
Ros
Blu
Sick
Belu

e-p scattering claims 1% error bars for 40 years now ⇒ limit reached?

Scatter of results suggests larger uncertainty.

Proton radius 2011

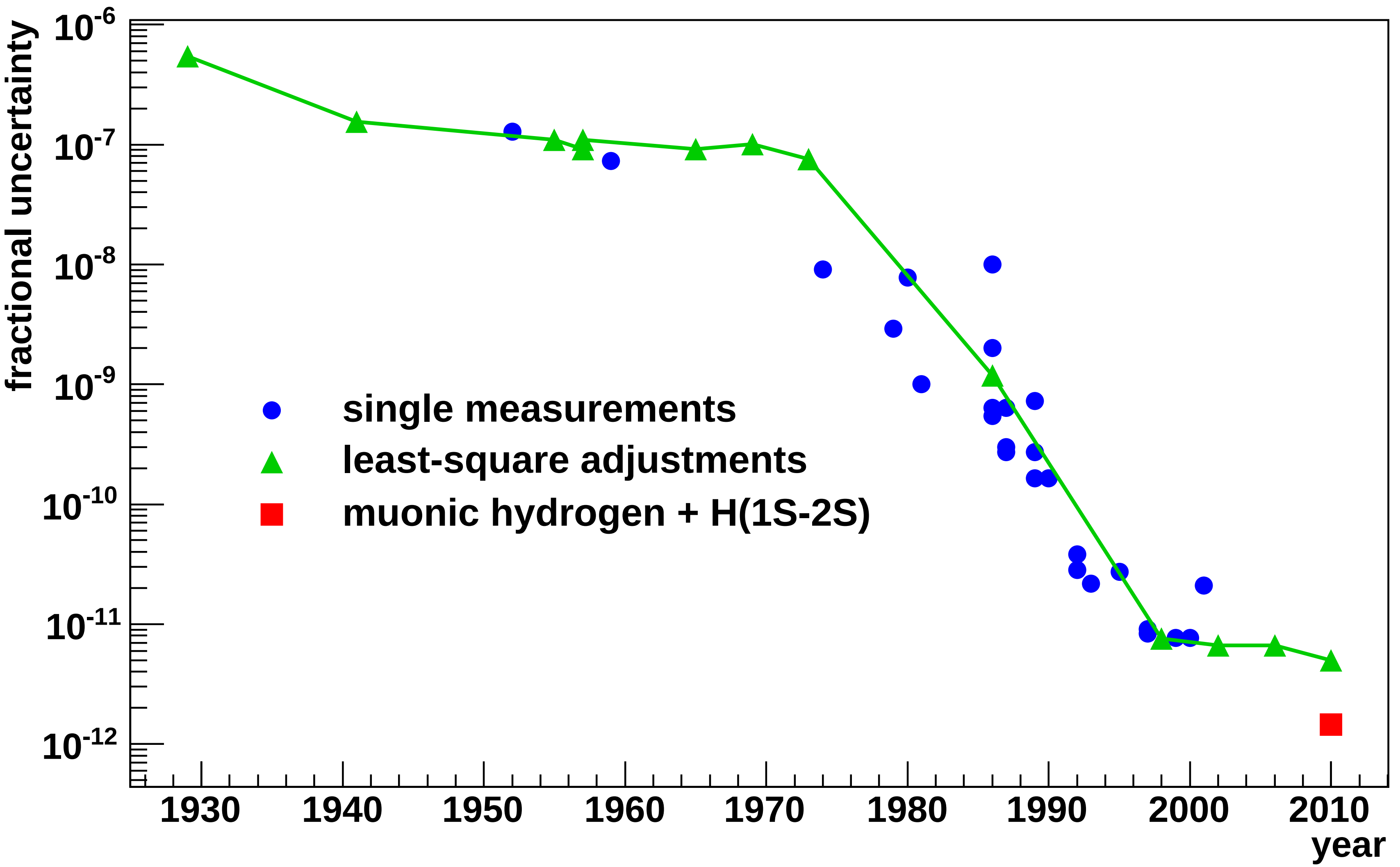
r_p=0.84184 (36)$_{exp}$(56)$_{theo}$ fm



R. Pohl *et al.*, Nature 466, 213 (2010).

Rydberg constant 2011

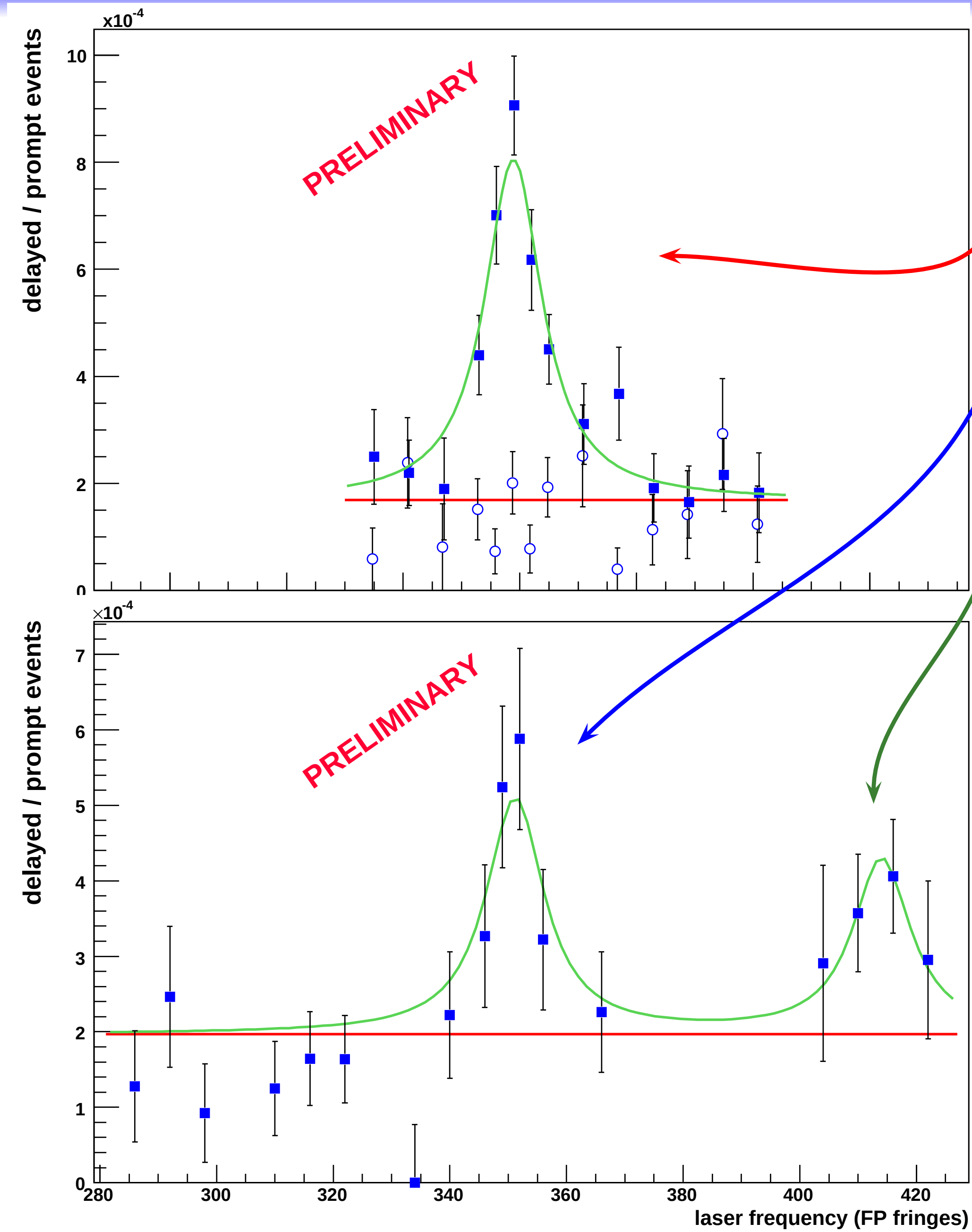
$$R_\infty = 10973731.568160(16)\,\mathrm{m}^{-1}$$ [1.5 parts in 10^{12}]



R. Pohl *et al.*, Nature 466, 213 (2010).

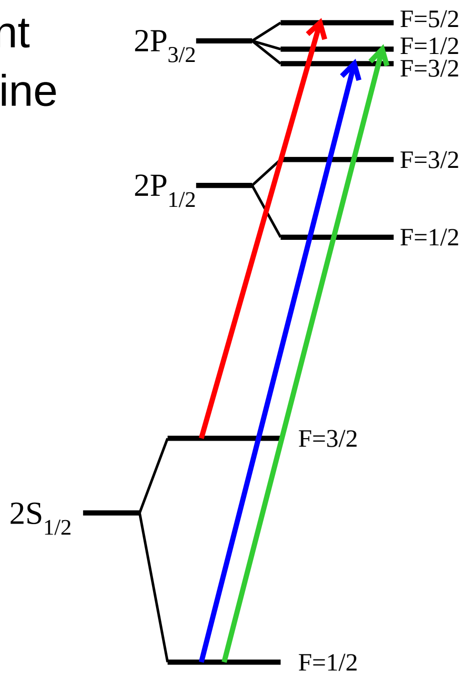
muonic deuterium

Muonic DEUTERIUM



2.5 resonances in muonic **deuterium**

- μd [$2S_{1/2}$(F=3/2) $\rightarrow$ $2P_{3/2}$(F=5/2)]
  20 ppm (stat., online)
- μd [$2S_{1/2}$(F=1/2) $\rightarrow$ $2P_{3/2}$(F=3/2)]
  45 ppm (stat., online)
- μd [$2S_{1/2}$(F=1/2) $\rightarrow$ $2P_{3/2}$(F=1/2)]
  70 ppm (stat., online)
  only 5σ significant
  identifies F=3/2 line



Summary

- muonic **hydrogen** $2S_{1/2}$(F=1) $\rightarrow$ $2P_{3/2}$(F=2) to 15 ppm (stat.+syst.)
  $\rightarrow$ r_p to 8×10^{-4} (experimental precision 4×10^{-4})
  $r_p = 0.84284 \pm 0.00058$ fm is ~~5σ~~ 6.8σ away from CODATA-20~~06~~ 10

  The proton is 4% smaller, and the Rydberg constant R_∞ is ~~4.9~~ 6.6 sigma off

- muonic **hydrogen** $2S_{1/2}$(F=0) $\rightarrow$ $2P_{3/2}$(F=1) to 18 ppm (to be published)
  exactly at the position deduced with our new r_p
  $\rightarrow$ 2S hyperfine splitting to 220 ppm
  $\rightarrow$ Zemach radius to a few % (radius of the magnetic moment distribution)

- muonic **deuterium** $2S_{1/2}$(F=3/2) $\rightarrow$ $2P_{3/2}$(F=5/2) to 20 ppm (stat., online)
  Theory: missing QED and nuclear structure corrections
  $\rightarrow$ deuteron charge radius and polarizability

- muonic **deuterium** $2S_{1/2}$(F=1/2) $\rightarrow$ $2P_{3/2}$(F=3/2 and F=1/2)
  $\rightarrow$ HFS, check calculations in μd

http://muhy.web.psi.ch

μp Lamb shift collaboration in 2009

F. KOTTMANN — ETH Zürich, Switzerland

A. ANTOGNINI, T.W. HÄNSCH, T. NEBEL,
R. POHL — MPQ, Garching, Germany

D. TAQQU — PSI, Switzerland

E.-O. Le BIGOT, F. BIRABEN, P. INDELICATO,
L. JULIEN, F. NEZ — Laboratoire Kastler Brossel, Paris, France

F.D. AMARO, J.M.R. CARDOSO, D.S. COVITA,
L.M.P. FERNANDES, J.A.M. LOPEZ, C.M.B. MONTEIRO,
J.M.F. DOS SANTOS, J.F.C.A. VELOSO — Department of Physics, Coimbra, Portugal

A. GIESEN, K. SCHUHMANN — Dausinger + Giesen, Stuttgart, Germany
T. GRAF — Institut für Strahlwerkzeuge, Stuttgart, Germany

C.-Y. KAO, Y.-W. LIU — National Tsing Hua University, Hsinchu, Taiwan

P. RABINOWITZ — Department of Chemistry, Princeton, USA

A. DAX, P. KNOWLES, L. LUDHOVA,
F. MULHAUSER, L. SCHALLER — former members, spent holidays at run 2009

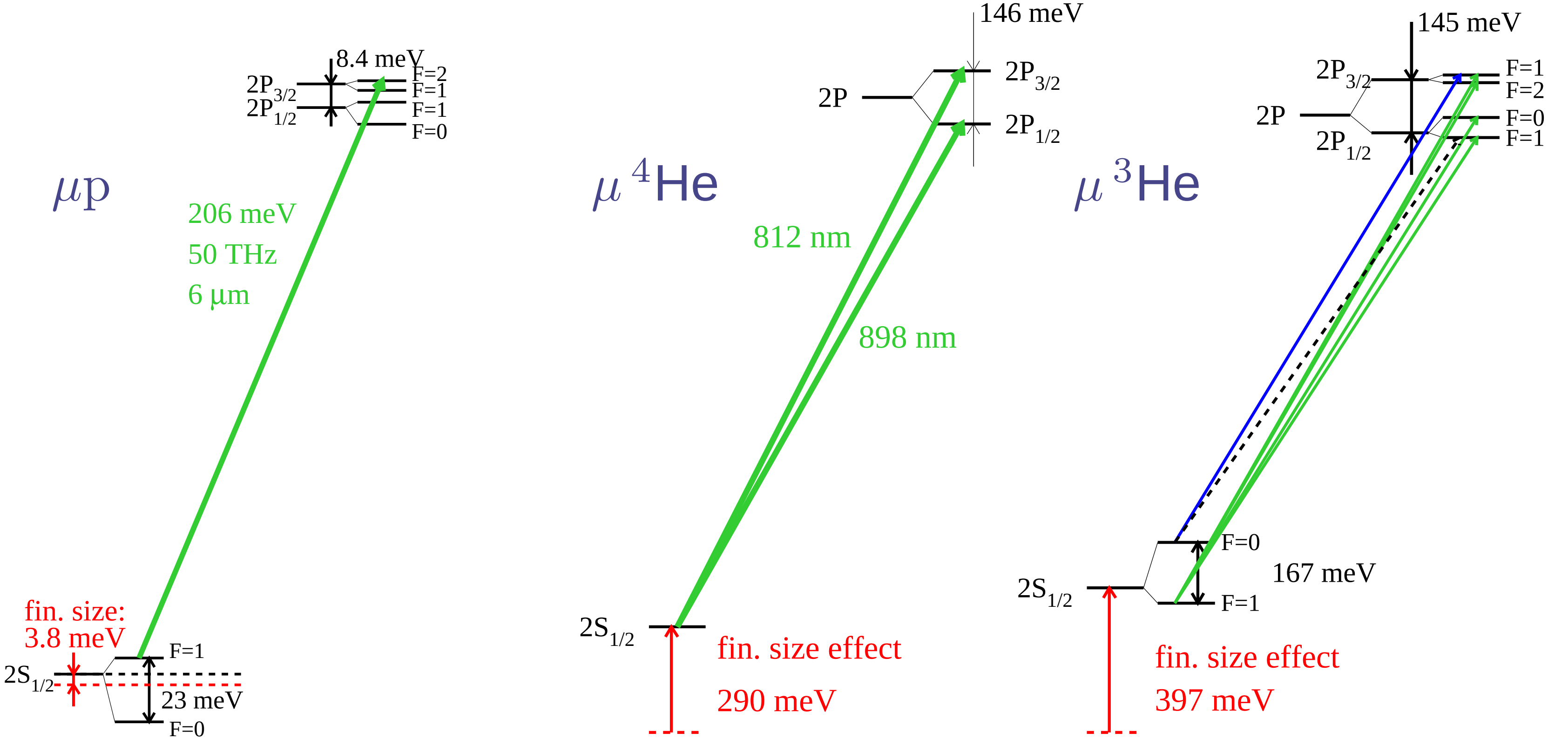
MPQ — Laboratoire Kastler Brossel, Physique quantique et applications — U C, Universidade de Coimbra — Paul Scherrer Institut, PSI — University of Stuttgart, Germany

Outlook: Lamb shift in muonic helium

- CREMA collaboration: Charge Radius Experiment with Muonic Atoms
- Exp. R10-01 approved at PSI in Feb. 2010
- Goal: Measure ΔE(2S-2P) in $\mu\,^4$He, $\mu\,^3$He
- $\Rightarrow$ alpha particle and helion charge radius to 3×10^{-4} (0.0005 fm)

Outlook: Lamb shift in muonic helium

- CREMA collaboration: Charge Radius Experiment with Muonic Atoms
- Exp. R10-01 approved at PSI in Feb. 2010
- Goal: Measure ΔE(2S-2P) in $\mu\,^4$He, $\mu\,^3$He
- ⇒ alpha particle and helion charge radius to 3×10^{-4} (0.0005 fm)



Outlook: Lamb shift in muonic helium

- CREMA collaboration: Charge Radius Experiment with Muonic Atoms
- Exp. R10-01 approved at PSI in Feb. 2010
- Goal: Measure ΔE(2S-2P) in $\mu\,^4$He, $\mu\,^3$He
- $\Rightarrow$ alpha particle and helion charge radius to 3×10^{-4} (0.0005 fm)
- aims:
  - help to solve the proton size puzzle
  - absolute charge radii of helion, alpha
  - low-energy effective nuclear models: ^{1}H, ^{2}D, ^{3}He, ^{4}He
  - QED test with He$^+$(1S-2S) [Udem @ MPQ, Eikema @ Amsterdam]

Outlook: Lamb shift in muonic helium

- CREMA collaboration: Charge Radius Experiment with Muonic Atoms
- Exp. R10-01 approved at PSI in Feb. 2010
- Goal: Measure ΔE(2S-2P) in μ^4He, μ^3He
- $\Rightarrow$ alpha particle and helion charge radius to 3×10^{-4} (0.0005 fm)
- aims:
  - help to solve the proton size puzzle
  - absolute charge radii of helion, alpha
  - low-energy effective nuclear models: ^{1}H, ^{2}D, ^{3}He, ^{4}He
  - QED test with He$^+$(1S-2S) [Udem @ MPQ, Eikema @ Amsterdam]
- identical muon beam
- similar laser, no Raman cell ($\rightarrow$ more pulse energy)
- similar, maybe better x-ray detectors (8.2 keV)
- event rate: 16-48 events per hour (not 6 per hour, μp)
- line with 300 GHz (1 nm!)

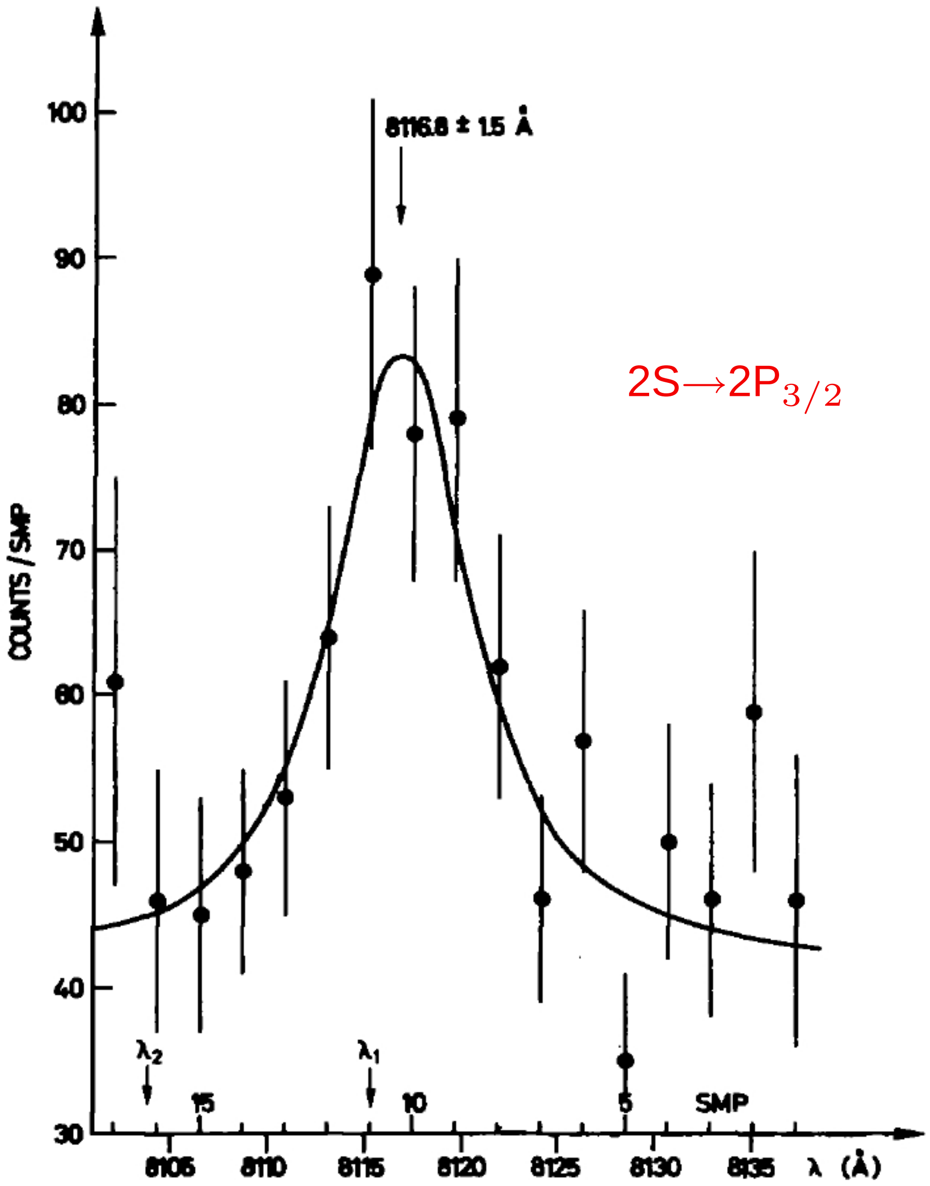
Outlook: Lamb shift in muonic helium

- CREMA collaboration: Charge Radius Experiment with Muonic Atoms
- Exp. R10-01 approved at PSI in Feb. 2010
- Goal: Measure ΔE(2S-2P) in $\mu\,^4$He, $\mu\,^3$He
- $\Rightarrow$ alpha particle and helion charge radius to 3×10^{-4} (0.0005 fm)
- aims:
  - help
  - abso
  - low-e … , ^{4}He
  - QED … na @ Amsterdam]
- identical
- similar laser, no Raman cell ($\longrightarrow$ more pulse energy)
- similar, maybe better x-ray detectors (8.2 keV)
- event rate: 16-48 events per hour (not 6 per hour, μp)
- line with 300 GHz (1 nm!)

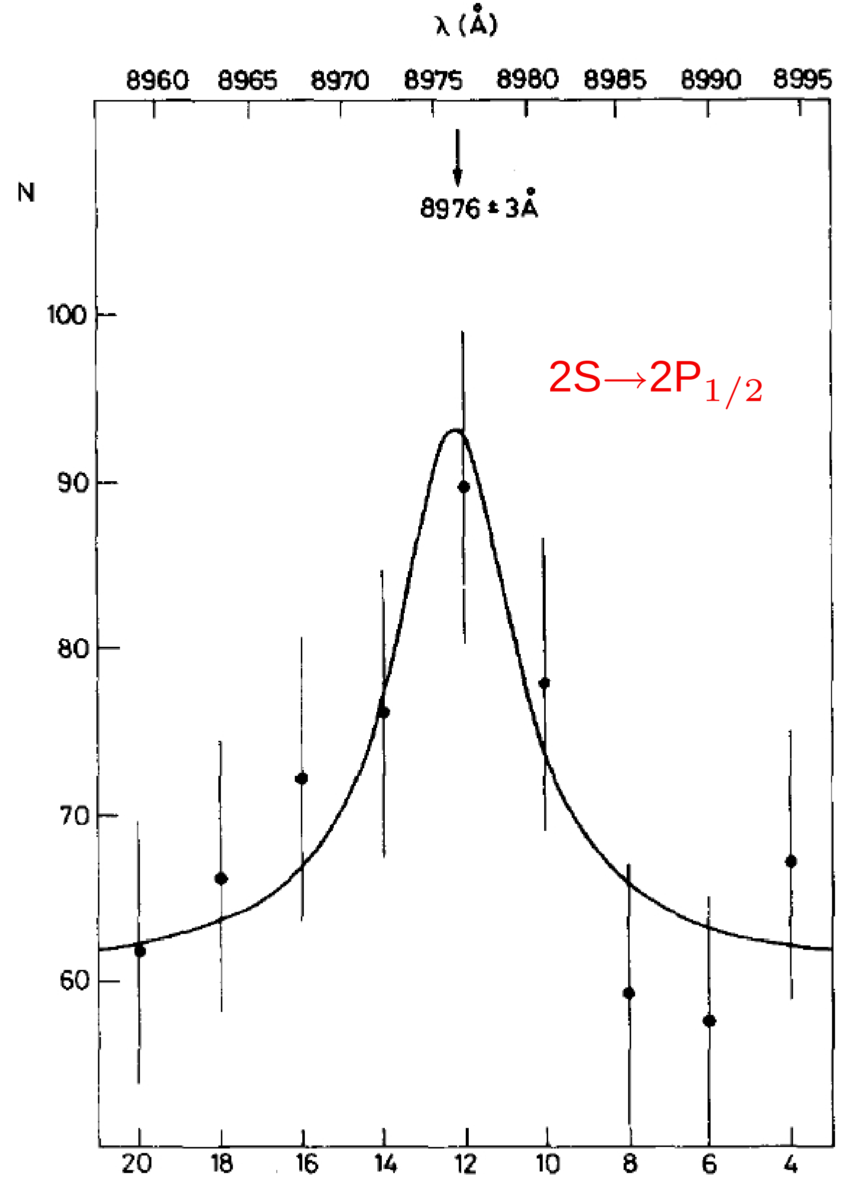
ERC Starting Grant for RP, 2011

1st beam time expected for 2013.

Old μHe$^+$ resonances



Carboni et al, Nucl. Phys. A273, 381 (1977)



Carboni et al, Phys. Lett. 73B, 229 (1978)

Proton Size Investigators thank you for your attention

Contributions to the μp Lamb shift

#	Contribution	Value	Unc.
3	Relativistic one loop VP	205.0282	
4	NR two-loop electron VP	1.5081	
5	Polarization insertion in two Coulomb lines	0.1509	
6	NR three-loop electron VP	0.00529	
7	Polarisation insertion in two and three Coulomb lines (corrected)	0.00223	
8	Three-loop VP (total, uncorrected)		
9	Wichmann-Kroll	-0.00103	
10	Light by light electron loop ((Virtual Delbrück)	0.00135	0.00135
11	Radiative photon and electron polarization in the Coulomb line $\alpha^2(Z\alpha)^4$	-0.00500	0.0010
12	Electron loop in the radiative photon of order $\alpha^2(Z\alpha)^4$	-0.00150	
13	Mixed electron and muon loops	0.00007	
14	Hadronic polarization $\alpha(Z\alpha)^4 m_r$	0.01077	0.00038
15	Hadronic polarization $\alpha(Z\alpha)^5 m_r$	0.000047	
16	Hadronic polarization in the radiative photon $\alpha^2(Z\alpha)^4 m_r$	-0.000015	
17	Recoil contribution	0.05750	
18	Recoil finite size	0.01300	0.001
19	Recoil correction to VP	-0.00410	
20	Radiative corrections of order $\alpha^n(Z\alpha)^k m_r$	-0.66770	
21	Muon Lamb shift 4th order	-0.00169	
22	Recoil corrections of order $\alpha(Z\alpha)^5 \frac{m}{M} m_r$	-0.04497	
23	Recoil of order α^6	0.00030	
24	Radiative recoil corrections of order $\alpha(Z\alpha)^n \frac{m}{M} m_r$	-0.00960	
25	Nuclear structure correction of order $(Z\alpha)^5$ (Proton polarizability)	0.015	0.004
26	Polarization operator induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$	0.00019	
27	Radiative photon induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$	-0.00001	
	Sum	206.0573	0.0045

Contributions to the μp Lamb shift

Contribution	our selection		Pachucki	Borie
Leading nuclear size contribution	-5.19745	$< r_p^2 >$	-5.1974	-5.1971
Radiative corrections to nuclear finite size effect	-0.0275	$< r_p^2 >$	-0.0282	-0.0273
Nuclear size correction of order $(Z\alpha)^6 < r_p^2 >$	-0.001243	$< r_p^2 >$		
Total $< r_p^2 >$ contribution	-5.22619	$< r_p^2 >$	-5.2256	-5.2244
Nuclear size correction of order $(Z\alpha)^5$	0.0347	$< r_p^3 >$	0.0363	0.0347
Nuclear size correction of order $(Z\alpha)^6 < r_p^4 >$	-0.000043	$< r_p^2 >^2$		

Contributions to the μp Lamb shift

Lamb shift: $\Delta E_{LS} = 206.0573(45) - 5.2262\, r_\mathrm{p}^2 + 0.0347\, r_\mathrm{p}^3$ meV

$u = 0.0045$ meV dominated by proton polarizability

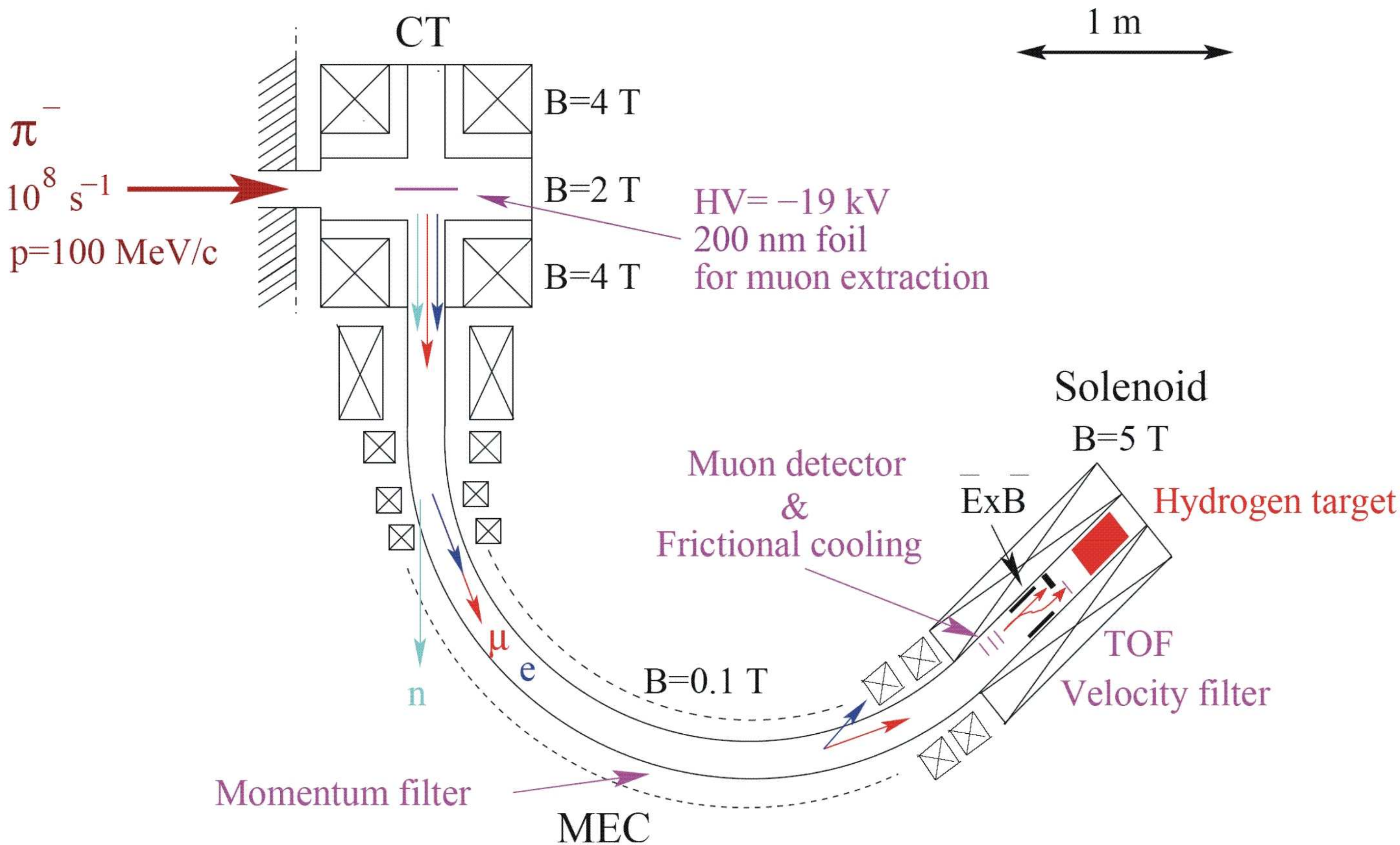
$2S$ Hyperfine structure: $\Delta E_{HFS}^{2S} = 22.8148\,(78)$ meV

using $R_Z = 1.022$ fm and scatter.

Fine structure: $\Delta E_{FS} = 8.352082$ meV

$2P_{3/2}$ Hyperfine structure: $\Delta E_{HFS}^{2P_{3/2}} = 3.392588$ meV

Muon beam line



r_p from electron scattering

PhD thesis J.C. Bernauer

- Rosenbluth cross section $\rightarrow$ Sachs form factor $\rightarrow r_\mathrm{p}$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Ros.}} = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} \frac{\varepsilon {G_E}^2 + \tau {G_M}^2}{\varepsilon(1+\tau)} \qquad \varepsilon = \left(1 + 2(1+\tau)\tan^2\frac{\theta}{2}\right)^{-1} \quad ; \quad \tau = \frac{Q^2}{4m_p^2}$$

G_E and G_M are the Fourier transforms of the charge and magnetization distributions

$G_E(0) = 1$ (charge), and $G_M(0) = \mu_p$ (magnetic moment)

$$\langle r_\mathrm{p}^2 \rangle = -6\hbar^2 \frac{dG_E(Q^2)}{dQ^2}\Big|_{\mathrm{Q}^2=0}$$ $\Rightarrow$ rms charge radius = slope of G_E at $Q^2 = 0$

extrapolation to $Q^2 \rightarrow 0$ required

$$Q^2[\ (\mathrm{GeV/c})^2] = \begin{cases} 6 \cdot 10^{-7} & (\mu\text{p}) \\ > 2 \cdot 10^{-2} & (\text{e-p scatt.}) \end{cases}$$

r_{p} from electron scattering

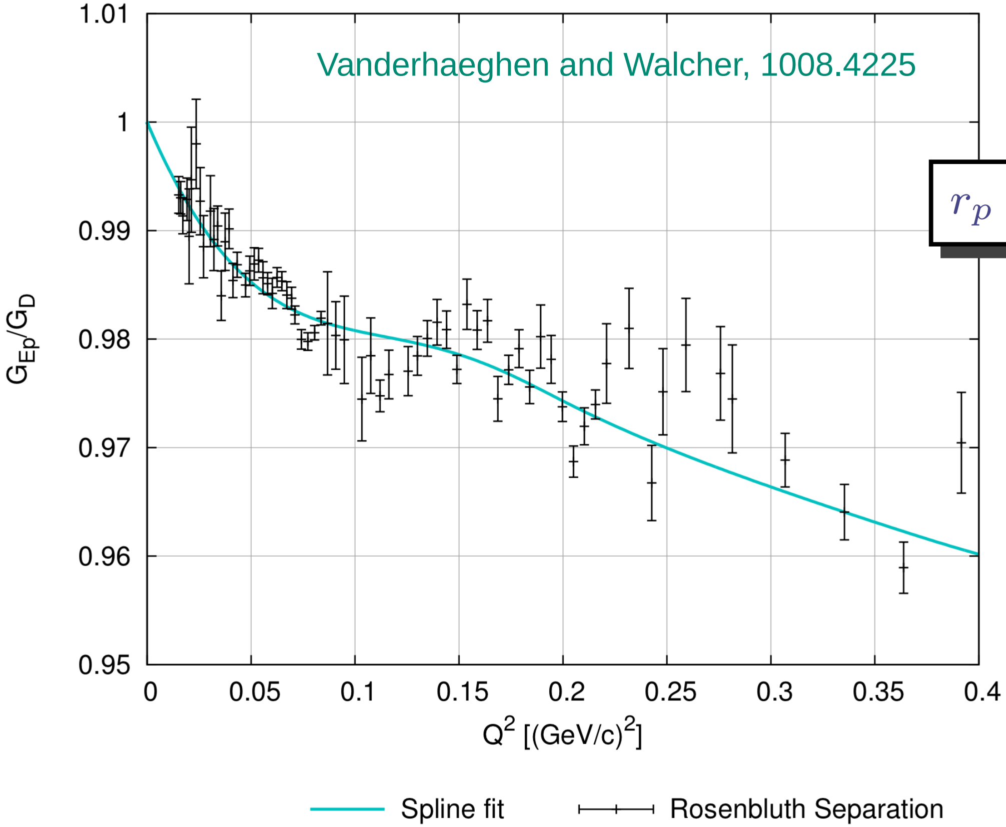
- Extrapolation non-trivial
  - Presence of “bump/dip” structure at lower Q^2 ?
  - Model assumption of the functional behavior of the form factor?
  - Normalization problems. Fitting with $G_E(Q^2 = 0) = 1 \rightarrow$ uncertainty underestimated.

r_{p} from new scattering data with 1% accuracy. Is that realistic?

r_p from electron scattering

- Extrapolation non-trivial
  - Presence of "bump/dip" structure at lower Q^2 ?
  - Model assumption of the functional behavior of the form factor?
  - Normalization problems. Fitting with $G_E(Q^2 = 0) = 1 \rightarrow$ uncertainty underestimated.

New Mainz data: G_E/G_{dipole} vs. $Q^2 \Leftarrow$ world's most accurate data at low Q^2



$$r_p = (0.879 \pm 0.005_{\mathrm{stat}} \pm 0.004_{\mathrm{syst}} \pm 0.005_{\mathrm{model}})\ \mathrm{fm}$$

Bernauer *et al.*, PRL **105**, 242001 (2010)

r_p from electron scattering

- Bernauer *et al.*, PRL **105**, 242001 (2010)

$$r_p = (0.879 \pm 0.005_{\rm stat} \pm 0.004_{\rm syst} \pm 0.005_{\rm model})\ \text{fm}$$

- Bernauer, PhD thesis (2010)

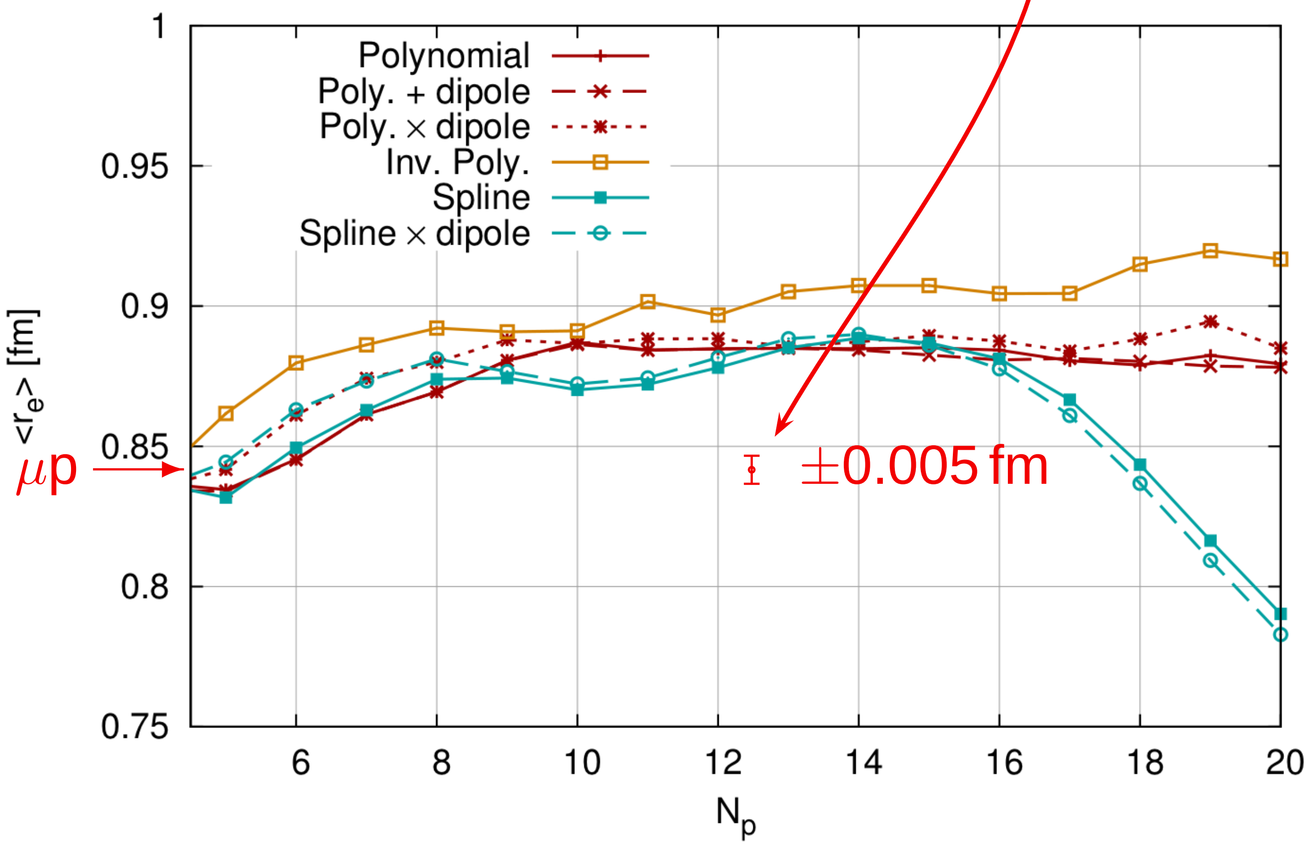


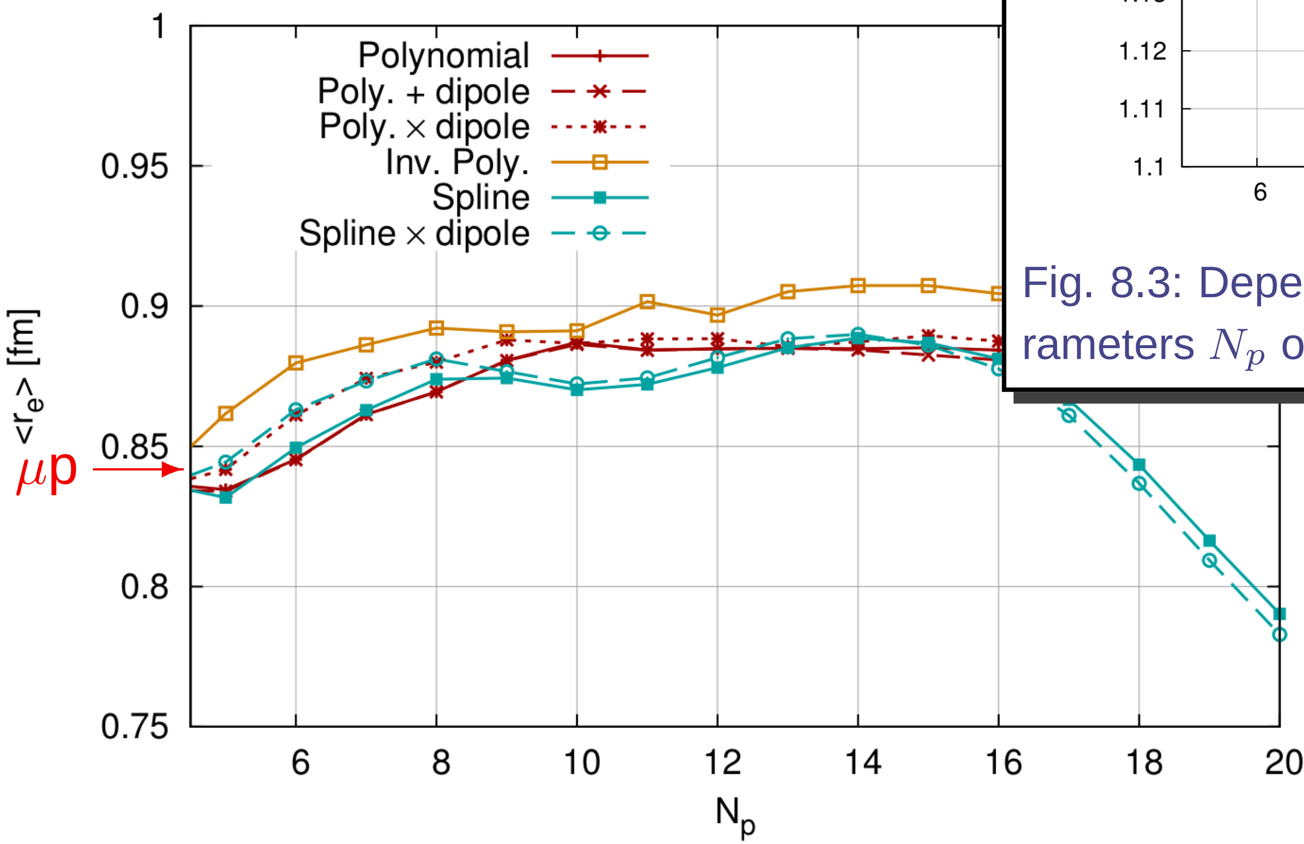
Fig. 9.21 (top):
The dependence of the extracted electric radius of the different flexible models on the number of parameters.

r_p from electron scattering

- Bernauer *et al.*, PRL **105**, 242001 (201

$$r_p = (0.879 \pm 0.005_{\rm stat} \pm 0.004_{\rm syst} \pm$$

- Bernauer, PhD thesis (2010)



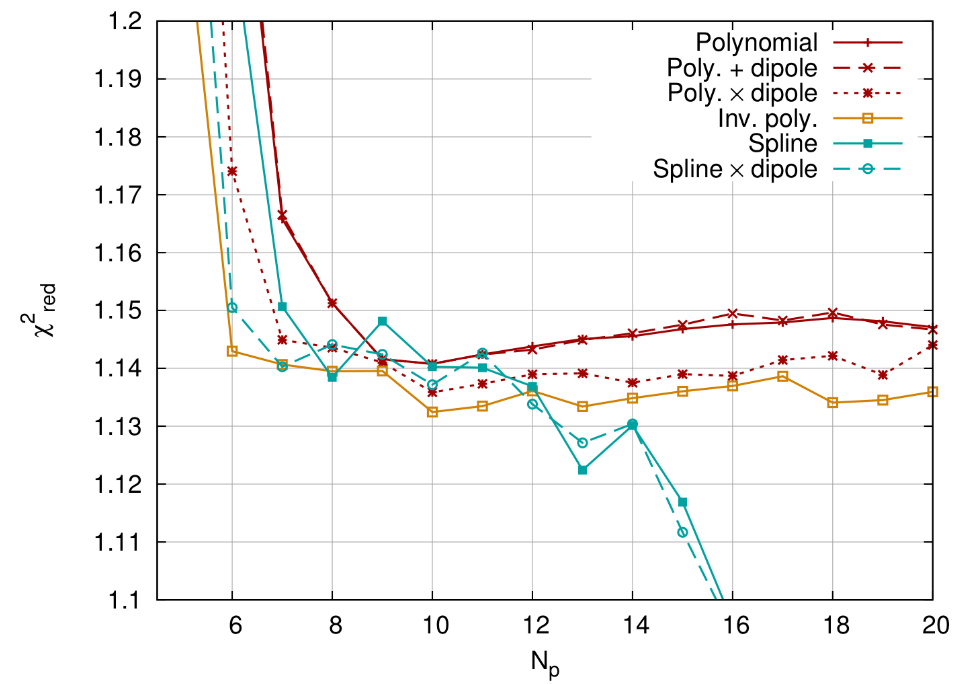
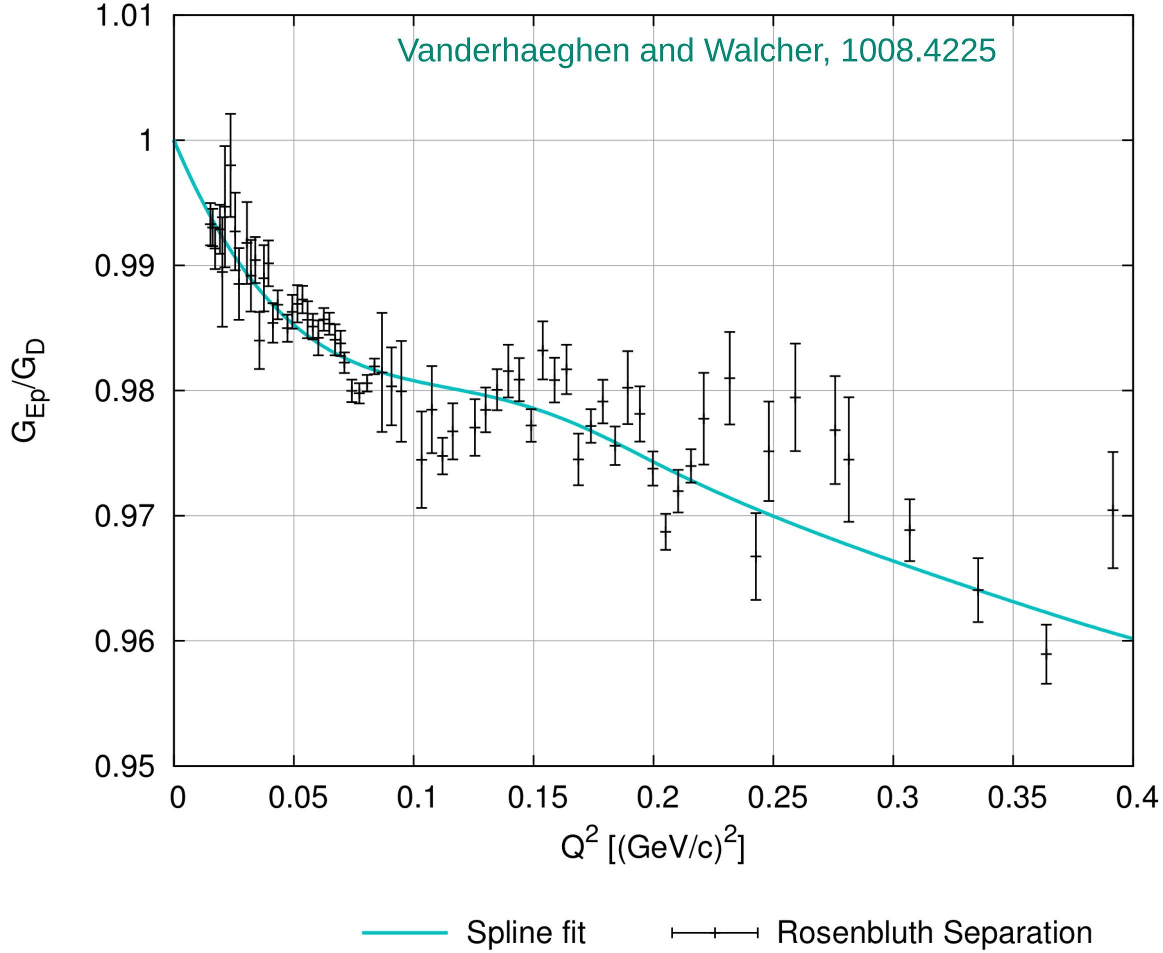


Fig. 8.3: Dependence of $\chi^2_{\rm red}$ on the number of parameters N_p of the form factor parametrizations...

Mainz scattering data at lowest Q^2

$$\langle r_p^2 \rangle = -6\hbar^2 \frac{dG_E(Q^2)}{dQ^2}\Big|_{Q^2=0}$$ $\Rightarrow$ rms charge radius = slope of G_E at $Q^2 = 0$

New Mainz data: G_E/G_{dipole} vs. Q^2



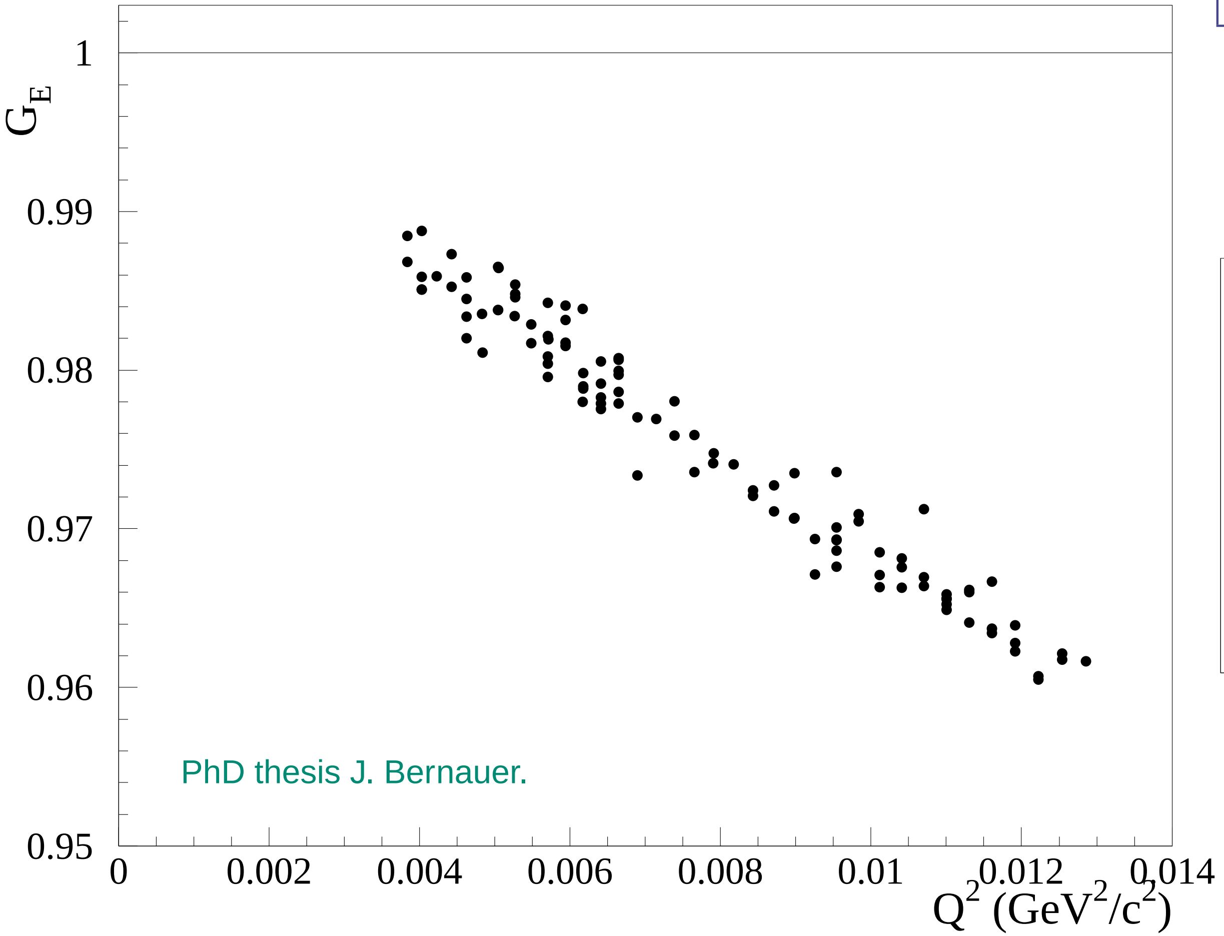
extrapolation to $Q^2 \to 0$ required

Mainz scattering data at lowest Q^2

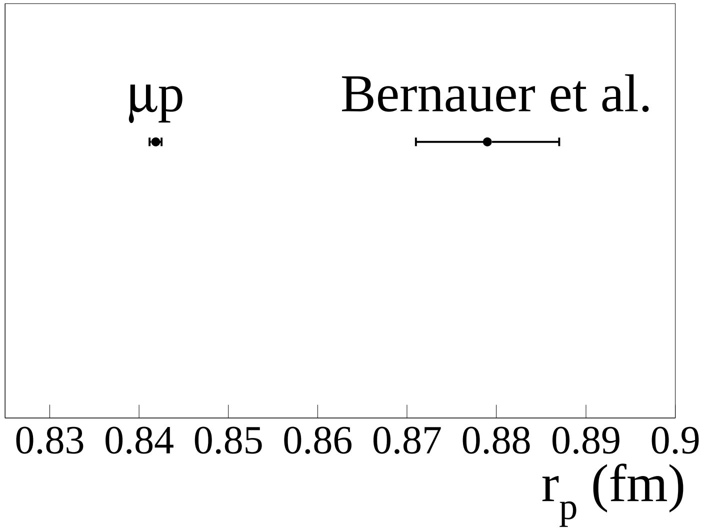
$$\langle r_p^2 \rangle = -6\hbar^2 \frac{dG_E(Q^2)}{dQ^2}\Big|_{Q^2=0}$$

$\Rightarrow$ rms charge radius = slope of G_E at $Q^2 = 0$

Lower half of 180MeV data



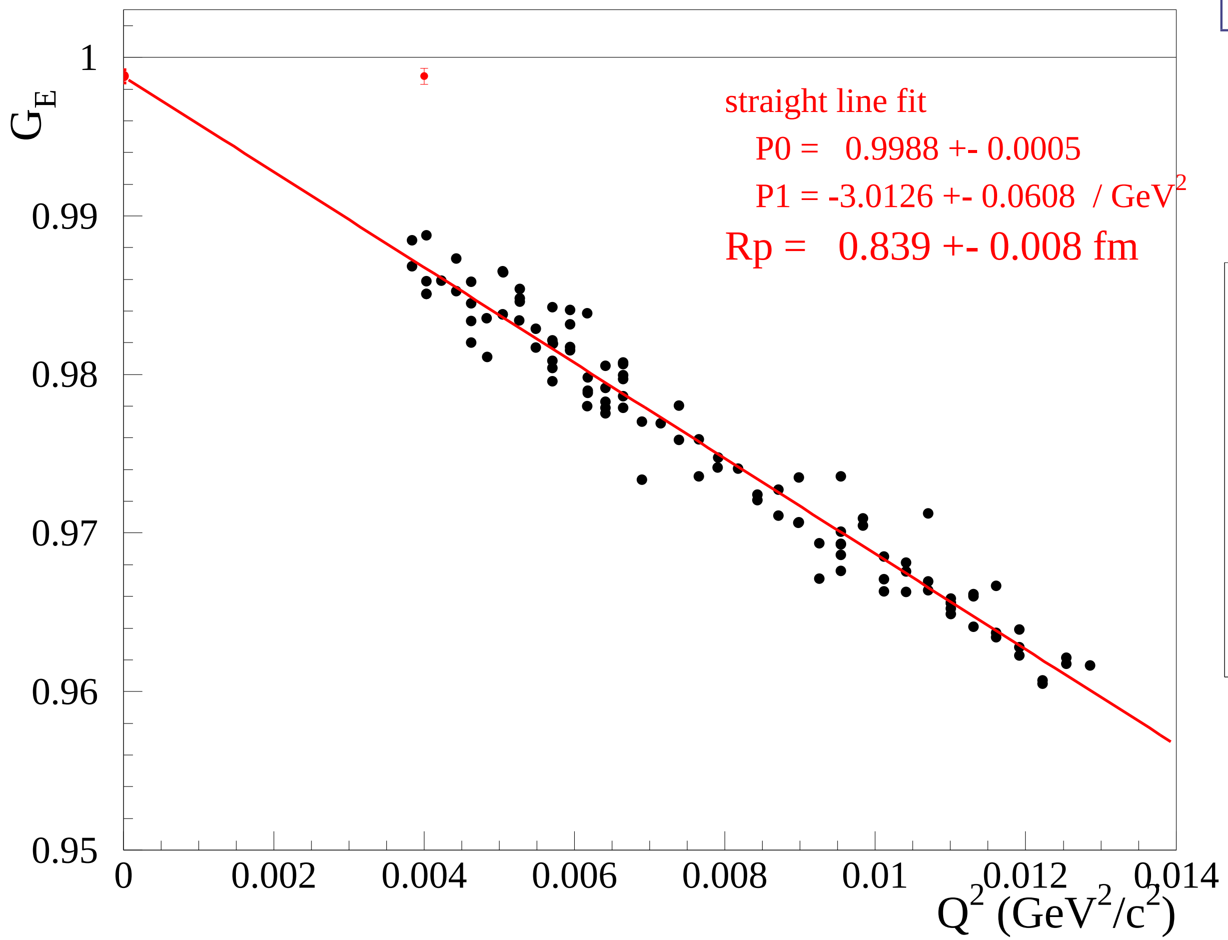
extrapolation to $Q^2 \rightarrow 0$ required



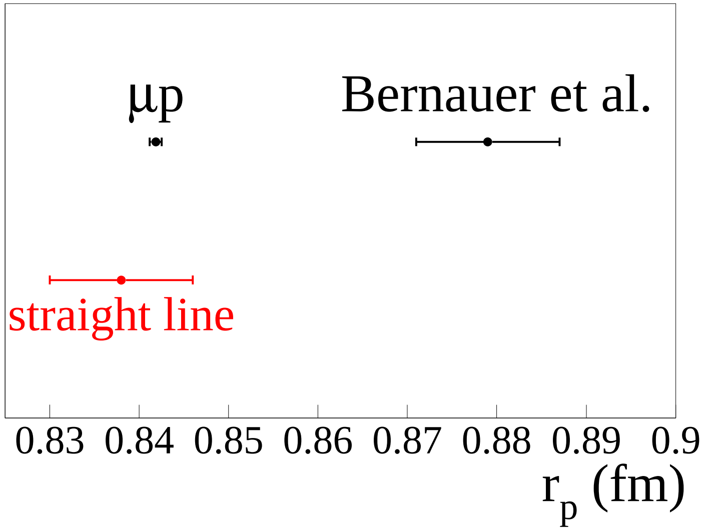
Mainz scattering data at lowest Q^2

$$\langle r_p^2 \rangle = -6\hbar^2 \frac{dG_E(Q^2)}{dQ^2}\Big|_{Q^2=0} \quad \Rightarrow \quad \text{rms charge radius} = \boxed{\text{slope of } G_E \text{ at } Q^2 = 0}$$

Fitting a straight line



extrapolation to $Q^2 \rightarrow 0$ required

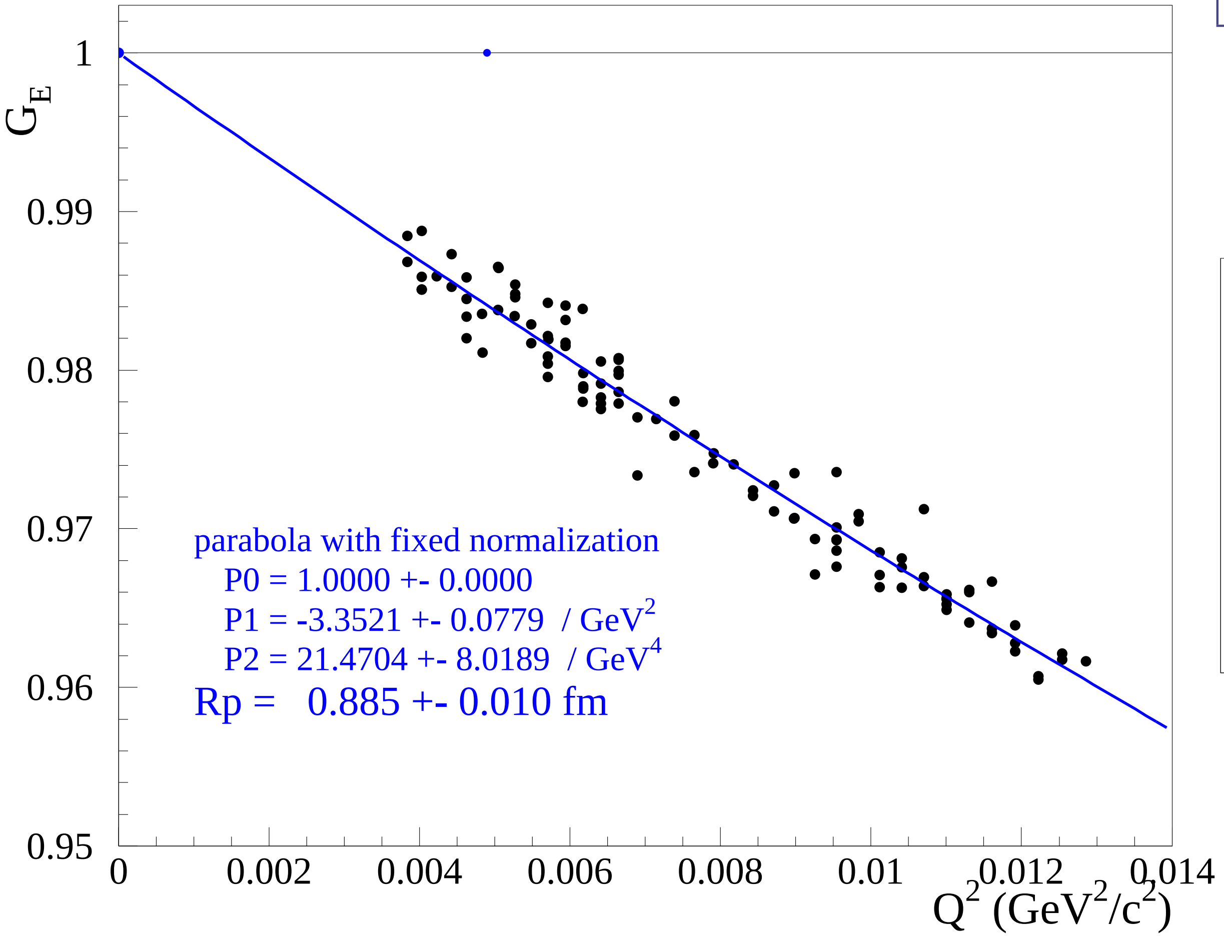


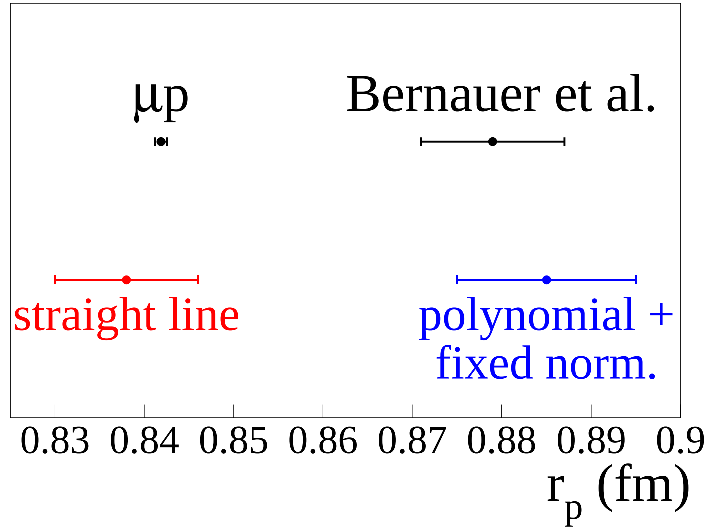
Mainz scattering data at lowest Q^2

$$\langle r_p^2 \rangle = -6\hbar^2 \frac{dG_E(Q^2)}{dQ^2}\bigg|_{Q^2=0}$$ $\Rightarrow$ rms charge radius = slope of G_E at $Q^2 = 0$

Polynomial, $G_E(0) = 1$

extrapolation to $Q^2 \rightarrow 0$ required





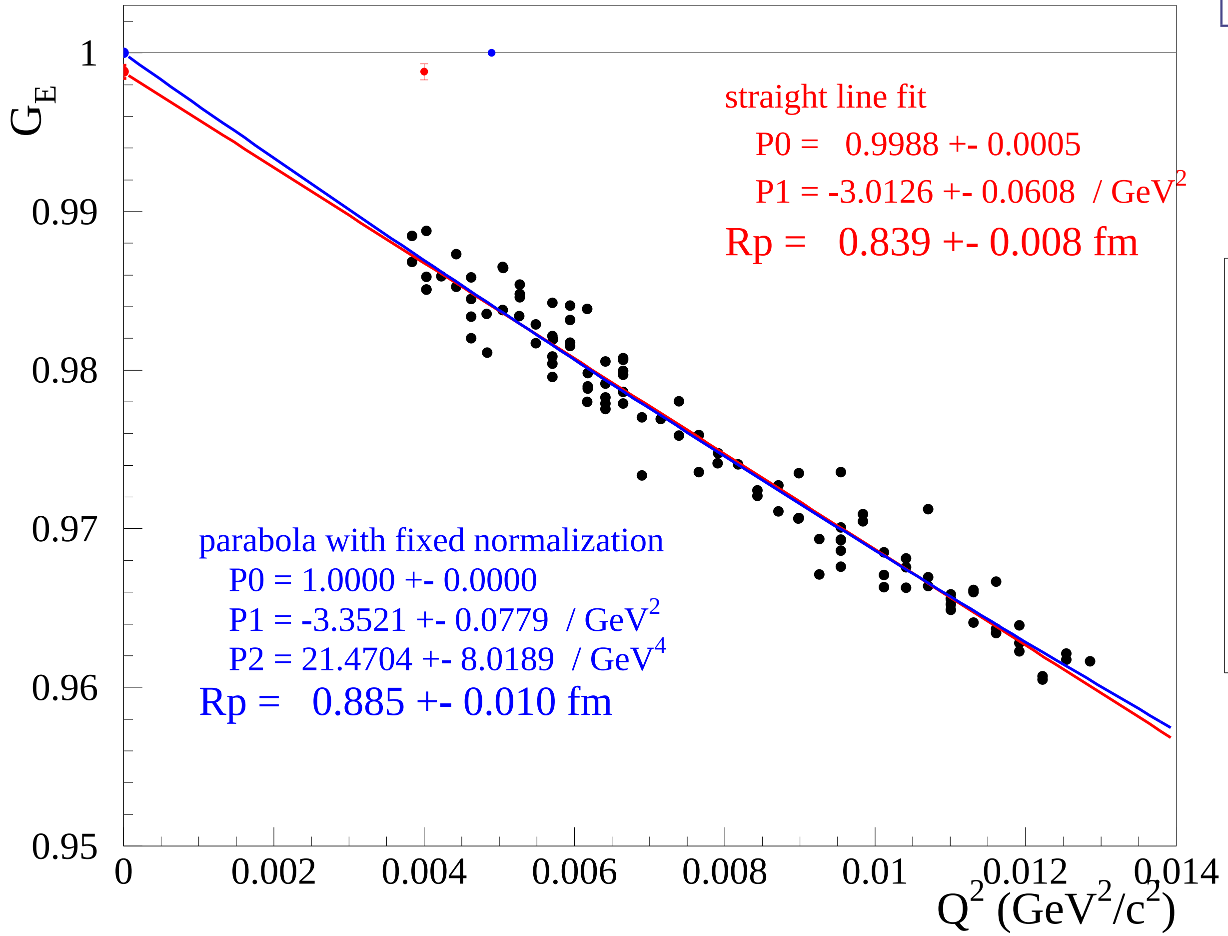
Mainz scattering data at lowest Q^2

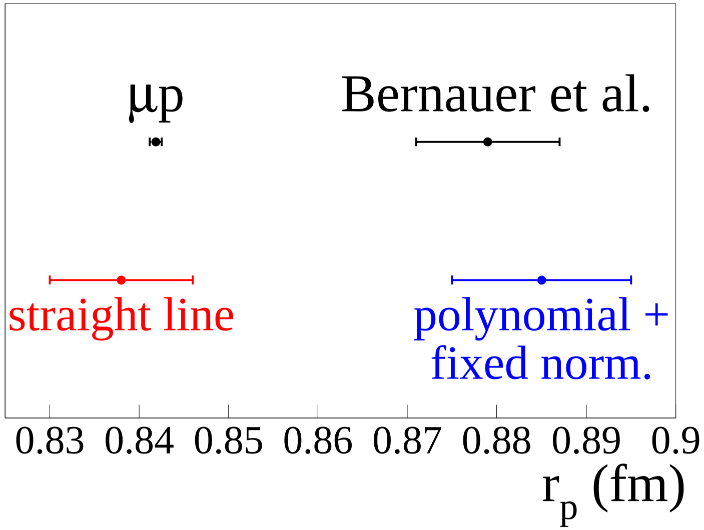
$$\langle r_p^2 \rangle = -6\hbar^2 \frac{dG_E(Q^2)}{dQ^2}\Big|_{Q^2=0}$$

$\Rightarrow$ rms charge radius = slope of G_E at $Q^2 = 0$

Extrapolation is subtle

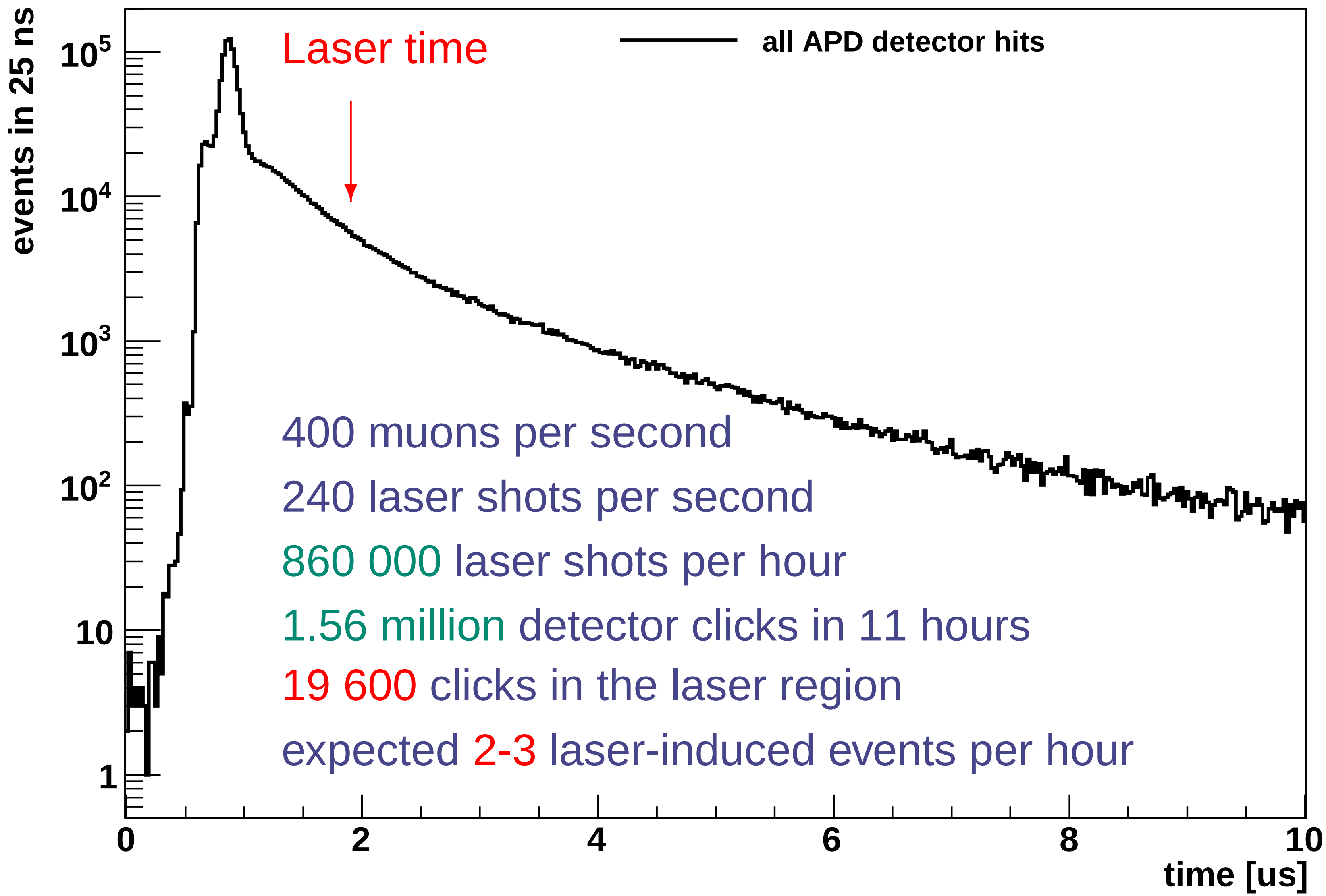
extrapolation to $Q^2 \to 0$ required





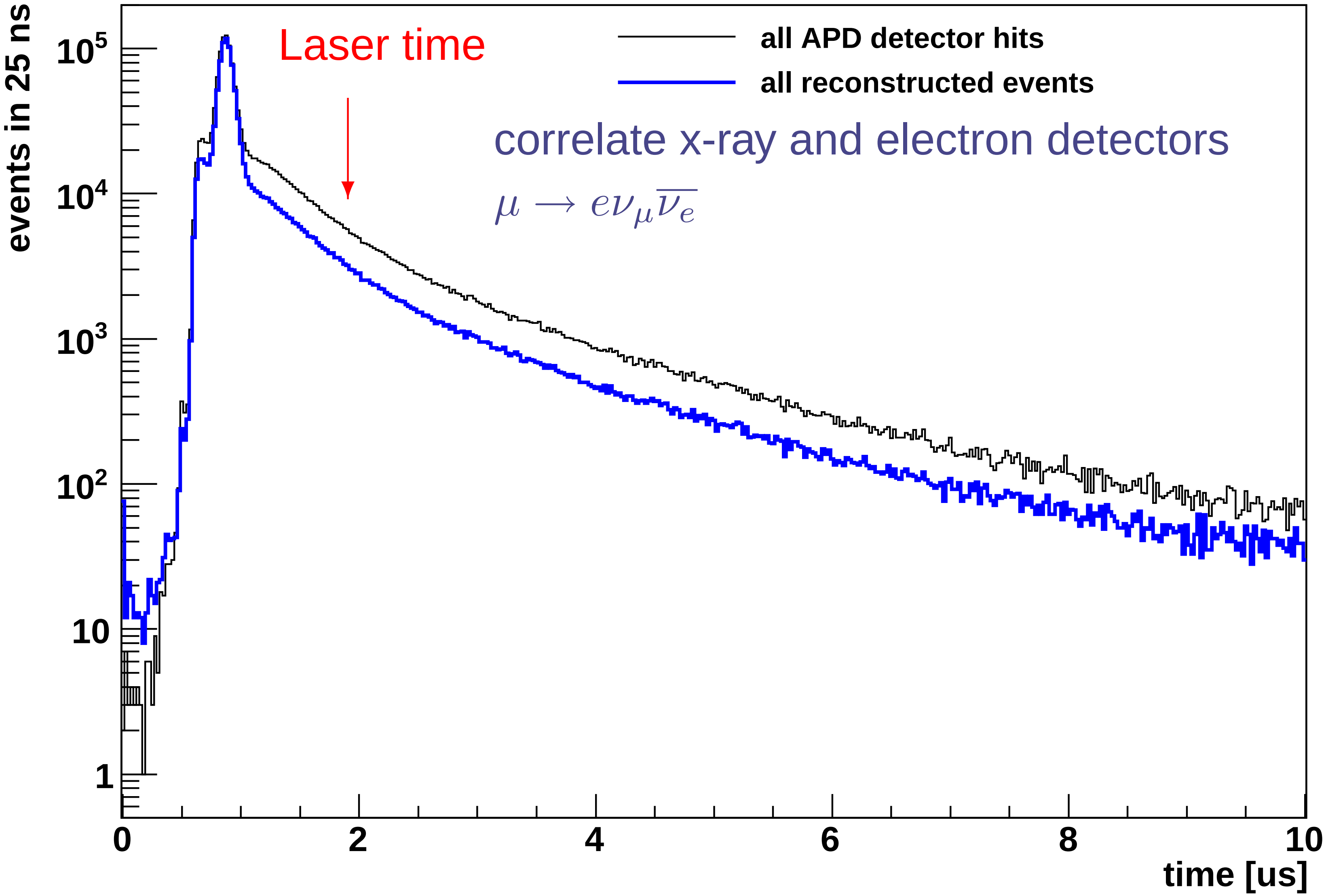
Data analysis

FP 900, 11 hours measurement



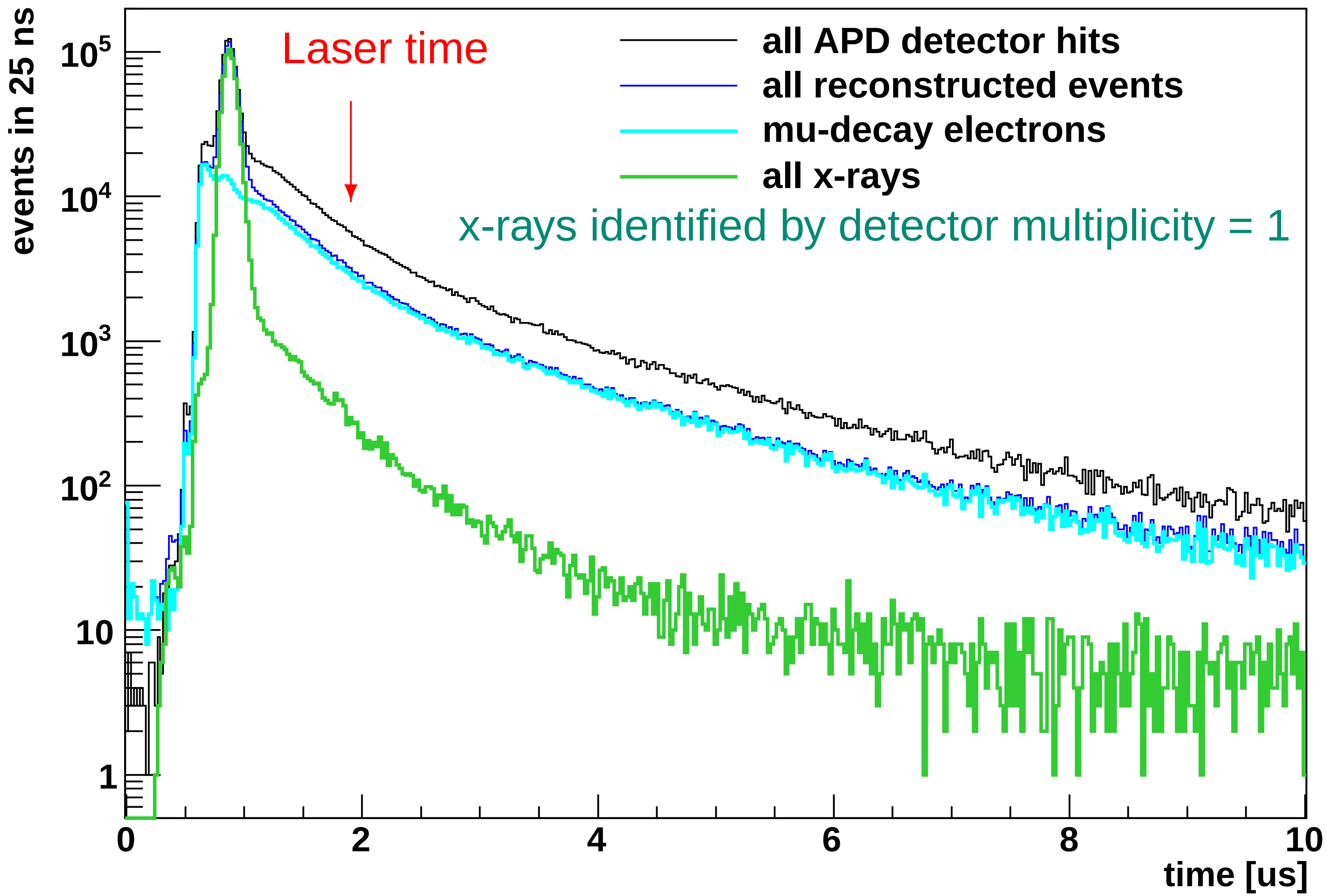
Data analysis

FP 900, 11 hours measurement



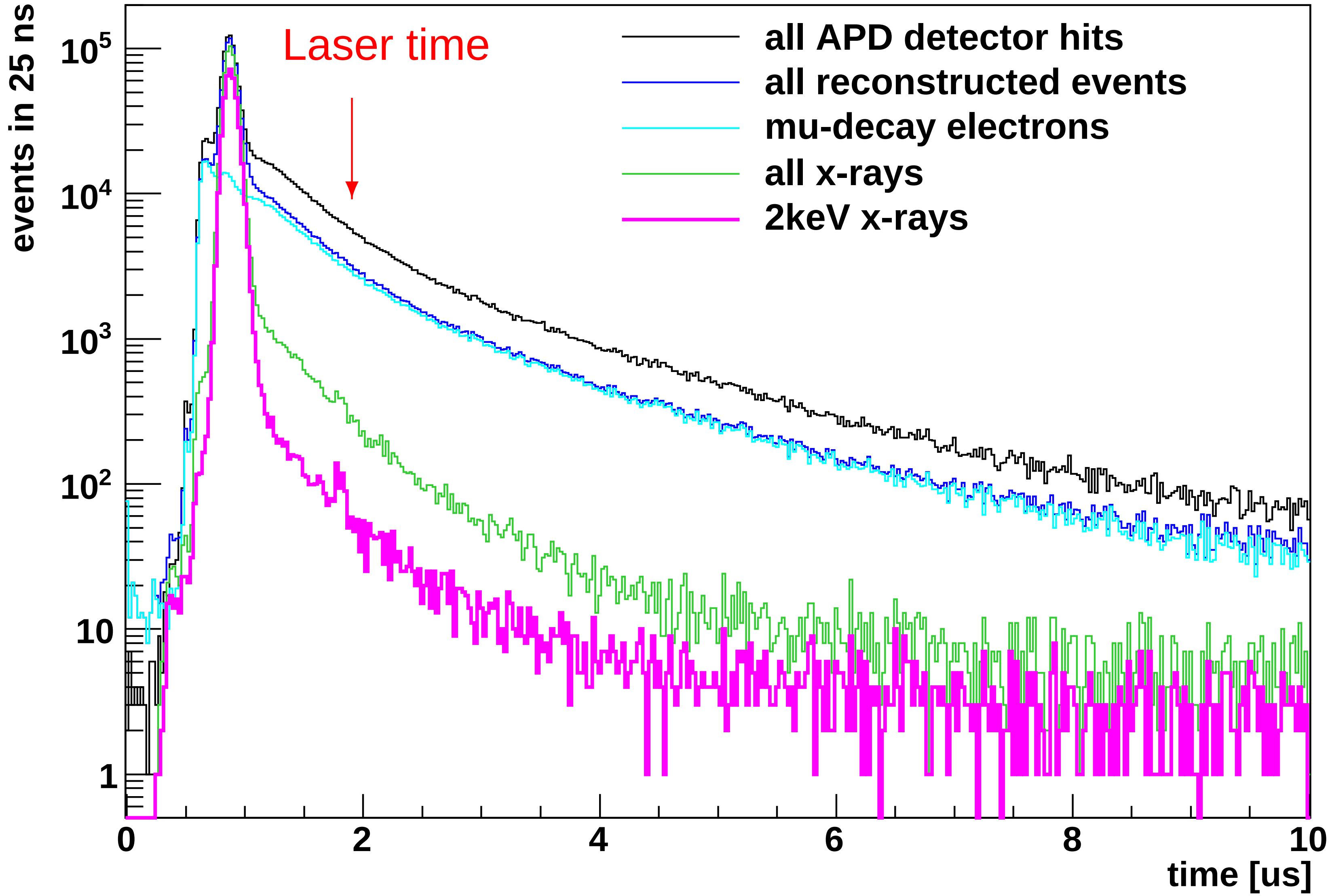
Data analysis

FP 900, 11 hours measurement



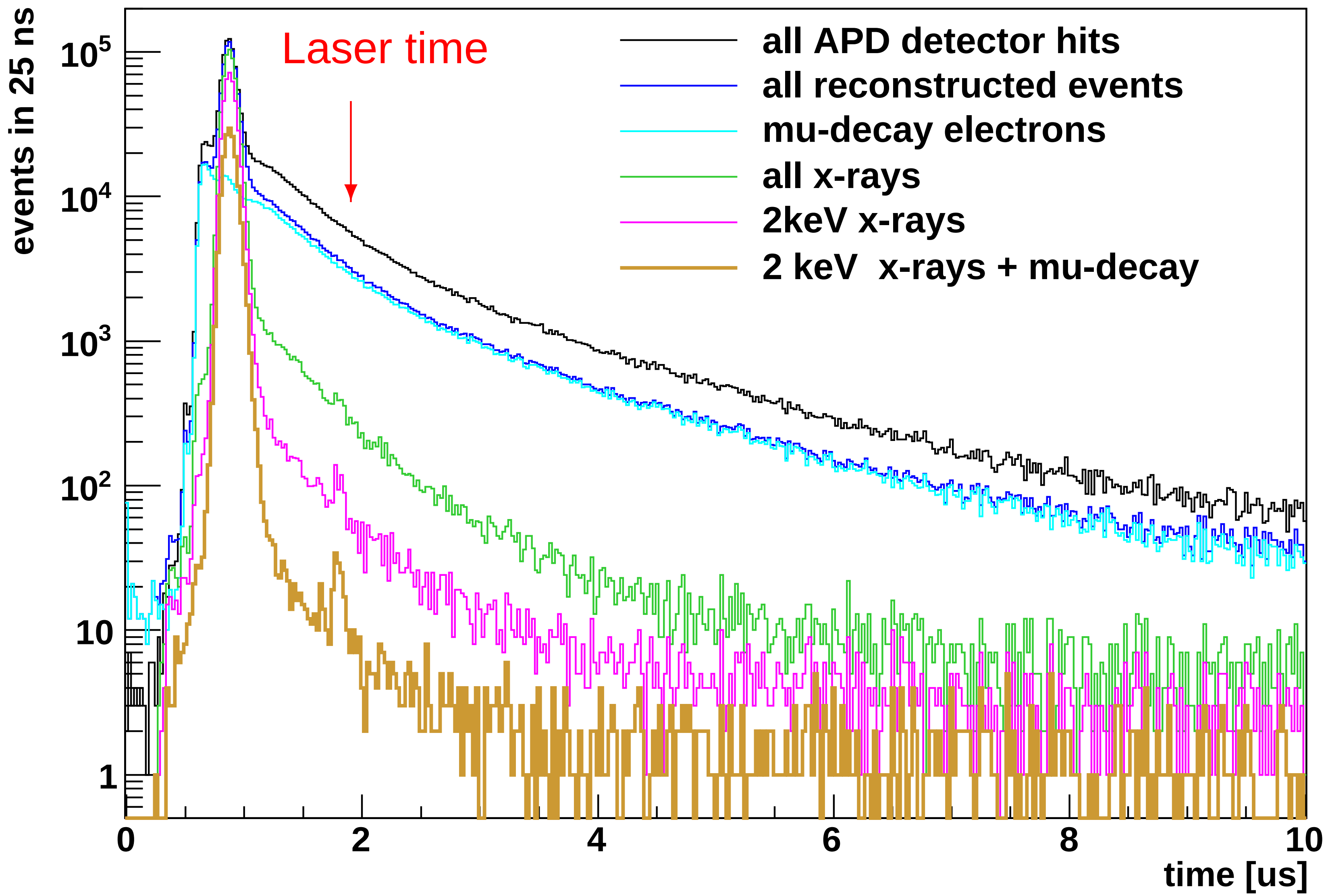
Data analysis

FP 900, 11 hours measurement



Data analysis

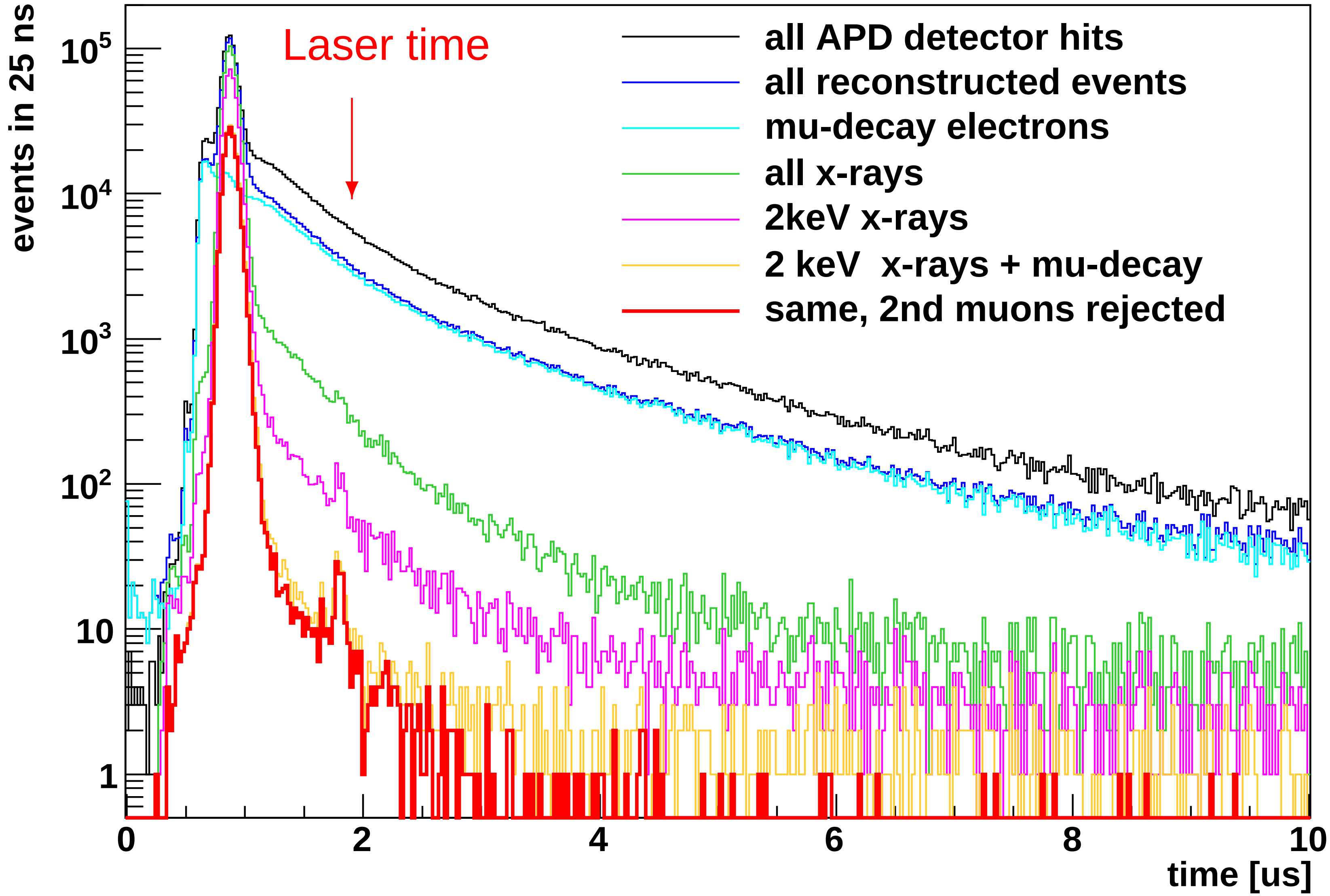
FP 900, 11 hours measurement



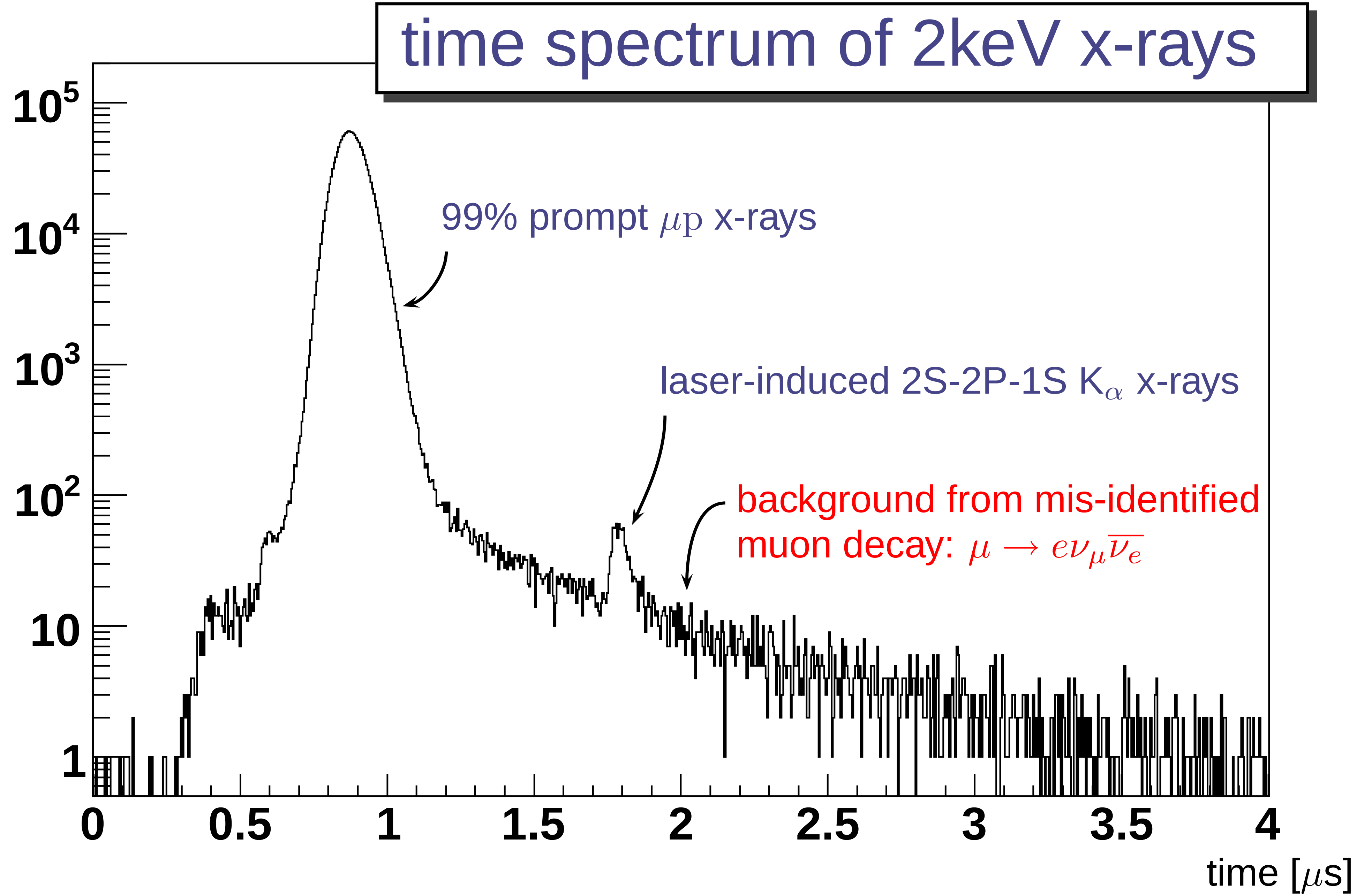
Data analysis

FP 900, 11 hours measurement

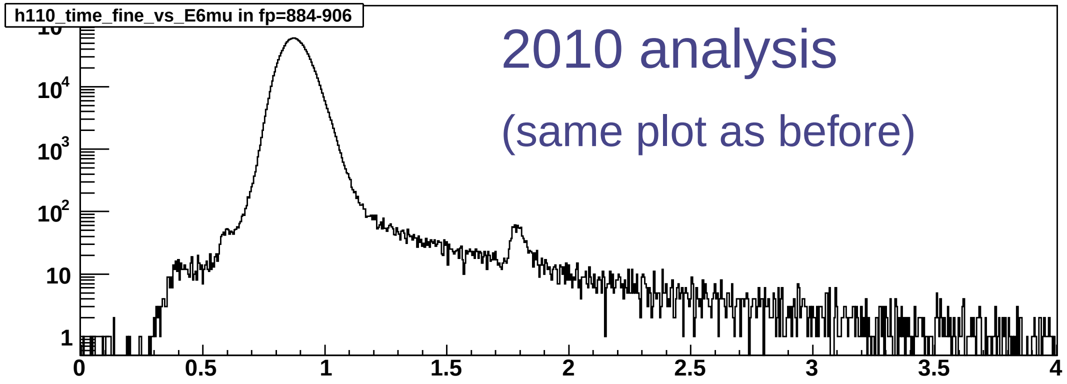
6 signal events per hour on resonance!
1 bgr. event/hour



Data analysis 2010



Data analysis 2011



golden event class:

2 keV x-ray , followed by a μ-decay electron

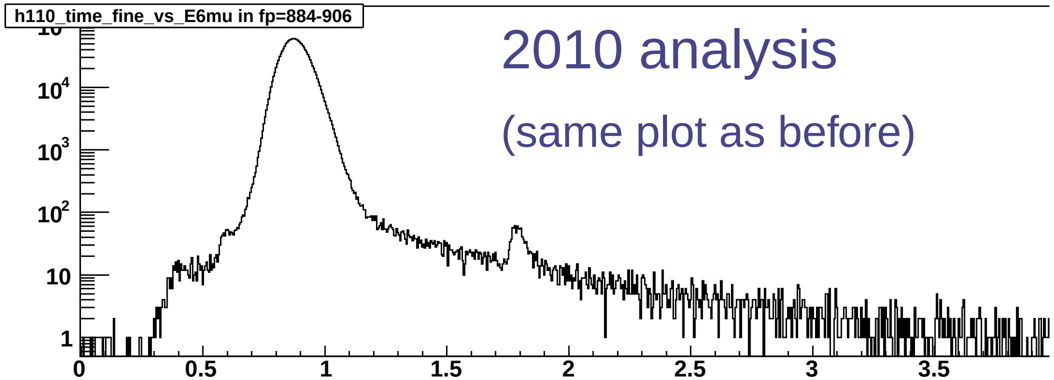
e.g. [0.2 . . . 5.2] μs after the x-ray

several ways to detect μ decay electrons

statistics vs. quality

Divide above data into....

Data analysis 2011



2010 analysis

(same plot as before)

golden event class:

2 keV x-ray , followed by a μ-decay ele

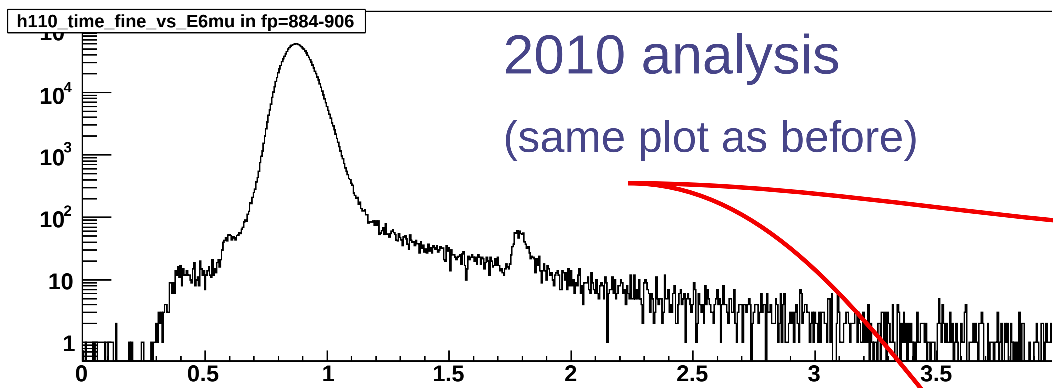
e.g. [0.2 . . . 5.2] μs after the x-ray

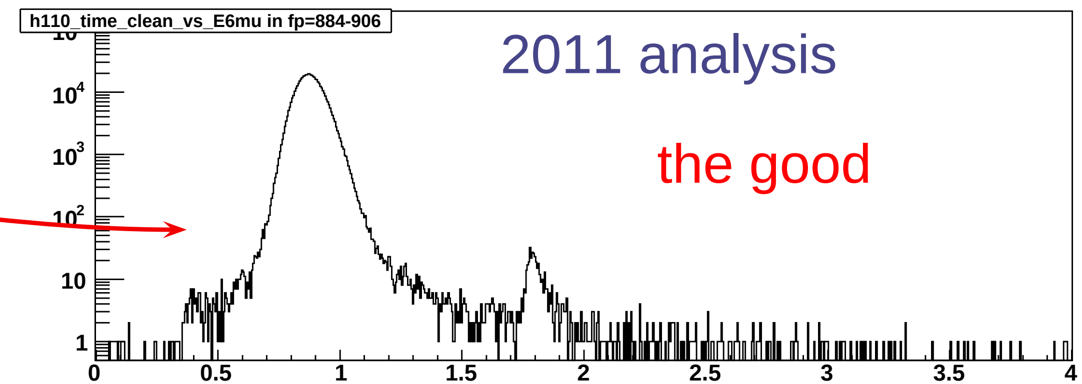
several ways to detect μ decay electror

statistics vs. quality

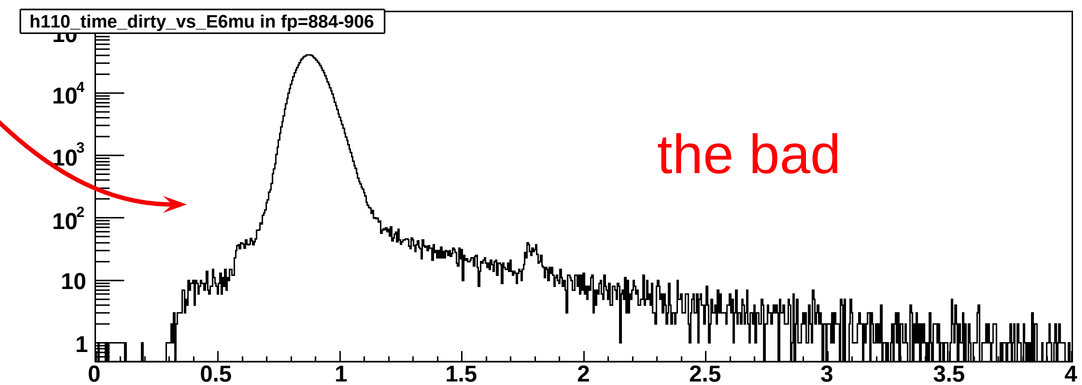
Divide above data into

Data analysis 2011

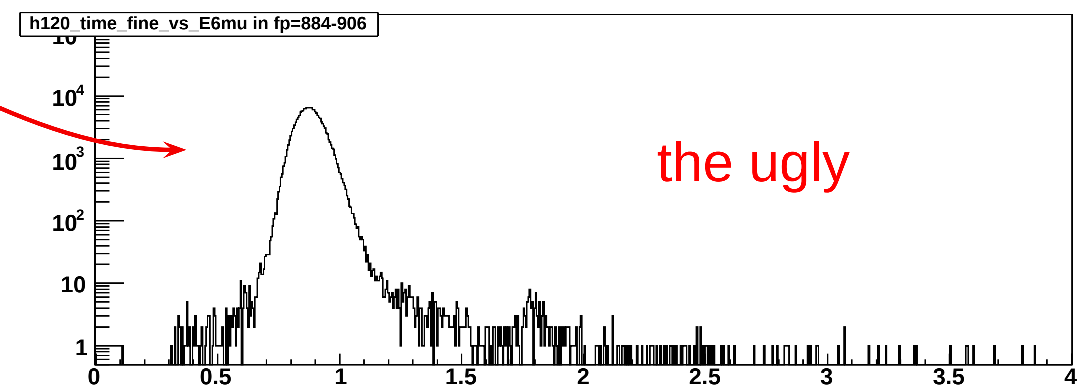




split “2010 data” (Nature)



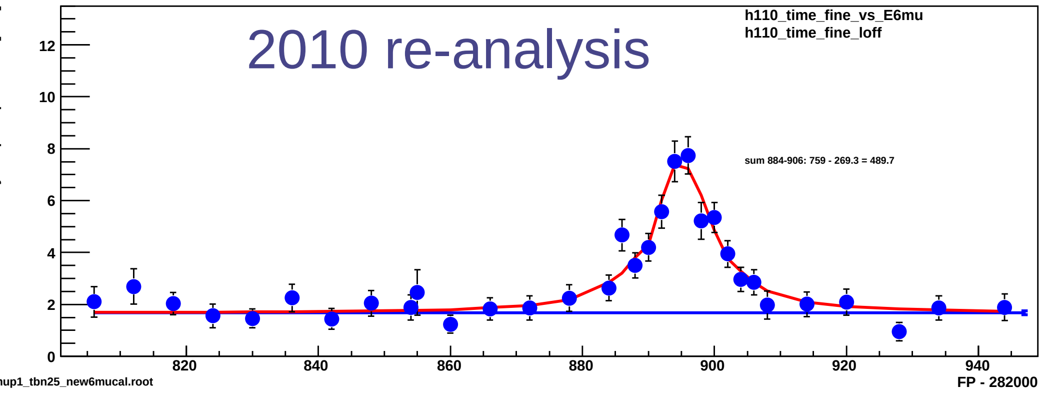
add previously discarded data



fit 3 resonances simultaneously

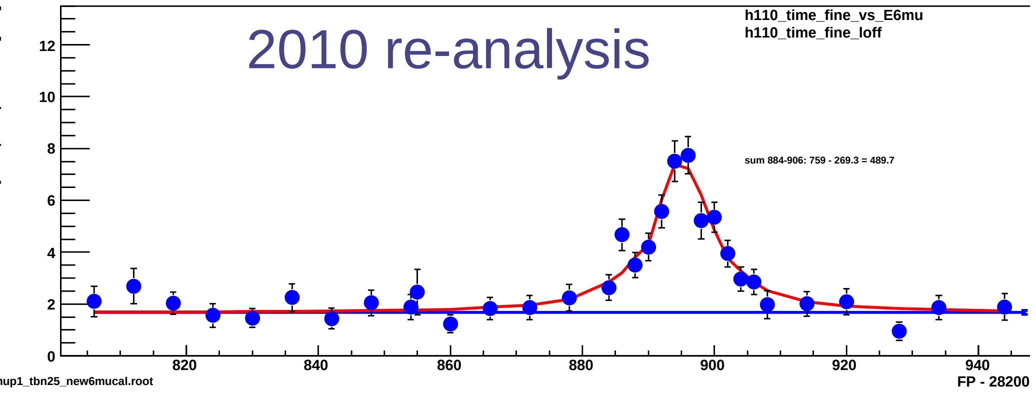
common position and width

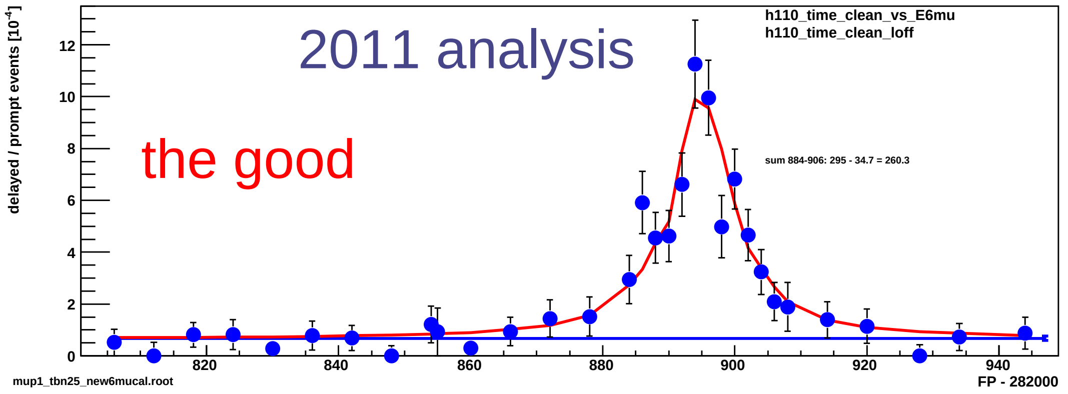
Data analysis 2011



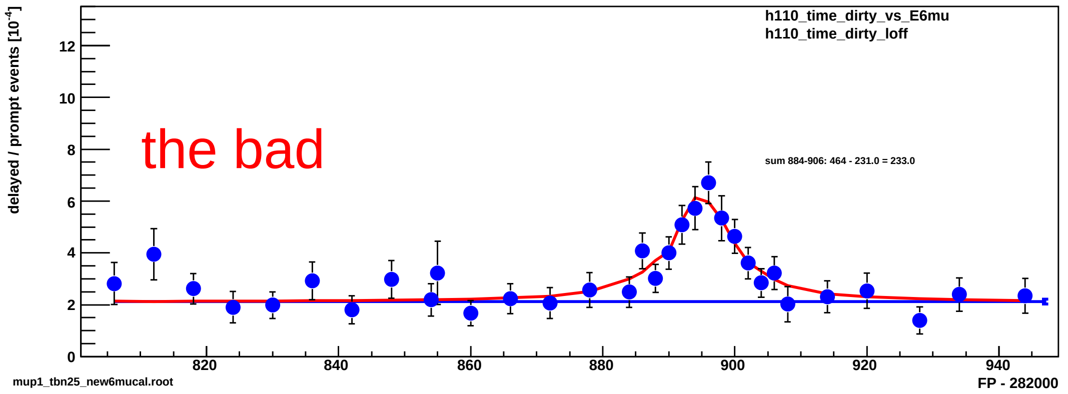
resonance plots

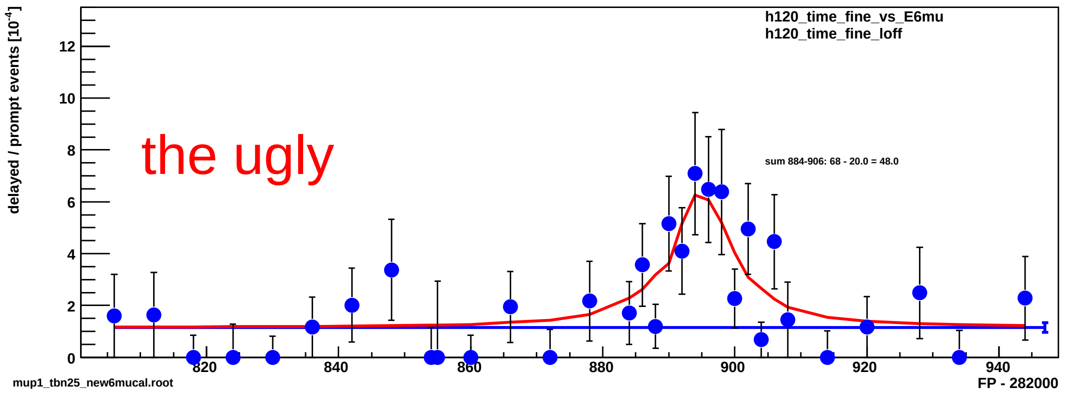
Data analysis 2011



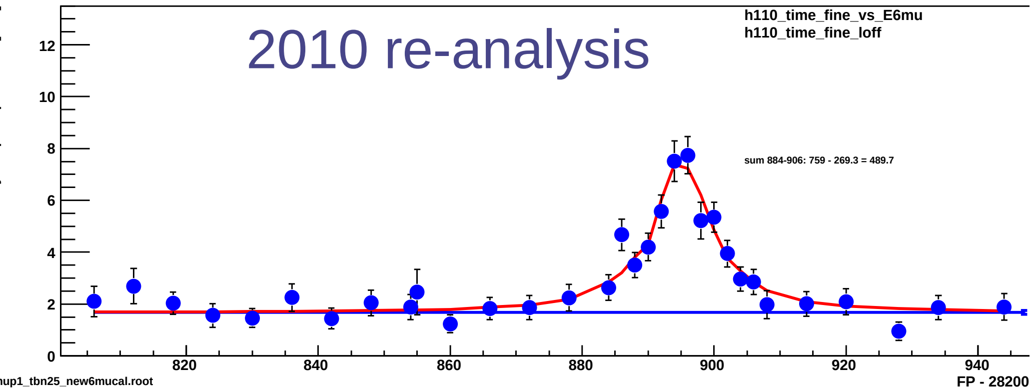


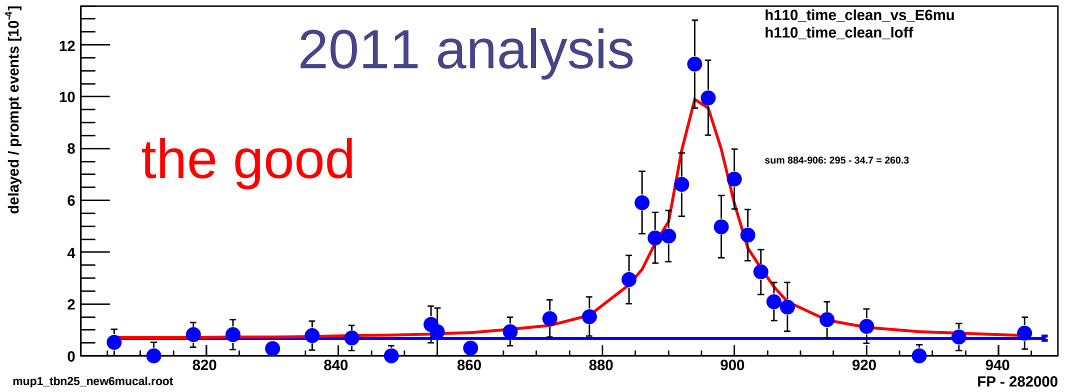
resonance plots

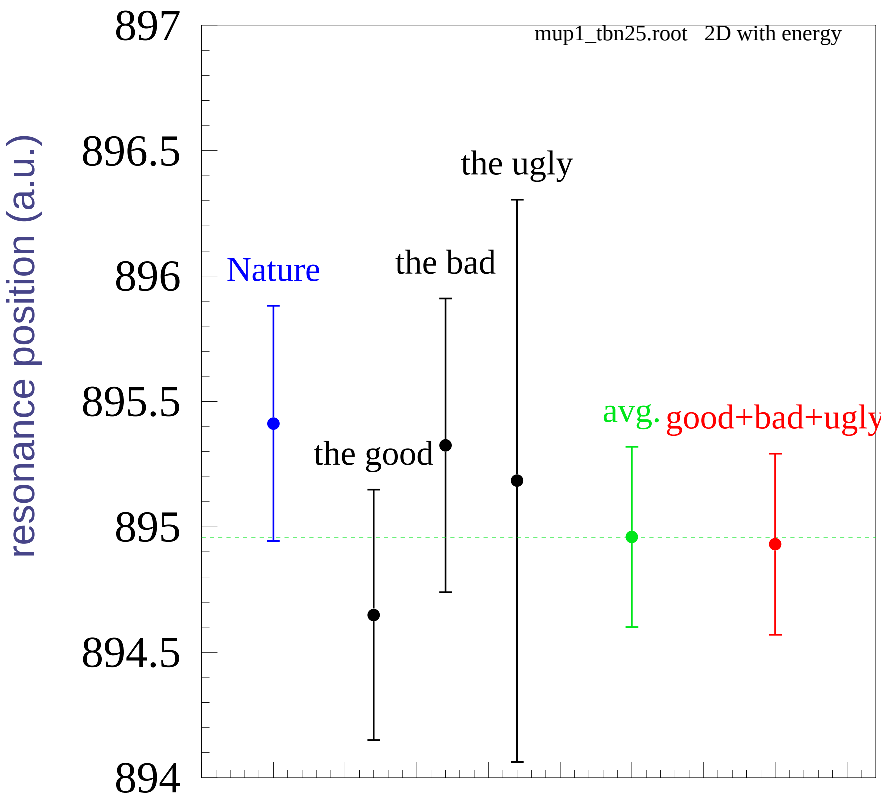


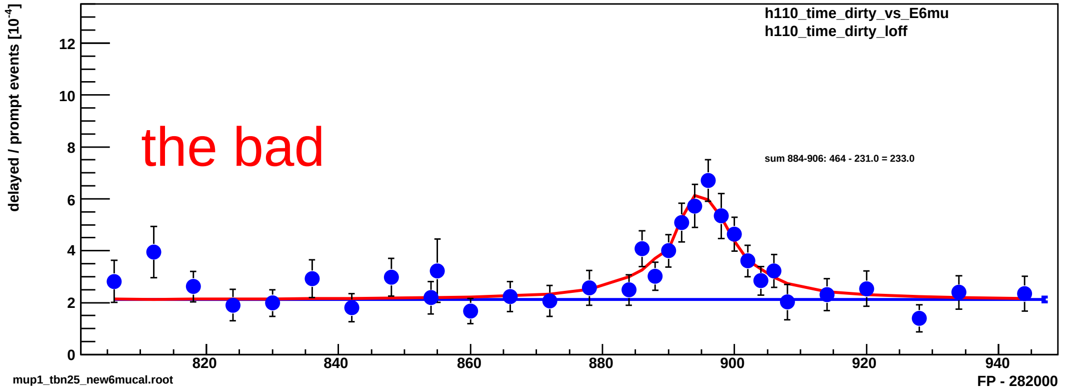


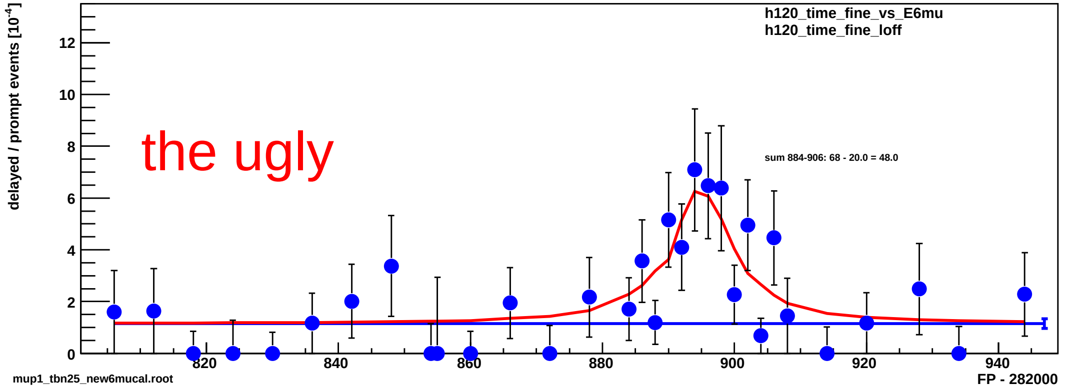
Data analysis 2011









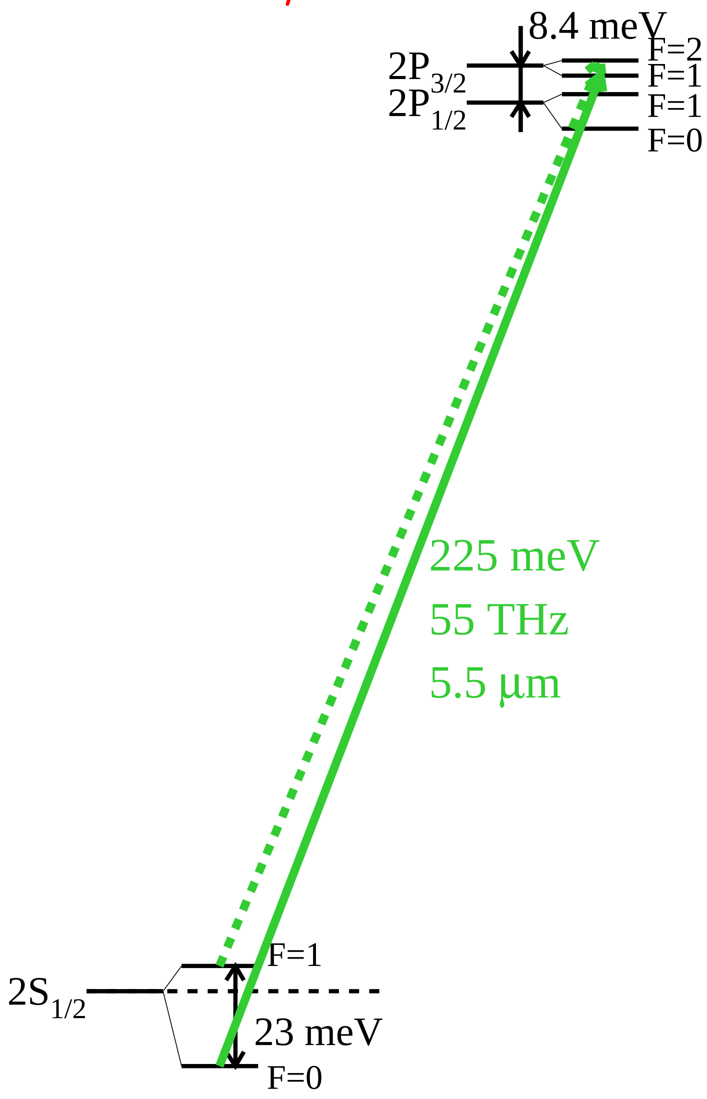


New data

2nd line in muonic hydrogen

μp ($2S_{1/2}$(F=0) $\rightarrow$ $2P_{3/2}$(F=1))

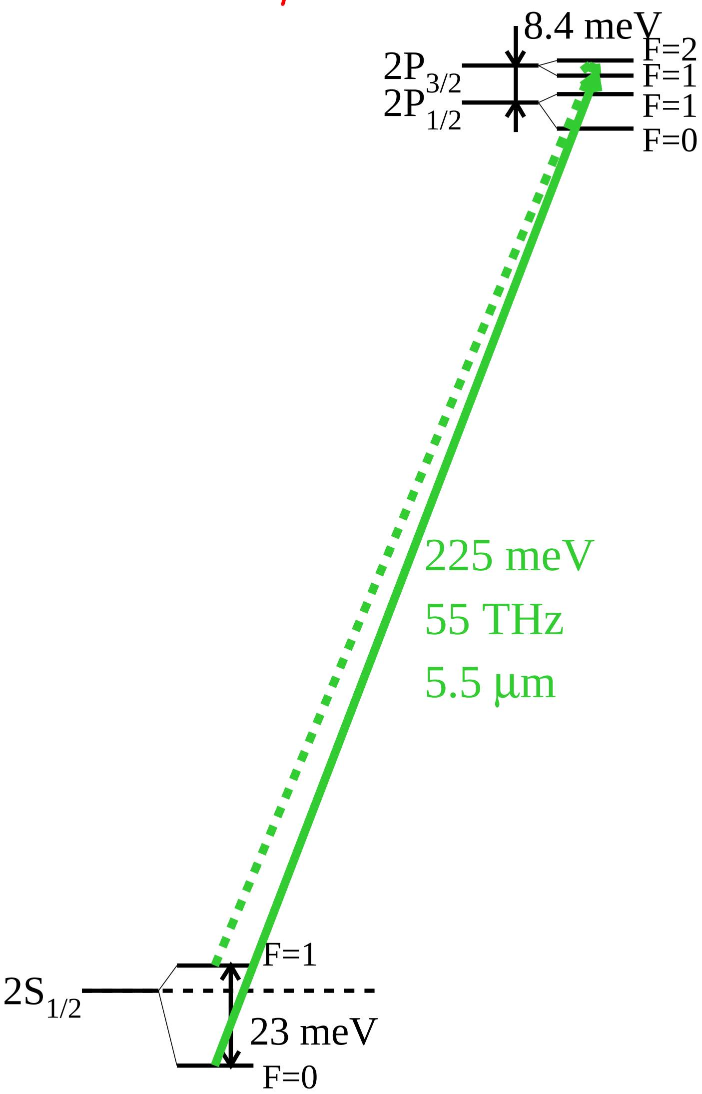
at $\lambda = 5.5\,\mu$m

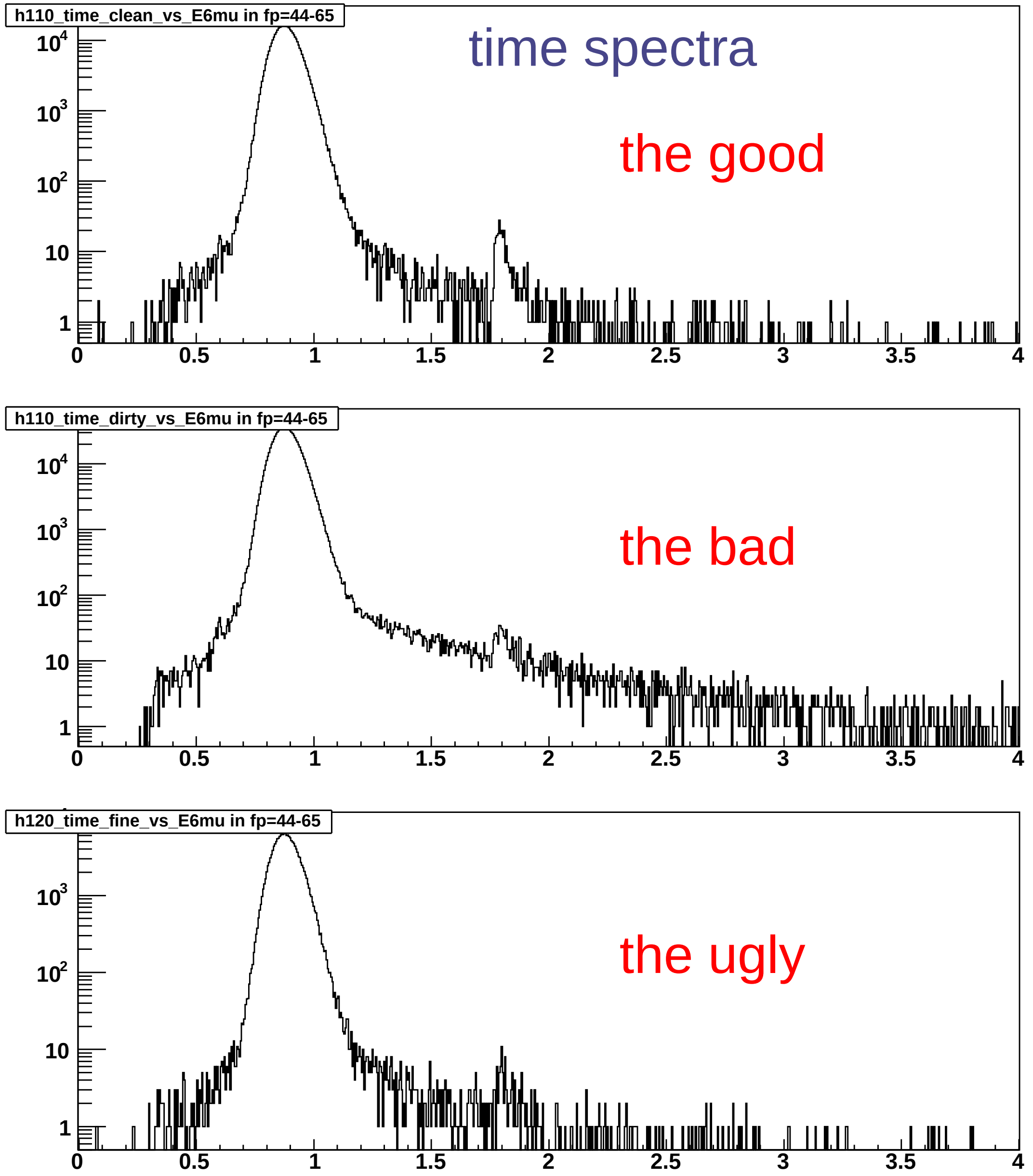


2nd line in muonic hydrogen

μp ($2S_{1/2}$(F=0) $\rightarrow$ $2P_{3/2}$(F=1))

at $\lambda = 5.5\ \mu$m

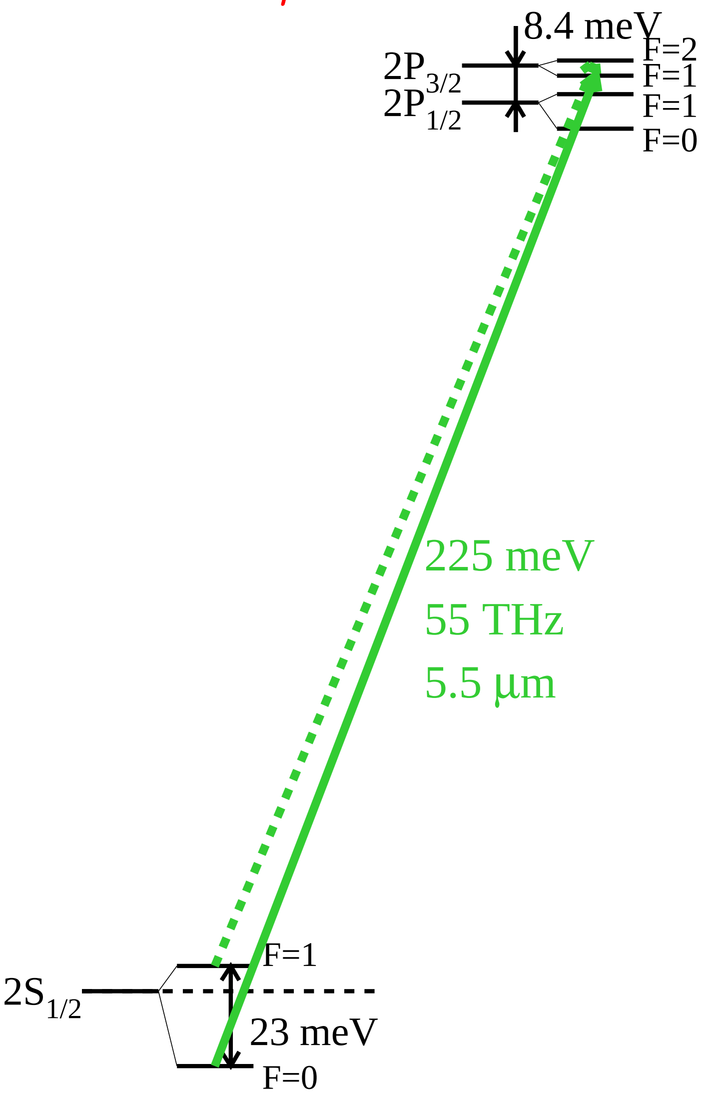


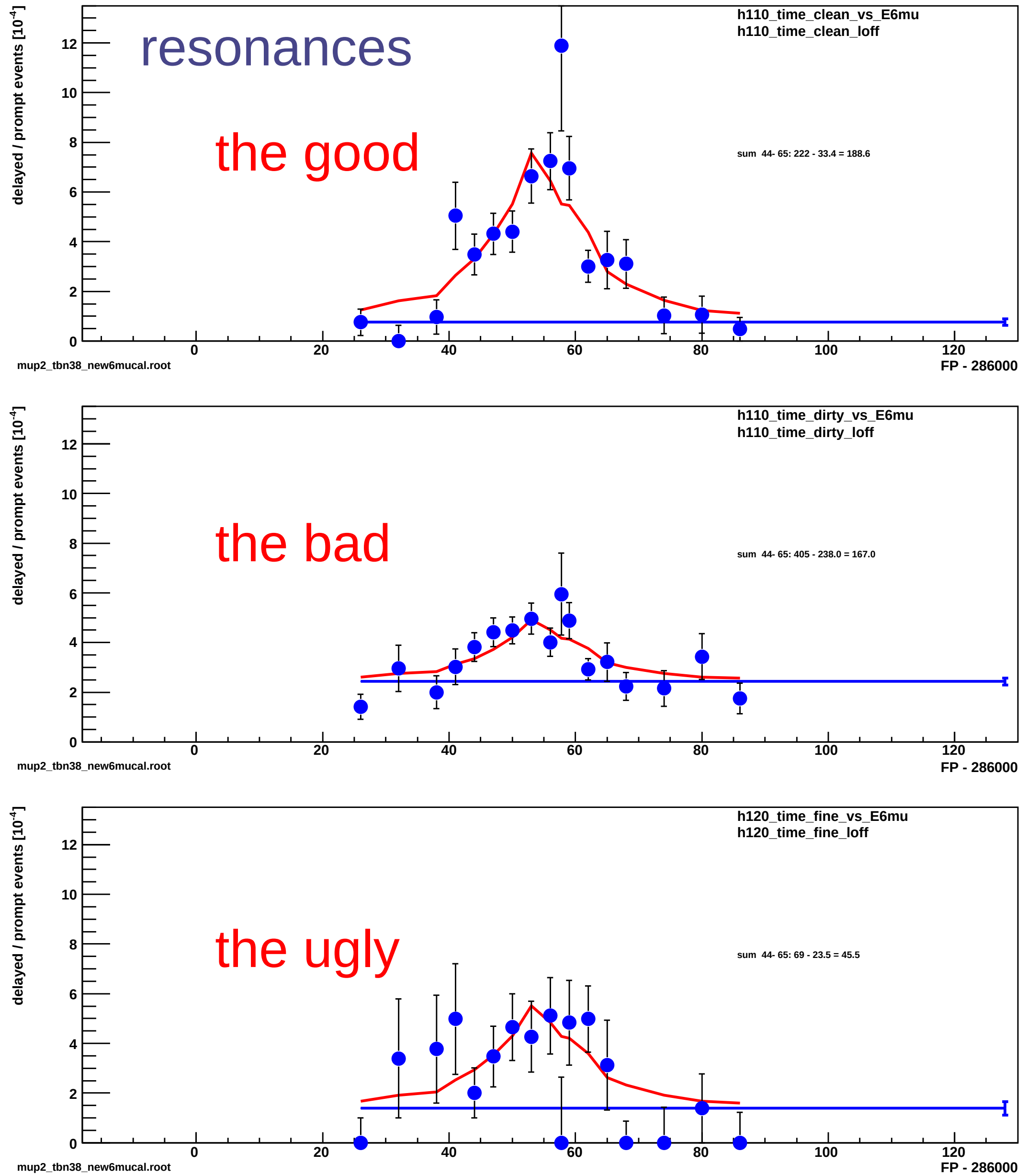


2nd line in muonic hydrogen

μp ($2S_{1/2}$(F=0) $\rightarrow$ $2P_{3/2}$(F=1))

at $\lambda = 5.5\,\mu$m

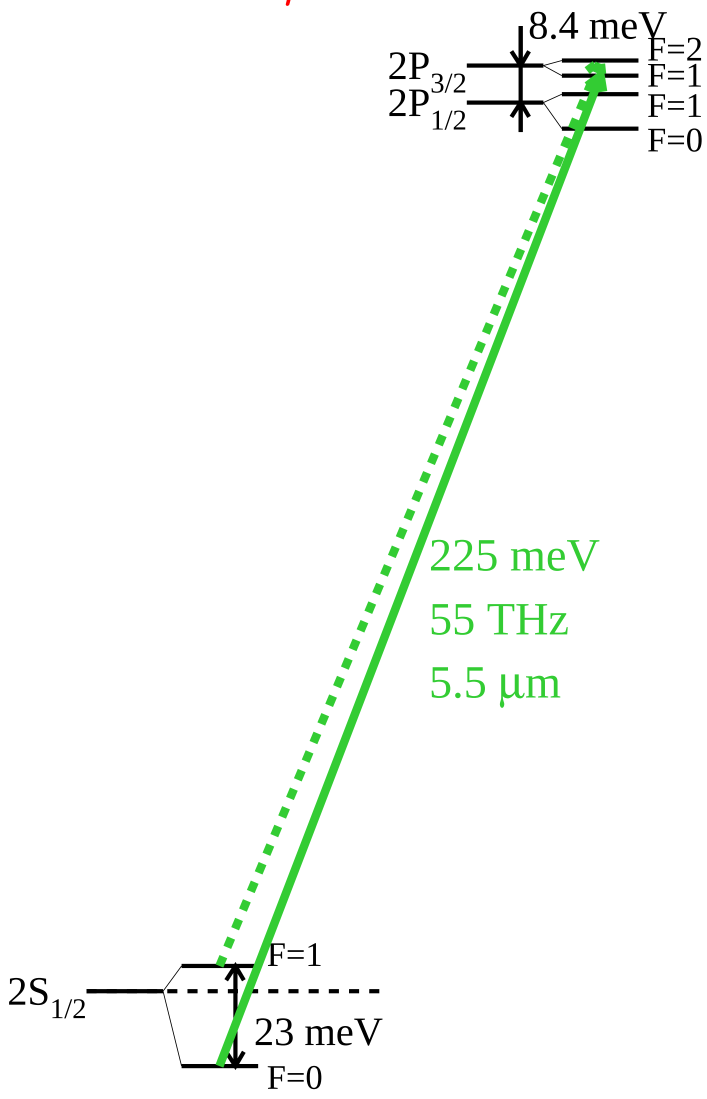


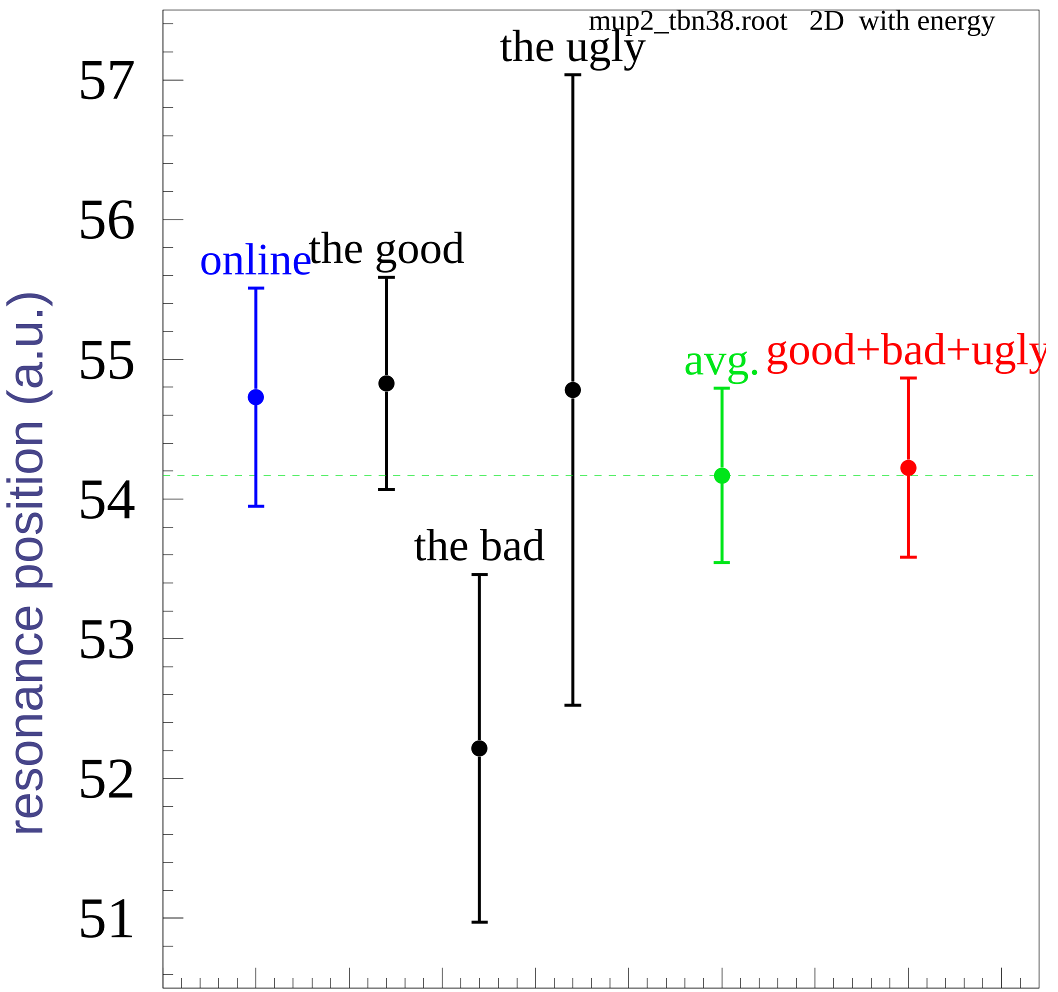


2nd line in muonic hydrogen

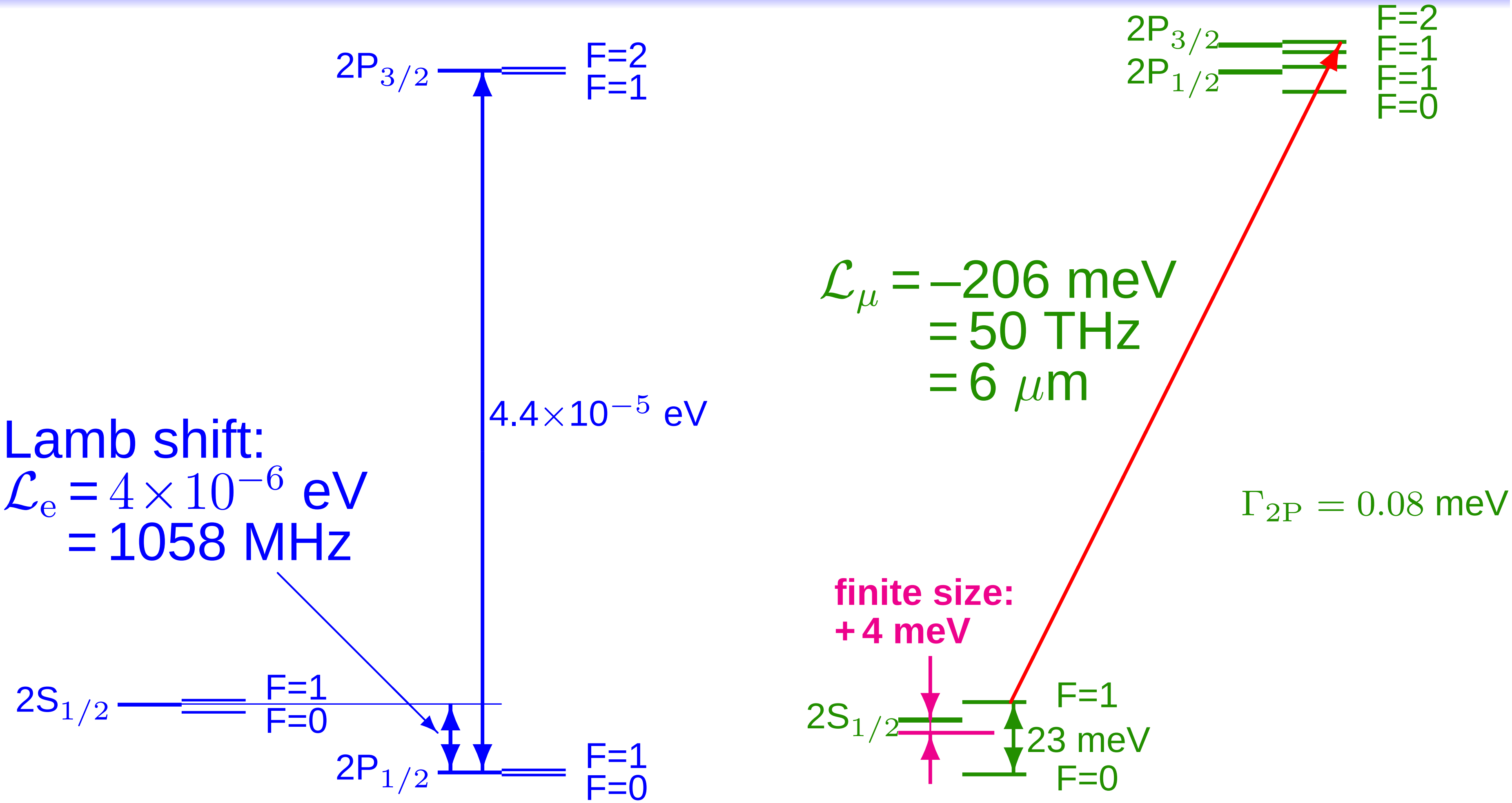
μp ($2S_{1/2}$(F=0) $\rightarrow$ $2P_{3/2}$(F=1))

at $\lambda = 5.5\,\mu$m





(n=2) - states of ep and μp



$a_o = 5\times10^{-11}$ m

ΔE_{2P-1S} = 10 eV

self energy = +1086 MHz

vac. pol. = – 27 MHz

$a_o^\mu = 3\times10^{-13}$ m

ΔE_{2P-1S} = 1900 eV

self energy = + 0.6 meV

vac. pol. = – 206 meV

Contributions to the $\mu^4\mathrm{He}^+$ Lamb shift

Contributions	ΔE (meV)
One-photon VP contribution, $\alpha(Z\alpha)^2$	1665.782
Two-loop VP contributions in first and second order PT, $\alpha^2(Z\alpha)^2$	15.188
Wichmann-Kroll correction	0.135
Three-loop VP in first and second order PT $\alpha^3(Z\alpha)^2$	0.138
Relativistic VP effects	-0.203
Hadronic VP	0.223
μ self-energy, μ VP, μ form factor corrections ($F_1'(0),\ F_2(0)$)	-11.243
Recoil corrections $(Z\alpha)^4$, $(Z\alpha)^5$, $(Z\alpha)^6$	-0.355
Radiative-recoil corrections	-0.040
Nuclear structure contribution of order $(Z\alpha)^4$: -105.322 $r_{\mathrm{He}}{}^2$	$-297.615\ (1.420)$
Nuclear structure contribution of order $(Z\alpha)^5$: 1.529 $r_{\mathrm{He}}{}^3$	$7.261\ (0.035)$
Nuclear structure and one- two-loop VP+ higher order nucl. structure	-2.357
Nuclear polarizability contribution	$3.100\ (0.600)$
Total splitting	1380.020 meV

μHe$^+$ Lamb shift, He nuclear radius

$$\Delta E(2P_{1/2} - 2S_{1/2}) = 1670.370(10)(600) - 105.322\, r_{\mathrm{He}}{}^2 + 1.529\, r_{\mathrm{He}}{}^3 \quad \mathrm{meV}$$
$$= 1380.020(10)^{\mathrm{th}}(600)^{\mathrm{pol}}(1420)^{\mathrm{fin.\ size}} \quad \mathrm{meV}$$

for r_{He} = 1.681(4) fm (Sick, 2008)

- Measure the transition frequency to 50 ppm ($\Leftrightarrow \Gamma/20 \Leftrightarrow 20$ GHz)
  - $\longrightarrow$ Determine r_{He} with $u_r = 1 \times 10^{-3}$ accuracy
    - Limited by nuclear polarizability
    - Polarizability was calculated in 1978 with $u_r = 20\%$
- Once He nuclear polarizability is calculated with $u_r = 5\%$
  - $\longrightarrow$ Determine r_{He} with $u_r = 3 \times 10^{-4}$ accuracy (10 times better than now)

Transitions in μ^3He$^+$ and μ^4He$^+$

Isotope	Transition	ΔE [meV]	λ [nm]	Pop.(η)	Mat. el. [a_μ]	$f_{a,b}$	$F_{\rm sat}$ [J/cm^2]	event rate [h^{-1}]
$\mu\,^4\mathrm{He}^+$	$2S_{1/2} - 2P_{3/2}$	1526	812	1	6	8/12	1.1	48
$\mu\,^4\mathrm{He}^+$	$2S_{1/2} - 2P_{1/2}$	1380	898	1	3	4/12	2.2	48
$\mu\,^3\mathrm{He}^+$	$2S_{1/2}^{F=0} - 2P_{1/2}^{F=1}$	1119	1108	1/4	3	1/12	2.1	—
$\mu\,^3\mathrm{He}^+$	$2S_{1/2}^{F=0} - 2P_{3/2}^{F=1}$	1294	958	1/4	6	2/12	1.1	12
$\mu\,^3\mathrm{He}^+$	$2S_{1/2}^{F=1} - 2P_{1/2}^{F=1}$	1286	964	3/4	2	2/12	3.2	22
$\mu\,^3\mathrm{He}^+$	$2S_{1/2}^{F=1} - 2P_{1/2}^{F=0}$	1344	923	3/4	1	1/12	6.4	13
$\mu\,^3\mathrm{He}^+$	$2S_{1/2}^{F=1} - 2P_{3/2}^{F=2}$	1436	863	3/4	5	5/12	1.3	36
$\mu\,^3\mathrm{He}^+$	$2S_{1/2}^{F=1} - 2P_{3/2}^{F=1}$	1461	849	3/4	1	1/12	6.4	16

adapted from A.P. Martynenko, PRA **76**, 0125505 (2007), with
$r_{^4\mathrm{He}}$= 1.681(4) fm from I. Sick, PRC **77**, 041302, (2008).
$r_{^3\mathrm{He}}$= 1.971(4) fm using ^{3}He-^{4}He isotope shift from P. Mueller et al., PRL **99**, 252501 (2007).

μHe$^+$ Signal rates

Effect	μp	$\mu\,^4$He$^+$	$\mu\,^4$He$^+$ / μp
	1 mbar	4 mbar	
Long lived $2S$-population	1.1%	2.2% *	2
$2S$ sub-level population	75%	100%	1.33
$2S$-lifetime (survival probability $= e^{-t_{\mathrm{Laser}}/\tau_{2S}}$)	1 μs	1.7 μs *	1.65
Muon stop in gas (trigger quality Q)	40%	60%	1.5
Muonic $2S$ atoms not drifting out of laser volume	80%	100%	1.25
Laser transition probability (only 20% for some weak transitions in $\mu\,^3$He$^+$)	30%	30%	1
Laser repetition rate	500 s^{-1}	500 s^{-1}	1
Detection of Lyman alpha X-ray	70%	70%	1
Total event rate increase: for $\mu\,^4$He$^+$			8
Total event rate increase: for $\mu\,^3$He$^+$ strong transition and triplet population			6
Total event rate increase: for $\mu\,^3$He$^+$ singlet population			2

* v. Arb *et al.*, Phys. Lett. B **136**, 232 (1984)

μp rate = 6 events/h $\longrightarrow$ $\mu\,^4$He$^+$ rate = 48 events/h

μHe^+ Statistics and Systematics

	$\mu\,^4He^+$	$\mu\,^3He^+$ (max)	$\mu\,^3He^+$ (min)	μp (max)
Event rate on resonance:	48 ev/h	36 ev/h	12 ev/h	6 ev/h
Background rate:	2.5 ev/h for all $\mu\,He^+$			1 ev/h

- We want to measure the transition with 50 ppm accuracy.

  This corresponds to $\Gamma/20$ = 20 GHz.
  - 500 events are sufficient $\Longleftrightarrow$ 1500 events on resonance
  - laser wavelength calibration: uncritical

    (we have determined the μp transition freq. to < 1 GHz)
- Systematics: will be < 5 GHz
  - AC/DC-Stark shift, Zeeman shift are all uncritical
  - Pressure shift, Doppler shift < 50 MHz.
  - Laser energy asymmetry left/right of centroid:

    $\rightarrow$ Need to measure the average laser energy with 3% accuracy